ISSN 1674-1056
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Citation: Chin. Phys. B . 2019, 28(2): 024207. **doi:** 10.1088/1674-1056/28/2/024207
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Controllable photon echo phase induced by modulated pulses and chirped beat detection*

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(Received 28 August 2018; revised manuscript received 14 October 2018; published online 10 January 2019)

In this paper, we propose a scheme for photon echo chirped detection process composed of additional modulation pulses to obtain controllable geometric phase. The geometric phases are observed and measured by a beat signal between the photon echo field and the chirped field. The chirped detection model reveals that the period of the beat signal increases as the chirped rate and delay time increase. Additionally, a two-fold relationship between the modulation phase and the echo shift phase is obtained. The numerical simulations accord with the theoretical results obtained by the finite difference time domain (FDTD) method.

Keywords: geometric phase, photon echo, chirped beat detection, Bloch sphere

PACS: 42.50.Md, 03.67.Lx **DOI:** 10.1088/1674-1056/28/2/024207

1. Introduction

A geometric phase is a phase change that is accumulated by the evolution of the wave function, which is an intrinsic property relating to the evolutionary path of an atom system.[1–3] After a cyclic evolution, geometric phase can be obtained from the phase information of the wave function and can be superimposed on the phase of the atom system, which can be observed in a closed two-level system. Unlike the dynamic phase, the geometric phase is only determined by the spatial solid angle formed by the wave function during system evolution, which is equivalent to the negative half of the established solid angle of space. Regardless of the initial state and the termination state, it is not related to special path in the evolution process. The advantages of geometric phase have attracted more and more research interest in recent years. In 2004, Tian *et al.* proposed the use of two pulses to acquire the geometric phase.[4] In 2007, Binder *et al.* demonstrated using three pulses to obtain the geometric phase.[5] However, geometric phase research is still in its infancy and the technique of detecting geometric phase by using photon echo is not perfect.[6] Therefore, it is meaningful to introduce geometric phase into the photon echo research[7–9] to detect the geometric phase and the phase change of the photon echo.

In this paper, the geometric phase analysis is introduced into the photon echo chirped detection process by using the designed modulation pulses. These geometric phases are observed and measured by a beat signal between the photon echo field and the chirped detection field. It can be found from the chirped detection model that the period of the beat signal increases as the chirped rate and delay time increase. A two-fold relationship between the modulation phase and the echo shift phase is demonstrated. It is shown that the simulation results accord with the theoretical results obtained by using the finite difference time domain (FDTD) method.

2. Echo signal detection

There are many methods to detect photon echo signal, and the use of chirped pulses to detect photon echo has previously been presented.[10,11] Assume that the input pulse can be written as

$$\Omega_{\rm in}(t)=\begin{cases}\Omega_{\rm c}\exp[{\rm i}(w_{\rm s}t+\frac{1}{2}\kappa t^2+\phi)]+{\rm c.c.},\\ \qquad t_0<t<t_1,\\ 0, \quad t<t_0 \ \text{ or } \ t>t_1,\end{cases}\tag{1}$$

where $\Omega_{\rm c}$ is the Rabi frequency of the chirped field, $w_{\rm s}$ is the starting frequency, ϕ is the field static phase, and κ is the chirped rate.

In an ideal case, the echo signal is a function of input signal with attenuation and delay. Therefore, it can be expressed as $\Omega_{\rm e}(t)=\xi\Omega_{\rm in}(t-\tau)$, where ξ is the echo efficiency. Substituting Eq. (1) into the above formula, the echo signal can be written as

$$\Omega_{\rm e}(t)=\begin{cases}\xi\Omega_{\rm c}\exp[{\rm i}(w_{\rm s}(t-\tau)+\frac{1}{2}\kappa(t-\tau)^2+\phi)]+{\rm c.c.},\\ \qquad t_0<t<t_1,\\ 0, \quad t<t_0 \ \text{ or } \ t>t_1.\end{cases}\tag{2}$$

The expressions for the echo field and the transmission field are as follows:

$$E_{\rm e}(t)=\Omega_{\rm e}(t)\hbar/\mu, \quad E_{\rm ref}(t)=\sigma[\Omega_{\rm in}(t)\hbar]/\mu,$$

*Project supported by the Tianjin Research Program of Application Foundation and Advanced Technology, China (Grant No. 15JCQNJC01100).
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where σ is the attenuation due to the average absorption over the frequency, $\hbar$ is the Planck constant, and μ is the dipole moment of a two-level transition. The beat signal of the detection field can be written as

$$E_{\text{beat}}(t) = \frac{\hbar|\Omega_{\text{c}}|^2}{\mu^2}[\xi^2+\sigma^2] \times [1+2\xi\sigma/(\xi^2+\sigma^2)\cos(\kappa\tau t+\Phi)], \quad (3)$$

where Φ is the oscillation phase.

Since the moment of the photon echo with different order is only related to the pulse delay, a formula for expressing the beat frequency signal of the reference field and echo field can be expressed as

$$E_{\text{beat}(i)}(t) = |E_{\text{ref}}(t)+E_{\text{ei}}(t)| = \frac{\hbar|\Omega_{\text{c}}|^2}{\mu^2}[\xi^2+\sigma^2][1+2\xi\sigma/(\xi^2+\sigma^2) \times \cos(\kappa\tau_i t+\Phi)], \quad i=1,2,3,\ldots,n. \quad (4)$$

Figure 1 shows the beat signals of the same echo signal at different chirped rates. The chirped rate from the top to the bottom is 1 MHz/μs, 2 MHz/μs, 3 MHz/μs, 4 MHz/μs, 5 MHz/μs, 8 MHz/μs, respectively, where $\Omega_{\text{c}} = 1.6$ MHz/μs, $\mu = 4\pi\times10^{-7}$ H/m, $\tau = 1$ μs, $\xi = 0.2$, and $\hbar = 6.626\times10^{-34}$ J·s. It can be found that the period of the beat signal increases as the chirped rate increases. In the following parameters, the chirped rate is set to be 8 MHz/μs.

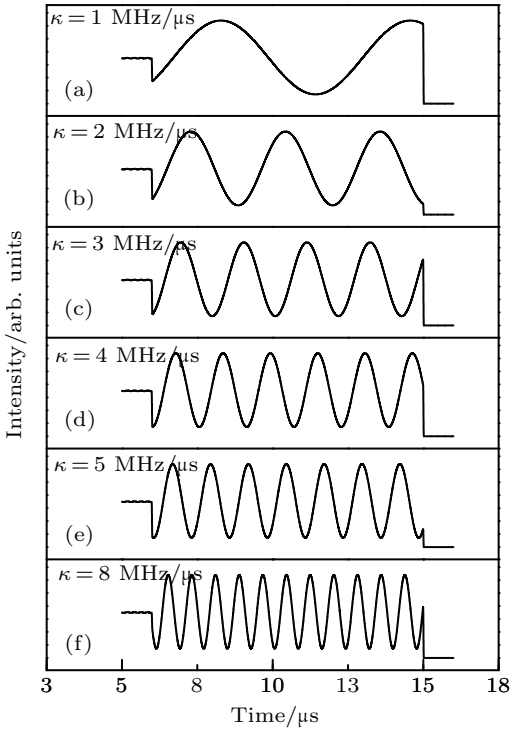


Fig. 1. Beat signals under different chirped rates.

Figure 2 shows the different beat signals for different time delays, where $\Omega_{\text{c}} = 1.6$ MHz/μs, $\mu = 4\pi\times10^{-7}$ H/m, $\kappa = 8$ MHz/μs, $\xi = 0.2$, and $\hbar = 6.626\times10^{-34}$ J·s. The time delays from the top to the bottom are 1 μs, 3 μs, 5 μs, 7 μs, 9 μs, and 11 μs, respectively. The red curve in Figs. 2(e) and 2(f) show the magnified images in the red square areas, respectively. It can be seen from the result that different echo beat signals have different periods under different time delays. This is due to the smaller oscillation frequency, namely the larger the delay time (the echo generated later), the smaller the period is. From the beat signal, the delay time can be used to distinguish the photon echo sequences.

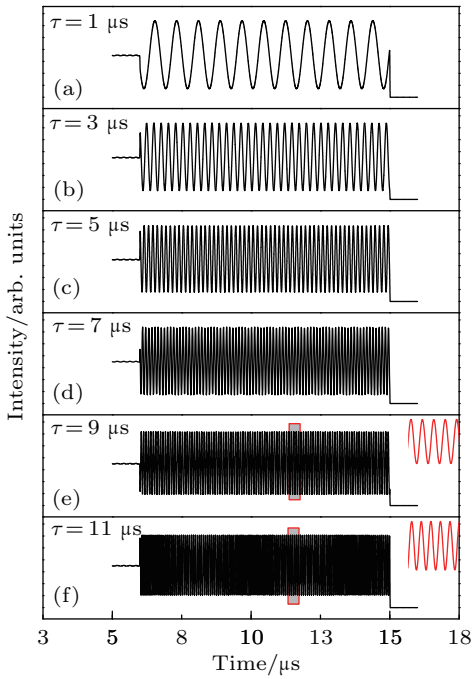


Fig. 2. Beat signals under different time delays.

3. Two-photon echo principle of additional modulated pulse

Before talking about the principle of photon echo under modulated pulse, we will briefly describe the principle to obtain the geometric phase under the modulated pulse. In a two-level system, the wave function can usually be described as

$$|\Psi\rangle = \mathrm{e}^{ir}\begin{pmatrix}\cos(\beta/2)\\ \mathrm{e}^{-\mathrm{i}\varphi}\sin(\beta/2)\end{pmatrix}. \quad (5)$$

Equation (5) contains the ground state $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and the excited state $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$, and the wave function can be represented by using a Bloch vector. For the energy level transitions of atoms, input pulses can drive the evolution of Bloch vectors, and the movement of the Bloch vector is controlled by the optical Bloch equation.

In the two-level system, there are two main ways to control the Bloch vector's geometry. The Bloch vector can rotate around the u-axis on the Bloch sphere by three modulation

pulses with pulse areas $\pi/2$, π, and $\pi/2$, respectively. Additionally, the Bloch vector can rotate around the w axis on the Bloch sphere by two modulation pulses with pulse areas π and π, respectively. These basic rotations can be realized purely by the controlled geometric phase change. These two forms can be expressed as

$$U_2(\delta) = \begin{pmatrix} \cos(\delta/2) & \sin(\delta/2) \\ -\sin(\delta/2) & \cos(\delta/2) \end{pmatrix},$$
$$U_3(\delta) = \begin{pmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{pmatrix}. \quad (6)$$

Since the cyclic evolution is usually accompanied by dynamic phases, the Bloch vector and the pull ratio vector remain perpendicular to each other in the system in the whole process, thereby eliminating the dynamic phase. Figure 3 shows a schematic diagram of a conventional two-pulse photon echo (Fig. 3(a)) and a two-pulse photon echo (Fig. 3(b)) after adding a modulated pulse.

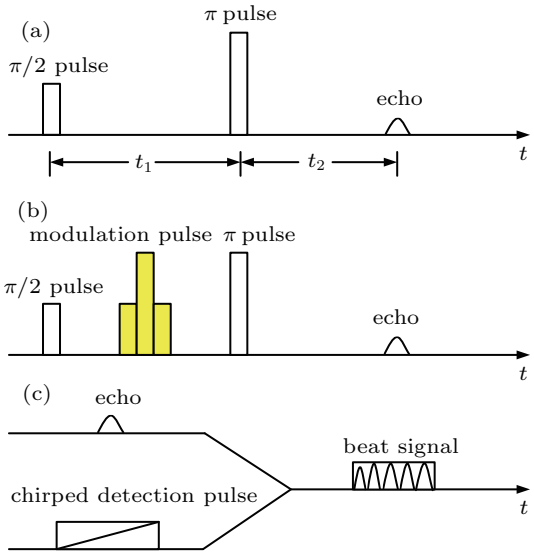


Fig. 3. Traditional two-pulse photon echo and photon echo with modulated pulses. (a) Traditional two-pulse photon echo; (b) two-pulse photon echo using modulated pulses; (c) detecting echoes with chirped pulses.

The traditional photon echo uses two pulses with pulse areas of $\pi/2$ and π, respectively. The interval between two pulses is t_1. Photon echoes are generated at t_2 after the second pulse ends, and the resulting echo signal is $E_e(t) \propto e^{-i\phi} \sin\beta$. Here the cyclic evolution rotating around the axis 2 is presented. Three modulation pulses are added between the two pulses, and their pulse areas are $\pi/2$, π, and $\pi/2$ (the total pulse area is 2π). The modulation phase is on the second pulse and its phase is δ, the resulting echo signal is $E_e(t) \propto e^{-i(\phi+\delta)} \sin\beta$. At the moment of photon echo formation, a chirped pulse is scanned to detect the photon echo and a beat signal is obtained (Fig. 3(c)). In this process, the action time of the chirped probe field must be greater than the time of the photon echo signal generation because only the echo can be generated to react with the chirped probe field, thereby generating the beat signal.

4. Bloch sphere description of modulated pulse photon echo

In the previous section, we explained the principle of modulated pulse photon echo. In this section, we mainly describe the echo performance by the added modulation pulse through using the Bloch sphere under a microscopic condition. The Bloch sphere is a unit with a two-dimensional spherical surface, and each pair of corresponding points corresponds to mutually orthogonal state vectors. The two ends of the Bloch sphere generally correspond to the ground state and the excited state of the atomic spin-up state and the spin-down state standard basal vector, respectively. Figure 4 illustrates the effects of modulated pulse for two-pulse photon echoes on Bloch spheres. At the beginning of the system, all of the atoms are in the ground state ($v = 0$, $w = -1$), First, an excitation pulse is applied to the two-level system with a pulse area of $\pi/2$. All atoms resonating within the pulse spectrum are caused to have a Bloch vector rotated by $\pi/2$ as shown in Fig. 4(a). The short excitation pulse is capable of uniformly covering the bandwidth of the entire non-uniform spectrum and has sufficient energy to act as a $\pi/2$ pulse for the entire two-level system. However, for a longer excitation pulse, there is a $\pi/2$ pulse effect when the pulse is near the center frequency, and it has almost no effect on the entire two-level system away from the center frequency. When the pulse action ends, all atoms are simultaneously at the same position on the Bloch sphere ($w = 0$, $v = -l$). The next step is to modulate the phase of the Bloch vector rotation process. A three-pulse modulation pulse is used, where neither of the first and third pulse carries the phase information and the second pulse has δ phase. Figure 4(b) shows the movement process. Before the start of action, all of the Rabi vectors $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$, and $\boldsymbol{\Omega}_3$ coincide with the u axis. After the phased pulse is applied, the partial Rabi vector $\boldsymbol{\Omega}_2$ is deflected, causing the original locus to change. After the end of the modulation pulse, the state of the atom returns to the original state due to 2π pulse effect, but during this operation the system acquires a geometric phase effect and the subsequent photon echo process. A short pulse with a pulse area of π is applied at time $t = t_1$, which causes all atoms to undergo a full semi-circle rotation such that all Bloch vectors rotate counterclockwise around the u-axis as shown in Fig. 4(c). However, because almost all Bloch vectors are rotated out of the v–w plane, they rotate along a semicircle that covers substantially the entire Bloch sphere. At time $t = t_2$, the amplitude of the new signal is obtained from the sum of all rotated Bloch vectors, and all Bloch vectors will re-phase and produce a light pulse. This pulse is the modulated two-pulse photon echo.

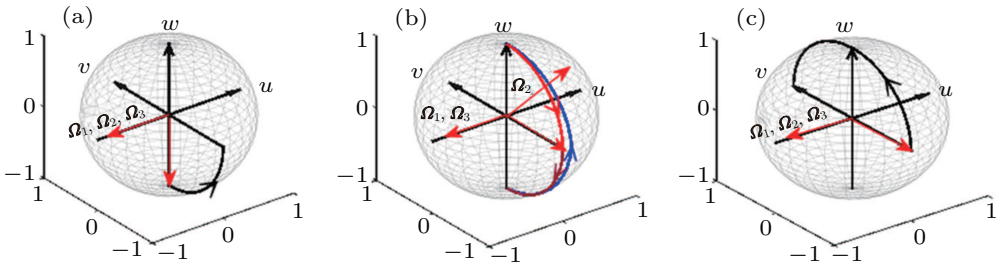


Fig. 4. Two-pulse photon echo process. (a) The first process of photon echo generation; (b) the second process of photon echo generation with modulation pulse; (c) the third process of photon echo generation.

5. Simulation results

In the previous part, we have already described the photon echo generation process by the modulation pulse, its simulation will be performed in this section. The pulse durations of the two short rectangular input pulses are both 200 ns, and the intermediate interval is 20 μs. The electric field amplitude of the first pulse is set to be 7.355×10^9 V/m, and the pulse area is $\pi/2$. The electric field amplitude of the second pulse is set to be 1.45×10^{10} V/m, and the pulse area is π. The pulse areas of the modulation pulse are $\pi/2$, π, and $\pi/2$, respectively. The resulting echo direction should be the same as the direction of the first pulse. After the end of the second pulse, a photon echo will be generated. At this time, a detection field with a chirped rate of 8 MHz/μs is used to detect the echo signal. The detection field parameters are as follows: $\Omega_c = 1.6$ MHz/μs, $w_s = 20$ MHz, $\phi = 0$ rad, and $\kappa = 8$ MHz/μs. The FDTD method algorithm is first applied to the electromagnetic field theory to treat the Maxwell equation. The algorithm is gradually transplanted into the optical field to study the photon echo phenomenon based on the semi-classical theory.[12] By using the FDTD method to simulate the beat detection signal, the beat signal between the photon echo and the chirped field is shown in Fig. 5. In Fig. 5, the values of modulation phase δ care 0, $\pi/4$, and $3\pi/4$, respectively, showing that the modulation pulse does change the phase of the echo signal without changing the signal shape and generation time of the echo signal. The relationship between the modulation phase and the echo phase is obtained through the fast Fourier transform method. As shown in Fig. 6, the points represent calculated values and the lines represent theoretical values. It can be concluded that the modulation phase and the echo phase belong to a double relationship. Figure 6 shows that relationship between echo phase shift and modulation phase is not completely linear, and the fitted value is 0.98785, which arises from the calculation method. In the simulation process in Matlab, because the time step size selected in the cycle calculation cannot be selected to be a minimum value, a certain deviation will be caused in the final result, and a nonlinear variation curve appears. The quantitative analysis of the relationship between time step size and fitted value is shown in Fig. 7. It can be seen that the fitted value decreases as the time step size increases. To make the results more accurate, we try to reduce the time step size. When the time step size is reduced to 3.125×10^{-5} μs, the results are obtained as shown in Fig. 6 and the fitted value is 0.98785.

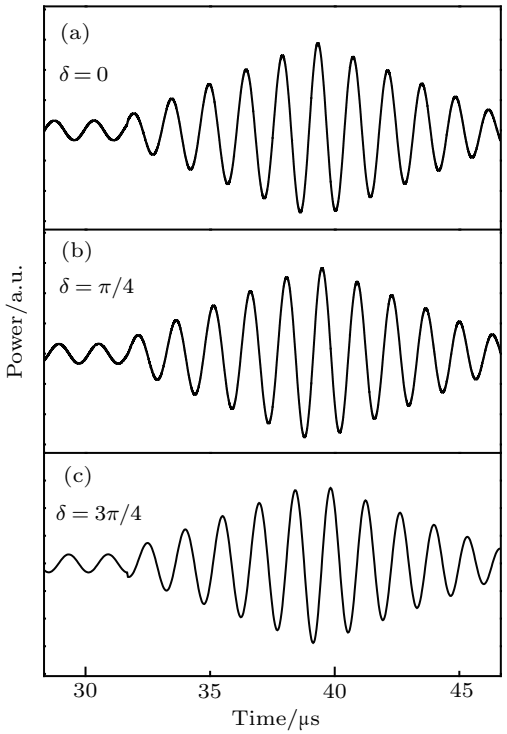


Fig. 5. Beat signals under different values of modulation phase with added modulation phases of (a) 0, (b) $\pi/4$, and (c) $3\pi/4$.

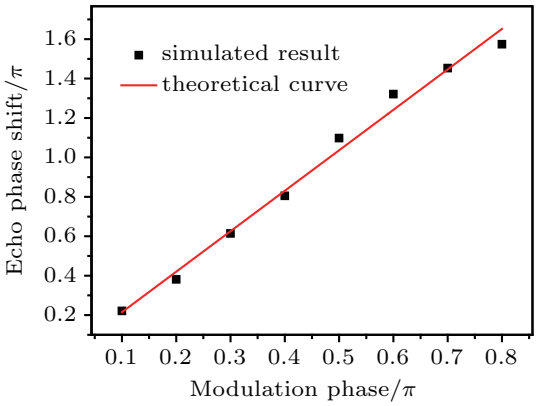


Fig. 6. Plot of echo phase shift versus modulation phase.

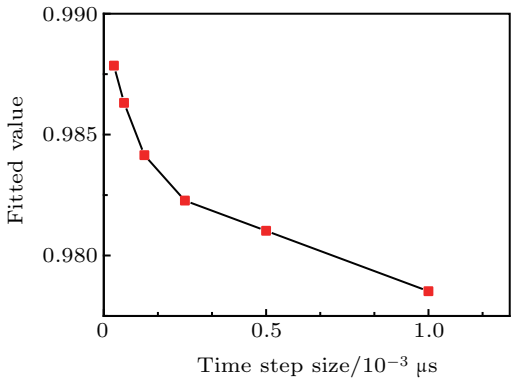


Fig. 7. Plot of fitted value versus time step size.

6. Summary

In this work, we propose a method and its implementation for photon echo chirped detection that is composed of additional modulation pulses to obtain controllable geometric phase. The accumulated geometric phases are observed and measured by a beat signal between the photon echo field and the chirped field. We have found that the period of the beat signal increases as the chirped rate and time delay increase. Furthermore, the delay time helps to distinguish the orders in which photon echoes are generated. The Bloch vector motion trajectory performed by the modulated two-pulse photon echo on the Bloch sphere is analyzed to verify the proposed method. By using the FDTD method to simulate the beat detection signal, a two-fold relationship between the modulation phase and the echo shift phase is demonstrated. The results are found to be the same as our expected phase shifts of the photon echo. This is conducive to the development of controllable photon echo technology.

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