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Amplitude and phase controlled absorption and dispersion of coherently driven five-level atom in double-band photonic crystal*

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The absorption–dispersion properties of a microwave-driven five-level atom embedded in an isotropic photonic bandgap (PBG) have been studied. Due to the singular density of modes (DOM) in the isotropic PBG and the dynamically coherence induced by the coupling fields, modified reservoir-induced transparency and quantum interference-induced transparency emerge simultaneously. Their interaction leads to ultra-narrow spectral structure. As a result of closed-loop configuration, these features can be manipulated by the amplitudes and relative phase of the coherently driven fields. The position and width of PBG also have an influence on the spectra. The theoretical studies can provide us with more efficient methods to control the atomic absorption–dispersion properties, which have applications in optical switching and slow light.

Keywords: isotropic photonic bandgap, absorption–dispersion spectra, phase and amplitude control, ultra-narrow spectral structure

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1. Introduction

It is well known that quantum coherence and interference effects can effectively manipulate the absorption and dispersion properties of atomic system. Agarwal has shown that the ordinary spontaneous decay of an excited degenerate V-type three-level atom can be modified due to the interference between the two atomic transitions.[1] Many interesting schemes have since been proposed to manipulate the interaction of light and atom via quantum coherence, such as electromagnetically induced transparency (EIT),[2,3] reservoir induced transparency,[4,5] EIT-enhanced four-wave mixing,[6,7] enhancement of the index of refraction without absorption,[8] slow light,[9] and so on.

In recent years, EIT has become one of the most significant subjects in quantum optics. Generally, EIT is achieved by using a second laser and the coherent interaction between the laser fields in atoms. However, a modified reservoir with rapidly varied density of modes (DOM) can also introduce transparency in the open atomic system, which does not require any external driving field to induce atomic coherence.[10–12] Photonic crystal (PC), as a new type material, is first discovered by John[13] and Yablonovitch[14] in the 1980s. This artificial periodic material has special density of states which can greatly affect the atomic action and the electromagnetic wave propagation with a range of frequencies classically forbidden in it.[15] Many interesting optical effects have been found in PCs; for example, giant Lamb shift,[16] localization of light,[17] photon-bound states,[18] suppression (or even complete cancellation), and enhancement of spontaneous emission.[19] Inspired by these investigations, it would be interesting to investigate the combined effects of coherent control by external driving fields and photon localization facilitated by a photonic bandgap (PBG). And then, much attention has been paid to the absorption and dispersion properties of atom embedded in PC. Paspalakis and his co-workers studied the steady-state transparency and the transient response of a Λ-type atom that decays spontaneously near a photonic band edge;[20,21] it was shown that the atom can become transparent to a probe laser field, even in the presence of other dissipative channels, which is originated from the square-root singularity of DOM of the PBG material at threshold. By studying the probe absorption–dispersion spectra of a driven three-level atom, Roshan *et al.* found that the atom is transparent to a probe laser field coupled to the free space transition and slow group velocities can be obtained near the transparency window.[22] Meanwhile, for a double V-type atom in a PBG structure, there are three types of transparency in the spectra, and it was shown that the absorption and dispersion properties of the system depend on the two types of quantum interference and DOM of the modified reservoir.[23] Du and co-workers investigated the dispersion and absorption of three-level atoms, and found that high-Q defect mode can induce transparency window in the absorptive spectra and lead to slow light.[24,25] Xu *et al.* mentioned a six-level atom embedded in double-band photonic crystals with a defect mode and found that the transparency windows can be controlled by the defect mode and the

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quantum interference effects.[26] Ding *et al.* proposed a five-level atom embedded in a double-band PC with defect mode in the bandgap, and the corresponding optical spectra of atom can be significantly modified.[27]

In this paper, we pay attention to the optical response of a five-level atomic system in double-band PC coupled by four fields, in which the one upper level is driven to two middle levels by two coupling laser fields and two middle levels are driven by a microwave laser to form a closed interaction contour. Meanwhile, another lower level is connected by a weak probe laser,which interacts with the free vacuum modes, and the last one of the atomic transitions decays spontaneously in isotropic PBG reservoir. Although the spontaneous emission and double EIT have been studied in a four-level atomic system with a closed interaction contour,[28–30] and the optical spectra of a five-level atomic system with various configurations embedded in PBG have been mentioned in a few papers,[31] the absorption–dispersion properties when a closed interaction contour is included in a five-level atom doped in the isotropic photonic crystal have not yet been studied carefully. With numerical calculations and analysis, we find that not only the amplitudes of applied laser fields but also the total closed interaction phase can modify the absorption–dispersion spectra. Moreover, the photonic bandgap width and resonant frequency splitting can also affect the positions and the lineshapes of the transparency windows.

This paper is organized as follows. In Section 2, we apply the time-dependent Schrödinger equation to analytically calculate the steady-state linear susceptibility of the atomic system. The general calculated results and their analysis are briefly discussed in Section 3. The major conclusions are summarized in Section 4.

2. Equations for the absorption and dispersion spectra

In this paper, we consider a five-level atomic system that consists of one excited state $|1\rangle$ and four ground states $|0\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$. The three upper transitions $|1\rangle \leftrightarrow |2\rangle$, $|2\rangle \leftrightarrow |3\rangle$, and $|1\rangle \leftrightarrow |3\rangle$ are coherently driven by three fields (two laser fields and one microwave field) with Rabi frequencies Ω_1, Ω_2, and Ω_3, respectively. In the following, we suppose that $\Omega_1 = |\Omega_1|\,\mathrm{e}^{\mathrm{i}\varphi_1}$, $\Omega_2 = |\Omega_2|\,\mathrm{e}^{\mathrm{i}\varphi_2}$, and $\Omega_3 = |\Omega_3|\,\mathrm{e}^{\mathrm{i}\varphi_3}$, in which φ_i (i = 1, 2, and 3) represents the phase of the corresponding coupling field. Here, the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ are electric-dipole-allowed. The Rabi frequencies are defined as $|\Omega_1| = d_{12}E_1/(2\hbar)$ and $|\Omega_3| = d_{13}E_3/(2\hbar)$ with d_{12} and d_{13} being the electric dipole moments of the corresponding transitions. E_1 and E_3 represent the amplitudes of the electric-component of the optical fields. The states $|2\rangle$ and $|3\rangle$ are usually the hyperfine levels of atom and their transition is electric-dipole-forbidden but magnetic-dipole-allowed. The Rabi frequency is expressed as $|\Omega_2| = \mu_{23}B_2/(2\hbar)$ with μ_{23} being the magnetic dipole moment and B_2 being the amplitude of magnetic-component of the microwave field. In the atomic system, the three transitions form a closed-loop configuration, and then the resulting absorption–dispersion spectra depend on the relative phase of the coupling fields. This type of closed-loop configuration can be realized in atomic systems and doped crystals.[32,33] The transition between levels $|1\rangle$ and $|4\rangle$ couples to the double-band isotropic PBG reservoir, while other transitions are assumed to be far away from the photonic bandgap and can be treated as in the free space, as shown in Fig. 1. In our discussion, we emphasize the absorption and dispersion properties of the transition $|1\rangle \leftrightarrow |0\rangle$.

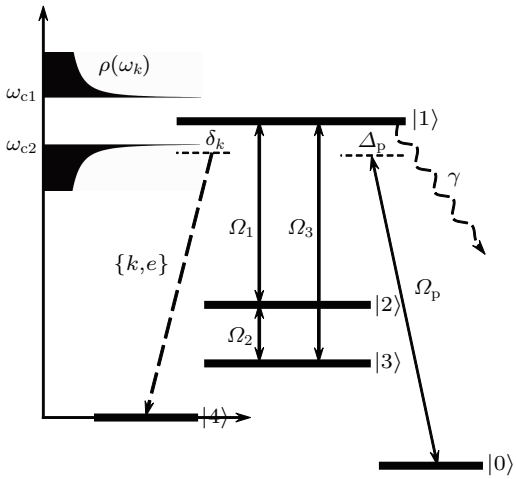


Fig. 1. Schematic diagram of a coherently driven five-level atom in a double-band isotropic PBG.

For double-band isotropic PBG reservoir, the dispersion relations near the photonic band edges are approximated by

$$\omega_k = \begin{cases} \omega_{c_1} + A_1(k-k_0)^2, & k > k_0, \\ \omega_{c_2} - A_2(k-k_0)^2, & k < k_0, \end{cases} \tag{1}$$

where $A_n \approx \omega_{cn}/k_0^2$ ($n = 1, 2$), and with ω_{c_1} and ω_{c_2} being the upper and lower frequencies of the double photonic band edges, respectively. The Hamiltonian describing the dynamics of this system in the interaction picture with the rotating wave approximation and the electric-dipole approximation can be written as

$$\begin{aligned} H_I = {} & \hbar(\Omega_1 |1\rangle\langle 2| + \Omega_2 |2\rangle\langle 3| + \Omega_3 |1\rangle\langle 3| + \Omega_p \mathrm{e}^{-\mathrm{i}\Delta_p t} |1\rangle\langle 0| \\ & + \sum_{ke} g_{ke}^{14} \mathrm{e}^{-\mathrm{i}\delta_k t} b_k |1\rangle\langle 4| + \mathrm{H.c.}) - \mathrm{i}\frac{\gamma}{2} |1\rangle\langle 1|, \end{aligned} \tag{2}$$

where $b_k^\dagger$ and b_k represent the creation and annihilation operators for the k-th vacuum mode with frequency ω_k, and the frequency-dependent coupling constant between the resonant transition and mode $\{ke\}$ of radiation field is $g_{ke}^{14} = \omega_{14} d_{14}/\hbar\sqrt{\hbar/(2\varepsilon_0\omega_k V_0)}\hat{e}_k \cdot \hat{u}_d$, in which d_{14} and $\hat{u}_d$ are the magnitude and unit vector of the atomic dipole moment, V_0 is the quantized volume, and $\hat{e}_k$ is polarization unit vector. The detuning of the spontaneously emitted photon in the isotropic

PC is defined as $\delta_k = \omega_k - \omega_{14}$. At time $t = 0$, this atom starts to interact with a probe field of frequency ω_p. $\Delta_p = \omega_p - \omega_{10}$ is the detuning frequency of the weak probe field from transition $|1\rangle \leftrightarrow |0\rangle$. Finally, γ denotes the spontaneous decay rate from the excited state $|1\rangle$ as a result of the interaction with vacuum.

The five-level atom is initially prepared in the lower level $|0\rangle$. The dynamics of the system can be described by using the Schrödinger equation. Then the state vector of the system at time t may be expressed as

$$|\Psi(t)\rangle = [A_0(t)|0\rangle + A_1(t)\,\mathrm{e}^{-\mathrm{i}\Delta_p t}|1\rangle + A_2(t)\,\mathrm{e}^{-\mathrm{i}\Delta_p t}|2\rangle + A_3(t)\,\mathrm{e}^{-\mathrm{i}\Delta_p t}|3\rangle]|\{0\}\rangle + \sum_{ke} A_{ke}(t)|4,\{ke\}\rangle. \quad (3)$$

The function $A_j(t)$ ($j = 0$, 1, 2, and 3) represents the probability amplitude to find the atom in state $|j\rangle$ with no photon decaying to the PBG reservoir. $A_{ke}(t)$ denotes the probability amplitude to find the atom in the ground state $|0\rangle$ and one photon of wave-vector k and polarization e is in the isotropic PC reservoir.

By substituting the Hamiltonian Eq. (2) and state vector Eq. (3) into the Schrödinger equation, we can derive the first-order differential equations for the amplitudes

$$\mathrm{i}\dot{A}_0(t) = \Omega_p A_1(t), \quad (4a)$$

$$\mathrm{i}\dot{A}_1(t) = \Omega_1 A_2(t) + \Omega_3 A_3(t) - \left(\mathrm{i}\frac{\gamma}{2} + \Delta_p\right)A_1(t) + \Omega_p A_0(t) + \sum_{ke} g_{ke}^{14}\,\mathrm{e}^{-\mathrm{i}\delta_k t - \mathrm{i}\Delta_p t}A_{ke}(t), \quad (4b)$$

$$\mathrm{i}\dot{A}_2(t) = -\Delta_p A_2(t) + \Omega_1^* A_1(t) + \Omega_2 A_3(t), \quad (4c)$$

$$\mathrm{i}\dot{A}_3(t) = -\Delta_p A_3(t) + \Omega_3^* A_1(t) + \Omega_2^* A_2(t), \quad (4d)$$

$$\mathrm{i}\dot{A}_{ke}(t) = g_{ke}^{14} A_1(t)\,\mathrm{e}^{\mathrm{i}\delta_k t - \mathrm{i}\delta t}. \quad (4e)$$

We proceed by performing a formal time integration of Eq. (4e), and then substitute the result into Eq. (4b) to obtain the integral–differential equations

$$\mathrm{i}\dot{A}_1(t) = \Omega_1 A_2(t) + \Omega_3 A_3(t) - \left(\mathrm{i}\frac{\gamma}{2} + \Delta_p\right)A_1(t) + \Omega_p A_0(t) - \mathrm{i}\int_0^t \mathrm{e}^{\mathrm{i}\Delta_p(t-t')}\cdot G(t-t')A_1(t')\,\mathrm{d}t', \quad (5)$$

where

$$G(t-t') = \sum_{ke}\left|g_{ke}^{14}\right|^2 \mathrm{e}^{-\mathrm{i}\delta_k(t-t')} \quad (6)$$

are the delayed Green functions in the photonic crystal reservoir. Following the previous work in Ref. [34], for the isotropic PBG reservoir, the Green's functions can be expressed as

$$G(t-t') = \frac{1}{2}\beta_{14}\left(\frac{\exp\{\mathrm{i}[-\delta_{c114}(t-t') + \pi/4]\}}{\sqrt{\pi(t-t')}} + \frac{\exp\{\mathrm{i}[-\delta_{c214}(t-t') - \pi/4]\}}{\sqrt{\pi(t-t')}}\right). \quad (7)$$

Here, the definitions of the parameters are

$$\beta_{14}^{3/2} = \frac{1}{2\pi\varepsilon_0}\frac{\omega_{14}^2 d_{14}^2 \omega_{c1}^{3/2}}{3\hbar c^3}, \quad \delta_{c114} = \omega_{c1} - \omega_{14}, \quad \delta_{c214} = \omega_{c2} - \omega_{14}. \quad (8)$$

δ_{c114} (δ_{c214}) represents the detuning frequencies between resonant transition frequency ω_{14} and the upper (lower) band edge frequencies ω_{c1} (ω_{c2}).

We aim to investigate the absorption–dispersion properties for a weak probe laser field. The equation of motion for the electric-field amplitude $E(z,t)$ with the slowly-varying-envelope approximation is described by the Maxwell equation

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial z}\right)E(z,t) = -\mathrm{i}\frac{\omega}{2c}\chi(\Delta_p)E(z,t), \quad (9)$$

where $\chi(\Delta_p)$ is the steady-state linear susceptibility of the medium and $v_g = c/[1 + \mathrm{Re}(\chi)/2 + (\omega/2)\partial\,\mathrm{Re}(\chi)/\partial\omega]$ is the group velocity of the laser pulse propagating in the medium. Since the transition $|1\rangle \leftrightarrow |0\rangle$ is treated as occurring in free space, the steady-state linear susceptibility is given by

$$\chi(\Delta_p) = -\frac{4\pi N|d_{10}|^2}{\Omega_p}A_0(t\to\infty)A_1^*(t\to\infty), \quad (10)$$

where N is the atomic density. We assume that the interaction between the probe laser and the atom is very weak ($\Omega_p \ll \beta_{14}, \gamma$), so that $A_0(t\to\infty) \approx 1$ is satisfied at all times. From Eqs. (4b), (4c), (4d), and (5), using the Laplace transform we can obtain

$$s\tilde{A}_1(s) = \frac{-\mathrm{i}\Omega_p}{(s - \mathrm{i}\Delta_p + \gamma/2) - \mathrm{i}\left(2|\Omega_1||\Omega_2||\Omega_3|\cos\theta + \Delta_p\Omega^2\right)/[|\Omega_2|^2 + (s - \mathrm{i}\Delta_p)^2] + \tilde{G}(s)}, \quad (11)$$

in which $\theta = \varphi_1 + \varphi_2 - \varphi_3$ is the relative phase of the three coupling fields. With the final value theorem, the long time behavior of the probability amplitude can be written as

$$A_1(t\to\infty) = \frac{-\mathrm{i}\Omega_p\cdot(|\Omega_2|^2 - \Delta_p^2)}{\mathrm{i}\Delta_p^3 + (\gamma/2)(|\Omega_2|^2 - \Delta_p^2) - 2\mathrm{i}|\Omega_1||\Omega_2||\Omega_3|\cos\theta - \mathrm{i}\Delta_p\Omega^2 + \tilde{G}(0)(|\Omega_2|^2 - \Delta_p^2)}, \quad (12)$$

where

$$\Omega^2 = |\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2, \quad (13)$$

$$\tilde{G}(s)=\frac{1}{2}\beta_{14}^{3/2}\left(\frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{s+\mathrm{i}(\delta_{\mathrm{c}114}-\Delta_{\mathrm{p}})}}+\frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{s+\mathrm{i}(\delta_{\mathrm{c}214}-\Delta_{\mathrm{p}})}}\right). \tag{14}$$

With Eqs. (10) and (12), we can derive the final expression of the steady-state linear susceptibility to calculate the absorption–dispersion spectra for several parameters of the atomic system, which is given by

$$\chi(\Delta_{\mathrm{p}})=-4\pi N|d_{10}|^2\frac{(\Delta_{\mathrm{p}}^2-|\Omega_2|^2)}{\Delta_{\mathrm{p}}^3-\Delta_{\mathrm{p}}\Omega^2-2|\Omega_1||\Omega_2||\Omega_3|\cos\theta-\mathrm{i}[\gamma/2+\tilde{G}^*(0)](\Delta_{\mathrm{p}}^2-|\Omega_2|^2)}. \tag{15}$$

3. Results and discussions

In this part, we study the influence of the photonic bandgap, the amplitudes, and the relative phase of three coupling laser fields on the absorption–dispersion spectra.

To investigate the effect of the amplitudes of the three coupling fields on the optical properties, we plot the probe absorption–dispersion spectra in form of $\mathrm{Im}(\chi)$ and $\mathrm{Re}(\chi)$ as a function of detuning frequency Δ_{p} for different amplitudes of three coupling fields with the other parameters set as $\beta_{14}=1$, $\gamma=1$, $\theta=0$, $\delta_{\mathrm{c}114}=1.0$, and $\delta_{\mathrm{c}214}=-1.0$. From Eq. (15), we can derive that when the detuning frequencies are $\Delta_{\mathrm{p}}=\pm\Omega_2$ and $\Delta_{\mathrm{p}}=\delta_{\mathrm{c}114},\delta_{\mathrm{c}214}$, the susceptibility $\chi(\Delta_{\mathrm{p}})$ turns to zero, which indicates vanishing absorption for the probe field, and then four transparency windows emerge in the absorption spectra. When there are no extra coupling fields as shown in Fig. 2(a), this five-level atomic system becomes a three-level Λ model, in which two transitions $|1\rangle\leftrightarrow|0\rangle$ and $|1\rangle\leftrightarrow|4\rangle$ are respectively coupled by the free vacuum reservoir and a modified isotropic PC reservoir. In the absorption spectra, it is found that the atom is transparent to the probe laser field with the detuning of $\Delta_{\mathrm{p}}=\pm1$. This type of transparency originates from the two square-root singularities of DOM of double-band PBG material at threshold. This effect is closely consistent with the results of a Λ-type atom embedded in isotropic single-band PBG.[20] In the case of $\Omega_1=1$, $\Omega_2=0$, and $\Omega_3=1$ as can be seen in Fig. 2(b), this five-level configuration changes to a tripod model which is driven by two strong laser fields and a weak probe laser field. The corresponding absorption spectra appear four symmetrical absorption peaks and three transparent points at the positions of $\Delta_{\mathrm{p}}=\pm1$ and $\Delta_{\mathrm{p}}=0$, which result from the peculiar DOM of the isotropic PBG and the destructive quantum interference effect, respectively. However, as the intensities of three coupling fields are set as $\Omega_1=1$, $\Omega_2=1$, and $\Omega_3=1$ in Fig. 2(c), and $\Omega_1=1$, $\Omega_2=2$, and $\Omega_3=2$ in Fig. 2(d), the three coupling fields form a closed loop which can lead to phase dependent interaction. When the relative phase is $\theta=0$, the absorption spectra become asymmetric. However, the transparency windows are still at the positions of $\Delta_{\mathrm{p}}=\pm\Omega_2$ and $\Delta_{\mathrm{p}}=\delta_{\mathrm{c}114},\delta_{\mathrm{c}214}$. In Fig. 2(c), the quantum interference induced transparency (at $\Delta_{\mathrm{p}}=\pm\Omega_2$) and the modified reservoir-induced transparency (at $\Delta_{\mathrm{p}}=\delta_{\mathrm{c}114},\delta_{\mathrm{c}214}$) are degenerated with each other. On the other hand, we also note that the dispersion curve has a very large slope around the transparency windows, therefore these configurations can be used to decrease the group velocity of light.

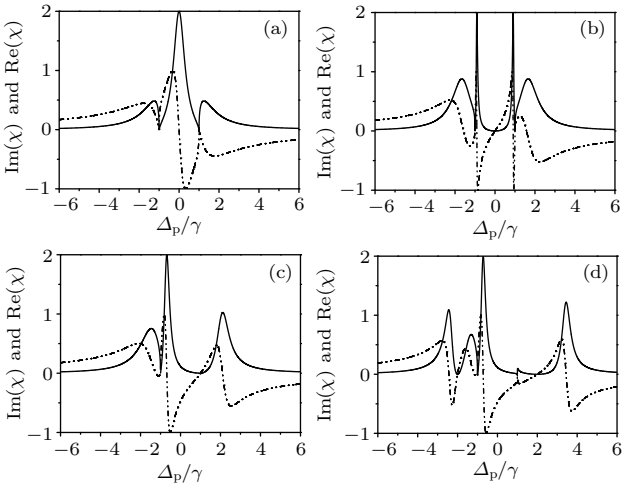


Fig. 2. The absorption $\mathrm{Im}(\chi)$ (solid lines) and dispersion $\mathrm{Re}(\chi)$ (dash–dot–dot lines) spectra (in arbitrary units) as a function of detuning frequency Δ_{p} for different amplitudes of three coupling fields. (a) $\Omega_1=0$, $\Omega_2=0$, and $\Omega_3=0$; (b) $\Omega_1=1$, $\Omega_2=0$, and $\Omega_3=1$; (c) $\Omega_1=1$, $\Omega_2=1$, and $\Omega_3=1$; and (d) $\Omega_1=1$, $\Omega_2=2$, and $\Omega_3=2$. Other parameters are set as: $\gamma=1$, $\beta_{14}=1$, $\theta=0$, $\delta_{\mathrm{c}114}=1.0$, and $\delta_{\mathrm{c}114}=-1.0$.

Due to the closed interaction loop, *i.e.*,

$$|1\rangle\xrightarrow{|\Omega_1|\mathrm{e}^{\mathrm{i}\varphi_1}}|2\rangle\xrightarrow{|\Omega_2|\mathrm{e}^{\mathrm{i}\varphi_2}}|3\rangle\xrightarrow{|\Omega_3|\mathrm{e}^{-\mathrm{i}\varphi_3}}|1\rangle,$$

the Autler–Townes splitting of state $|1\rangle$ depends on the closed interaction phase θ of the three coupling fields. Therefore, we can obtain a phase-dependent linear susceptibility. For the case that $\gamma=1$, $\beta_{14}=1$, $\Omega_1=1$, $\Omega_2=2$, $\Omega_3=1$, $\delta_{\mathrm{c}114}=1.0$, and $\delta_{\mathrm{c}214}=-1.0$, the probe absorption and dispersion as a function of probe detuning frequency Δ_{p} for different collective phases are displayed in Fig. 3. It is obvious that three transparency windows can be observed at the positions of $\Delta_{\mathrm{p}}=\pm1,2$ as shown in Fig. 3(a). By diagonalizing the Hamiltonian of the coherently driven closed-loop, the energies of the three dressed-state levels are $E_+=1+\sqrt{3}$, $E_0=1-\sqrt{3}$, and $E_-=-2$, respectively. However, the eigenvector corresponding to $E_-=-2$ is $|-\rangle=(|2\rangle-|3\rangle)/\sqrt{2}$, which has no component of $|1\rangle$. Then there are only two possible probe transitions in the dressed-state picture. The destructive quantum interference between them gives rise to vanishing absorption at $\Delta_{\mathrm{p}}=2$. In free space, the two dressed states correspond to two distinct absorption peaks. Nevertheless, as a result of the square-root singularity at the DOM threshold in the

isotropic PBG reservoir, two new transparency windows occur near the photonic band edge for the probe field with detunings of $\Delta_p = \pm 1$. In addition, two small absorption peaks arise near by the transparency points. When the relative phase of the coupling fields is $\theta = \pi$, the absorption and dispersion curves are similar with the case of $\theta = 0$ (see Fig. 3(d)). But the absorption–dispersion spectra in Figs. 3(a) and 3(d) are opposite, *i.e.*, $\chi(\Delta_p)\big|_{\theta=0} = \chi(-\Delta_p)\big|_{\theta=\pi}$. Figures 3(b) and 3(c) display the optical spectra when $\theta = \pi/4$ and $\theta = \pi/2$, respectively. The medium is transparent to the probe laser field at four positions of $\Delta_p = \pm 1, \pm 2$. In these conditions of $\theta \neq n\pi$ (n is an integer), there are three probe transitions in the dressed-state picture, among which the quantum interference results in two transparency windows at $\Delta_p = \pm 2$, while the other two windows at $\Delta_p = \pm 1$ are induced by the modified reservoir. When giving an insight into the formula of linear susceptibility, we can see that the line shape of absorption is symmetric in the case of $\theta = (n + 1/2)\pi$ (n is an integer) as shown in Fig. 3(c). Comparing Fig. 2(d) with Fig. 3(a), we can observe that different intensities of Ω_3 change the number of the transparent windows.

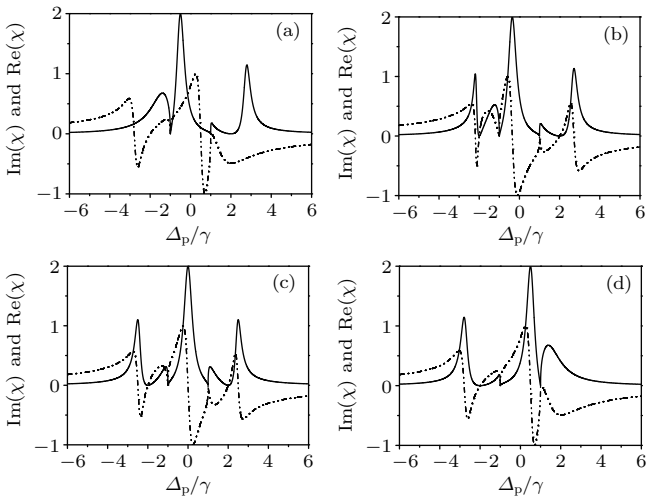


Fig. 3. The absorption $\mathrm{Im}(\chi)$ (solid lines) and dispersion $\mathrm{Re}(\chi)$ (dash-dot-dot lines) spectra (in arbitrary units) as a function of detuning frequency Δ_p for different phases of three coupling fields. (a) $\theta = 0$; (b) $\theta = \pi/4$; (c) $\theta = \pi/2$; and (d) $\theta = \pi$. Other parameters are set as: $\gamma = 1$, $\beta_{14} = 1$, $\Omega_1 = 1$, $\Omega_2 = 2$, $\Omega_3 = 1$, $\delta_{c114} = 1.0$, and $\delta_{c214} = -1.0$.

From this analysis, we can see that the square-root singularity at the DOM threshold in an isotropic PBG plays an important role in the reservoir induced transparency. For more discussions, we plot the susceptibility in the case of different widths of PBG and the detuning of atomic resonant frequency from the center of the double bandgap, as shown in Fig. 4. The parameters are set as $\gamma = 1$, $\beta_{14} = 1$, $\Omega_1 = 1$, $\Omega_2 = 2$, $\Omega_3 = 1$, and $\theta = \pi/2$. Based on the results above, we can conclude that there must be two transparency windows at the position of $\Delta_p = \pm\Omega_2$ when $\theta \neq n\pi$ (n is an integer), as shown in Figs. 4(a) and 4(b). The vanishing absorption is attributed to the laser fields induced quantum coherence. However, the width and position of the double PBG can effectively affect the other two transparency windows (at $\Delta_p = \delta_{c114}, \delta_{c214}$) and modify the line-shapes of absorption peaks. Generally, there are three transitions in the dressed-state picture, which corresponds to three absorption peaks in vacuum reservoir. Their positions are determined by the energies of Autler-Townes splitting, *i.e.*, $E_\pm = \pm 2.45$ and $E_0 = 0$ with the given parameters. If the center frequency of double PBG is equal to the atomic frequency, the absorption spectrum is symmetric (see Fig. 3(c)). While in the case of $\delta_{c114} = 0$ and $\delta_{c214} = -3$, one of the dressed state (with eigenvalue $E_0 = 0$) is resonant with the upper band edge, and the central absorption peak splits into two peaks by one reservoir induced transparency window. Then, the spectrum becomes asymmetric as shown in Fig. 4(a). The effect of width of PBG on the absorption spectra is also obvious. Figure 4(b) shows the case of $\delta_{c114} = 0.5$ and $\delta_{c214} = -0.5$. We can see that the central absorption peak is narrowed compared with Fig. 3(c).

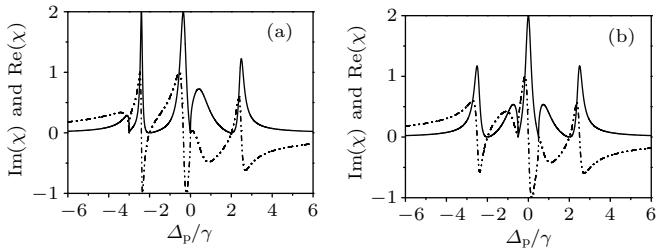


Fig. 4. The absorption $\mathrm{Im}(\chi)$ (solid lines) and dispersion $\mathrm{Re}(\chi)$ (dash–dot–dot lines) spectra (in arbitrary units) as a function of detuning frequency Δ_p for different bandgap widths. (a) $\delta_{c114} = 0$, $\delta_{c214} = -3$; (b) $\delta_{c114} = 0.5$, $\delta_{c214} = -0.5$. Other parameters are set as: $\gamma = 1$, $\beta_{14} = 1$, $\Omega_1 = 1$, $\Omega_2 = 2$, $\Omega_3 = 1$, and $\theta = \pi/2$.

Interacting dark resonances can lead to the emergence of sharp spectral features between two electromagnetically induced transparency windows.[35] In the considered atomic system interacting with isotropic PBG, two additional transparency windows arise. If the width of PBG is narrow enough, a central narrow structure between the two modified reservoir-induced transparency windows is obtained as shown in Fig. 5(a). The absorption peak can be much less than the natural line-width. Moreover, we can also achieve sub-natural feature by using the interaction between the electromagnetically induced transparency window and the modified reservoir-induced transparency window (see Fig. 5(b)).

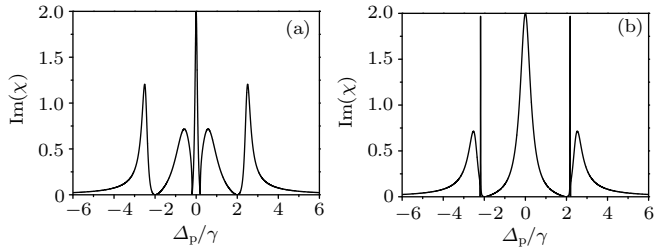


Fig. 5. The absorption spectra (in arbitrary units) as a function of detuning frequency Δ_p. (a) $\delta_{c114} = 0.2$, $\delta_{c214} = -0.2$; (b) $\delta_{c114} = 2.2$, $\delta_{c214} = -2.2$. Other parameters are set as: $\gamma = 1$, $\beta_{14} = 1$, $\Omega_1 = 1$, $\Omega_2 = 2$, $\Omega_3 = 1$, and $\theta = \pi/2$.

Finally, we consider the effects of different atom–reservoir resonant coupling constant β_{14} on the absorption

property of the atomic system. The absorption spectra as a function of detuning frequency Δ_p are plotted in Fig. 6. Compared with Fig. 3(c), we can observe that the atom-reservoir resonant coupling constant β_{14} does not affect the transparent windows but it can change the line-shape of absorption peaks. In more detail, for small value of β_{14}, PBG has little influence on the absorption spectrum; the positions, widths, and intensities are similar to the case in a vacuum reservoir. However, as β_{14} becomes large, the intensities of sideband absorptions peaks decrease and their line widths are broadened, while the central peak is narrowed with intensity unchanged.

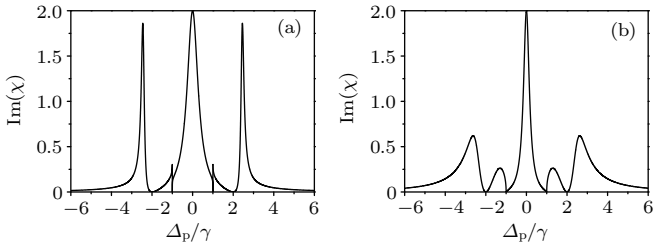


Fig. 6. The absorption $\mathrm{Im}(\chi)$ spectra (in arbitrary units) as a function of detuning frequency Δ_p for different β_{14} values. (a) $\beta_{14} = 0.2$; (b) $\beta_{14} = 2$. Other parameters are set as: $\gamma = 1$, $\Omega_1 = 1$, $\Omega_2 = 2$, $\Omega_3 = 1$, $\theta = \pi/2$, $\delta_{c114} = 1.0$, and $\delta_{c214} = -1.0$.

4. Conclusion

In conclusion, we have studied the absorption–dispersion spectra of a microwave- driven five-level atom embedded in the double-band anisotropic photonic crystal. With the help of a Laplace transform and final-value theorem, the expression of linear susceptibility is derived and numerically investigated. The resulting absorption- dispersion depends on the relative phase and amplitude of the coupling fields, and can be controlled by the width and position of the PBG as well. In this scheme, there are two types of transparencies which, respectively, result from the fields dynamically induced quantum interference and the square root singularity at the DOM threshold in isotropic PBG. The interaction between the modified reservoir induced transparency and electromagnetically induced transparency can lead to the emergence of ultra-narrow spectral feature. These theoretical studies can provide us with further flexibility to control the interactions between light and atom, especially the optical properties of atoms, which may have applications in optical switching and slow light.

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