ISSN 1674-1056
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Citation: Chin. Phys. B . 2019, 28(2): 020301. **doi:** 10.1088/1674-1056/28/2/020302
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Error-detected single-photon quantum routing using a quantum dot and a double-sided microcavity system*

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(Received 14 October 2018; revised manuscript received 5 December 2018; published online 10 January 2019)

Based on a hybrid system consisting of a quantum dot coupled with a double-sided micropillar cavity, we investigate the implementation of an error-detected photonic quantum routing controlled by the other photon. The computational errors from unexpected experimental imperfections are heralded by single photon detections, resulting in a unit fidelity for the present scheme, so that this scheme is intrinsically robust. We discuss the performance of the scheme with currently achievable experimental parameters. Our results show that the present scheme is efficient. Furthermore, our scheme could provide a promising building block for quantum networks and distributed quantum information processing in the future.

Keywords: quantum routing, error-detected quantum computation, quantum information processing

PACS: 03.67.Hk, 03.67.Pp, 03.67.–a **DOI:** 10.1088/1674-1056/28/2/020301

1. Introduction

Quantum networks are the main physical platforms for scalable quantum information processing (QIP) and quantum communication.[1–6] A quantum router, which is a quantum-mechanical counterpart of the classical router, can take advantage of quantum dynamics and phenomena such as quantum superposition or entanglement. It is used to direct a signal qubit to its desired signal mode controlled by the state of a control qubit while keeping the signal qubit intact. In some cases, one can use a classical optical pulse to control the spatial mode of signals.[7] Nevertheless, it is necessary that both the control and signal be quantum for a fully functional quantum router. A quantum router is not only important in quantum networks but is also very important in QIP, such as in quantum network manipulation,[8] quantum message authentication,[9] quantum secure direct communication,[10–16] quantum entanglement generation,[17,18] and hyperentangled-state manipulation.[19,20] In particular, a quantum router can work as quantum wave combiner in duality quantum computer,[21–23] which is very important in designing quantum algorithms.[24–27] In recent years, the construction of photonic quantum routers has been intensively investigated. In 2013, Lemr *et al.*[28,29] presented protocols of quantum routing with linear optics. Two similar linear optical processes, named "quantum jointing" and "quantum splitting," have been demonstrated.[30] Various single-photon routing schemes based on photonic qubits,[31–33] cross-Kerr nonlinearity,[34] atom–cavity,[35–37] coupled resonators,[38,39] or optomechanical systems[40] have been proposed or demonstrated.[41]

Recently, some schemes[42–46] have been proposed for the construction of single-photon quantum routers based on the nonlinear interaction of the photon-atom in a cavity quantum electrodynamics (QED) system. However, in a realistic condition, the inevitable interactions between the quantum system with the environment result in imperfect nonlinear interaction, which will suppress the fidelity of the quantum operation or bring secondary pollution to the system.[47–49] Although one could use error correction schemes to solve this problem, this would again consume more resources.[50] Therefore, finding an error-detected method for quantum routing is of considerable significance in quantum networks and computation. In 2012, Li *et al.*[51] proposed a robust atom–photon entangling gate based on the scattering of photons off single emitters in one dimensional waveguides. Then, Li *et al.*[52] and Borges *et al.*[53] proposed two similar proposals for error-rejecting quantum computing via cavity-assisted photon scattering, respectively. In 2016, another proposal for error-detected generating and complete analysing of hyperentangled Bell states with quantum dot (QD)-double-sided system was proposed.[54] Subsequently, several error-rejecting schemes

*Project supported by the Scientific Research Foundation of Shanxi Institute of Technology (Grant No. 201706001), the Fund for Shanxi "1331 Project" Key Subjects Construction, the China Postdoctoral Science Foundation (Grant No. 2017M612411), the Education Department Foundation of Henan Province, China (Grant No. 18A140009), and the National Natural Science Foundation of China (Grant Nos. 61821280, 11604190, and 61465013).

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for quantum repeaters,[55–60] quantum logic gates,[61–65] and entanglement generation[66] have been proposed, in which computational errors are heralded by single photon detections, resulting a unity operation fidelity, which is a highly desirable feature for quantum communication or computation.

Recent work has shown that the electron spin in a GaAs/InAs-based charged QD is a promising candidate for solid-state qubit, and holds great promise in QIP and quantum networks.[67–71] The electron-spin coherence time of a charged QD can be maintained for more than 3 μs[72,73] and the electron spin-relaxation time can be longer (∼ms).[74,75] Moreover, it is comparatively easy to embed a QD in a solid-state cavity and it can be easily manipulated and initialized.[76–79] Based on a singly charged QD inside an optical resonant cavity, many interesting schemes for QIP have been proposed.[80–84]

In this paper, inspired by recent progress, we propose a practical scheme for single-photon quantum routing controlled by the other photon that circumvents local errors. Here, we use a system consists of a single QD coupled with a doubled-sided micropillar cavity functioning as a drop-filter structure. In this scheme, the signal photon can be directed to its desired signal mode controlled by the control photon qubit, with the original signal photon being unaffected. Furthermore, imperfect nonlinear interaction resulting from weak couplings, atomic decay into undesired modes or frequency mismatches of the incident single-photon pulses may decrease the probability of success. The fidelity of the present scheme, or the present scheme works in an error-detected way, makes it far different from the previous schemes based on cavity-QED.[42–46] Therefore, we assess the feasibility of the present scheme. The results show that the scheme can operate effectively by considering current or near-future techniques and it allows implementation in a more practical parameter regime for QDs.

2. Input–output relation for a single photon with a QD-cavity coupling system

Consider a quantum system that consists of a QD coupled to a micropillar cavity consisting of two Bragg reflector mirrors, as shown in Fig. 1(a). Generally speaking, as shown in Ref. [70], in the single charged GaAs/InAs QD associated with the dipole transitions, the quantum system has four relevant electronic levels. The confinement potential of the QD is much tighter in the growth direction than that in the transverse direction, so we can define the growth direction as the quantization axis (z axis) for angular momentum. The spin of the holes are $J_z = \pm(3/2)(|\Downarrow\rangle|\Uparrow\rangle)$ and the spin of the single electron states are $J_z = \pm(1/2)(|\downarrow\rangle|\uparrow\rangle)$. An exciton X^- consisting of two electrons bound to one heavy hole with negative charges can then be created [as shown in Fig. 1(b)].

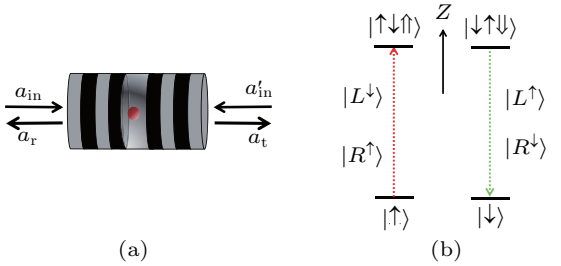


Fig. 1. Schematic setup of the composite system and the single photon cavity input-output process. (a) A QD is trapped in a double-sided cavity. (b) The relevant electron spin energy level structure and the transitions of a QD.

According to the Pauli's principle, the electron-spin state does not interact with the hole spin because the two electrons have total spin zero, which is a singlet state. Therefore, we can achieve two optically allowed transitions between the electron states and the exciton states ($|\uparrow\rangle \leftrightarrow |\uparrow\downarrow\Uparrow\rangle$ and $|\downarrow\rangle \leftrightarrow |\downarrow\uparrow\Downarrow\rangle$) by involving a photon whose spin is $s_z = +1$ or $s_z = -1$. Therefore, the Heisenberg equations of motion of the cavity mode and the lowing operator are[70]

$$\begin{aligned}
\frac{\mathrm{d}\hat{a}}{\mathrm{d}t} &= \left[\mathrm{i}(\omega-\omega_{\rm c})+\kappa+\frac{\kappa_{\rm s}}{2}\right]\hat{a}-g\sigma_- \\
&\quad -\sqrt{\kappa}\hat{a}_{\rm in}-\sqrt{\kappa}\hat{a}'_{\rm in}+\hat{\xi}, \\
\frac{\mathrm{d}\sigma_-}{\mathrm{d}t} &= \left[\mathrm{i}(\omega-\omega_{X^-})-\frac{\gamma}{2}\right]\sigma_- - g\sigma_z\hat{a}+\hat{\varsigma}, \\
\hat{a}_r &= \hat{a}_{\rm in}+\sqrt{\kappa}\hat{a}, \quad \hat{a}_t = \hat{a}'_{\rm in}+\sqrt{\kappa}\hat{a},
\end{aligned} \tag{1}$$

here $\hat{\xi}$ and $\hat{\varsigma}$ are the noise operators needed to conserve the commutation relations. γ represents decay rate of the QD, while κ and $\kappa_{\rm s}$ describe the cavity damping rate and the cavity leakage rate, respectively. In the weak-excitation approximation, we can adiabatical eliminate the cavity mode and lead to the reflection and transmission coefficients as[69,70]

$$\begin{aligned}
r(\omega) &= 1+t(\omega), \\
t(\omega) &= \frac{\kappa[\mathrm{i}(\omega_{X^-}-\omega)+\gamma/2]+g^2}{[\mathrm{i}(\omega_{\rm c}-\omega)+\kappa+\kappa_{\rm s}/2][\mathrm{i}(\omega_{X^-}-\omega)+\gamma/2]+g^2}.
\end{aligned} \tag{2}$$

By setting $g = 0$, the reflection and transmission coefficients for a cold cavity with the QD uncoupled to the cavity can be written as

$$\begin{aligned}
r_0(\omega) &= \frac{\mathrm{i}(\omega_{\rm c}-\omega)+\kappa_{\rm s}/2}{\mathrm{i}(\omega_{\rm c}-\omega)+\kappa+\kappa_{\rm s}/2}, \\
t_0(\omega) &= -\frac{\kappa}{\mathrm{i}(\omega_{\rm c}-\omega)+\kappa+\kappa_{\rm s}/2}.
\end{aligned} \tag{3}$$

Equations (2) and (3) indicate that an incident photon follows the rules for optical transitions in a realistic QD-cavity system:[70]

$$\begin{aligned}
&|L^{\uparrow},\uparrow\rangle \to t_0|L^{\uparrow},\uparrow\rangle + r_0|R^{\downarrow},\uparrow\rangle, \\
&|L^{\uparrow},\downarrow\rangle \to r|R^{\downarrow},\downarrow\rangle + t|L^{\uparrow},\downarrow\rangle, \\
&|L^{\downarrow},\uparrow\rangle \to r|R^{\uparrow},\uparrow\rangle + t|L^{\downarrow},\uparrow\rangle, \\
&|L^{\downarrow},\downarrow\rangle \to t_0|L^{\downarrow},\downarrow\rangle + r_0|R^{\uparrow},\downarrow\rangle, \\
&|R^{\uparrow},\uparrow\rangle \to r|L^{\downarrow},\uparrow\rangle + t|R^{\uparrow},\uparrow\rangle,
\end{aligned}$$

$$
\begin{aligned}
&|R^{\uparrow},\downarrow\rangle \rightarrow t_0|R^{\uparrow},\downarrow\rangle + r_0|L^{\downarrow},\downarrow\rangle,\\
&|R^{\downarrow},\uparrow\rangle \rightarrow t_0|R^{\downarrow},\uparrow\rangle + r_0|L^{\uparrow},\uparrow\rangle,\\
&|R^{\downarrow},\downarrow\rangle \rightarrow r|L^{\uparrow},\downarrow\rangle + t|R^{\downarrow},\downarrow\rangle. \qquad (4)
\end{aligned}
$$

Here $|R\rangle$ and $|L\rangle$ denote the right- and left-circular polarizations along the propagation direction with respect to the z axis, respectively. The superscript arrow $\uparrow$ or $\downarrow$ represents that the direction of the photon is parallel or antiparallel to the z axis of the electron spin.

Next, we consider an error-detected circuit unit for the interaction between the circularly polarized photon and the QD-cavity system, as shown in Fig. 2, which is constructed with a 50:50 beam splitter (BS), two half-wave plates (Hs), and a QD double-sided cavity system.

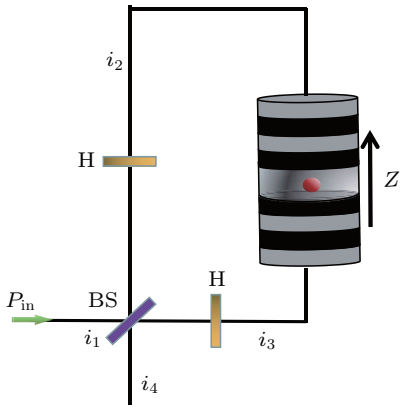


Fig. 2. Schematic illustration of the error-detected circuit unit for the interaction between the circularly polarized photon and the QD-cavity system. BS is a 50:50 beam splitter that performs the spatial-mode Hadamard operation $[|i_1\rangle \rightarrow (1/\sqrt{2})(|i_2\rangle + |i_3\rangle), |i_4\rangle \rightarrow (1/\sqrt{2})(|i_2\rangle - |i_3\rangle)]$ on the photon. H denotes a half-wave plate which is set to 22.5° to induce the Hadamard transformations on the polarization of photons as $|R\rangle \rightarrow (1/\sqrt{2})(|R\rangle + |L\rangle)$, $|L\rangle \rightarrow (1/\sqrt{2})(|R\rangle - |L\rangle)$.

Suppose a photon in the state $|\varphi\rangle_p = |R\rangle$ is injected into the error-detected circuit unit, and the QD is initialized to $|\varphi^+\rangle = (1/\sqrt{2})(|\uparrow\rangle + |\downarrow\rangle)$. First, the incident photon is divided into two spatial modes by the BS and then transmits the Hs from each spatial mode. The state of the whole system composed of the photon and the QD in the cavity is changed from the state $|\Phi\rangle_0 = (1/\sqrt{2})|R\rangle|i_1\rangle \otimes (|\uparrow\rangle + |\downarrow\rangle)$ into the state $|\Phi\rangle_1$. Here $|\Phi\rangle_1$ is

$$
\begin{aligned}
|\Phi\rangle_1 = \; & \frac{1}{2}[(|R^{\downarrow}\rangle + |L^{\downarrow}\rangle)|i_2\rangle + (|R^{\uparrow}\rangle + |L^{\uparrow}\rangle)|i_3\rangle]\\
& \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle). \qquad (5)
\end{aligned}
$$

After the interaction between the photon and the QD-cavity system, the state of the composite system becomes

$$
\begin{aligned}
|\Phi\rangle_2 = \; & \frac{1}{2\sqrt{2}}[(t_0 + r_0)|R^{\downarrow}\rangle|i_3\rangle + (t + r)|L^{\downarrow}\rangle|i_3\rangle\\
& + (t + r)|R^{\uparrow}\rangle|i_2\rangle + (t_0 + r_0)|L^{\uparrow}\rangle|i_2\rangle]|\uparrow\rangle\\
& + \frac{1}{2\sqrt{2}}[(t_0 + r_0)|R^{\uparrow}\rangle|i_2\rangle + (t + r)|L^{\uparrow}\rangle|i_2\rangle\\
& + (t + r)|R^{\downarrow}\rangle|i_3\rangle + (t_0 + r_0)|L^{\downarrow}\rangle|i_3\rangle]|\downarrow\rangle. \qquad (6)
\end{aligned}
$$

Subsequently, the photon is reflected by the mirrors, and then passes through Hs again, the state is given

$$
\begin{aligned}
|\Phi\rangle_3 = \; & \frac{1}{4}[(t + r + t_0 + r_0)|R^{\downarrow}\rangle|i_3\rangle\\
& + (t + r + t_0 + r_0)|R^{\uparrow}\rangle|i_2\rangle\\
& - (t + r - t_0 - r_0)|L^{\downarrow}\rangle|i_3\rangle\\
& + (t + r - t_0 - r_0)|L^{\uparrow}\rangle|i_2\rangle]|\uparrow\rangle\\
& + \frac{1}{4}[(t + r + t_0 + r_0)|R^{\downarrow}\rangle|i_3\rangle\\
& + (t + r + t_0 + r_0)|R^{\uparrow}\rangle|i_2\rangle\\
& + (t + r - t_0 - r_0)|L^{\downarrow}\rangle|i_3\rangle\\
& - (t + r - t_0 - r_0)|L^{\uparrow}\rangle|i_2\rangle]|\downarrow\rangle. \qquad (7)
\end{aligned}
$$

Finally, the photon would interfere at the BS, and the state evolves to

$$
|\Phi\rangle_4 = D|R\rangle|i_1\rangle|\varphi^+\rangle + T|L\rangle|i_4\rangle|\varphi^-\rangle. \qquad (8)
$$

Here $|\varphi^-\rangle = (1/\sqrt{2})(|\uparrow\rangle - |\downarrow\rangle)$, $D = (1/2)(t + r + t_0 + r_0)$, and $T = (1/2)(t + r - t_0 - r_0)$ are the reflection and the transmission coefficients of the error-detected circuit unit, respectively.

Similarly, in the other case where the QD is initially in the state $|\varphi^-\rangle$, the evolution process of the interaction between a right-circularly polarized photon and the circuit unit in the realistic condition can be described as

$$
|R\rangle|\varphi^-\rangle \rightarrow D|R\rangle|i_1\rangle|\varphi^-\rangle + T|L\rangle|i_4\rangle|\varphi^+\rangle. \qquad (9)
$$

If the photon is reflected from the circuit unit with probability of D^2, the polarization of the photon and the state of the QD would not change. If the photon is transmitted from the error-detected circuit unit with probability of T^2, the polarization of the photon is flipped and the superposition state of the QD is changed from $|\varphi^+\rangle$ to $|\varphi^-\rangle$ or from $|\varphi^-\rangle$ to $|\varphi^+\rangle$.

3. Error-detected single-photon quantum routing controlled by the other photon

The function of a quantum router is to direct a signal qubit to its desired spatial mode controlled by the state of a control qubit while keeping the signal qubit unchanged. With the faithful process described above, we now show how to implement a quantum router between two photons. Here the control qubit is encoded in polarization degree of freedom of a single photon as $|\varphi\rangle_c = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$. The signal qubit attached initially to the other photon is also polarization encoded as $|\varphi\rangle_s = \alpha|R\rangle + \beta|L\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. The QD is initially prepared in state $|\varphi\rangle_1 = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$.

As illustrated in Fig. 3, our scheme for implementing a quantum router between two photons can be achieved with three steps.

Step 1 A control photon p_c in the state $|\varphi\rangle_c$ is launched into the polarization beam splitter (PBS1) which transmits the photon in the right-circular polarization $|R\rangle$ and reflects the photon in the left-circular polarization $|L\rangle$, respectively. The wave packet in the left-circular polarization $|L\rangle$ would pass through X_1 and the circuit unit with the reflection coefficient D and transmission coefficient T, while the wave packet in the right-circular polarization $|R\rangle$ would pass through the circuit unit directly. After the photon passes the circuit unit, the composite system composed of the QD and the control photon evolves to

$$\begin{aligned}|\Psi\rangle_1' = \frac{1}{\sqrt{2}}(&D|R\rangle_c|\varphi^+\rangle|i_1\rangle+\underline{T|L\rangle_c|\varphi^-\rangle|i_4\rangle}\\&+T|L\rangle_c|\varphi^-\rangle|i_5\rangle+\underline{D|R\rangle_c|\varphi^+\rangle|i_6\rangle}).\end{aligned}\tag{10}$$

Here, $|i_n\rangle$ represents the output spatial modes (see Fig. 3). The wave packets in spatial mode $|i_4\rangle$ will be detected by the D1, and wave packets in $|i_3\rangle$ will be guided into $|i_6\rangle$ and be detected by the D2 (these two terms are underlined in Eq. (10)). Therefore, when neither D1 nor D2 clicks, the final state of the composite system composed of the QD and the control photon will be

$$|\Psi\rangle_1 = D|R\rangle_c|\varphi^+\rangle|i_1\rangle+T|L\rangle_c|\varphi^-\rangle|i_5\rangle.\tag{11}$$

Here, we have omitted the global coefficient. In the following, we will omit the spatial mode of the control photon p_c since its spatial mode would stay unchanged.

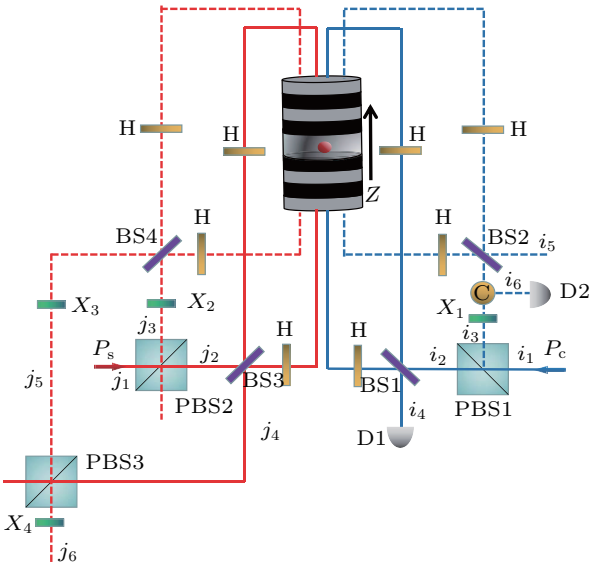


Fig. 3. Schematic diagram of the error-detected single-photon quantum router for the implementation of signal-photon quantum routing controlled by a control photon. PBS$_i$ ($i = 1, 2, 3$) is a circularly polarized beam splitter which transmits the photon in the right-circular polarization $|R\rangle$ and reflects the photon in the left-circular polarization $|L\rangle$, respectively. D is a single photon detector and C is an optical circulator which can guide the photon to an appropriate path. X_i ($i = 1, 2, 3, 4$) is a half-wave plate which is used to perform a polarization bit-flip operation $\sigma_x^p = |R\rangle\langle L| + |L\rangle\langle R|$ on the photon passing through it.

Step 2 In this round, the signal photon (label P_s) in $|\varphi\rangle_s$ is injected into PBS2 from $|j_1\rangle$, the wave packet in the left-circular polarization $|L\rangle$ would pass through X_2 and the circuit unit, and then pass through X_2 in spatial mode $|j_3\rangle$ and X_3 in spatial mode $|j_5\rangle$, respectively. While the wave packet in the right-circularly polarization $|R\rangle$ would pass through the circuit unit directly. Similar to step 1, before the wave packets reach PBS2 and PBS3, the state of the composite system is given

$$\begin{aligned}|\Psi\rangle_2 = {}& D|R\rangle_c(\alpha D|R\rangle_s|\varphi^+\rangle|j_2\rangle+\alpha T|L\rangle_s|\varphi^-\rangle|j_4\rangle\\&+\beta D|L\rangle_s|\varphi^+\rangle|j_3\rangle+\beta T|R\rangle_s|\varphi^-\rangle|j_5\rangle)\\&+T|L\rangle_c(\alpha D|R\rangle_s|\varphi^-\rangle|j_2\rangle+\alpha T|L\rangle_s|\varphi^+\rangle|j_4\rangle\\&+\beta D|L\rangle_s|\varphi^-\rangle|j_3\rangle+\beta T|R\rangle_s|\varphi^+\rangle|j_5\rangle).\end{aligned}\tag{12}$$

Then, the two wave packets in the right- and left-circular polarization rejoin after passing through PBS2 and PBS3, the evolution result is

$$\begin{aligned}|\Psi\rangle_3 = {}& D^2|R\rangle_c|\varphi^+\rangle|j_1\rangle(\alpha|R\rangle_s+\beta|L\rangle_s)\\&+DT|R\rangle_c|\varphi^-\rangle|j_6\rangle(\alpha|L\rangle_s+\beta|R\rangle_s)\\&+TD|L\rangle_c|\varphi^-\rangle|j_1\rangle(\alpha|R\rangle_s+\beta|L\rangle_s)\\&+T^2|L\rangle_c|\varphi^+\rangle|j_6\rangle(\alpha|L\rangle_s+\beta|R\rangle_s).\end{aligned}\tag{13}$$

Step 3 Then, we should measure the QD in orthogonal basis $\{|\varphi^+\rangle, |\varphi^-\rangle\}$. If the measurement result is $|\varphi^+\rangle$, then the state of the system is

$$\begin{aligned}|\Psi\rangle_4 = {}& D^2|R\rangle_c|j_1\rangle(\alpha|R\rangle_s+\beta|L\rangle_s)\\&+T^2|L\rangle_c|j_6\rangle(\alpha|L\rangle_s+\beta|R\rangle_s).\end{aligned}\tag{14}$$

If the outcome of the measurement is $|\varphi^-\rangle$, then we obtain

$$\begin{aligned}|\Psi'\rangle_4 = {}& DT|R\rangle_c|j_6\rangle(\alpha|L\rangle_s+\beta|R\rangle_s)\\&+TD|L\rangle_c|j_1\rangle(\alpha|R\rangle_s+\beta|L\rangle_s).\end{aligned}\tag{15}$$

These two equations are generally a superposition state of two modes in ports $|j_6\rangle$ and $|j_1\rangle$ with the two-photon state is changed from $|\varphi\rangle_c \otimes |\varphi\rangle_s$ to $|\Psi\rangle_4$ or $|\Psi'\rangle_4$. We then perform a bit-flip operation on the signal photon in $|j_6\rangle$ spatial mode by letting it pass through X_4, as shown in Fig. 3. Consequently, the signal photon can be directed to the output port $|j_6\rangle$, output port $|j_1\rangle$ or both, controlled by the control photon qubit, while the original signal-photon qubit is unchanged. Specifically, when the measurement result is $|\varphi^+\rangle$, if the control photon is set to $|R\rangle_c$, then the signal photon is directed into spatial mode $|j_1\rangle$ with its state unchanged, if the control photon is set to $|L\rangle_c$, the signal photon is directed into spatial mode $|j_6\rangle$ with its state unchanged, if the control photon is set to $|\varphi\rangle_c = (1/\sqrt{2})(|R\rangle + |L\rangle)$, the state of the signal photon is still unchanged, but can be directed into spatial modes $|j_1\rangle$ and $|j_6\rangle$. When the measurement result of the QD spin is $|\varphi^-\rangle$, then the case is similar. Therefore the purpose of the

signal-photon quantum routing is achieved. It should noted that the state of the output signal photon is independent of the reflection coefficients r_m and transmission coefficients t_m because they only appear as a global coefficient in Eqs. (14) and (15). By this means, the present scheme does not suffer from unexpected system detrimental and works in an error-detected way.

4. Success probability and experimental feasibilities

In the previous quantum routing schemes based cavity-QED,[42–46] system loss, such as photon loss, cavity decay, atomic spontaneous emission, and imperfect coupling efficiency may reduce the fidelity of the schemes and induce computational errors. Therefore, these schemes prefer to work in strong-coupling condition $g^2 \gg \kappa\gamma$ for high fidelity. In contrast, in our scheme, because the reflection and transmission coefficients r_m and t_m only appear in the global coefficient, the errors coming from the experimental imperfections would only decrease the success probability rather than the fidelity of the scheme.

In step 2, the total probability that the signal photon output from either port $|j_6\rangle$ or port $|j_1\rangle$ is

$$\eta = D^4 + 2T^2D^2 + T^4, \tag{16}$$

which is essentially the success probability of the quantum routing.

The total success probability η of the quantum routing is shown in Fig. 4 as a function of the side leakage κ_s/κ and the normalized coupling strength g/κ. When the probe photon is resonant to the cavity, $\eta = 0.978$ and $\eta = 0.522$ can be achieved in the regime of resonance scattering with $g/\kappa = 3$ and the Purcell regime with $g/\kappa = 0.5$, respectively, for $\gamma/\kappa = 0.1$ and $\kappa_s = 0$. When provided with a highly efficient single-photon source generating 10000 single-photons/s,[86] we may accomplish the quantum routing within a short time.

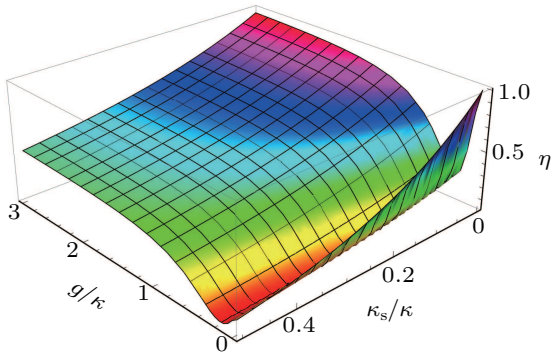


Fig. 4. The success probability of the quantum router vs the side leakage κ_s/κ and the normalized coupling strength g/κ. Here $\gamma/\kappa = 0.1$ is taken.

In addition, single photon pulse linewidth usually introduces additional infidelity in the previous schemes.[42–46] However, it will not introduce computational errors for the present scheme because one does with a monochromatic photon wave packet.[52] We plot the success probability of our scheme as a function of the linewidth Δ/κ as shown in Fig. 5. One can find that when $\kappa_s = 0$, $\Delta \in [-0.1\kappa, 0.1\kappa]$, the success probability $\eta > 0.939$ for $g/\kappa = 3$, and is relatively insensitive to the deviation of the linewidth Δ. Considering the decrease of success probability because of computational errors, and the practical input-output coupling efficiency is less than 1,[87] our scheme may be more efficient and practical than the previous ones.[42–46]

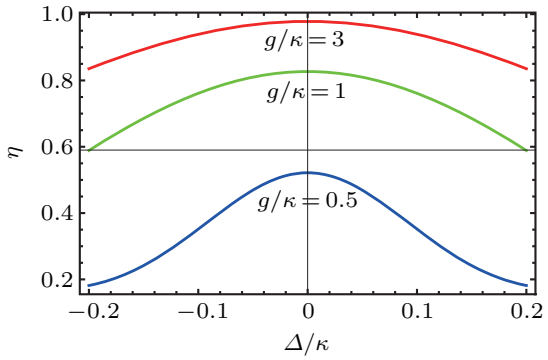


Fig. 5. The success probability of the quantum router *versus* the pulse linewidth Δ/κ. Here $\gamma/\kappa = 0.1$ is taken. Here the red line, green line and blue line represent $g/\kappa = 3$, $g/\kappa = 1$, and $g/\kappa = 0.5$, respectively.

As an emitter, QD in a microcavity is appealing as the technique of fast preparing the superposition state of an electron spin in a charged QD,[76,88] fast manipulating the electron spin in a charged QD,[77,78,89,90] and detecting the state of the electron spin in a charged QD[91] has been realized experimentally. Meanwhile, spin decoherence would affect the fidelity by the amount of $F = [1 + \exp(-\Delta t/T_2^e)]/2$.[70] Here, Δt and T_2^e are the time interval between input photons and the electron spin coherence time. In experiment, the order of magnitude of Δt and T_2^e can reach ns and μs.[73] While the QD spin dephasing have an effect on the fidelity by the amount of $F_2 = [1 - \exp(-\tau/T_2^e)]$, which can be neglected as the order of magnitude of cavity photon life time τ is ps.[92,93] The parameters above for calculation of success probability are based on the current experiments. Thereby the present scheme is efficient under practical experimental conditions. It should point out that, although our proposal is detailed with the QD-cavity system, it could also be implemented with atom–cavity[94] or nitrogen–vacancy–cavity system.[95]

5. Summary

In summary, by using a QD-double-sided microcavity system and linear optical elements, we present a scheme for implementing single-photon quantum routing in an error-detected way. In the present scheme, the computational errors are eliminated by single photon detections, while experimental defections would decrease the success probability rather than

the fidelity. Therefore this scheme is inherently robust and more practical than the previous schemes. All of these advantages suggest that the present scheme may be used in practical large-scale QIP and in the construction of future complex quantum networks.

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