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Damped electrostatic ion acoustic solitary wave structures in quantum plasmas with Bohm potential and spin effects

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Nonlinear properties of ion acoustic solitary waves are studied in the case of dense magnetized plasmas. The degenerate electrons with relative density effects from their spin states in the same direction and from equally probable up and down spinning states are taken up separately. Quantum statistical as well as quantum tunneling effects for both types of electrons are taken. The ions have large inertia and are considered classically, whereas the electrons are degenerate. The collisions of ions and electrons with neutral atoms are considered. We derive the deformed Korteweg de–Vries (DKdV) equation for small amplitude electrostatic potential disturbances by employing the reductive perturbation technique. The Runge–Kutta method is applied to solve numerically the DKdV equation. The analytical solution of DKdV is also presented with time dependence. We discuss the profiles for velocity, amplitude, and time variations in solitons for the cases when all the electrons are spinning in the same direction and for the case when there is equal probability of electrons having spin up and spin down. We have found that the wave is unstable because of the collisions between neutral gas molecules and the charged plasmas particles in the presence of degenerate electrons.

Keywords: degenerate plasma, collisions, spin-up, spin-down

PACS: 52.20.Fs, 52.25.Ya, 52.30.Ex, 52.35.–g **DOI:** 10.1088/1674-1056/28/1/015202

1. Introduction

The ion acoustic wave is the basic mode of plasma waves in collisionless as well as collisional plasmas. In collisionless plasmas, the pressure of electrons provides the restoring force, whereas the inertia comes from the mass of ions. The species couple with each other through electrostatic forces. In collisional plasmas, the collisions between different types of particles can also couple the dynamics of plasma species. This leads to more complicated dynamics of ions in the ion acoustic waves generated in collisional plasmas.[1,2]

Many authors have researched the ion acoustic waves propagating in collisional plasmas. Different aspects of ion acoustic waves in collisional plasmas were studied by Vranjes and Poedts.[3] They pointed out that the collisions with the neutral atoms modify the ion acoustic wave mode. This mode becomes evanescent at low density and then reappears at high density. The neutrals are dragged along electrons and ions at high density and the three fluids propagate together. The neutrals and plasma species are strongly coupled such that these two different fluids contribute towards the propagation of the wave together.

There are numerous laboratory applications of quantum plasmas. Some examples of their presence are quantum Coulomb crystals, inertial confinement fusion, as well as ultra small electronic devices. The dense plasma systems are also found in astrophysical objects such as white dwarfs, and even in more dense environments of neutron stars and pulsars.[4–9] The quantum effects in plasma appear when the Fermi temperature associated with the plasma number density exceeds the system temperature. The other condition is that the de Broglie wave length should be comparable to the inter-particle distance. Highly dense plasmas are created in atomic clusters which are irradiated by femtosecond laser pulses, where quantum effects appear and electron–ion collisions result in heating the plasmas.[10–12] Ghosh and Chakrabarti[13] studied the nonlinear electrostatic structures with electron–ion collisions and found that when the time scale of the electron–ion collisions is smaller than the time scale for the frequency mode oscillation of the nonlinear wave, the structures are described by the deformed Korteweg de–Vries (DKdV) equation.

The solitons with weak dissipation in quantum plasmas were investigated by Ghosh.[14] The collisions of neutral atoms with ions and electrons were taken into account. The author found that a solitary structure can get excited in the dissipative plasma system when the ion-plasma frequency is sufficiently high as compared to the ion-neutral collision frequency.

The damped solitary wave structures having degenerate attributes of ultra-relativisitic electrons, with nonrelativistic ions, were investigated by Ahmad *et al.*[15] The authors considered the neutral atom–electron and neutral atom–ion collisions using simple relaxation mechanism and concluded that the solitary wave is completely damped for $\tau \to \infty$.

In the above cited works, quantum statistical effects as

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well as quantum tunneling of fermions were taken into account in the presence of electron–neutrals and ion–neutrals collisions. The aim of this work is to include the spin effects of fermions in the presence of collisions. The spin effects play an important role in dense plasmas where they are used to measure more precisely the plasma number density as compared to the situations employed in the conventional techniques. Spin effects further contribute towards the measurement of magnetic field in association with plasma waves as well as in the resonance line of plasmas and its Doppler broadening width.[16,17]

The paper is arranged as follows. Section 2 consists of the basic set of equations that govern the dynamics of ions and electrons, written with normalization. In Section 3, we derive the DKdV equation by using the method of reductive perturbation. Section 4 deals with DKdV numerical solution by applying the Runge–Kutta method. The time dependent analytical solution is also presented. Section 5 summarizes our findings in this work.

2. Basic equations in dense plasma with spinning electrons

We consider, in this work, the dense magnetized homogeneous plasmas comprised of ions and electrons. We take the electrons with spin-up and spin-down as separate species.[18] The collisions of neutral atoms with electrons and ions are also included. The equations that govern the dynamics of ions and degenerate electrons (in spin-up as well as spin-down states) in the presence of electric and magnetic fields are given as follows.

The ion continuity equation is

$$\partial_t n_{\rm i} + \boldsymbol{\nabla}\cdot(n_{\rm i}\boldsymbol{v}_{\rm i}) = 0, \tag{1}$$

where for the ion species, $n_{\rm i}$ is the density and $v_{\rm i}$ is the velocity. The equation for momentum conservation is

$$\partial_t \boldsymbol{v}_{\rm i} + (\boldsymbol{v}_{\rm i}.\boldsymbol{\nabla})\boldsymbol{v}_{\rm i} = \frac{e}{m_{\rm i}}[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v}_{\rm i}\times\boldsymbol{B})] - \nu_{\rm in}\boldsymbol{v}_{\rm i}. \tag{2}$$

Since ions have large inertia as compared to electrons, we consider the ions as non-degenerate. Here $\boldsymbol{E}$ represents the electric field, e the charge, c the speed of light, $m_{\rm i}$ the mass of the ions, and B the magnetic field intensity, while $\nu_{\rm in}$ is the collision frequency between ions and neutral atoms.

The electrons are degenerate and the effects of quantum statistics along with quantum tunneling and spin are also included in their dynamic equations. The momentum conservation equation for the spin-up electrons is

$$\begin{aligned}&\partial_t \boldsymbol{v}_{\rm e,up} + (\boldsymbol{v}_{\rm e,up}.\boldsymbol{\nabla})\boldsymbol{v}_{\rm e,up}\\ &= -\frac{e}{m_{\rm e}}\left[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v}_{\rm e,up}\times\boldsymbol{B})\right] - \frac{1}{m_{\rm e}n_{\rm e,up}}\boldsymbol{\nabla}p_{\rm Fe,up}\\ &\quad + \left(\frac{1}{9}\right)\frac{\hbar^2}{2m_{\rm e}^2}\boldsymbol{\nabla}\left(\frac{1}{\sqrt{n_{\rm e,up}}}\nabla^2\sqrt{n_{\rm e,up}}\right) - \nu_{\rm e,up}\boldsymbol{v}_{\rm e,up}. \end{aligned}\tag{3}$$

The Fermi pressure for the spin-up electrons is

$$p_{\rm Fe,up} = \frac{m_{\rm e}v_{\rm Fe,up}^2}{5n_{\rm e0,up}^{2/3}}n_{\rm e,up}^{5/3},$$

where the Fermi speed of the spin-up electrons is defined as $v_{\rm Fe,up}^2 = \hbar^2(3\pi^2 n_{0,\rm up})^{2/3}/m_{\rm e}^2$ and the term $\sim\hbar^2$ is for the Bohm potential. $\nu_{\rm e,up}$ is the collision frequency of electrons in spin-up state and is defined as $\nu_{\rm e,up} = \nu_{\rm ei,up} + \nu_{\rm en,up}$, where $\nu_{\rm ei,up}$ is the collision frequency due to ions and $\nu_{\rm en,up}$ is the frequency of collision with neutrals.[18]

The spin-down electrons are considered separately and their momentum conservation equation is

$$\begin{aligned}&\partial_t \boldsymbol{v}_{\rm e,down} + (\boldsymbol{v}_{\rm e,down}.\boldsymbol{\nabla})\boldsymbol{v}_{\rm e,down}\\ &= -\frac{e}{m_{\rm e}}\left[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{v}_{\rm e,down}\times\boldsymbol{B})\right] - \frac{1}{m_{\rm e}n_{\rm e,down}}\boldsymbol{\nabla}p_{\rm Fe,down}\\ &\quad + \frac{1}{9}\frac{\hbar^2}{2m_{\rm e}^2}\boldsymbol{\nabla}\left(\frac{1}{\sqrt{n_{\rm e,down}}}\boldsymbol{\nabla}^2\sqrt{n_{\rm e,down}}\right) - \nu_{\rm e,down}\boldsymbol{v}_{\rm e,down}. \end{aligned}\tag{4}$$

The Fermi pressure for the spin-down electrons is

$$p_{\rm Fe,down} = \frac{m_{\rm e}v_{\rm Fe,down}^2}{5n_{\rm e0,down}^{2/3}}n_{\rm e,down}^{5/3},$$

where the Fermi speed of the spin-down electrons $v_{\rm Fe,down}^2 = \hbar^2(3\pi^2 n_{0,\rm down})^{2/3}/m_{\rm e}^2$ and $\nu_{\rm e,down}$ is the sum of collision frequencies of electrons in spin-down state with ions and neutrals. The Poisson equation in this case is

$$\nabla^2\Phi = 4\pi e\left(n_{\rm e,up} + n_{\rm e,down} - n_{\rm i}\right). \tag{5}$$

We assume that the wave is propagating obliquely such that $\boldsymbol{\nabla} = (\partial_x, \boldsymbol{\partial}_y, \partial_z)$ and the magnetic field is oriented in the z direction so that $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{z}}$.

We have scaled temporal coordinates by ion cyclotron frequency $\omega_{\rm ci}$ while spatial coordinates by ion Larmor radius $\rho_{\rm i}$. The densities of plasma species are scaled by their respective densities in equilibrium. The normalization of the electrostatic potential is done by $e\Phi/2k_{\rm B}T_{\rm F}$, where $T_{\rm F}$ denotes the Fermi temperature of electrons such that $k_{\rm B}T_{\rm F} = \frac{\hbar^2(3\pi^2 n_0)^{2/3}}{2m_{\rm e}}$ with $k_{\rm B}$ the Boltzmann constant. It is worth to mention here that we have scaled our equation with respect to the total density of electrons. We have normalized the dynamics with respect to up and down dynamics in case of electrons.

For ions, the continuity equation, after normalization, is now

$$\partial_t n_{\rm i} + \partial_x(n_{\rm i}v_{\rm ix}) + \partial_y(n_{\rm i}v_{\rm iy}) + \partial_z(n_{\rm i}v_{\rm iz}) = 0. \tag{6}$$

The spatial components of the ion momentum conservation equations, after normalization, are

$$\partial_t v_{\rm ix} + \left(v_{\rm ix}\partial_x + v_{\rm iy}\partial_y + v_{\rm iz}\partial_z + \Omega^{-1}\mu_{\rm in}\right)v_{\rm ix} = -\partial_x\Phi + v_{\rm iy}, \tag{7}$$

$$\partial_t v_{iy} + \left(v_{ix}\partial_x + v_{iy}\partial_y + v_{iz}\partial_z + \Omega^{-1}\mu_{in}\right) v_{iy} = -\partial_y \Phi - v_{ix}, \quad (8)$$

$$\partial_t v_{iz} + \left(v_{ix}\partial_x + + v_{iy}\partial_y + v_{iz}\partial_z + \Omega^{-1}\mu_{in}\right) v_{iz} = -\partial_z \Phi. \quad (9)$$

In the above equations, $\mu_{in} = \nu_{in}/\omega_{pi}$, and $\Omega = \omega_{ci}/\omega_{pi}$.

The x component of the spin-up electron momentum conservation equation is

$$\begin{aligned} &6(2\delta_{e,up})^{2/3} n_{e,up}^{-1/3}\partial_x n_{e,up}^{-1/3} + 18\Omega_{e,up}^{-1}\mu_{up} v_{ex,up} \\ &= 18\partial_x \Phi - 18 v_{ey,up} + \Omega^2 H^2 \partial_x \left((n_{e,up})^{-1/2} \partial_x^2 \sqrt{n_{e,up}} \right), \quad (10) \end{aligned}$$

where $\mu_{up} = \nu_{up}/\omega_{pi}$, $\Omega_{e,up} = \omega_{ci}/\omega_{pe,up}$, and $H = \dfrac{\hbar\omega_{pe}}{2k_B T_{Fe}}$. The y component of the spin-up electron momentum conservation equation is

$$\begin{aligned} &6(2\delta_{e,up})^{2/3} n_{e,up}^{-1/3}\partial_y n_{e,up}^{-1/3} + 18\Omega_{e,up}^{-1}\mu_{up} v_{ey,up} \\ &= 18\partial_y \Phi + 18 v_{ex,up} + \Omega^2 H^2 \partial_y \left((n_{e,up})^{-1/2} \partial_y^2 \sqrt{n_{e,up}} \right), \quad (11) \end{aligned}$$

whereas the z component is given by

$$\begin{aligned} &6(2\delta_{e,up})^{2/3} n_{e,up}^{-1/3}\partial_z n_{e,up}^{-1/3} + 18\Omega_{e,up}^{-1}\mu_{up} v_{ez,up} \\ &= 18\partial_z \Phi + \Omega^2 H^2 \partial_z \left((n_{e,up})^{-1/2} \partial_z^2 \sqrt{n_{e,up}} \right). \quad (12) \end{aligned}$$

The respective components of the spin-down electron momentum conservation equations are

$$\begin{aligned} 18\partial_x \Phi &= 18 v_{ey,down} + 6(2\delta_{e,down})^{2/3} n_{e,down}^{-1/3}\partial_x n_{e,down}^{-1/3} \\ &\quad - 18\Omega_{e,down}^{-1}\mu_{down} v_{ex,down} \\ &\quad + \Omega^2 H^2 \partial_x \left((n_{e,down})^{-1/2} \partial_x^2 \sqrt{n_{e,down}} \right), \quad (13) \\ 18\partial_y \Phi &= -18 v_{ex,down} + 6(2\delta_{e,down})^{2/3} n_{e,down}^{-1/3}\partial_y n_{e,down}^{-1/3} \\ &\quad - 18\Omega_{e,down}^{-1}\mu_{down} v_{ey,down} \\ &\quad + \Omega^2 H^2 \partial_y \left((n_{e,down})^{-1/2} \partial_y^2 \sqrt{n_{e,down}} \right), \quad (14) \\ 18\partial_z \Phi &= 6(2\delta_{e,down})^{2/3} n_{e,down}^{-1/3}\partial_z n_{e,down}^{-1/3} \\ &\quad - 18\Omega_{e,down}^{-1}\mu_{down} v_{ez,down} \\ &\quad + \Omega^2 H^2 \partial_y \left((n_{e,down})^{-1/2} \partial_y^2 \sqrt{n_{e,down}} \right), \quad (15) \end{aligned}$$

where $\mu_{down} = \nu_{down}/\omega_{pi}$ and $\Omega_{e,down} = \omega_{ci}/\omega_{pe,down}$.

The Poisson equation in these cases is normalized as

$$\Omega^2 \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right) \Phi = \left(\delta_{e,up} n_{e,up} + \delta_{e,down} n_{e,down} - n_i\right), \quad (16)$$

where $\Omega = \omega_{ci}/\omega_{pi}$, $\delta_{e,up} = n_{e0,up}/n_{i0}$, and $\delta_{e,down} = n_{e0,down}/n_{i0}$.

We assume the perturbation of the form $\exp \mathrm{i}\,(k_x x + k_y y + k_z z - \omega t)$, where ω is the frequency, we obtain the following dispersion relation:

$$a\omega^4 - b\omega^2 + c = 0,$$

where

$$\begin{aligned} a &= k^2 + \frac{\delta_{up}}{(2\delta_{e,up})^{2/3} + H^2 k^2/36} + \frac{\delta_{down}}{(2\delta_{e,down})^{2/3} + H^2 k^2/36}, \\ b &= k^2 + \Omega^2 k^2 + \frac{\delta_{up}}{(2\delta_{e,up})^{2/3} + \Omega^2 H^2 k^2/36} \\ &\quad + \frac{\delta_{down}}{(2\delta_{e,down})^{2/3} + \Omega^2 H^2 k^2/36} \\ &\quad + k^2(\Omega^{-1}\mu_{in} + \Omega_{e,up}^{-1}\mu_{up} + \Omega_{e,down}^{-1}\mu_{down}), \\ c &= \Omega^2 k_z^2. \end{aligned}$$

In the absence of collision, we obtain the dispersion relation as that in Ref. [19].

3. Derivation and solution of DKdV equation

In this section, we are going to derive the DKdV equation using the reductive perturbation technique.[20,21] The variables for space and time are stretched as

$$\xi = \varepsilon^{1/2}(l_x x + l_y y + l_z z - \lambda t), \quad \tau = \varepsilon^{3/2} t.$$

Here ε characterizes the nonlinearity extent and is taken as a small expansion parameter $(0 < \varepsilon < 1)$. Direction cosines l_x, l_y, and l_z obey the condition $l_x^2 + l_y^2 + l_z^2 = 1$, while λ denotes the phase velocity of the wave, which is to be calculated later. We also assume that the wave is propagating at angle θ $(l_z = \cos\theta)$, which is between the directions of the wave propagation and the external magnetic field.[22]

Perturbed quantities n_i, v_{ix}, v_{iy}, v_{iz}, Φ, and $\mu_{i,e}$ are expanded in powers of ε in the following form:

$$\begin{aligned} n_{i,e} &= 1 + \varepsilon n_{i,e}^{(1)} + \varepsilon^2 n_{i,e}^{(2)} + \cdots, \\ v_{ie,x} &= \varepsilon^{3/2} v_{ie,x}^{(1)} + \varepsilon^2 v_{ie,x}^{(2)} + \cdots, \\ v_{ie,y} &= \varepsilon^{3/2} v_{ie,y}^{(1)} + \varepsilon^2 v_{ie,y}^{(2)} + \cdots, \\ v_{ie,z} &= \varepsilon v_{ie,z}^{(1)} + \varepsilon^2 v_{ie,z}^{(2)} + \cdots, \\ \Phi &= \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \cdots, \\ \mu_{i,e} &= \varepsilon^{3/2} \nu_{0i,e}. \end{aligned} \quad (17)$$

Now substituting this perturbation scheme in Eqs. (6)–(16), we obtain the relations in terms of single variable

$$\lambda n_i^{(1)} = l_z v_{iz}^{(1)}. \quad (18)$$

The $\sim \varepsilon^{3/2}$ terms in the z component for the ion momentum conservation equation are

$$l_z \Phi^{(1)} = \lambda v_{iz}^{(1)}. \quad (19)$$

The $\sim \varepsilon$ terms of the Poisson equation are

$$\delta_{e,up} n_{e,up}^{(1)} + \delta_{e,down} n_{e,down}^{(1)} = n_i^{(1)}. \quad (20)$$

The $\sim \varepsilon^{3/2}$ terms in the z component of the momentum conservation equation for the spin-up electrons are

$$4\Phi^{(1)} = (2\delta_{e,up})^{2/3} n_{e,up}^{4/3}, \quad (21)$$

while terms of $\sim \varepsilon^{3/2}$ in the z component of the spin-down electron momentum conservation equation are

$$4\Phi^{(1)} = (2\delta_{\mathrm{e,down}})^{2/3} n_{\mathrm{e,down}}^{4/3}. \tag{22}$$

Equations (18)–(22) are solved to obtain the phase velocity for the ion-acoustic wave propagating in plasmas that is magnetized and possesses both spin-up and spin-down states of degenerate electrons

$$3\lambda^2(\delta_{\mathrm{e,up}}^{1/3} + \delta_{\mathrm{e,down}}^{1/3}) = 2^{2/3} l_z^2. \tag{23}$$

As seen from Eq. (23), there is an alteration in phase speed for the wave due to the variation in degenerate electrons density with obliqueness parameter.

The terms $(\varepsilon^{5/2})$ in the ion continuity equation lead to

$$\lambda \partial_\xi n_{\mathrm{i}}^{(2)} - l_z \partial_\xi v_{\mathrm{iz}}^{(2)} = \partial_\tau n_{\mathrm{i}}^{(1)} + l_z \partial_\xi v_{\mathrm{iz}}^{(1)} + l_z \partial_\xi \left(n_{\mathrm{i}}^{(1)} v_{\mathrm{iz}}^{(1)} \right). \tag{24}$$

The terms $(\varepsilon^{5/2})$ in the z component of the equation for the ion momentum yield

$$-l_z \partial_\xi \Phi^{(2)} - \lambda \partial_\xi v_{\mathrm{iz}}^{(2)} = \partial_\tau v_{\mathrm{iz}}^{(1)} + l_z v_{\mathrm{iz}}^{(1)} \partial_\xi v_{\mathrm{iz}}^{(1)} - \Omega^{-1} \nu_{\mathrm{i0}} v_{\mathrm{iz}}^{(1)}. \tag{25}$$

The second higher order terms $\left(\varepsilon^2\right)$ in the Poisson equation are

$$\delta_{\mathrm{e,up}} n_{\mathrm{e,up}}^{(2)} + \delta_{\mathrm{e,down}} n_{\mathrm{e,down}}^{(2)} - n_{\mathrm{i}}^{(2)} = \Omega^2 \partial_\xi^2 \Phi^{(1)}. \tag{26}$$

For the z components of the spin-up and spin-down electron momenta, the next order $(\varepsilon^{5/2})$ terms are

$$\begin{aligned} & 9\partial_\xi \Phi^{(2)} - 3(2\delta_{\mathrm{e,up}})^{2/3} \partial_\xi n_{\mathrm{e,up}}^{(2)} \\ = & -(2\delta_{\mathrm{e,up}})^{2/3} n_{\mathrm{e,up}}^{(1)} \partial_\xi n_{\mathrm{e,up}}^{(1)} \\ & -9\Omega_{\mathrm{e,up}}^{-1} \nu_{\mathrm{e0,up}} v_{\mathrm{ez,up}}^{(1)} - \Omega^2 l_z^2 \frac{H^2}{4} \partial_\xi^3 n_{\mathrm{e,up}}^{(1)}, \end{aligned} \tag{27}$$

$$\begin{aligned} & 9\partial_\xi \Phi^{(2)} - 3(2\delta_{\mathrm{e,down}})^{2/3} \partial_\xi n_{\mathrm{e,down}}^{(2)} \\ = & -(2\delta_{\mathrm{e,down}})^{2/3} n_{\mathrm{e,down}}^{(1)} \partial_\xi n_{\mathrm{e,down}}^{(1)} \\ & -9\Omega_{\mathrm{e,down}}^{-1} \nu_{\mathrm{e0,down}} v_{\mathrm{ez,down}}^{(1)} - \Omega^2 l_z^2 \frac{H^2}{4} \partial_\xi^3 n_{\mathrm{e,down}}^{(1)}. \end{aligned} \tag{28}$$

Equations (24)–(28) are solved with the phase velocity value obtained in Eq. (23). This leads to the DKdV equation for ion-acoustic waves with dissipation effects in magnetized plasma having non-degenerate ions but degenerate electrons with both the spin states

$$\partial_\tau \Phi^{(1)} + A\Phi^{(1)} \partial_\xi \Phi^{(1)} + B\partial_\xi^3 \Phi^{(1)} + C\Phi^{(1)} = 0, \tag{29}$$

where the respective coefficients for nonlinearity, dispersion, and dissipation are written as

$$A = \frac{l_z^2}{2\lambda} \left[3 + \frac{1}{3} (\delta_{\mathrm{e,up}} + \delta_{\mathrm{e,down}}) \right],$$

$$B = \Omega^2 \frac{\lambda^3}{2l_z^2} \left(1 - \frac{H^2}{36} \right),$$

$$C = \frac{\lambda^2}{2l_z} \left[\frac{2l_z}{\lambda^2} \Omega^{-1} \nu_{\mathrm{i0}} + \Omega_{\mathrm{e,up}}^{-1} \frac{\delta_{\mathrm{e,up}}}{(2\delta_{\mathrm{e,up}})^{2/3}} \nu_{\mathrm{e0,up}} + \Omega_{\mathrm{e,down}}^{-1} \frac{\delta_{\mathrm{e,down}}}{(2\delta_{\mathrm{e,down}})^{2/3}} \nu_{\mathrm{e0,down}} \right].$$

In the absence of dissipation, nonlinearity balances the dispersion, which is due to the quantum tunneling effects and solitons are retrieved. The dissipation in the system appears due to the collisions between neutrals and plasma species.

4. Results and discussion

4.1. Numerical approach

For numerical studies of Eq. (29), we define the density polarization of degenerate electrons as follows:[19,25]

$$\delta = \frac{n_{\mathrm{e0,up}} - n_{\mathrm{e0,down}}}{n_{\mathrm{e0,up}} + n_{\mathrm{e0,down}}}, \quad \delta \in [0, 1], \tag{30}$$

where $\delta = 0$ is the case when the electrons in spin-up state have equal probability to the ones in spin-down state. On the other hand, $\delta = 1$ is the case when all electrons have unidirectional spin. Thus, we write $\delta_{\mathrm{up}} = n_{\mathrm{e0,up}}/n_{\mathrm{i0}} = (1+\delta)/2$ and $\delta_{\mathrm{down}} = n_{\mathrm{e0,down}}/n_{\mathrm{i0}} = (1-\delta)/2$.

Let $\zeta = \alpha\xi + U_0\tau$, here α is the wave number and is considered as 1 and U_0 is the velocity of the nonlinear structure, then equation (29) becomes

$$U_0 \partial_\zeta \Phi^{(1)} + A\alpha\Phi^{(1)} \partial_\zeta \Phi^{(1)} + B\alpha^3 \partial_\zeta^3 \Phi^{(1)} + C\Phi^{(1)} = 0. \tag{31}$$

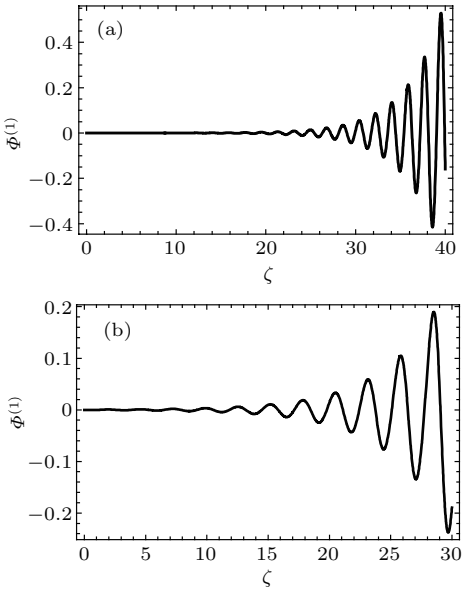


Fig. 1. The oscillatory curve is shown for the cases of (a) $\delta = 1$ and (b) $\delta = 0$. The other plasma parameters are $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{\mathrm{i0}} = 0.5$, $\nu_{\mathrm{e0,up}} = 0.05$, and $\nu_{\mathrm{e0,down}} = 0.09$.

We have solved Eq. (31) by applying the Runge–Kutta method for the plasma parameters $H = 0.15$ which represents the plasma density 1.5×10^{27} cm^{-3}.[26] The initial values are $(\Phi^{(1)}, \partial_\chi \Phi^{(1)}, \partial_\chi^2 \Phi^{(1)}) = (0, 0, \sigma)$ as $\chi \to \infty$ where σ denotes

a very small quantity. For the $\delta = 1$ case, we observe that the nonlinear wave exhibits an oscillatory behavior at the beginning of independent variable, and at a certain point the wave varies exponentially as shown in Fig. 1(a). It is also observed that for the case with $\delta = 0$, i.e., when the electrons equally populate both spin states, the solitary wave structure oscillates with a smaller amplitude as illustrated in Fig. 1(b).

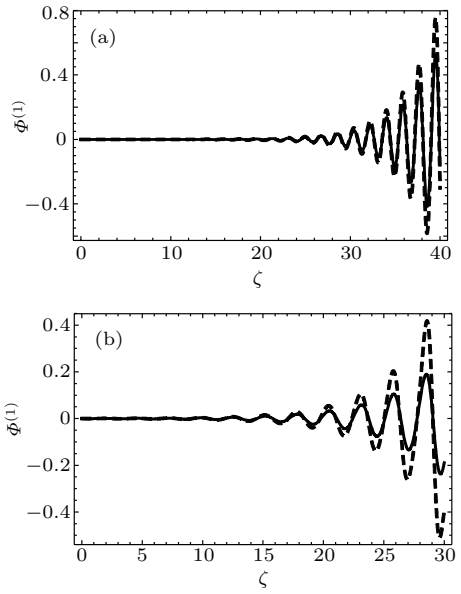


Fig. 2. The variations in the amplitude of oscillations are shown with increasing ν_{i0}: $\nu_{i0} = 0.5$ (bold curve) and $\nu_{i0} = 0.53$ for the cases of (a) $\delta = 1$ and (b) $\delta = 0$, whereas the other plasma parameters are $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{e0,up} = 0.05$, and $\nu_{e0,down} = 0.09$.

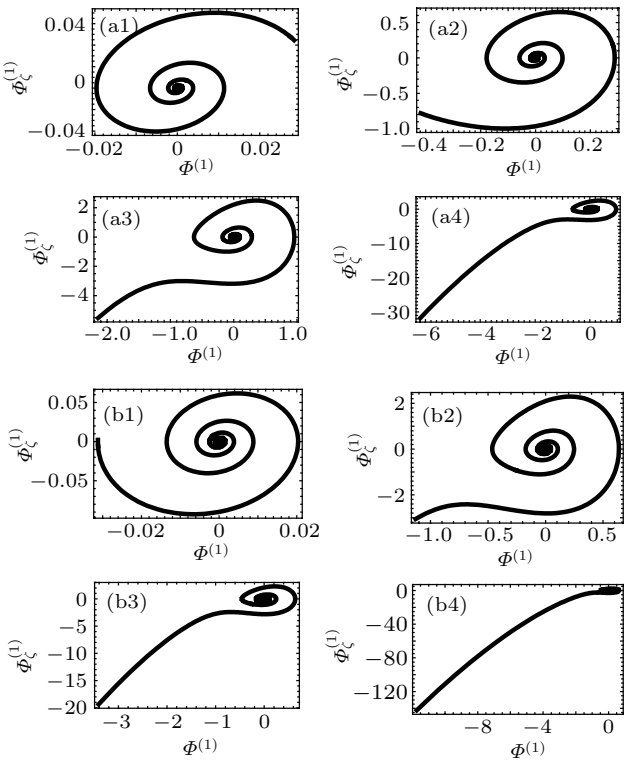


Fig. 3. Plots of $\Phi^{(1)}$ and $\Phi^{(1)}_{\zeta}$ for the cases of (a) $\delta = 1$ and (b) $\delta = 0$ with other plasma parameters as $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{e0,up} = 0.05$, and $\nu_{e0,down} = 0.09$.

We observe that by increasing the collisions between neutrals and plasma species, the amplitude of the oscillation increases, as shown in Figs. 2(a) and 2(b). It is noticed that the oscillatory wave amplitude is enhanced when all degenerate electrons have an unidirectional spin as compared to the case when the electrons possess both spin polarizations. However, the amplitude grows in both cases with the passage of time. This fact indicates that the wave is unstable after a critical point, with increasing value of the independent variable χ. To check whether the initial conditions are unstable or not, we plot the graphs at $(0,\sigma,0)$, $(\sigma,0,0)$, and $(0,0,\sigma)$ showing the variations of $\Phi^{(1)}$ and $\partial_{\chi}\Phi^{(1)}$, which confirms that $(0,0,0)$ is an unstable focus, as was observed in Ref. [22] for the ion–electron dust case in classical plasmas. This fact has been demonstrated in Fig. 3. We observe that for the case $\delta = 0$, the wave becomes unstable earlier, in comparison to the case $\delta = 1$.

4.2. Analytical approach

If we ignore the collisions between plasma species and the neutrals, then $C = 0$ and we obtain KdV equation

$$\partial_{\tau}\Phi^{(1)} + A\Phi^{(1)}\partial_{\xi}\Phi^{(1)} + B\partial^{3}_{\xi}\Phi^{(1)} = 0, \tag{32}$$

$$\Phi^{(1)} = \Psi_0 \operatorname{sech}^2\left(\frac{\xi - U\tau}{\Delta}\right), \tag{33}$$

where Ψ_0 represents the peak amplitude and Δ is the width of soliton with U as the pulse speed, so that

$$\Psi_0 = \frac{3U}{A}, \quad \Delta = \sqrt{\frac{4B}{U}}. \tag{34}$$

Equation (29) is not analytically solvable because the energy of soliton is not conserved due to weak collisions. However, for the case of weak collisions, a simple perturbative analysis gives a solution with time dependence[23,24]

$$\Phi^{(1)}(\eta,\tau) = \Psi_{\mathrm{m}}(\tau)\operatorname{sech}^2\left(\frac{\eta - U(\tau)}{\Delta(\tau)}\right), \tag{35}$$

where $U(\tau) = \frac{A}{3}\int_0^t \Psi_0(\tau)\,\mathrm{d}\tau$ is the soliton velocity, $\Psi_{\mathrm{m}}(\tau)$ represents the time dependent peak amplitude, and $\Delta(\tau)$ denotes the width of the single pulsed soliton, so that

$$U(\tau) = U_0 \exp\left(\frac{-4C\tau}{3}\right), \tag{36}$$

$$\Psi_{\mathrm{m}}(\tau) = \frac{3U_0}{A}\exp\left(\frac{-4C\tau}{3}\right), \tag{37}$$

$$\Delta(\tau) = \sqrt{\frac{4B}{U_0}}\exp\left(\frac{2C\tau}{3}\right). \tag{38}$$

We also investigate the variations in time dependent velocity amplitude and the width of soliton profile. It is observed that for $\delta = 1$, the degenerate electrons attain higher velocities initially and then decay with the passage of time. On the other hand, when the electrons are populated in both spin

states, they attain smaller values of velocities. This fact is depicted in Fig. 4. We observe that the time dependent amplitude decays sharply, when all the electrons are spinning in the same direction. For this case, the soliton also attains high amplitude initially and then decays. On the other hand, the exponential decay in the soliton amplitude is smaller when the electrons populate both spin states, as shown in Fig. 5. The solitary structures broaden with the passage of time. The solitary wave has higher width and flourishes sharply with the passage of time, for the case $\delta = 1$. The soliton profile attains smaller initial values when the electrons are spinning in both directions and the profile grows exponentially, as displayed in Fig. 6.

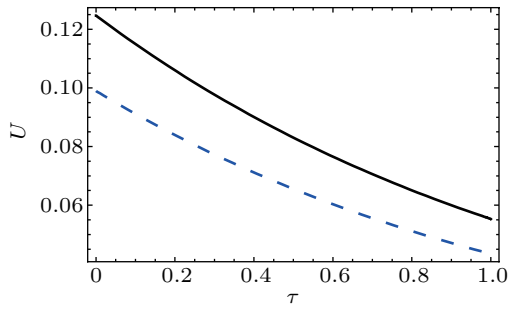


Fig. 4. Analytical variations in the velocity of soliton profile are shown for the cases of $\delta = 1$ (bold curve) and $\delta = 0$ (dash curve) and the other plasma parameters are $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{e0,up} = 0.05$, and $\nu_{e0,down} = 0.09$.

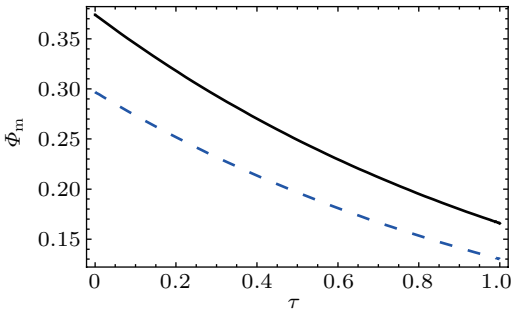


Fig. 5. Decay in the amplitude of soliton profile is depicted for the cases of $\delta = 1$ (bold curve) and $\delta = 0$ (dash curve) with the other plasma parameters being $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{e0,up} = 0.05$, and $\nu_{e0,down} = 0.09$.

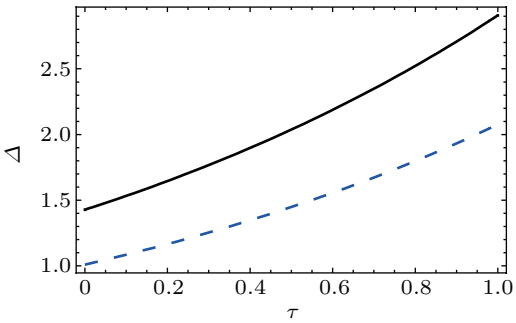


Fig. 6. Growth in the width of soliton profile is displayed for the cases of $\delta = 1$ (bold curve) and $\delta = 0$ (dash curve) with the other plasma parameters taken as $\Omega = 0.87$, $H = 0.15$, $l_z = 0.7$, $\nu_{e0,up} = 0.05$, and $\nu_{e0,down} = 0.09$.

5. Summary

We studied small amplitude damped ion acoustic solitary waves with three spatial component dense plasmas. We also derived the DKdV equation using the method for reductive perturbation. The numerical solution of the DKdV equation predicts that instability exists due to collisions between plasma species and neutrals. We have also observed that the time dependent speed and amplitude of the soliton profile decays sharply, when all the electrons are spinning in the same direction. The width enhances slowly for the situation when the degenerate electrons have equally populated both spin-up and spin-down states. Such results are relevant in astrophysical plasma environments where the spin of electrons has to be included, along with collisions, to describe highly dense systems. Our findings are also applicable to measure the plasma density and the width of Doppler broadening of resonance line of plasmas where the electron spin effects play an important role.[17,27]

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