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Periodically modulated interaction effect on transport of Bose–Einstein condensates in lattice with local defects*

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We theoretically investigate the periodically modulated interaction effect on the propagation properties of a traveling plane wave in a Bose–Einstein condensate (BEC) trapped in a deep annular lattice with local defects both analytically and numerically. By using the two-mode ansatz and the tight-binding approximation, a critical condition for the system preserving the superfluidity is obtained analytically and confirmed numerically. We find that the coupled effects of periodic modulated atomic interactions, the quasi-momentum of the plane wave, and the defect can control the superfluidity of the system. Particularly, when we consider the periodic modulation in the system with single defect, the critical condition for the system entering the superfluid regime depends on both the defect and the momentum of the plane wave. This is different from the case for the system without the periodic modulation, where the critical condition is only determined by the defect. The modulation and quasi-momentum of the plane wave can enhance the system entering the superfluid regime. Interestingly, when the modulated amplitude/frequency, the defect strength, and the quasi-momentum of the plane wave satisfy a certain condition, the system will always be in the superfluid region. This engineering provides a possible means for studying the periodic modulation effect on propagation properties and the corresponding dynamics of BECs in disordered optical lattices.

Keywords: Bose–Einstein condensate, periodic modulation, superfluidity, lattice with defects

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1. Introduction

Recently, the external modulation of the system parameters in time has attracted great interest in the study of Bose–Einstein condensates (BEC)[1–7] and opens interesting possibilities of controlling the quantum engineering. Especially, modulation fixing on the atomic interaction[8,9] (or scattering length) for the BEC periodically with laser beams[10] has been realized in the experiment[11] by means of optical Feshbach resonance[12–15] and has also been studied in different systems previously.[16–18] Interestingly, due to the modulation, the number of bosons allowed to tunnel for a two-mode BEC could be precisely controlled[19] and the defect-free Mott states in optical lattice occurred.[20] On the other hand, the successful control of ultracold atoms in an optical lattice has made it an ideal playground to explore a variety of fascinating quantum phenomena.[21–25] Especially, the tunneling of atoms of the BEC in an optical lattice can be controlled by the potential barrier.[26–29] Optical lattices provide a possible method to investigate ultracold atoms, but small random impurities or defects always exist, which can be induced by additional lasers and/or magnetic fields.[30–36] In a common physical system, the defects can be spatially localized or extended.[30,31] Particularly, the competition between defect and atomic interaction plays a crucial role in transportation property of the system and induces rich phenomena, for example, the defect can inhibits the plane waves transportation[37–39] and the propagation of plane waves experiences a crossover from a superfluid regime to a normal regime.[26,30,39–41] Thus, the transportation properties of the disordered nonlinear discrete system have become a challenging issue. However, the periodic modulation effect on the transportation properties of BEC in an optical lattice with defects has not been explored theoretically.

In this paper, we investigate the periodic modulation effect on the propagation properties of BEC in a deep annular lattice with local defect both analytically and numerically. By using the two-mode ansatz and the tight-binding approximation, we find that there exists a critical condition for system preserving the superfluidity. The coupled effects of periodic modulated atomic interactions strength Γ (the periodic modulation strength Γ is the proportion of modulated amplitude A to modulated frequency ω, i.e., $\Gamma = A/\omega$) and quasi-momentum k of the plane wave can control the superfluidity of the system. Particularly, when there is no periodic modulation, we find whether the system enters the superfluid regime or not in the case of a single impurity is only dependent on the strength of the defect, while in Gaussian defect the superfluid regime is not only related to the defect, but also to the quasi-momentum k. However, when the periodic modulation is considered, we

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find that the critical condition for the system entering the superfluid regime is not only in relation to the defect, but also to the periodic modulated amplitude, the modulated frequency, and the momentum k of the plane wave. Importantly, the modulated amplitude, frequency, and quasi-momentum can enhance the system entering the superfluid regime, which means when the modulated amplitude/frequency, the defect strength, and the quasi-momentum of the plane wave satisfy a particular condition, the system can more easily enter or even always in the superfluid regime.

2. Model

We consider bosons confined in an one-dimensional optical lattice with defects in the presence of periodic modulation of atomic interaction. For a sufficiently deep lattice, considering the tight-binding approximation, the system can be described by the dimensionless discrete nonlinear Schrödinger equations with defects as[30,31]

$$\mathrm{i}\frac{\partial \psi_n}{\partial t} = -\frac{1}{2}(\psi_{n-1}+\psi_{n+1}) + (\varepsilon_n + g(t)|\psi_n|^2)\psi_n, \tag{1}$$

where the physical variables are rescaled as $x \sim x/k_1$, $t \sim t\sqrt{m/\hbar k_1^2}$, $\psi_n \sim \psi_n\sqrt{k_1^3/2\pi}$, here k_1 is the wave vector of the lattice. ψ_n is the wave function in the n-th site of the lattice, $n = 1,2,\ldots,N$ (the number of sites), and the norm $\sum_n |\psi_n|^2 = N$ is conserved. The dimensionless defect ε_n may be created with additional laser and/or magnetic fields and can be spatially localized or extended,[30] which is proportional to any external field superimposed on the lattice. For a lattice without impurity, ε_n is a constant, for a defected lattice, ε_n in each lattice is different and expresses the defect distribution. $g(t) = g_0 + A\cos\omega t$, here

$$g_0 = \frac{4\pi\hbar^2}{mJ}\frac{a}{(2\pi)^{3/2}l_\perp^2 l_0}$$

is induced by the atomic contact interaction, with a being the corresponding interatomic s-wave scattering length, m being the particle mass, and

$$J = \frac{4}{\sqrt{\pi}}s^{3/4}\mathrm{e}^{-2\sqrt{s}}E_\mathrm{R}.$$

$E_\mathrm{R} = 2\hbar^2\pi^2/(m\lambda^2)$ is the recoil energy of the optical lattice, and s is the strength of the optical lattice. $l_\perp = \sqrt{\hbar/m\omega_\perp}$ is the oscillator length associated with a vertical harmonic confinement with $\omega_\perp$ being the vertical trapping frequency. $l_0 = \lambda/2\pi s^{1/4}$ is the oscillator length of the lattice potential with λ being the laser wavelength. In our study, we focus on the case with $g_0 > 0$ and $g > 0$, i.e., only consider the case of repulsive interaction. Here $A\cos(\omega t)$ is the time dependent term, which results from the periodic modulation of the s-wave scattering length a, A is the modulated amplitude, and ω is the modulated frequency.

Under high frequency approximation $\omega \gg \max\{g_0, A\}$, i.e., $A/\omega \ll 1$, the driving field varies so rapidly that the wave function is not able to respond to this variation appreciably and thus can be approximately described by the slowly varying function of time. We can take the transformation[42]

$$\psi_n = \exp\left(-\frac{\mathrm{i}A\sin(\omega t)|\varphi_n|^2}{\omega}\right)\varphi_n, \tag{2}$$

and the formula

$$J_0\left(\frac{A}{\omega}(|\varphi_n|^2 - |\varphi_{n-1}|^2)\right) = \frac{1}{T}\int_{-T/2}^{T/2}\mathrm{e}^{\mathrm{i}\frac{A\sin(\omega t)(|\varphi_n|^2-|\varphi_{n-1}|^2)}{\omega}}\mathrm{d}t. \tag{3}$$

Then by averaging out the high-frequency terms,[42,43] equation (1) reduces to

$$\begin{aligned}\mathrm{i}\frac{\partial\varphi_n}{\partial t} =& -\frac{1}{2}J_0\left(\frac{A}{\omega}(|\varphi_n|^2 - |\varphi_{n-1}|^2)\right)\varphi_{n-1} \\ &- \frac{1}{2}J_0\left(\frac{A}{\omega}(|\varphi_n|^2 - |\varphi_{n+1}|^2)\right)\varphi_{n+1} \\ &+ (\varepsilon_n + g_0|\varphi_n|^2)\varphi_n,\end{aligned} \tag{4}$$

where $J_0(x)$ is the zeroth-order Bessel function. Compared to Eq. (1), without periodic modulation of the atomic interactions, the tunneling coefficient is a constant, i.e., when the modulated amplitude $A = 0$, $J_0(0) \equiv 1$. However, when the periodic modulation is considered, the tunneling term is occupation dependent, i.e., $J_\mathrm{eff} = J_0((A/\omega)\Delta n)$, where $\Delta n = |\varphi_n|^2 - |\varphi_{n-1}|^2$. Clearly, the tunneling processes can be controlled by the modulated strength, i.e., with some suitable modulated amplitude A and modulated frequency ω, the system will more easily enter into the superfluid regime.

The Lagrangian of Eq. (4) is

$$\mathcal{L} = \sum_n \mathrm{i}\dot{\varphi}_n\varphi_n^* - H, \tag{5}$$

where the Hamiltonian in Eq. (5) is

$$\begin{aligned}H =& \sum_n -\frac{1}{2}\left[J_0\left[\frac{A}{\omega}\left(|\varphi_{n+1}|^2 - |\varphi_n|^2\right)\right]\varphi_n\varphi_{n+1}^*\right. \\ &\left. + J_0\left[\frac{A}{\omega}(|\varphi_n|^2 - |\varphi_{n+1}|^2)\right]\varphi_n^*\varphi_{n+1}\right] \\ &+ \varepsilon_n|\varphi_n|^2 + \frac{g_0}{2}|\varphi_n|^4.\end{aligned} \tag{6}$$

3. Superfluidity with defects

In order to investigate the dynamical transportation properties of the condensates in the optical lattice with defects ε_n, we study the propagation of plane wave $\varphi_n(t=0) \sim \mathrm{e}^{\mathrm{i}kn}$ in an annular optical lattice, i.e., with periodic boundary conditions and $k = 2\pi l/N$, l being an integer $(l = 0,\ldots,N-1)$. It

is known that when $\cos(k) < 0$, the system becomes modulationally unstable.[30,31] So we consider the case of $\cos(k) > 0$. The angular momentum of the system is defined as

$$L(t) = \mathrm{i}\sum_n (\varphi_n \varphi^*_{n+1} - \text{c.c.}). \tag{7}$$

The angular momentum $L(t)$ oscillates between the initial values L_0 and $-L_0$, corresponding to the plane waves with wave vectors k and $-k$. The different rotational states with opposite wave vectors are degenerate in the absence of impurities. However, the defects split the degeneracy by coupling the two k and $-k$ waves, which is similar to the tunneling barrier in a double-well potential between the left and right localized states. For this case, the relative population of the two waves oscillates according to an effective Josephson Hamiltonian.[31] Therefore, when the impurity ε is small and the energy split introduced by the impurity is smaller than the energy gap between different rotational states, as to avoid mixing with other states, one can use a two-mode ansatz to investigate the dynamical evolution of the wave functions[30,31]

$$\varphi_n(t) = A(t)\,\mathrm{e}^{\mathrm{i}kn} + B(t)\,\mathrm{e}^{-\mathrm{i}kn}, \tag{8}$$

here we set $A,B = \sqrt{n_{A,B}(t)}\,\mathrm{e}^{\mathrm{i}\phi_{A,B}(t)}$, $z = n_A - n_B$, and $\phi = \phi_A - \phi_B$.

Inserting Eq. (8) into Eq. (7), the angular momentum can be obtained

$$L(t) = 2Nz\sin k. \tag{9}$$

Obviously, the angular momentum is proportional to z. Equation (9) indicates that, the wave is completely reflected for $\langle z\rangle = 0$, i.e., $\langle L\rangle = 0$ (the $\langle\ldots\rangle$ stands for a time average), then the system is in the normal regime. While the wave is only partially reflected by the defect and others preserve passing though the defect for $\langle z\rangle \neq 0$, i.e., $\langle L\rangle \neq 0$, then the system is in the superfluid regime. The observation of a persistent current is associated with the superfluid regime.[30] Here, we focus on the effect of the periodic modulation of atomic interaction on transportation properties in the lattices with single and Gaussian type defects.

3.1. The single defect

Firstly, we study a single defect

$$\varepsilon_n = \varepsilon\delta_{n,\bar{n}} \tag{10}$$

at the site $\bar{n}$ and consider the propagation of the plane wave. By inserting Eq. (8) into Eq. (5), using $\sum_n \mathrm{e}^{2\mathrm{i}kn} = 0$ and $A/\omega \ll 1$, then

$$\sum_n J_0\left(\frac{A}{\omega}(\Delta n)\right) \approx N\left(1 - \frac{2A^2}{\omega^2} n_A n_B \sin^2 k\right),$$

$$\frac{\mathcal{L}}{N} = -n_A\dot{\phi}_A - n_B\dot{\phi}_B - \left(\frac{2A^2}{\omega^2}\cos k\sin^2 k + g_0\right) n_A n_B - \frac{2\varepsilon}{N}\sqrt{n_A n_B}\cos(\phi_A - \phi_B + 2k\bar{n}). \tag{11}$$

Applying the Euler–Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\mathcal{L}}{\partial\dot{q}_i} = \frac{\partial\mathcal{L}}{\partial q_i},$$

where the variational parameters $q_i(t) = n_{A,B}, \phi_{A,B}$, we obtain

$$\dot{z} = -\frac{2\varepsilon}{N}\sqrt{1-z^2}\sin\phi,$$
$$\dot{\phi} = \frac{2\varepsilon}{N}\frac{z}{\sqrt{1-z^2}}\cos\phi + \left(\frac{2A^2}{\omega^2}\cos k\sin^2 k + g_0\right) z, \tag{12}$$

with the replacement $\phi + 2k\bar{n} \to \phi$. By using $\dot{z} = -\partial H/\partial\phi$ and $\dot{\phi} = \partial H/\partial z$, the energy of the system is

$$H = \left(\frac{A^2}{\omega^2}\cos k\sin^2 k + \frac{g_0}{2}\right) z^2 - \frac{2\varepsilon}{N}\sqrt{1-z^2}\cos\phi. \tag{13}$$

The propagation properties of the waves are closely related to the system energy. If z cannot reach the value 0, i.e., $\langle z\rangle \neq 0$, the system is in the superfluid regime. To prevent the system from reaching the state $\langle z\rangle = 0$, the initial energy of the system H_0 should be larger than the energy of this state, i.e., $H_0 \geqslant H(z=0)$. In the initial state, $z(0) = 1$ and $\phi(0) = 0$, the initial energy is

$$H_0 = \frac{1}{2}\left(g_0 + \frac{2A^2}{\omega^2}\cos k\sin^2 k\right).$$

Because

$$H(z=0) = -\frac{2\varepsilon}{N}\cos\phi \leqslant 2\varepsilon/N,$$

if

$$H_0 = \frac{1}{2}\left(g_0 + \frac{2A^2}{\omega^2}\cos k\sin^2 k\right) \geqslant 2\varepsilon/N,$$

i.e.,

$$g_0 \geqslant \frac{4\varepsilon}{N} - \frac{2A^2}{\omega^2}\cos k\sin^2 k,$$

then $H_0 \geqslant H(z=0)$ should be satisfied for all values of ϕ. That is, when

$$g_0 \geqslant \frac{4\varepsilon}{N} - \frac{2A^2}{\omega^2}\cos k\sin^2 k,$$

z cannot reach the value 0 and the system will be in a superfluid regime. Thus, the critical conditions for preserving the superfluidity can be obtained as

$$g_\mathrm{c} = \frac{4\varepsilon}{N} - 2\Gamma^2\cos k\sin^2 k, \tag{14}$$

where the periodic modulation strength $\Gamma = A/\omega \ll 1$. Physically, the defect will destroy and reflect the propagation of traveling plane waves. However, the interatomic interactions provide an effective energy barrier which can preserve the

traveling wave through the defect, if the condition Eq. (13) is satisfied. That is, there exists a critical g_c, when $g_0 > g_c$, the traveling wave can preserve passing though the defect coherently, then the system is in the superfluid state. When $g_0 < g_c$, however, the plane wave will be reflected by the defect, then the system is in the normal region. Equation (13) indicates that the superfluidity of the system is related to the discreteness, defect, modulated amplitude, and quasi-momentum of plane wave. When $\Gamma = 0$, then the critical value $g_c = 4\varepsilon/N$ for the system in superfluid regime without periodic modulation is derived,[26,30,31] which means that the critical condition is only related to the defects ε. However, when $\Gamma \neq 0$, i.e., when the periodic modulation of the atomic interaction is considered, the critical condition g_c is determined by the modulation strength Γ, quasi-momentum of wave k, and defects ε, which means that the system entering the superfluid regime is not only in relation to the defects ε but also to the periodic modulated amplitude A, the modulated frequency ω, and the momentum k of the plane wave. That is, one can control the transportation properties with the external modulation of the atomic interaction and the momentum of the plane wave.

Importantly, equation (14) indicates that the coupled effects of defect ε, modulated strength Γ, and quasi-momentum k determine the critical value g_c and control the superfluidity of the system. The competition between the defect, modulated strength, and quasi-momentum of the plane wave provides a critical scattering length for maintaining the superfluidity. Direct numerical simulation of Eq. (1) as show in Fig. 1 indicates that, there exists a critical value g_c, when $g_0 < g_c$, $L(t)/L_0$ oscillates around 0 (see Fig. 1(a)), which means $\langle L(t)\rangle/L_0 = 0$ (see Fig. 1(b)), thus the system enters a normal regime; when $g_0 > g_c$, $L(t)/L_0$ oscillates between 0 and 1 (see Fig. 1(a)), that is $\langle L(t)\rangle/L_0 \neq 0$ (see Fig. 1(b)), thus the system is in a superfluid regime. Particularly, when $g_0 = g_c$, $L(t)$ asymptotically approaches 0. The critical value g_c for this transition is determined by the defect, the periodic modulation strength (modulated amplitude and frequency), and the quasi-momentum of the plane wave.

Figure 2 shows the critical value g_c against the quasi-momentum k and the modulated strength Γ for the single defect. Clearly, we find that the critical scattering length g_c strongly depends on the quasi-momentum k and the modulated strength Γ. When $k \to 0$ or $\pi/2$, the critical value g_c is irrelevant to modulated strength Γ due to $\cos k \sin^2 k = 0$ (see Eq. (13)). The g_c decreases with the increase of Γ, which means that the increase of periodically modulated amplitude A or the decrease of modulated frequency ω can enhance the system entering the superfluid regime. Particularly, there exits a critical $k_c = \pi/3$ (which can be obtained with $\partial g_c/\partial k = 0$, that is independence of Γ), at $k = k_c$, g_c is minimum, and when $k \to k_c$, g_c changes sharply with Γ, which means for the special value of k, g_c is highly sensitive to Γ, i.e., the system is much easier to enter the superfluid regime. Furthermore, from Eq. (13), when $k = k_c$ and $\Gamma = \Gamma_c = [2\varepsilon/(\sqrt{3}N)]^{1/2}$, $g_c = 0$. Then, when $\Gamma > \Gamma_c$ and $k_m < k < k_M$ (k_m and k_M are determined by $\cos^3 k - \cos k + 2\varepsilon/(N\Gamma^2) = 0$), $g_c \equiv 0$, and the system always passes through the defect. That is, for some range of the modulated strength Γ and momentum of the plane wave, the system is always in the superfluid state. To confirm the analytical results, numerical results obtained by direct numerical simulation of Eq. (1) are also shown in Fig. 2. We find that the analytical results qualitatively agree with the numerical results.

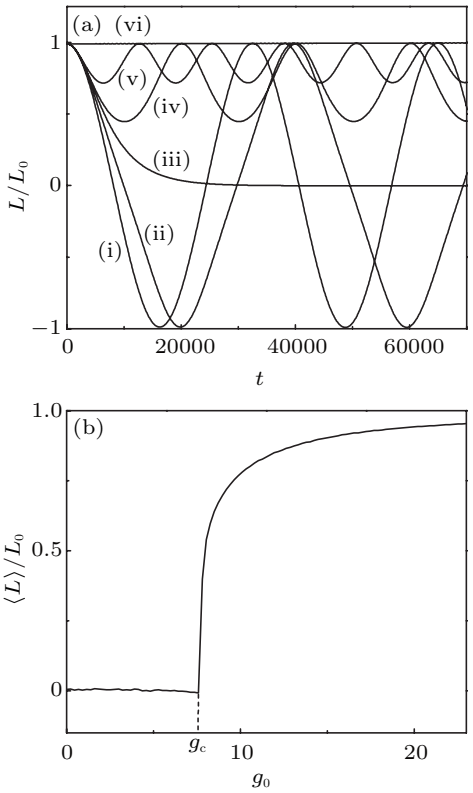


Fig. 1. (a) The normalized angular momentum $L(t)/L_0$ vs. time for $g_0/a_0 = 2, 6, 7, 9, 12, 200$ corresponding respectively to (i)–(vi). (b) The average value of the normalized angular momentum $\langle L(t)\rangle/L_0$ vs. the scattering length g_0. Parameters in the simulation are $N = 100$, $\varepsilon = 0.01$, $l = 20$, and $\Gamma = 0.01$.

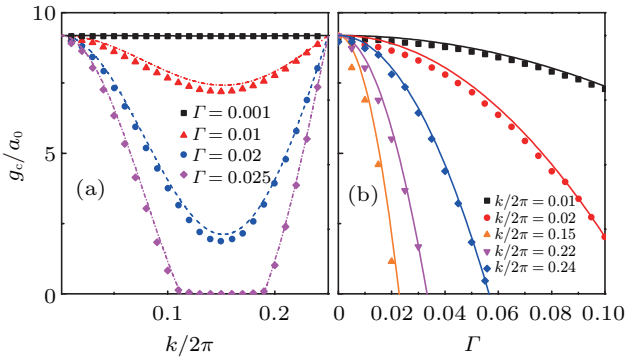


Fig. 2. The critical scattering length g_c as a function of quasi-momentum k for different Γ (a) and as a function of modulated strength Γ for different k (b) with a single defect. Here a_0 is the Bohr radius, $N = 100$, and $\varepsilon = 0.01$. The points are numerical simulations of Eq. (1) and the lines are the analytical results of Eq. (13).

3.2. The Gaussian defect

Here we consider a Gaussian defect with the width σ centered on the site $\bar{n}$

$$\varepsilon_n = \frac{\varepsilon}{\sqrt{\pi}\sigma} \mathrm{e}^{-(n-\bar{n})^2/\sigma^2}. \tag{15}$$

For sufficiently large N and $\sigma \gtrsim 1$, we will have $\sum_n \varepsilon_n \approx \int \mathrm{d}n\varepsilon_n = \varepsilon$.[30] In the same way as the case of a single defect, the effective Hamiltonian is

$$H \approx \left(\frac{A^2}{\omega^2}\cos k \sin^2 k + \frac{g_0}{2}\right) z^2 - \frac{2\varepsilon \mathrm{e}^{-k^2\sigma^2}}{N}\sqrt{1-z^2}\cos\phi. \tag{16}$$

We can clearly see that the system with a Gaussian defect is similar to the system of a single defect, then the critical g_c of the Gaussian defect for supporting the superfluid is

$$g_c = \frac{4\varepsilon_{\mathrm{eff}}}{N} - 2\Gamma^2 \cos k \sin^2 k, \tag{17}$$

with an effective defect $\varepsilon_{\mathrm{eff}} = \varepsilon \mathrm{e}^{-k^2\sigma^2}$. Hence one can expect that the effects of defect $\varepsilon_{\mathrm{eff}}$, modulated strength Γ, and quasi-momentum k on the superfluidity of the system are similar to the case of the single defect. The coupled effects of periodic modulated amplitude A, modulated frequency ω, and quasi-momentum k on critical scattering length g_c are similar to the single defect case. However, for the Gaussian defect, the relation between g_c and k is more complex, then the critical condition for supporting the superfluid regime is much different to the single defect one.

For the Gaussian defect, the critical scattering length g_c against quasi-momentum k and the modulated strength Γ is plotted in Fig. 3. Just like the case of single defect, we can see that the critical scattering length g_c changes with quasi-momentum k and modulated strength Γ. The critical scattering length g_c decreases with increasing modulated strength Γ, thus, with a large modulated strength, the superfluid can be more easily preserved due to the relatively small critical scattering length g_c. However, compared to the single defect case shown in Fig. 2, the variation of g_c against k has very different character for the Gaussian defect case. As k increases, the critical scattering length g_c decreases with quasi-momentum k monotonously, and finally g_c reaches zero at $k \geq k_c$, where k_c strongly depends on Γ and is determined by $g_c = 4\varepsilon \mathrm{e}^{-k^2\sigma^2}/N - 2\Gamma^2 \cos k \sin^2 k = 0$, which means that the system will always pass through the defect when $k > k_c$. The coupling of the periodic modulation and the Gauss defect can enhance the system into the superfluid state.

Here, we have investigated the periodic modulation effect on transportation properties with Gaussian defect. It is shown that, without the modulation, the system with a Gaussian defect has a larger scattering length than that for the system with a single defect, which means that the system with Gaussian defect is difficult to enter the superfluid regime. However, when the modulation is considered for the Gaussian defect, the quasi-momentum k and modulated strength Γ can both enhance the system entering the superfluid regime.

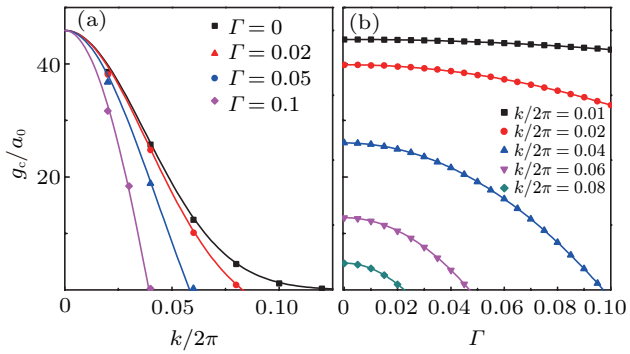


Fig. 3. The critical scattering length g_c against (a) quasi-momentum k and (b) modulated strength Γ in Gaussian defect. Here a_0 is the Bohr radius, and $N = 100$, $\varepsilon = 0.05$, $\sigma = 3$. The points are numerical simulations of Eq. (1) and the lines are the analytical results of Eq. (17).

4. Conclusion

We have studied the periodic modulation effect on propagation properties of BEC in an optical lattice with defects both analytically and numerically. Within a two-mode ansatz and the tight-binding approximation, the coupled effects of the modulated strength (the periodic modulation strength is the proportion of modulated amplitude to modulated frequency) and quasi-momentum on transportation properties are investigated both for a single defect and a Gaussian defect. The result shows that the superfluid state can exist beyond a critical scattering length, which is influenced by the periodic modulated amplitude, the modulated frequency, the quasi-momentum of the plane wave, and the defects dramatically. We show that, the modulated amplitude/frequency and quasi-momentum of the plane wave can enhance the system entering the superfluid regime, especially, when the modulated amplitude/frequency and quasi-momentum of the plane wave satisfy a particular condition, the system can more easily enter or even always in the superfluid regime. This engineering provides a possible means for studying and controlling the transport properties of BECs in disordered optical lattice. Experimentally, defects in an optical lattice can be generated by exposing the lattice to speckle lasers, adding an incommensurate lattice-forming laser, and by other means. Meanwhile, techniques for adjusting the scattering length globally have been crucial to many experimental achievements and can be implemented by a spatially inhomogeneous external magnetic field in the vicinity of a Feshbach resonance. Then, the properties of a traveling plane wave in a defected lattice can be observed.

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