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Analytical treatment of Anderson localization in a chain of trapped ions experiencing laser Bessel beams*

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Trapped ions, under electromagnetic confinement and Coulomb repulsion, can behave as non-interacting particles in one-dimensional lattices. Here we explore analytically the possible effects regarding Anderson localization in a chain of trapped ions experiencing laser Bessel beams. Under an experimentally feasible condition, we predict an analytical form of the energy-dependent mobility edges, which is verified to be in good agreement with the exact numerical results except for the top band. Some other important properties regarding the phonon localization in the ion chain are also discussed both analytically and numerically. Our results are relevant to experimental observation of localization–delocalization transition in the ion trap and helpful for deeper understanding of the rich phenomena due to long-range phonon hopping.

Keywords: mobility edge, localization, eigenstates, critical values

PACS: 03.75.Lm, 05.50.+q, 64.60.Cn, 63.20.–e **DOI:** 10.1088/1674-1056/28/1/010306

1. Introduction

Anderson localization is a fundamental quantum phenomenon due to interference in disordered systems.[1] As a universal feature in physics, Anderson localization has been predicted in various systems, e.g., light propagation in quasiperiodic waveguide[2] and matter waves in quasiperiodic and controlled disordered systems.[3,4] Anderson localization or related effects has always been a hot research topic.[3–9] For example, under some restrictions such as nearest-neighbor hopping, Aubry–André(AA) model[10] has been proposed to verify that quasiperiodic potentials can also lead to Anderson localization. Further work along this line can be found in Refs. [11]–[13]. Although either quasiperiodic or disordered systems can cause exponential localization, there is a fundamental difference in between.[14] Localization always happens for any strength of disorder in one-dimensional models,[15–17] whereas there exist localization–delocalization phase transitions (LDPTs) in some quasiperiodic systems.[17–19] Therefore, for the quasiperiodic model, critical values and mobility edges are very important issues and have attracted much attention.[19–24]

In this paper, we investigate analytically the energy-dependent mobility edges and other characteristics regarding a quasiperiodic model using a chain of trapped ions confined stably and equidistantly in an anharmonic ion trap.[25] Trapped-ion systems can behave as non-interacting one-dimensional lattices for quantum computation and quantum simulation, in which the vibrational degrees of freedom of the ions act as the data bus to transfer quantum information in the chain and entangle the separate ions. These quantized vibrations of the ions, i.e., phonons, can be well controlled for oriented movement and aggregation. The former reflects the fundamental process of heat conduction,[26–28] and the latter is relevant to Anderson localization.[15,29]

2. Model and mobility edge

We assume a chain of ions trapped in an anharmonic ion trap, and these ions only move along the x axis. With the application of Bessel standing waves, the phonon localization can be induced as in Ref. [29]. In this situation, there are phonon hoppings between any two of the ions via the long-range Coulomb interaction, and the corresponding Hamiltonian for this phonon system is given by ($\hbar = 1$)[29]

$$H = \sum_l \Omega_l a_l^\dagger a_l + \sum_l \sum_{l>j} T_{i,j}(a_l^\dagger a_j + \text{H.c}), \tag{1}$$

with

$$\begin{aligned} \Omega_l &= \sqrt{\omega^2 - \Omega_{\text{A}}^2 \cos^2(2\pi\chi l + \delta)}, \\ T_{l,j} &= \frac{C}{|l-j|^3 \sqrt{\Omega_l \Omega_j}}, \end{aligned} \tag{2}$$

where $a_k^\dagger$ (a_k) is the creation (annihilation) operator of the phonon regarding the k-th ion, Ω_l is the effective frequency of the phonons on the ion l, and $T_{l,j}$ represents the phononic hopping between the lth and jth ions. Ω_A is the effective strength

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regarding the Bessel standing waves of the applied lasers, and C is the constant determined by the charge of each ion and the ions's spacing. $\omega = \sqrt{\omega_x^2 - 2C\sum_{j\neq l}\frac{1}{|j-l|^3}}$ is an effective frequency, reflecting the balance between the potential confinement and the Coulomb repulsion. For generality, we assume a large number of ions in the ion chain of our interest, for which we can ignore the edge effect of the chain. As such, we may replace $\sum_{j\neq l}\frac{1}{|j-l|^3}$ in ω by the Riemann zeta function $2\zeta(3)$.

We focus on the condition of $\omega^2 \gg \Omega_A^2$ below, which is easily satisfied in ion trap experiments. We rewrite Ω_l and $T_{l,j}$ using Taylor's expansion,

$$\begin{aligned}\Omega_l &\simeq \sqrt{\omega^2 - \Omega_{\rm A}^2/2} - \frac{\Omega_{\rm A}^2}{4\sqrt{\omega^2-\Omega_{\rm A}^2/2}}\cos(4\pi\chi l + 2\delta),\\ T_{l,j} &\simeq \frac{C}{|l-j|^3\sqrt{\omega^2-\Omega_{\rm A}^2/2}},\end{aligned} \tag{3}$$

which includes the quasi-periodic term regarding $\frac{\Omega_A^2}{4\sqrt{\omega^2-\Omega_A^2/2}}$, but neglects the higher-order terms. We show below that equation (1) with the reduced forms of Ω_l and $T_{l,j}$ could present some interesting features regarding the phonon localization of the ion chain, such as the analytical expression of the energy-dependent mobility edges and the LDPT critical properties independent of ω. Since the ion trap system involving long-range interaction is much more complicated than the Aubry–André model, our analytical discussion would be very helpful for understanding the rich phenomena observed in ion trap experiments. Previous studies have shown that the mobility edges are hard to obtain analytically in the model with the hopping terms damping as the power law,[21] even for a self-dual power-law model.[17] In comparison with the model in Ref. [21], where the mobility edge takes the form of $\cosh(p) = \frac{E+t}{V}$ with $p = \ln(t_1/t_2)$ and $t = t_1\exp(p)$, our model is different, but with similar characteristics in Anderson localization.[29] Hence, we may define t_n similarly by $T_{l,j}$ with $n = |l-j|$, that is, $t_n = \frac{C}{|n|^3\sqrt{\omega^2-\Omega_{\rm A}^2/2}}$. As such, we have $p = \ln(t_1/t_2) = \ln(8)$ and $t = \frac{8C}{\sqrt{\omega^2-\Omega_{\rm A}^2/2}}$. Besides, under the condition of $\omega^2 \gg \Omega_{\rm A}^2$, the average value of Ω_l is reduced to $\sqrt{\omega^2-\Omega_{\rm A}^2/2}$. Hence, the mobility edge in our model takes the following form (see Appendix A for detail):

$$\cosh(p) = \frac{E - \sqrt{\omega^2-\Omega_{\rm A}^2/2} + \frac{8C}{\sqrt{\omega^2-\Omega_{\rm A}^2/2}}}{\frac{\Omega_{\rm A}^2}{4\sqrt{\omega^2-\Omega_{\rm A}^2/2}}}. \tag{4}$$

As presented below, numerical treatment of Eq. (1) confirms that equation (4) indeed describes the mobility edges in a very good approximation, which even works for some special cases, such as $\omega = 5.1\sqrt{C}$ that is beyond the condition of $\omega^2 \gg \Omega_{\rm A}^2$.

To verify Eq. (4), we need to numerically diagonalize the Hamiltonian of Eq. (1). With the obtained eigenvalues and eigenstates, we employ the inverse participation ratio (IPR) to describe the localization–delocalization transition. To this end, we first assume the normalized eigenstates $\Psi^{(m)} = [\psi_1^{(m)}, \psi_2^{(m)}, \ldots, \psi_N^{(m)}]^{\rm T}$, where the superscript m represents the $m-1$ excited states (with $m = 1$, the ground state) and $\psi_l^{(m)}$ is the probability amplitude for the phonon at site l in the ion chain (N is the number of the ions under consideration). The IPR is defined as[19]

$$\mathrm{IPR}^{(m)} = \sum_{l=1}^{N}\left|\psi_l^{(m)}\right|^4, \tag{5}$$

which is a finite value for localization and tends to zero in the case of delocalization.

The numerical results of the IPR are plotted in Fig. 1. As a comparison, we consider the characteristic parameters of both the forms of Eq. (2) and Eq. (3). Besides, the results of Eq. (4) are also plotted in Fig. 1 to see how well this analytical form of the mobility edges works. The comparison of the panels (a) and (c) with (b) and (d) in Fig. 1 shows a good agreement, implying that the mobility edges obtained from Eq. (4) present a qualitatively accurate boundary for localization and delocalization, since they are in good agreement with the IPR numerical results for different ω, except for the top band. Meanwhile, the comparison indicates that equation (3) is an excellent approximation of Eq. (2) under the condition of $\omega^2 \gg \Omega_A^2$.

As we know, the mobility edge is a boundary to separate the localization and delocalization phases. In our case, the localization (delocalization) appears below (above) the boundary, as demonstrated in Fig. 1. To fully understand the analytical form in Eq. (4), we have to find out the reason of the deviation from the numerical results of Eq. (3) in the top band. The approximation made by truncation in Taylor's series expansion is obviously not the main reason, because we have seen Eq. (3) to be the excellent approximation of Eq. (2) in the numerical comparison in Fig. 1. The possible reason might be that the hopping interaction in Eq. (4) is of the form of $\exp -p\,|l-j|$, which is different from the real hopping interaction $T_{l,j} \sim \frac{1}{|l-j|^3}$. The top band is more sensitive to this difference than other parts of the bands. Another reason is from the fact that the linear mobility edge in Eq. (4) cannot describe the energy-dependent mobility edge in our model,[21] which is typically nonlinear, especially in the case of a large value $t_1/t_2 = 8$.

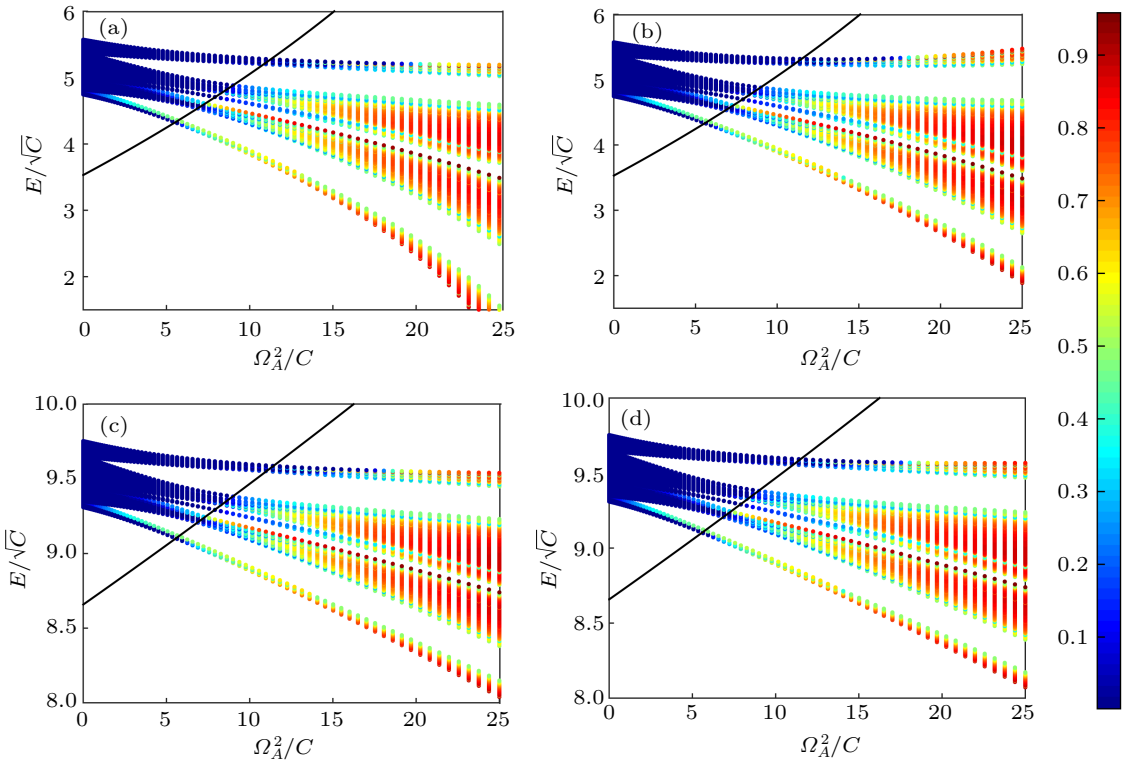


Fig. 1. Energy eigenstates of Eq. (1) with the parameter definition of Eq. (2) (left panels) and Eq. (3) (right panels) for 1000 trapped ions and $\chi = (1+\sqrt{5})/2$. (a) $\omega = 5.1\sqrt{C}$, (b) $\omega = 5.1\sqrt{C}$, (c) $\omega = 9.5\sqrt{C}$ and (d) $\omega = 9.5\sqrt{C}$. The color bar indicates the magnitude of the IPR. The black solid line in each panel is the mobility edge obtained from Eq. (3).

3. Effects of ω on the critical values of eigenstates

Under the condition of $\omega^2 \gg \Omega_A^2$, we may also find some other interesting properties regarding the LDPT critical behavior. Figure 2 presents the critical behavior regarding the eigenstates of the system (Eqs. (1) and (2)) using the IPR, which looks insensitive to the change of ω, even for the case of $\omega = 7\sqrt{C}$ beyond the condition of $\omega^2 \gg \Omega_A^2$. The reason could be speculated from the reduced forms in Eq. (3) that both the strengths of disorder and hopping turn to be weaker with increasing ω, implying a counteraction with each other for the phonon localization. A more strict proof for this point can be found in Appendix B.

To further clarify this point, we introduce the information entropy $S^{(m)}$ [30,31] as follows:

$$S^{(m)} = -\sum_{l=1}^{N} \left|\psi_l^{(m)}\right|^2 \ln\left(\left|\psi_l^{(m)}\right|^2\right). \tag{6}$$

When the information entropy $S^{(m)}$ takes its minimum value $[S^{(m)} = 0]$, the state will be localized in a single site. On the other hand, when the information entropy $S^{(m)}$ is in its maximum value $[S^{(m)} = \ln(N)]$, the state will be distributed in all the sites with equiprobability.[24] In addition, $S^{(m)}$ changes severely around the critical point of the phase transition in the system.[24] Using $S^{(m)}$, we may verify more clearly that the critical behavior is indeed independent of ω. In our treatment, we have randomly chosen 4 eigenstates to investigate the effect of ω by Eqs. (1) and (2), as shown in Fig. 3. By comparing the cases using different values of ω, we find that, although $S^{(m)}$ is more sensitive to ω than $\left|\Psi^{(m)}\right|$, particularly in the critical region of the localization–delocalization phase transition, no obvious discrepancy occurs in the comparison, except for the higher excited state (e.g., $S^{(900)}$ in the panel (d)) which is beyond the condition of $\omega^2 \gg \Omega_A^2$. This indicates that both the reduced parameter definitions in Eq. (3) and the form of mobility edges in Eq. (4) work for the values of ω in our model.

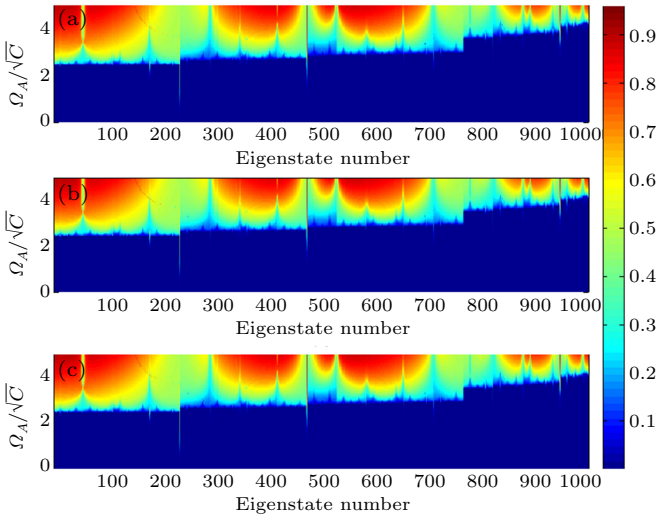


Fig. 2. IPRs for different eigenstates regarding the parameter definition of Eq. (2) in variation of Ω_A, where 1000 trapped ions are considered and $\chi = \sqrt{5}+1/2$. (a) $\omega = 7\sqrt{C}$, (b) $\omega = 10\sqrt{C}$, (c) $\omega = 30\sqrt{C}$.

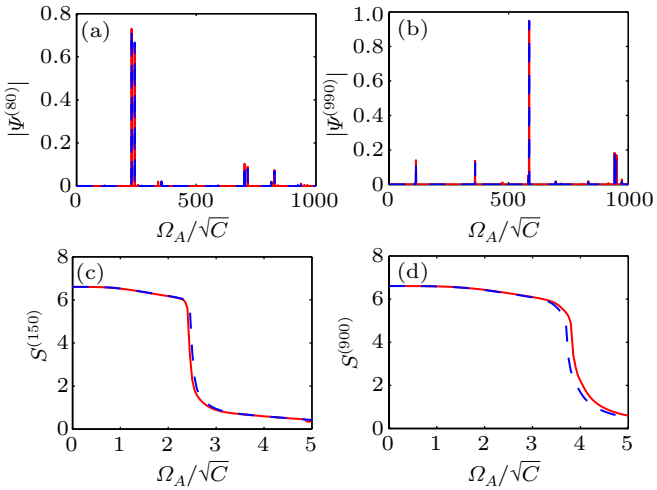


Fig. 3. $|\Psi^{(m)}|$ and $S^{(m)}$ for 1000 trapped ions based on the parameter definition of Eq. (2), where $\chi=(1+\sqrt{5})/2$. (a) $|\Psi^{(80)}|$ with $\Omega_{\rm A}=4.5\sqrt{C}$, $\omega=5.2\sqrt{C}$ (red solid line) and $11\sqrt{C}$ (blue dashed line), (b) $|\Psi^{(990)}|$ with the same parameter values as in panel (a). (c) $S^{(150)}$ with $\omega=5.5\sqrt{C}$ (red solid line) and $18\sqrt{C}$ (blue dashed solid) (d) $S^{(900)}$ with the same parameter values as in panel (c).

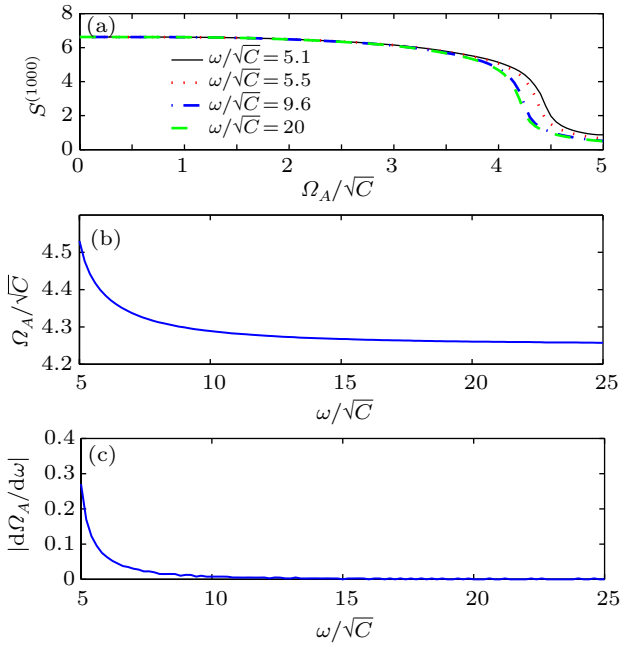


Fig. 4. (a) $S^{(1000)}$ versus $\Omega_{\rm A}$ for different values of ω. (b) Ω_A versus ω for different values of ω, where we choose $S^{(1000)}=2$. (c) The derivative of the values in panel (b).

4. Conclusion

In conclusion, we have carried out analytical investigation for the Anderson localization in a quasi-periodic model using a chain of trapped ions. Since the condition of $\omega^2\gg\Omega_{\rm A}^2$ is feasible experimentally, our work is tightly related to experimental realization of Anderson localization in ion traps. Our main result is Eq. (4), the analytic form of the mobility edges, which is useful for deeply understanding the localization properties in such a long-range hopping model. In addition, the reduced forms in Eq. (3) also helps considering experimental parameters in a more appropriate way. For example, a larger value of ω is more advantageous for ion trapping. Since the occurrence of LDPT is insensitive to ω, we may try to increase ω in future experimental implementation.

Appendix A

The eigen-energy for the Hamiltonian (1) is given by

$$Ea_l=\Omega_l a_l+\sum_{l\neq j}T_{l,j}a_j.$$

Assume $|l-j|=n$, under the condition of $\omega^2\gg\Omega_{\rm A}^2$, we write Ω_l as

$$\Omega_l\simeq\sqrt{\omega^2-\Omega_{\rm A}^2/2}-\frac{\Omega_A^2}{4\sqrt{\omega^2-\Omega_{\rm A}^2/2}}\cos(4\pi\chi l+2\delta),$$

and the hopping interaction for phonons between the lth and jth ions is

$$T_{l,j}\simeq\frac{C}{|n|^3\sqrt{\omega^2-\Omega_{\rm A}^2/2}}.$$

Then, the above eigen-energy for our model can be rewritten as

$$[(E'+t)+V\cos(4\pi\chi l+2\delta)]a_l=\sum_n t_n(a_{l+n}+a_{l-n})+ta_l,$$

with the hopping interaction

$$t_n=\frac{C}{|n|^3\sqrt{\omega^2-\Omega_{\rm A}^2/2}},$$

the disordered potential

$$V=\frac{\Omega_{\rm A}^2}{4\sqrt{\omega^2-\Omega_{\rm A}^2/2}},$$

and the effective eigen-energy $E'=E-\sqrt{\omega^2-\Omega_A^2/2}$. Here t is an energy parameter to be determined, which would determine the localization transitions.

To get the value of t, we employ the method in Ref. [21]. In our model, since the phonon hopping interaction would be realized via the Coulomb interaction, the nearest-neighbor interaction t_1 and the next nearest-neighbor interaction t_2 govern the system. As a result, we define $p=\ln(t_1/t_2)=\ln(8)$, and the energy parameter is given by $t=t_1\exp(p)=8t_1$. Then, the mobility edge can be written as

$$\cosh(p)=\frac{E'+t}{V}=\frac{E-\sqrt{\omega^2-\Omega_A^2/2}+\dfrac{8C}{\sqrt{\omega^2-\Omega_{\rm A}^2/2}}}{\dfrac{\Omega_{\rm A}^2}{4\sqrt{\omega^2-\Omega_{\rm A}^2/2}}}.$$

Appendix B

Under the condition of $\omega^2\gg\Omega_A^2$, we have the reduced Hamiltonian with the parameters of the forms in Eq. (3). We

prove below that the eigenstates of this Hamiltonian are independent of ω.

In this case, the Hamiltonian includes the following elements:

$$H_a(l,l) = \sqrt{\omega^2 - \Omega_{\rm A}^2/2} - \frac{\Omega_{\rm A}^2}{4\sqrt{\omega^2 - \Omega_{\rm A}^2/2}} \cos(4\pi\chi l + 2\delta)$$

and

$$H_a(l,j) = H_a(j,l) = \frac{C}{|l-j|^3 \sqrt{\omega^2 - \Omega_{\rm A}^2/2}}.$$

We assume the eigenstates and eigenvalues to be $\Phi^{(m)} = \left[\phi_1^{(m)}, \phi_2^{(m)}, \ldots \phi_l^{(m)}, \ldots\right]$ and $\varepsilon^{(m)}$. From $H_a\Phi^{(m)} = \varepsilon^{(m)}\Phi^{(m)}$, we have

$$\sum_{j\neq l} \frac{C\phi_j^{(m)}}{|l-j|^3 \sqrt{\omega^2 - \Omega_{\rm A}^2/2}} - \frac{\Omega_{\rm A}^2 \phi_l^{(m)}}{4\sqrt{\omega^2 - \Omega_{\rm A}^2/2}} \cos(4\pi\chi l + 2\delta)$$
$$= \left(\varepsilon^{(m)} - \sqrt{\omega^2 - \Omega_{\rm A}^2/2}\right) \phi_l^{(m)}, \tag{B1}$$

which can be reduced to

$$\sum_{j\neq l} \frac{C\phi_j^{(m)}}{|l-j|^3} - \frac{1}{4}\Omega_{\rm A}^2 \phi_l^{(m)} \cos(4\pi\chi l + 2\delta)$$
$$= \sqrt{\omega^2 - \Omega_{\rm A}^2/2} \left(\varepsilon^{(m)} - \sqrt{\omega^2 - \Omega_{\rm A}^2/2}\right) \phi_l^{(m)}. \tag{B2}$$

Equation (B2) can be simply rewritten as $B_a\Phi^{(m)} = \sqrt{\omega^2 - \Omega_{\rm A}^2/2}(\varepsilon^{(m)} - \sqrt{\omega^2 - \Omega_A^2/2})\Phi^{(m)}$ with $B_a(l,j) = B_a(j,l) = \frac{C}{|l-j|^3}$, $B_a(l,l) = -\frac{1}{4}\Omega_{\rm A}^2 \cos(4\pi\chi l + 2\delta)$. Since B_a is independent of ω, the right-hand side term $\sqrt{\omega^2 - \Omega_{\rm A}^2/2}\left(\varepsilon^{(m)} - \sqrt{\omega^2 - \Omega_{\rm A}^2/2}\right)$, as the eigenvalue, should turn to be irrelevant to ω after counteraction, and the eigenstate $\Phi^{(m)}$ is definitely independent of ω. This can also be found in Fig. 4.

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