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Noiseless linear amplification for the single-photon entanglement of arbitrary polarization–time-bin qudit*

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Single-photon entanglement (SPE) is an important source in quantum communication. In this paper, we put forward a single-photon-assisted noiseless linear amplification protocol to protect the SPE of an arbitrary polarization–time-bin qudit from the photon transmission loss caused by the practical channel noise. After the amplification, the fidelity of the SPE can be effectively increased. Meanwhile, the encoded polarization–time-bin features of the qudit can be well preserved. The protocol can be realized under the current experimental conditions. Moreover, the amplification protocol can be extended to resist complete photon loss and partial photon loss during the photon transmission. After the amplification, we can not only increase the fidelity of the target state, but also solve the decoherence problem simultaneously. Based on the above features, our amplification protocol may be useful in future quantum communication.

Keywords: single-photon entanglement, noiseless linear amplification, polarization–time-bin qudit, complete and partial photon loss

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1. Introduction

Entanglement is an important source in quantum communication. In the past few decades, entanglement has been widely used in quantum teleportation,[1–4] quantum secure direct communication,[5–12] quantum key distribution,[13–16] and so on.[17–24] Among the various entanglement forms, single-photon entanglement (SPE) with the form of $(|1_A 0_B\rangle + |0_A 1_B\rangle)/\sqrt{2}$ (A and B are two distant locations) is the simplest but most important entanglement mode. SPE has important applications in cryptography[25,26] and quantum repeater protocols.[27,28] For example, in 2010, Salart *et al.* reported that the purification of SPE could be applied in remote quantum communication based on quantum repeaters.[28] Recently, the Guerreiro group used SPE combined with superconducting detectors to demonstrate the quantum nonlocality and related device-independent applications.[29] In previous applications of SPE, the information was often encoded in the spatial modes (A and B) of the single-photon qubit. Actually, besides the spatial-mode, there are several other degrees of freedom (DOFs) in the photonic system which can become sources to encode quantum information, such as polarization, time-bin, frequency, and orbit-angular-momentum. In recent years, the qudits which encode a higher d-dimensional quantum state ($d > 2$) in single photons have attracted much attention in the quantum communication field due to their superior properties over qubits. For instance, it has been proved that in quantum key distribution, encoding secret keys in qudits rather than qubits can increase security against eavesdropping attacks and the noise threshold during the communication.[30–32] Among the various DOFs in the photonic system, polarization and time-bin are widely used in quantum communication. Polarization is easy to manipulate and measure,[33–35] and the time-bin entanglement is a robust form of optical quantum information. Time-bin entanglement has been successfully used in the transmission of qubits over hundreds of kilometers[36–39] and in teleportation using real-world fiber networks.[40,41] For example, in 2013, Inagaki *et al.* reported the distribution of time-bin entangled photon pairs over 300 km in an optical fiber.[39] Combining the advantages of polarization and time-bin entanglement, the polarization–time-bin qudit is attractive in the future quantum communication field. In 2018, Yoo *et al.* reported their experimental preparation and characterization of four-dimensional polarization–time-bin quantum states.[42] Based on the above research, in this paper, we consider the SPE of the polarization–time-bin qudit, which has potential application in future quantum secure communication.

Unfortunately, during the distribution in a practical quan-

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tum channel, the single-photon qudit may be lost due to the channel noise. Photon transmission loss is a fundamental limitation in quantum communication, and will make the communication fidelity decrease exponentially with the channel length.[43] Noiseless linear amplification (NLA) is an effective method to overcome the photon transmission loss problem, and was first proposed by Ralph and Lund in 2009.[44] Since then, a large number of NLA protocols have been proposed successively.[45–64] In 2010, Gisin *et al.* proposed a theoretical heralded qubit amplifier for single-photon qubits. They first used this heralded qubit amplifier in device-independent quantum key distribution (DI-QKD) to lengthen the communication distance.[45] In 2012, Osorio *et al.* realized the heralded noiseless amplification of the single-photon qubit in linear optics in an experiment.[49] Later, Zhang *et al.* first used Gisin's amplifier to protect the two-mode SPE.[50] In 2015, Zhou put forward the first nonlinear recyclable amplification protocol for SPE with the help of cross-Kerr nonlinearity.[56] In 2017, Monteiro *et al.* proposed a heralded amplification protocol of SPE and they adopted this amplification protocol in the DI-QKD. They showed that by exploiting the amplification protocol, they can maintain high-fidelity entangled states over loss-equivalent distances longer than 50 km.[57] Besides single-photon-assisted NLA protocols, in 2013, Scott *et al.* proposed an entanglement-assisted NLA protocol of a single-photon qubit.[58] Later, considering that ideal auxiliary single photons and auxiliary photon entanglement are difficult to obtain under current experimental conditions, the group of Sheng extended the entanglement-assisted amplification protocol to protect the SPE under imperfect auxiliary photon states generated by a current spontaneous parametric down-conversion source.[59–61] Although the amplification of SPE has been widely discussed, the previous NLA protocols cannot be used to protect the SPE of the polarization–time-bin qudit from photon loss. In this paper, we put forward an NLA protocol for protecting the SPE of the polarization–time-bin qudit assisted with some auxiliary polarized single photons. This NLA protocol only requires some common linear optical devices, which makes it realizable using current experimental technology. After the amplification, the polarization–time-bin encoding features of the single photon can be perfectly retained. Moreover, we will discuss the application of our NLA protocol to resist complete photon loss and partial photon loss. After performing the NLA protocol, we can not only increase the fidelity of the SPE, but also solve the decoherence problem in the spatial mode simultaneously.

The paper is organized as follows. In Section 2, we explain the NLA protocol for the SPE of an arbitrary polarization–time-bin qudit in detail. In Section 3, we extend the NLA protocol to solve complete photon loss and partial photon loss in spatial mode simultaneously. In Sections 4 and 5, we present a discussion and give the conclusion.

2. NLA for the SPE of arbitrary polarization–time-bin qudit

In this section, we will explain the amplification protocol for the SPE of an arbitrary polarization–time-bin qudit. We assume that the photon source S generates an arbitrary polarization–time-bin qudit with the form

$$\begin{aligned}|\psi^{\text{in}}\rangle &= (\alpha|H\rangle+\beta|V\rangle)\otimes(\delta|S\rangle+\eta|L\rangle)\\ &= \alpha\delta|H^S\rangle+\alpha\eta|H^L\rangle+\beta\delta|V^S\rangle+\beta\eta|V^L\rangle,\end{aligned}\tag{1}$$

where $|H\rangle$ represents the horizontal polarization and $|V\rangle$ represents the vertical polarization. $|S\rangle$ and $|L\rangle$ mean the short and long time-bins, respectively. The four coefficients α, β, δ, and η meet $|\alpha|^2+|\beta|^2=1$ and $|\delta|^2+|\eta|^2=1$. Here, we suppose that all the four coefficients are real for simplicity. The polarization–time-bin qudit $|\psi^{\text{in}}\rangle$ is distributed to Alice (A) and Bob (B) with equal probability, which can create a maximally entangled SPE as

$$|\Phi\rangle_{AB}=\frac{1}{\sqrt{2}}(|\psi^{\text{in}}\rangle_{a_1}|0\rangle_{g_1}+|0\rangle_{a_1}|\psi^{\text{in}}\rangle_{g_1}).\tag{2}$$

Unfortunately, after transmission through a practical noisy quantum channel, the qudit may be completely lost with the probability of $1-F$, which makes the SPE degrade into a mixed state as

$$\rho_{\text{in}}=F|\Phi\rangle_{AB}\langle\Phi|+(1-F)|\text{vac}\rangle\langle\text{vac}|.\tag{3}$$

The schematic principle of this protocol is shown in Fig. 1. In order to realize the amplification, Alice and Bob should perform the same operations simultaneously. In detail, they need to prepare two auxiliary single photons in the spatial modes b_1, c_1, e_1, and f_1, respectively. One is in $|H\rangle$ and the other is in $|V\rangle$. Alice makes the two auxiliary photons in b_1 (c_1) mode pass through $\text{PBS}_{2\text{a}}$ and $\text{PBS}_{3\text{a}}$ ($\text{PBS}_{6\text{a}}$ and $\text{PBS}_{7\text{a}}$), successively. Bob makes the two auxiliary photons in e_1 (f_1) mode pass through $\text{PBS}_{2\text{b}}$ and $\text{PBS}_{3\text{b}}$ ($\text{PBS}_{6\text{b}}$ and $\text{PBS}_{7\text{b}}$) successively. PBS represents a polarization beam splitter, which can totally transmit the photon in $|H\rangle$ and reflect the photon in $|V\rangle$. Alice and Bob should precisely control the path length of the long (L) and short (S) paths, which makes the time-bins of the photons in $|H\rangle$ and $|V\rangle$ be precisely $|S\rangle$ and $|L\rangle$, respectively. As a result, with the help of PBSs, Alice and Bob are able to create an auxiliary photon state as $|\varphi_1\rangle=|H^S\rangle_{b_2}\otimes|V^L\rangle_{b_2}\otimes|H^S\rangle_{c_2}\otimes|V^L\rangle_{c_2}\otimes|H^S\rangle_{e_2}\otimes|V^L\rangle_{e_2}\otimes|H^S\rangle_{f_2}\otimes|V^L\rangle_{f_2}$.

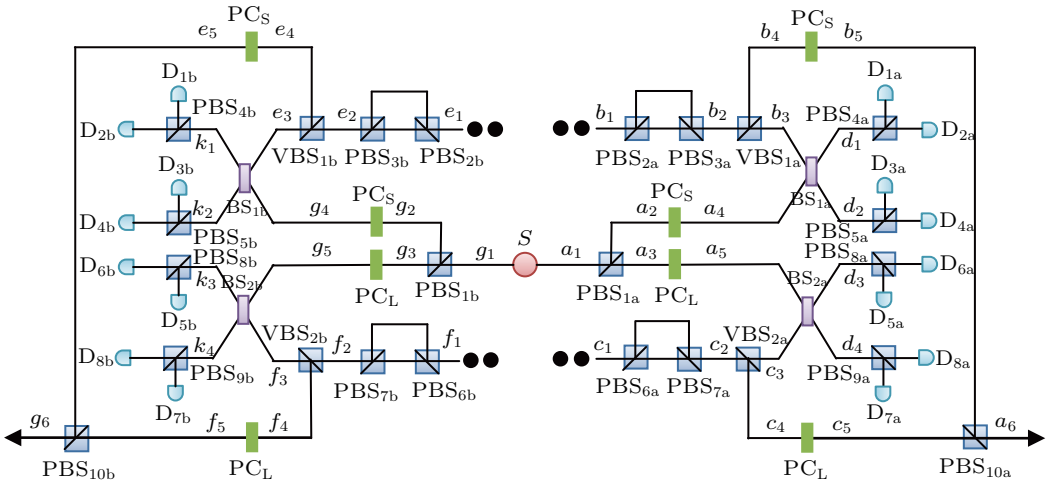


Fig. 1. The basic principle of the NLA protocol for the SPE of arbitrary polarization–time-bin qudit. The protocol requires some auxiliary polarized single photons and some common linear optical devices. Here, BS, PBS, and VBS represent the 50:50 beam splitter, polarization beam splitter, and variable beam splitter, respectively. A Pockel cell (PC) is used to reverse the polarization of photons. The D_{ia} and D_{ib} ($i = 1,2,\ldots,8$) represent the photon-number-resolving detectors.

Then, Alice and Bob make the two auxiliary photons in each of the four spatial modes b_1, c_1, e_1, and f_1 enter a VBS with the transmittance of t. Here, we name the four VBSs as $\mathrm{VBS_{1a}}$, $\mathrm{VBS_{2a}}$, $\mathrm{VBS_{1b}}$, and $\mathrm{VBS_{2b}}$, respectively. After the VBSs, the auxiliary photon state will evolve to

$$\begin{aligned}|\varphi_2\rangle \;=\; & (\sqrt{t}|H^S 0\rangle_{b_3b_4} + \sqrt{1-t}|0H^S\rangle_{b_3b_4}) \\ & \otimes(\sqrt{t}|V^L 0\rangle_{b_3b_4} + \sqrt{1-t}|0V^L\rangle_{b_3b_4}) \\ & \otimes(\sqrt{t}|H^S 0\rangle_{c_3c_4} + \sqrt{1-t}|0H^S\rangle_{c_3c_4}) \\ & \otimes(\sqrt{t}|V^L 0\rangle_{c_3c_4} + \sqrt{1-t}|0V^L\rangle_{c_3c_4}) \\ & \otimes(\sqrt{t}|H^S 0\rangle_{e_3e_4} + \sqrt{1-t}|0H^S\rangle_{e_3e_4}) \\ & \otimes(\sqrt{t}|V^L 0\rangle_{e_3e_4} + \sqrt{1-t}|0V^L\rangle_{e_3e_4}) \\ & \otimes(\sqrt{t}|H^S 0\rangle_{f_3f_4} + \sqrt{1-t}|0H^S\rangle_{f_3f_4}) \\ & \otimes(\sqrt{t}|V^L 0\rangle_{f_3f_4} + \sqrt{1-t}|0V^L\rangle_{f_3f_4}) \\ =\; & [t|H^S V^L\rangle_{b_3} + (1-t)|H^S V^L\rangle_{b_4} \\ & + \sqrt{t(1-t)}(|H^S V^L\rangle_{b_3b_4} + |V^L H^S\rangle_{b_3b_4})] \\ & \otimes[t|H^S V^L\rangle_{c_3} + (1-t)|H^S V^L\rangle_{c_4} \\ & + \sqrt{t(1-t)}(|H^S V^L\rangle_{c_3c_4} + |V^L H^S\rangle_{c_3c_4})] \\ & \otimes[t|H^S V^L\rangle_{e_3} + (1-t)|H^S V^L\rangle_{e_4} \\ & + \sqrt{t(1-t)}(|H^S V^L\rangle_{e_3e_4} + |V^L H^S\rangle_{e_3e_4})] \\ & \otimes[t|H^S V^L\rangle_{f_3} + (1-t)|H^S V^L\rangle_{f_4} \\ & + \sqrt{t(1-t)}(|H^S V^L\rangle_{f_3f_4} + |V^L H^S\rangle_{f_3f_4})]. \end{aligned} \tag{4}$$

On one hand, we first consider that the photonic qudit is not lost with the probability of F. In this case, the whole photon state is $|\Psi_1\rangle = |\Phi\rangle_{AB} \otimes |\varphi_2\rangle$. Alice and Bob make the single photon in a_1 and g_1 modes pass through $\mathrm{PBS_{1a}}$ and $\mathrm{PBS_{1b}}$, respectively, which makes $|\Phi\rangle_{AB}$ in Eq. (2) become

$$\begin{aligned}|\Phi_1\rangle_{AB} \;=\; & \frac{1}{\sqrt{2}}[(\alpha\delta|H^S\rangle_{a_3} + \alpha\eta|H^L\rangle_{a_3} + \beta\delta|V^S\rangle_{a_2} \\ & + \beta\eta|V^L\rangle_{a_2})|0\rangle_{g_1} + |0\rangle_{a_1}(\alpha\delta|H^S\rangle_{g_3} + \alpha\eta|H^L\rangle_{g_3} \\ & + \beta\delta|V^S\rangle_{g_2} + \beta\eta|V^L\rangle_{g_2})]. \end{aligned} \tag{5}$$

Then, they make the photons in both a_2 and g_2 modes pass through the PC_S, and the photons in both a_3 and g_3 modes pass through the PC_L. The PC_S (PC_L) can swap the polarization under the temporal mode S (L). After that, $|\Phi_1\rangle_{AB}$ further evolves to

$$\begin{aligned}|\Phi_2\rangle_{AB} \;=\; & \frac{1}{\sqrt{2}}[(\alpha\delta|H^S\rangle_{a_5} + \alpha\eta|V^L\rangle_{a_5} + \beta\delta|H^S\rangle_{a_4} \\ & + \beta\eta|V^L\rangle_{a_4})|0\rangle_{g_1} + |0\rangle_{a_1}(\alpha\delta|H^S\rangle_{g_5} + \alpha\eta|V^L\rangle_{g_5} \\ & + \beta\delta|H^S\rangle_{g_4} + \beta\eta|V^L\rangle_{g_4})]. \end{aligned} \tag{6}$$

Afterwards, Alice and Bob let the photons in a_4b_3, a_5c_3, g_4e_3, and g_5f_3 modes enter four 50:50 BSs, respectively. We mark the four BSs as $\mathrm{BS_{1a}}$, $\mathrm{BS_{2a}}$, $\mathrm{BS_{1b}}$, and $\mathrm{BS_{2b}}$, respectively, which can make

$$\begin{aligned} |1\rangle_{a_4} &= \frac{1}{\sqrt{2}}(|1\rangle_{d_1} + |1\rangle_{d_2}), \quad |1\rangle_{b_3} = \frac{1}{\sqrt{2}}(|1\rangle_{d_1} - |1\rangle_{d_2}), \\ |1\rangle_{a_5} &= \frac{1}{\sqrt{2}}(|1\rangle_{d_3} + |1\rangle_{d_4}), \quad |1\rangle_{c_3} = \frac{1}{\sqrt{2}}(|1\rangle_{d_3} - |1\rangle_{d_4}), \\ |1\rangle_{g_4} &= \frac{1}{\sqrt{2}}(|1\rangle_{k_1} + |1\rangle_{k_2}), \quad |1\rangle_{e_3} = \frac{1}{\sqrt{2}}(|1\rangle_{k_1} - |1\rangle_{k_2}), \\ |1\rangle_{g_5} &= \frac{1}{\sqrt{2}}(|1\rangle_{k_3} + |1\rangle_{k_4}), \quad |1\rangle_{f_3} = \frac{1}{\sqrt{2}}(|1\rangle_{k_3} - |1\rangle_{k_4}). \end{aligned} \tag{7}$$

Based on the Hong–Ou–Mandel bunching effect, Alice and Bob should make the photons in the two input modes arrive at the BS at the same time. In this way, the parties must accurately adjust the length of each line. After the BSs, $|\Phi_2\rangle_{AB}$ in Eq. (6) and the auxiliary photon state $|\varphi_2\rangle$ in Eq. (4) will turn into

$$\begin{aligned}|\Phi_3\rangle_{AB} \;=\; & \frac{\alpha}{2}(\delta|H^S\rangle_{d_3} + \delta|H^S\rangle_{d_4} + \eta|V^L\rangle_{d_3} + \eta|V^L\rangle_{d_4}) \\ & + \frac{\beta}{2}(\delta|H^S\rangle_{d_1} + \delta|H^S\rangle_{d_2} + \eta|V^L\rangle_{d_1} + \eta|V^L\rangle_{d_2}) \\ & + \frac{\alpha}{2}(\delta|H^S\rangle_{k_3} + \delta|H^S\rangle_{k_4} + \eta|V^L\rangle_{k_3} + \eta|V^L\rangle_{k_4}) \end{aligned}$$

$$
\begin{aligned}
&+\frac{\beta}{2}(\delta|H^S\rangle_{k_1}+\delta|H^S\rangle_{k_2}+\eta|V^L\rangle_{k_1}+\eta|V^L\rangle_{k_2}), \quad (8)\\
|\varphi_3\rangle \;=\; &\Bigg[\frac{t}{2}(|H^SV^L\rangle_{d_1}+|H^SV^L\rangle_{d_2}-|H^SV^L\rangle_{d_1d_2}\\
&-|V^LH^S\rangle_{d_1d_2})+(1-t)|H^SV^L\rangle_{b_4}\\
&+\frac{\sqrt{t(1-t)}}{\sqrt{2}}(|H^SV^L\rangle_{d_1b_4}-|H^SV^L\rangle_{d_2b_4}\\
&+|V^LH^S\rangle_{d_1b_4}-|V^LH^S\rangle_{d_2b_4})]\\
&\otimes\Bigg[\frac{t}{2}(|H^SV^L\rangle_{d_3}+|H^SV^L\rangle_{d_4}-|H^SV^L\rangle_{d_3d_4}\\
&-|V^LH^S\rangle_{d_3d_4})+(1-t)|H^SV^L\rangle_{c_4}\\
&+\frac{\sqrt{t(1-t)}}{\sqrt{2}}(|H^SV^L\rangle_{d_3c_4}-|H^SV^L\rangle_{d_4c_4}\\
&+|V^LH^S\rangle_{d_3c_4}-|V^LH^S\rangle_{d_4c_4})\Bigg]\\
&\otimes\Bigg[\frac{t}{2}(|H^SV^L\rangle_{k_1}+|H^SV^L\rangle_{k_2}-|H^SV^L\rangle_{k_1k_2}\\
&-|V^LH^S\rangle_{k_1k_2})+(1-t)|H^SV^L\rangle_{e_4}\\
&+\frac{\sqrt{t(1-t)}}{\sqrt{2}}(|H^SV^L\rangle_{k_1e_4}-|H^SV^L\rangle_{k_2e_4}\\
&+|V^LH^S\rangle_{k_1e_4}-|V^LH^S\rangle_{k_2e_4})\Bigg]\\
&\otimes\Bigg[\frac{t}{2}(|H^SV^L\rangle_{k_3}+|H^SV^L\rangle_{k_4}-|H^SV^L\rangle_{k_3k_4}\\
&-|V^LH^S\rangle_{k_3k_4})+(1-t)|H^SV^L\rangle_{f_4}\\
&+\frac{\sqrt{t(1-t)}}{\sqrt{2}}(|H^SV^L\rangle_{k_3f_4}-|H^SV^L\rangle_{k_4f_4}\\
&+|V^LH^S\rangle_{k_3f_4}-|V^LH^S\rangle_{k_4f_4})\Bigg]. \quad (9)
\end{aligned}
$$

Table 1. The successful photon detection results of Alice. Here, the • means our protocol is successful under the detection result (each of the detectors registers exactly one photon).

	$D_{1a}D_{2a}$	$D_{1a}D_{4a}$	$D_{2a}D_{3a}$	$D_{3a}D_{4a}$
$D_{5a}D_{6a}$	•	•	•	•
$D_{5a}D_{8a}$	•	•	•	•
$D_{6a}D_{7a}$	•	•	•	•
$D_{7a}D_{8a}$	•	•	•	•

Next, Alice and Bob respectively detect the output photons in the $d_1d_2d_3d_4$ and $k_1k_2k_3k_4$ modes with the help of the PBSs and single-photon detectors D_{ia} and D_{ib} $(i=1,2,\ldots,8)$. We can deduce that $|H^S\rangle_{d1(d3)}$, $|H^S\rangle_{d2(d4)}$, $|H^S\rangle_{k1(k3)}$, and $|H^S\rangle_{k2(k4)}$ will be respectively detected by $D_{2a(6a)}$, $D_{4a(8a)}$, $D_{2b(6b)}$, and $D_{4b(8b)}$, while $|V^L\rangle_{d1(d3)}$, $|V^L\rangle_{d2(d4)}$, $|V^L\rangle_{k1(k3)}$, and $|V^L\rangle_{k2(k4)}$ will be respectively detected by $D_{1a(5a)}$, $D_{3a(7a)}$, $D_{1b(5b)}$, and $D_{3b(7b)}$. It is worth noting that all the photon detectors D_{ia} and D_{ib} $(i=1,2,\ldots,8)$ are required to be able to distinguish the input photon number. After the photon detection, Alice and Bob can determine whether the protocol is successful based on the photon detection results. Here, we only list the 16 successful detection results of Alice in Table 1. The successful detection results of Bob are the same as those of Alice. In this way, we can see that there are a total of $16\times16=256$ successful cases in our protocol.

Here, we take the detection results of $D_{1a}D_{2a}D_{5a}D_{6a}D_{1b}D_{2b}D_{5b}D_{6b}$ each registering exactly one photon for example. Under the detection results, the whole-photon state $|\Phi_3\rangle_{AB}\otimes|\varphi_3\rangle$ will collapse to

$$
\begin{aligned}
&|\Phi_4\rangle_{AB}\\
=\;&\frac{\alpha}{\sqrt{2}}(\delta|H^S\rangle_{c_4}+\eta|V^L\rangle_{c_4})+\frac{\beta}{\sqrt{2}}(\delta|H^S\rangle_{b_4}+\eta|V^L\rangle_{b_4})\\
&+\frac{\alpha}{\sqrt{2}}(\delta|H^S\rangle_{f_4}+\eta|V^L\rangle_{f_4})+\frac{\beta}{\sqrt{2}}(\delta|H^S\rangle_{e_4}+\eta|V^L\rangle_{e_4}), \quad (10)
\end{aligned}
$$

with the probability of $t^7(1-t)/256$.

Then, Alice and Bob make the photons in b_4e_4 modes pass through the PC_S and the photons in c_4f_4 modes pass through the PC_L. After that, the state $|\Phi_4\rangle_{AB}$ will turn into

$$
\begin{aligned}
&|\Phi_5\rangle_{AB}\\
=\;&\frac{\alpha}{\sqrt{2}}(\delta|H^S\rangle_{c_5}+\eta|H^L\rangle_{c_5})+\frac{\beta}{\sqrt{2}}(\delta|V^S\rangle_{b_5}+\eta|V^L\rangle_{b_5})\\
&+\frac{\alpha}{\sqrt{2}}(\delta|H^S\rangle_{f_5}+\eta|H^L\rangle_{f_5})+\frac{\beta}{\sqrt{2}}(\delta|V^S\rangle_{e_5}+\eta|V^L\rangle_{e_5}). \quad (11)
\end{aligned}
$$

Finally, Alice makes the output photons in b_5c_5 modes pass through PBS_{10a} and Bob makes the output photons in e_5f_5 modes pass through PBS_{10b}. These operations make $|\Phi_5\rangle_{AB}$ finally evolve to

$$
\begin{aligned}
&|\Phi^{\rm out}\rangle_{AB}\\
=\;&\frac{1}{\sqrt{2}}(\alpha\delta|H^S\rangle_{a_6}+\alpha\eta|H^L\rangle_{a_6}+\beta\delta|V^S\rangle_{a_6}+\beta\eta|V^L\rangle_{a_6}\\
&+\alpha\delta|H^S\rangle_{g_6}+\alpha\eta|H^L\rangle_{g_6}+\beta\delta|V^S\rangle_{g_6}+\beta\eta|V^L\rangle_{g_6})\\
=\;&\frac{1}{\sqrt{2}}(|\psi^{\rm in}\rangle_{a_6}|0\rangle_{g_6}+|0\rangle_{a_6}|\psi^{\rm in}\rangle_{g_6}), \quad (12)
\end{aligned}
$$

which has the same form of $|\Phi\rangle_{AB}$.

If we get one of the other 255 successful detection results, we can also finally obtain the output state the same as $|\Phi^{\rm out}\rangle_{AB}$ with the help of the phase-flip operation. Therefore, the total success probability for Alice and Bob obtaining $|\Phi^{\rm out}\rangle_{AB}$ is

$$
P_1=F\frac{t^7(1-t)}{256}\times256=Ft^7(1-t). \quad (13)
$$

On the other hand, we consider the case in which the single photon is lost with the probability of $1-F$. Under this case, the input state is the vacuum state and the whole photon state is the auxiliary photon state $|\varphi_2\rangle$. After the auxiliary photons pass through the BSs, it can be found that only the item $t|H^SV^L\rangle_{b_3}\otimes t|H^SV^L\rangle_{c_3}\otimes t|H^SV^L\rangle_{e_3}\otimes t|H^SV^L\rangle_{f_3}$ can lead to successful photon detection results. In detail, after the BSs, such item will evolve to On the other hand, we consider the case in which the single photon is lost with the probability of $1-F$. Under this case, the input state is the vacuum state and the whole photon state is the auxiliary photon state $|\varphi_2\rangle$. After

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$$\begin{aligned}|\varphi_4\rangle \;=\; & \frac{t}{2}(|H^SV^L\rangle_{d_1}+|H^SV^L\rangle_{d_2}-|H^SV^L\rangle_{d_1d_2}-|V^LH^S\rangle_{d_1d_2})\\ & \otimes\frac{t}{2}(|H^SV^L\rangle_{d_3}+|H^SV^L\rangle_{d_4}-|H^SV^L\rangle_{d_3d_4}-|V^LH^S\rangle_{d_3d_4})\\ & \otimes\frac{t}{2}(|H^SV^L\rangle_{k_1}+|H^SV^L\rangle_{k_2}-|H^SV^L\rangle_{k_1k_2}-|V^LH^S\rangle_{k_1k_2})\\ & \otimes\frac{t}{2}(|H^SV^L\rangle_{k_3}+|H^SV^L\rangle_{k_4}-|H^SV^L\rangle_{k_3k_4}-|V^LH^S\rangle_{k_3k_4}),\end{aligned} \tag{14}$$

which can lead to the 256 successful detection results described above. Under any one of the 256 detection results, $|\varphi_4\rangle$ will finally collapse to the vacuum state.

From Eqs. (4) and (14), the success probability of finally obtaining the vacuum state is t^8. Therefore, under the case where the initial single photon is lost, the success probability of our protocol is

$$P_2=(1-F)t^8. \tag{15}$$

Until now, the entire amplification process is completed. When the protocol is successful, Alice and Bob will share a new mixed state

$$\rho_{\rm out}=F'|\Phi^{\rm out}\rangle_{AB}\langle\Phi^{\rm out}|+(1-F')|{\rm vac}\rangle\langle{\rm vac}|, \tag{16}$$

with a total success probability of

$$P=P_1+P_2=Ft^7(1-t)+(1-F)t^8. \tag{17}$$

The fidelity of the target state $|\Phi^{\rm out}\rangle_{AB}$ is

$$F'=\frac{P_1}{P_1+P_2}=\frac{Ft^7(1-t)}{Ft^7(1-t)+(1-F)t^8}=\frac{F(1-t)}{F-2Ft+t}. \tag{18}$$

The fidelity F' of the new mixed state does not relate to the four coefficients α, β, δ, and η, but is only determined by the fidelity F of the initial input mixed state and the transmittance t of the VBSs. We define the amplification factor g as

$$g=\frac{F'}{F}=\frac{1-t}{F-2Ft+t}. \tag{19}$$

To realize the amplification, we need $F'>F$, that is, $g>1$. It can be calculated that as long as we provide suitable VBSs with $t<1/2$, we can obtain $g>1$.

3. Application of the amplification protocol to resist complete photon loss and partial photon loss

Interestingly, besides realizing the amplification of SPE, our protocol can be extended to resist decoherence in the spatial DOF. This means our protocol can simultaneously protect the SPE of the polarization–time-bin qudit from complete transmission photon loss and partial photon loss caused by environmental noise.

Here, we suppose that the photon source S generates a polarization–time-bin qudit with the form of $|\psi^{\rm in}\rangle$ in Eq. (1). The qudit is sent to Alice and Bob to generate a maximally entangled SPE. Unfortunately, the environmental noise may cause both photon transmission loss and decoherence in the spatial DOF. As a result, after the photon transmission, Alice and Bob share a mixed state as

$$\rho_{\rm in1}=F|\Phi'\rangle_{AB}\langle\Phi'|+(1-F)|{\rm vac}\rangle\langle{\rm vac}|, \tag{20}$$

where the state $|\Phi'\rangle_{AB}$ is a less entangled SPE with the form

$$|\Phi'\rangle_{AB}=a|\psi^{\rm in}\rangle_{a_1}|0\rangle_{g_1}+b|0\rangle_{a_1}|\psi^{\rm in}\rangle_{g_1}. \tag{21}$$

Here, a and b are two entanglement coefficients, which meet $|a|^2+|b|^2=1$. We also assume that the values of a and b are real for simplicity. Moreover, we need to know the values of the two coefficients in advance, which can be obtained by measuring a large number of the target state.

Our protocol can be extended to increase the fidelity of the target state, while recovering the two entanglement coefficients a and b to be the same. To achieve this goal, we only need to adjust the transmittance of the four VBSs. Here, we define the transmittance of $\rm VBS_{1a}$, $\rm VBS_{2a}$ to be $t_{\rm a}$ and that of $\rm VBS_{1b}$, $\rm VBS_{2b}$ to be $t_{\rm b}$. If the single photon is not lost, after the BSs, the single photon state and auxiliary photon state can be written as

$$\begin{aligned}&|\Phi_3'\rangle_{AB}\\ =\;& \frac{a\alpha}{\sqrt{2}}(\delta|H^S\rangle_{d_3}+\delta|H^S\rangle_{d_4}+\eta|V^L\rangle_{d_3}+\eta|V^L\rangle_{d_4})\\ &+\frac{a\beta}{\sqrt{2}}(\delta|H^S\rangle_{d_1}+\delta|H^S\rangle_{d_2}+\eta|V^L\rangle_{d_1}+\eta|V^L\rangle_{d_2})\\ &+\frac{b\alpha}{\sqrt{2}}(\delta|H^S\rangle_{k_3}+\delta|H^S\rangle_{k_4}+\eta|V^L\rangle_{k_3}+\eta|V^L\rangle_{k_4})\\ &+\frac{b\beta}{\sqrt{2}}(\delta|H^S\rangle_{k_1}+\delta|H^S\rangle_{k_2}+\eta|V^L\rangle_{k_1}+\eta|V^L\rangle_{k_2}),\end{aligned} \tag{22}$$

$$\begin{aligned}|\varphi_3'\rangle \;=\; & \Bigg[\frac{t_{\rm a}}{2}(|H^SV^L\rangle_{d_1}+|H^SV^L\rangle_{d_2}-|H^SV^L\rangle_{d_1d_2}\\ & -|V^LH^S\rangle_{d_1d_2})+(1-t_{\rm a})|H^SV^L\rangle_{b_4}\\ & +\frac{\sqrt{t_{\rm a}(1-t_{\rm a})}}{\sqrt{2}}(|H^SV^L\rangle_{d_1b_4}-|H^SV^L\rangle_{d_2b_4}\\ & +|V^LH^S\rangle_{d_1b_4}-|V^LH^S\rangle_{d_2b_4})\Bigg]\\ & \otimes\Bigg[\frac{t_{\rm a}}{2}(|H^SV^L\rangle_{d_3}+|H^SV^L\rangle_{d_4}-|H^SV^L\rangle_{d_3d_4}\\ & -|V^LH^S\rangle_{d_3d_4})+(1-t_{\rm a})|H^SV^L\rangle_{c_4}\\ & +\frac{\sqrt{t_{\rm a}(1-t_{\rm a})}}{\sqrt{2}}(|H^SV^L\rangle_{d_3c_4}-|H^SV^L\rangle_{d_4c_4}\end{aligned}$$

$$
\begin{aligned}
&+|V^L H^S\rangle_{d_3c_4}-|V^L H^S\rangle_{d_4c_4})\Big]\\
&\otimes\Big[\frac{t_b}{2}(|H^S V^L\rangle_{k_1}+|H^S V^L\rangle_{k_2}-|H^S V^L\rangle_{k_1k_2}\\
&-|V^L H^S\rangle_{k_1k_2})+(1-t_b)|H^S V^L\rangle_{e_4}\\
&+\frac{\sqrt{t_b(1-t_b)}}{\sqrt{2}}(|H^S V^L\rangle_{k_1e_4}-|H^S V^L\rangle_{k_2e_4}\\
&+|V^L H^S\rangle_{k_1e_4}-|V^L H^S\rangle_{k_2e_4})\Big]\\
&\otimes\Big[\frac{t_b}{2}(|H^S V^L\rangle_{k_3}+|H^S V^L\rangle_{k_4}-|H^S V^L\rangle_{k_3k_4}\\
&-|V^L H^S\rangle_{k_3k_4})+(1-t_b)|H^S V^L\rangle_{f_4}\\
&+\frac{\sqrt{t_b(1-t_b)}}{\sqrt{2}}(|H^S V^L\rangle_{k_3f_4}-|H^S V^L\rangle_{k_4f_4}\\
&+|V^L H^S\rangle_{k_3f_4}-|V^L H^S\rangle_{k_4f_4})\Big]. \qquad (23)
\end{aligned}
$$

After the photon detection, we also take the detection results of $D_{1a}D_{2a}D_{5a}D_{6a}D_{1b}D_{2b}D_{5b}D_{6b}$ each registering one photon for example. Combining Eqs. (22) and (23), the whole state $|\Phi_3'\rangle_{AB}\otimes|\varphi_3'\rangle$ will collapse to

$$
\begin{aligned}
|\Phi_4'\rangle_{AB} \;=\;& a\alpha t_b^2 t_a\sqrt{t_a(1-t_a)}(\delta|H^S\rangle_{c_4}+\eta|V^L\rangle_{c_4})\\
&+a\beta t_b^2 t_a\sqrt{t_a(1-t_a)}(\delta|H^S\rangle_{b_4}+\eta|V^L\rangle_{b_4})\\
&+b\alpha t_a^2 t_b\sqrt{t_b(1-t_b)}(\delta|H^S\rangle_{f_4}+\eta|V^L\rangle_{f_4})\\
&+b\beta t_a^2 t_b\sqrt{t_b(1-t_b)}(\delta|H^S\rangle_{e_4}+|\eta|V^L\rangle_{e_4}). \qquad (24)
\end{aligned}
$$

Here, by adjusting the two entanglement coefficients of the spatial modes to be the same, we can control the transmittance of the four VBSs to meet $a\alpha t_b^2 t_a\sqrt{t_a(1-t_a)}=b\alpha t_a^2 t_b\sqrt{t_b(1-t_b)}$. This transmittance requirement can be simplified as

$$a^2 t_b(1-t_a)=b^2 t_a(1-t_b). \qquad (25)$$

As the values of a and b are known, t_b can be expressed as a function of t_a. That is to say, as long as the value of t_a is fixed, we can deterministically obtain the value of t_b. Under this case, the state in Eq. (24) can finally evolve to maximally entangled SPE with the form

$$|\Phi_1'\rangle_{AB}=\frac{1}{\sqrt{2}}(|\psi^{\rm in}\rangle_{a_6}|0\rangle_{g_6}+|0\rangle_{a_6}|\psi^{\rm in}\rangle_{g_6}). \qquad (26)$$

The success probability can be written as

$$P_1' \;=\; F[a^2 t_b^4 t_a^3(1-t_a)+b^2 t_a^4 t_b^3(1-t_b)]. \qquad (27)$$

On the other hand, if the initial single photon is lost, by selecting the item corresponding to one of the 256 successful detection results, we can also obtain the vacuum state with the probability of

$$P_2'=(1-F)t_a^4 t_b^4. \qquad (28)$$

In this way, after operating the protocol, Alice and Bob can finally share a new mixed state as

$$\rho_{\rm out1}=F''|\Phi_1'\rangle_{AB}\langle\Phi_1'|+(1-F'')|{\rm vac}\rangle\langle{\rm vac}|. \qquad (29)$$

The total success probability can be calculated as

$$P'=P_1'+P_2', \qquad (30)$$

and the fidelity F'' equals

$$F''=P_1'/P'. \qquad (31)$$

Similar to in Section 2, by adjusting the transmittance t_a, we can also make $F''>F$ and realize the amplification.

4. Discussion

This paper mainly describes a simple and efficient amplification protocol for the SPE of an arbitrary polarization–time-bin qudit. This protocol only needs some auxiliary single photons in $|H\rangle$ or $|V\rangle$ polarization. After performing the protocol, we can judge whether the protocol is successfully heralded by the single-photon detection results. As long as the protocol is successful, we can effectively increase the fidelity of SPE by adjusting the transmittance of the VBSs. Moreover, after the amplification, the encoded polarization and time-bin features of the single-photon qudit can be well preserved and cannot be leaked to either Alice or Bob. This amplification protocol is based on linear optics, which can be implemented under the current experimental condition. Meanwhile, the distilled new mixed state with high fidelity can be retained and used in other applications. Moreover, we also discuss the application of our protocol by adjusting the transmittance of the four VBSs, and we can protect the SPE of the polarization–time-bin qudit from complete photon loss and partial photon loss, simultaneously.

It is important to discuss the fidelity F', the amplification factor g, and the total success probability P of the protocol. From Eqs. (17)–(19), it can be found that the three factors have nothing to do with the coefficients α, β, δ, and η, but are only determined by the transmittance t of the VBSs and the fidelity F of the initial input state. We calculate the values of F', g, and P as a function of the transmittance t under the initial fidelity $F=0.2, 0.4, 0.6, 0.8$, respectively. As shown in Figs. 2 and 3, the amplification can be realized when $t<1/2$. The smaller t is, the higher value F' and g will achieve. When t is close to 0, F' is close to 1 and g is close to $1/F$. In Fig. 3, when $t=1/2$, the four curves intersect at one point of $g=1$. On the other hand, as shown in Fig. 4, the lower value of t leads to the lower P. When t is close to 0, P is also close to 0. As the amplification is realized only when $t<1/2$, we control the scale of t to be $[0,0.5]$. When t is 0.5, the maximum value of P is $1/256$. The success probability of our protocol is lower than that of the previous amplification protocol.[63,64]

The reason is that for preserving the polarization and time-bin features of the qudit, we have to use more auxiliary photons and perform more parity checks. As the success probability of the parity check is less than 1, the more parity checks lead to the lower total success probability of the protocol. On the other hand, it can be found that there is a trade-off between P and F'. In practical application, we should consider both factors and select suitable VBSs.

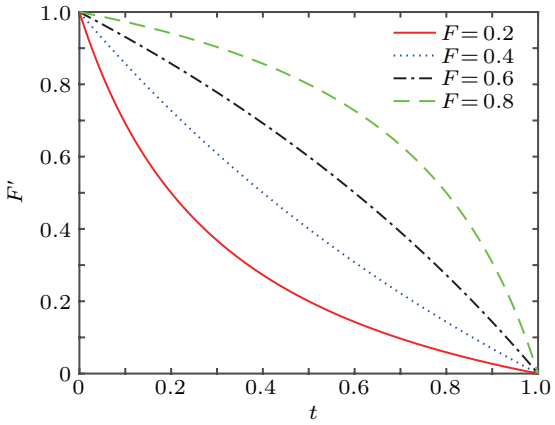


Fig. 2. The fidelity F' as a function of the transmittance t of the VBSs with different initial fidelity $F = 0.2, 0.4, 0.6, and 0.8$. The smaller t is, the higher value F' will achieve. When $t \to 0$, F' under arbitrary initial fidelity F will be close to 1.

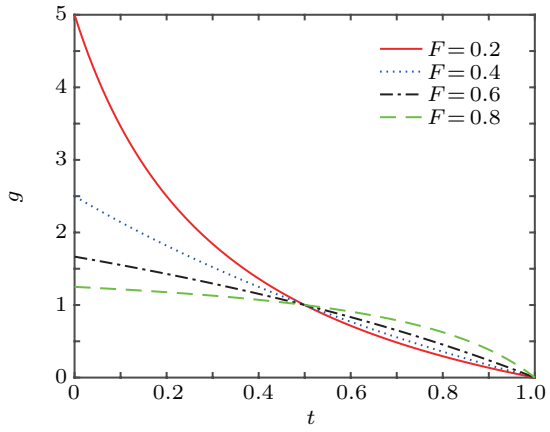


Fig. 3. The amplification factor g as a function of the transmittance t with different initial fidelity $F = 0.2, 0.4, 0.6, and 0.8$. When $t = 0.5$, the four curves intersect at one point of $g = 1$, which indicates that the fidelity of the distilled new mixed state equals that of the initial mixed state. The amplification only occurs under $t < 1/2$.

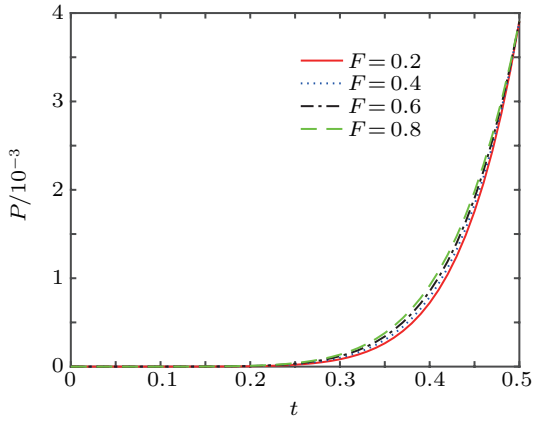


Fig. 4. The total success probability P as a function of the transmittance t with different initial fidelity $F = 0.2, 0.4, 0.6, and 0.8$. A higher value of t leads to a higher value of P. Therefore, there is a trade-off between F' and P. A higher F' leads to a lower P.

5. Conclusion

In summary, we propose an efficient NLA protocol to protect the SPE of an arbitrary polarization–time-bin qudit assisted by some auxiliary polarized photons, some linear optical devices, and photon-number-resolving detectors. In this protocol, Alice and Bob individually perform similar operations in her/his location simultaneously. Each of them adopts some PBSs and Pockel cells to convert the original photon state into two polarization components, which will be individually amplified simultaneously. During the operation, with the help of a 50:50 BS and single-photon detection, the parties can judge whether the protocol is successful according to the photon detection results. When the protocol is successful, the parties can finally distill a new mixed state with higher fidelity by adjusting the transmittance of the VBS. The amplification protocol can be realized under current experimental conditions. Moreover, after the amplification, the encoded polarization and time-bin features of the single-photon qudit can be well preserved and cannot be leaked to either Alice or Bob. We also discuss the application of our amplification protocol to resist complete photon loss and partial photon loss problems during photon transmission. After the amplification, Alice and Bob can not only increase the fidelity of the target state, but also solve the decoherence problem. Based on the above features, our amplification protocol may be useful in the future quantum communication field.

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