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Direct measurement of the concurrence of hybrid entangled state based on parity check measurements*

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The hybrid entangled state is widely discussed in quantum information processing. In this paper, we propose the first protocol to directly measure the concurrence of the hybrid entangled state. To complete the measurement, we design parity check measurements (PCMs) for both the single polarization qubit and the coherent state. In this protocol, we perform three rounds of PCMs. The results show that we can convert the concurrence into the success probability of picking up the correct states from the initial entangled states. This protocol only uses polarization beam splitters, beam splitters, and weak cross-Kerr nonlinearities, which is feasible for future experiments. This protocol may be useful in future quantum information processing.

Keywords: hybrid entangled state, quantum computation, parity measurement, concurrence

PACS: 03.67.Dd, 03.67.Hk, 03.65.Ud **DOI:** 10.1088/1674-1056/28/1/010301

1. Introduction

With the development of quantum information technology, quantum entanglement is found to be an important physical resource that plays a significant role and has been widely used in quantum protocols,[1] such as quantum teleportation,[2,3] quantum key distribution (QKD),[4–6] quantum secure direct communication,[7–11] quantum machine learning,[12–15] and other important protocols.[16–20]

Quantum entanglement can be divided into many types. For example, in discrete variable systems, the typical entanglement is the two-photon polarization entanglement.[21] If the quantum state can simultaneously entangle in two degrees of freedom, such as polarization–momentum and polarization–time-bin, such an entangled state is called a hyperentangled state, which has also been widely discussed in both theory and experiment.[22–32] Recently, another kind of entanglement, called hybrid entanglement, has also been used in quantum information processing.[33–45] The hybrid entanglement means that the entanglement is generated between different degrees of freedom of a particle pair. A typical hybrid entanglement is the microscopic–macroscopic hybrid entanglement, which is useful for loophole-free Bell-type inequality tests. It is also a useful source in QKD, quantum teleportation, and quantum computation.[45]

Usually, the entanglement is difficult to characterize directly. How to measure or quantify entanglement in both theory and experiment is one of the fundamental questions in the field of quantum information and quantum physics theory. In recent years, people have conducted extensive and deep studies on the theory of entanglement quantification, and many different measurement methods have been proposed, such as the entanglement of formation (EOF), partial entropy entanglement, relative entropy entanglement, negativity, and so on.[46–51] The EOF was firstly proposed by Bennett *et al.* in 1996.[46] Subsequently, the EOF of any two-qubit state was proved to be related to the concurrence (C),[52,53] which can be defined as

$$C(\rho) = \max\{0, \lambda_1, -\lambda_2, -\lambda_3, -\lambda_4\}, \tag{1}$$

where λ_i $(i = 1,2,3,4)$ are the non-negative eigenvalues of the Hermitian matrix $R = \sqrt{\sqrt{\rho}\widetilde{\rho}\sqrt{\rho}}$ in descending order and $\widetilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. Here ρ^* is the complex conjugate of ρ, and σ_y is the usual Pauli operator. For example, the concurrence of the two-qubit pure state

$$|\varphi\rangle = \lambda_0|00\rangle + \lambda_1|01\rangle + \lambda_2|10\rangle + \lambda_3|11\rangle \tag{2}$$

can be written as

$$C(\varphi) = |\langle\varphi|\sigma_y \otimes \sigma_y|\varphi\rangle| = 2|\lambda_0\lambda_3 - \lambda_1\lambda_2|. \tag{3}$$

There are some important progresses in the measurement of the concurrence.[54–62] For example, in 2006, Walborn *et al.* used the hyperentanglement encoded in the polarization and momentum degrees of freedom to measure the concurrence of the two-qubit polarization pure state.[54] In 2007, Romero

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et al. proposed a direct measurement scheme for the concurrence of atomic two-qubit pure states.[55] Zhang *et al.* proposed an efficient measurement method for the concurrence of two-photon polarization entangled pure states by using cross-Kerr nonlinearity media.[56,57] In 2014, an optimal approach for measuring the atom entanglement was proposed.[59] In 2015, the first approach for measuring the hyperentanglement was discussed.[60]

Although several protocols have been proposed for measuring the entanglement, none discusses the approach of measuring the hybrid entanglement. In this paper, we will propose the first protocol to measure the hybrid entangled state directly. We will exploit the cross-Kerr nonlinearity and 50:50 beam splitter (BS) to construct the parity check measurement (PCM) for the polarization qubit and coherent state, respectively. This paper is organized as follows. In Section 2, we introduce the key elements: the PCMs for both the polarization qubit and the coherent state. In Section 3, we explain the protocol for the direct concurrence measurement of the hybrid entangled states. In Section 4, we present the discussion and conclusion.

2. Two-qubit and coherent state PCM

In our protocol, firstly, we use the cross-Kerr nonlinearity to complete the PCM.[63] Cross-Kerr nonlinearity is a powerful tool widely used in different quantum information processing protocols.[63–73] In Fig. 1, the polarization BS (PBS) can transmit the horizontally polarized photon ($|H\rangle$) and reflect the vertically polarized photon ($|V\rangle$). The effect of each cross-Kerr nonlinear medium is to introduce a phase shift ($\pm\theta$) to the coherent probe beam $|\alpha\rangle_{\rm p}$ when one photon passes through the medium. The total phase shift of $|\alpha\rangle_{\rm p}$ is proportional to the number of signal photons passing through the media.

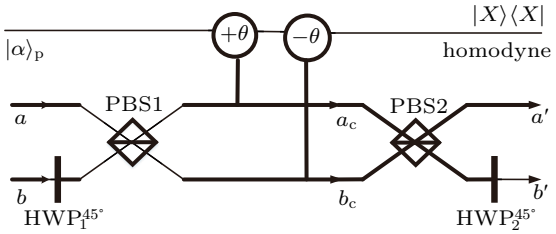


Fig. 1. Schematic diagram of the PCM. The function of the PCM is to distinguish the states $|HV\rangle$ and $|VH\rangle$ from $|HH\rangle$ and $|VV\rangle$. The PBS can transmit the horizontally polarized photon ($|H\rangle$) and reflect the vertically polarized photon ($|V\rangle$). HWP^{45° is a half-wave plate placed in 45°, whose function is $|V\rangle \to |H\rangle$ and $|H\rangle \to |V\rangle$. A photon in $a_{\rm c}$ mode introduces $+\theta$ on the coherent probe beam $|\alpha\rangle_{\rm p}$, while a photon in $b_{\rm c}$ introduces $-\theta$ on $|\alpha\rangle_{\rm p}$. $|X\rangle\langle X|$ represents an X quadrature homodyne measurement.

It is assumed that the two input photons in the signal modes a and b are $|\psi\rangle_{sa} = a_0|H\rangle_a + a_1|V\rangle_a$ and $|\psi\rangle_{sb} = b_0|H\rangle_b + b_1|V\rangle_b$ with $|a_0|^2 + |a_1|^2 = 1$ and $|b_0|^2 + |b_1|^2 = 1$. When the photons in the signal modes a and b successively pass through the $\mathrm{HWP}_1^{45^\circ}$, PBS1, and cross-Kerr nonlinear media shown in Fig. 1, and then exit through PBS2 and $\mathrm{HWP}_2^{45^\circ}$, the evolution of the whole system state can be expressed as

$$\begin{aligned}
&|\psi\rangle_{sa}|\psi\rangle_{sb}|\alpha\rangle_{\rm p}\\
=\ &(a_0|H\rangle_a + a_1|V\rangle_a)\\
&\otimes(b_0|H\rangle_b + b_1|V\rangle_b)\otimes|\alpha\rangle_{\rm p}\\
\xrightarrow{\mathrm{HWP}_1^{45^\circ}}\ &(a_0b_0|HV\rangle_{ab} + a_0b_1|HH\rangle_{ab}\\
&+a_1b_0|VV\rangle_{ab} + a_1b_1|VH\rangle_{ab})\otimes|\alpha\rangle_{\rm p}\\
\xrightarrow{\mathrm{PBS1,Kerr-medias}}\ &(a_0b_1|HH\rangle_{a_{\rm c}b_{\rm c}} + a_1b_0|VV\rangle_{a_{\rm c}b_{\rm c}})\otimes|\alpha\rangle_{\rm p}\\
&+a_0b_0|HV\rangle_{b_{\rm c}}|\alpha{\rm e}^{-2{\rm i}\theta}\rangle_{\rm p}\\
&+a_1b_1|VH\rangle_{a_{\rm c}}|\alpha{\rm e}^{2{\rm i}\theta}\rangle_{\rm p}\\
\xrightarrow{\mathrm{PBS2,HWP}_2^{45^\circ}}\ &(a_0b_1|HV\rangle_{a'b'} + a_1b_0|VH\rangle_{a'b'})\otimes|\alpha\rangle_{\rm p}\\
&+a_0b_0|HH\rangle_{a'b'}|\alpha{\rm e}^{-2{\rm i}\theta}\rangle_{\rm p}\\
&+a_1b_1|VV\rangle_{a'b'}|\alpha{\rm e}^{2{\rm i}\theta}\rangle_{\rm p}. \qquad (4)
\end{aligned}$$

The coherent state $|\alpha\rangle_{\rm p}$ is measured by the X quadrature homodyne measurement to determine whether there is a phase change.[63] In the homodyne measurement, the phase changes $\pm\theta$ are indistinguishable. If the coherent state $|\alpha\rangle_{\rm p}$ has a phase change of $\pm2\theta$, the photon state in the output signal mode will collapse to an even state $|HH\rangle_{a'b'}$ or $|VV\rangle_{a'b'}$, and if there is no phase change, the photon state in the output signal mode will collapse to an odd state $|HV\rangle_{a'b'}$ or $|VH\rangle_{a'b'}$.

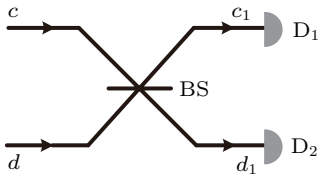


Fig. 2. Schematic diagram of the principle of the PCM. Two coherent states in spatial modes c and d pass through a 50:50 BS. The photons in the output modes c_1 and d_1 are detected by the photon detectors D1 and D2, respectively.

Second, we use a BS to complete the PCM for the coherent state, as shown in Fig. 2. We suppose that two coherent states with the form of $|\pm\alpha\rangle$ in the spatial modes c and d pass through the BS, respectively. The BS can make the two coherent states evolve to

$$\begin{aligned}
&\mathrm{BS}(|\alpha\rangle|\alpha\rangle) \to |\sqrt{2}\alpha\rangle|0\rangle,\\
&\mathrm{BS}(|\alpha\rangle|-\alpha\rangle) \to |0\rangle|\sqrt{2}\alpha\rangle,\\
&\mathrm{BS}(|-\alpha\rangle|\alpha\rangle) \to |0\rangle|-\sqrt{2}\alpha\rangle,\\
&\mathrm{BS}(|-\alpha\rangle|-\alpha\rangle) \to |-\sqrt{2}\alpha\rangle|0\rangle. \qquad (5)
\end{aligned}$$

3. Direct measurement of the concurrence of a hybrid entangled state

A hybrid entangled state that is combined with the single photon and the coherent state can be written as

$$|\varphi\rangle_{ab} = \chi_1|H\rangle_a|\alpha\rangle_b + \chi_2|H\rangle_a|-\alpha\rangle_b$$

$$+\chi_3|V\rangle_a|\alpha\rangle_b+\chi_4|V\rangle_a|-\alpha\rangle_b, \tag{6}$$

with $|\chi_1|^2+|\chi_2|^2+|\chi_3|^2+|\chi_4|^2=1$. In Eq. (6), we should let $|\alpha\rangle$ be large enough to ensure $\langle-\alpha|\alpha\rangle\to 0$.

The schematic principle of the concurrence measurement protocol is shown in Fig. 3. In order to measure the entanglement, we require two copies of the original hybrid entangled state. Here the first copy in the spatial modes a and b is expressed as $|\varphi\rangle_{ab}$, and the second copy in the spatial modes c and d is expressed as $|\varphi\rangle_{cd}$. State $|\varphi\rangle_{ab}$ combined with $|\varphi\rangle_{cd}$ can be written as

$$\begin{aligned}
&|\psi\rangle_{abcd}\\
&=|\varphi\rangle_{ab}\otimes|\varphi\rangle_{cd}\\
&=\chi_1^2|H\rangle_a|\alpha\rangle_b|H\rangle_c|\alpha\rangle_d+\chi_2^2|H\rangle_a|-\alpha\rangle_b|H\rangle_c|-\alpha\rangle_d\\
&\quad+\chi_3^2|V\rangle_a|\alpha\rangle_b|V\rangle_c|\alpha\rangle_d+\chi_4^2|V\rangle_a|-\alpha\rangle_b|V\rangle_c|-\alpha\rangle_d\\
&\quad+\chi_1\chi_2(|H\rangle_a|\alpha\rangle_b|H\rangle_c|-\alpha\rangle_d+|H\rangle_a|-\alpha\rangle_b|H\rangle_c|\alpha\rangle_d)\\
&\quad+\chi_1\chi_3(|H\rangle_a|\alpha\rangle_b|V\rangle_c|\alpha\rangle_d+|V\rangle_a|\alpha\rangle_b|H\rangle_c|\alpha\rangle_d)\\
&\quad+\chi_1\chi_4(|H\rangle_a|\alpha\rangle_b|V\rangle_c|-\alpha\rangle_d+|V\rangle_a|-\alpha\rangle_b|H\rangle_c|\alpha\rangle_d)\\
&\quad+\chi_2\chi_3(|H\rangle_a|-\alpha\rangle_b|V\rangle_c|\alpha\rangle_d+|V\rangle_a|\alpha\rangle_b|H\rangle_c|-\alpha\rangle_d)\\
&\quad+\chi_2\chi_4(|H\rangle_a|-\alpha\rangle_b|V\rangle_c|-\alpha\rangle_d+|V\rangle_a|-\alpha\rangle_b|H\rangle_c|-\alpha\rangle_d)\\
&\quad+\chi_3\chi_4(|V\rangle_a|\alpha\rangle_b|V\rangle_c|-\alpha\rangle_d+|V\rangle_a|-\alpha\rangle_b|V\rangle_c|\alpha\rangle_d).
\end{aligned} \tag{7}$$

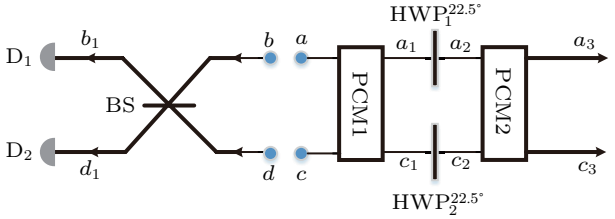


Fig. 3. Schematic diagram showing the principle of the direct concurrence measurement of the hybrid entangled state. It consists of three PCMs, which have been described in Section 2. The structure of both PCM1 and PCM2 are shown in Fig. 1. In each round of the PCM, we pick up the odd parity states. We perform a Hadamard operation on the photon in spatial modes a_1 and c_1 with the help of HWP$^{22.5^\circ}$. HWP$^{22.5^\circ}$ is a half-wave plate oriented at 22.5°, whose function is $|H\rangle\to(|H\rangle+|V\rangle)/\sqrt{2}$ and $|V\rangle\to(|H\rangle-|V\rangle)/\sqrt{2}$.

As shown in Fig. 3, we let the coherent states in spatial modes b and d pass through the BS to make a PCM. The initial state becomes the following state:

$$\begin{aligned}
|\psi_1\rangle_{ab_1cd_1}=\;&\chi_1^2|H\rangle_a|\sqrt{2}\alpha\rangle_{b_1}|H\rangle_c|0\rangle_{d_1}\\
&+\chi_2^2|H\rangle_a|-\sqrt{2}\alpha\rangle_{b_1}|H\rangle_c|0\rangle_{d_1}\\
&+\chi_3^2|V\rangle_a|\sqrt{2}\alpha\rangle_{b_1}|V\rangle_c|0\rangle_{d_1}\\
&+\chi_4^2|V\rangle_a|-\sqrt{2}\alpha\rangle_{b_1}|V\rangle_c|0\rangle_{d_1}\\
&+\chi_1\chi_2(|H\rangle_a|0\rangle_{b_1}|H\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|H\rangle_a|0\rangle_{b_1}|H\rangle_c|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_1\chi_3(|H\rangle_a|\sqrt{2}\alpha\rangle_{b_1}|V\rangle_c|0\rangle_{d_1}\\
&+|V\rangle_a|\sqrt{2}\alpha\rangle_{b_1}|H\rangle_c|0\rangle_{d_1})\\
&+\chi_1\chi_4(|H\rangle_a|0\rangle_{b_1}|V\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|H\rangle_c|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_2\chi_3(|H\rangle_a|0\rangle_{b_1}|V\rangle_c|-\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|H\rangle_c|\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_2\chi_4(|H\rangle_a|-\sqrt{2}\alpha\rangle_{b_1}|V\rangle_c|0\rangle_{d_1}\\
&+|V\rangle_a|-\sqrt{2}\alpha\rangle_{b_1}|H\rangle_c|0\rangle_{d_1})\\
&+\chi_3\chi_4(|V\rangle_a|0\rangle_{b_1}|V\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|V\rangle_c|-\sqrt{2}\alpha\rangle_{d_1}).
\end{aligned} \tag{8}$$

Then, the coherent states in the output modes b_1 and d_1 are detected by the photon detectors D1 and D2, respectively. By selecting the items that make the spatial mode b_1 have no photon, we will obtain the following entangled state:

$$\begin{aligned}
|\psi_1\rangle_{ab_1cd_1}=\;&\frac{1}{\sqrt{N_1}}[\chi_1\chi_2(|H\rangle_a|0\rangle_{b_1}|H\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|H\rangle_a|0\rangle_{b_1}|H\rangle_c|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_1\chi_4(|H\rangle_a|0\rangle_{b_1}|V\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|H\rangle_c|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_2\chi_3(|H\rangle_a|0\rangle_{b_1}|V\rangle_c|-\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|H\rangle_c|\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_3\chi_4(|V\rangle_a|0\rangle_{b_1}|V\rangle_c|\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_a|0\rangle_{b_1}|V\rangle_c|-\sqrt{2}\alpha\rangle_{d_1})],
\end{aligned} \tag{9}$$

with $N_1=2(|\chi_1|^2+|\chi_3|^2)(|\chi_2|^2+|\chi_4|^2)$. The probability of obtaining the state in Eq. (9) from the initial state in Eq. (7) can be expressed as $P_1=N_1$. Subsequently, we let the photons in spatial modes a and c pass through PCM1. After selecting odd parity terms from Eq. (9), the state will become the following state:

$$\begin{aligned}
|\psi_2\rangle_{a_1b_1c_1d_1}=\;&\frac{1}{\sqrt{N_2}}[\chi_1\chi_4(|H\rangle_{a_1}|0\rangle_{b_1}|V\rangle_{c_1}|\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_{a_1}|0\rangle_{b_1}|H\rangle_{c_1}|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\chi_2\chi_3(|H\rangle_{a_1}|0\rangle_{b_1}|V\rangle_{c_1}|-\sqrt{2}\alpha\rangle_{d_1}\\
&+|V\rangle_{a_1}|0\rangle_{b_1}|H\rangle_{c_1}|\sqrt{2}\alpha\rangle_{d_1})],
\end{aligned} \tag{10}$$

with $N_2=2(|\chi_1|^2|\chi_4|^2+|\chi_2|^2|\chi_3|^2)$. The probability of obtaining the state of Eq. (10) from the state of Eq. (9) can be expressed as $P_2=N_2/N_1$.

Next, we perform Hadamard operations on the photons in spatial modes a_1 and c_1. Therefore, the hybrid entangled state in Eq. (10) will evolve to

$$\begin{aligned}
|\psi_3\rangle_{a_2b_1c_2d_1}=\;&\frac{\chi_1\chi_4+\chi_2\chi_3}{2\sqrt{N_2}}(|H\rangle_{a_2}|H\rangle_{c_2}-|V\rangle_{a_2}|V\rangle_{c_2})\\
&\otimes(|0\rangle_{b_1}|\sqrt{2}\alpha\rangle_{d_1}+|0\rangle_{b_1}|-\sqrt{2}\alpha\rangle_{d_1})\\
&+\frac{\chi_1\chi_4-\chi_2\chi_3}{2\sqrt{N_2}}(|H\rangle_{a_2}|V\rangle_{c_2}-|V\rangle_{a_2}|H\rangle_{c_2})\\
&\otimes(|0\rangle_{b_1}|-\sqrt{2}\alpha\rangle_{d_1}-|0\rangle_{b_1}|\sqrt{2}\alpha\rangle_{d_1}).
\end{aligned} \tag{11}$$

In order to achieve the task of measuring the concurrence of the hybrid entanglement state, we let the photons in a_2

and c_2 modes pass through PCM2. After selecting odd parity terms, we will obtain the following state:

$$|\psi_3\rangle_{a_3b_1c_3d_1} = \frac{1}{2}(|H\rangle_{a_3}|V\rangle_{c_3} - |V\rangle_{a_3}|H\rangle_{c_3}) \otimes(|0\rangle_{b_1}|-\sqrt{2}\alpha\rangle_{d_1} - |0\rangle_{b_1}|\sqrt{2}\alpha\rangle_{d_1}). \quad (12)$$

We use $P_3 = (|\chi_1\chi_4| - |\chi_2\chi_3|)^2/N_2$ to represent the probability of obtaining the state in Eq. (12) from the state in Eq. (11). Finally, we obtain the total probability of obtaining the state in Eq. (12) from the initial state of the hybrid entangled state

$$P_{\text{total}} = P_1P_2P_3 = (|\chi_1\chi_4| - |\chi_2\chi_3|)^2. \quad (13)$$

According to Eqs. (3) and (13), we can find that the concurrence of the hybrid entangled state is

$$C(\varphi) = 2|\lambda_0\lambda_3 - \lambda_1\lambda_2| = 2\sqrt{P_{\text{total}}}. \quad (14)$$

In the above discussion, to obtain the concurrence of the hybrid entangled state, we perform three PCMs. In the first round of PCMs, our aim is to choose the items that make the spatial mode b_1 have no photon. In the second and third rounds of PCMs, our aim is to obtain the odd parity state from the hybrid entangled state. Using this method, the measurement of concurrence can be directly converted to the total success probability of the three PCMs. In a practical experiment, we should perform the PCMs many times to obtain the success probability to calculate the concurrence.

4. Discussion and conclusion

In this paper, we propose a protocol to directly measure the entanglement concurrence of a hybrid entangled state. Obviously, the key components of the protocol are PBSs, BSs, and weak cross-Kerr nonlinearity. In the first round of PCM, the BS plays the key role of selecting the appropriate item. As shown in Fig. 3, we choose the items that make spatial mode b_1 have no photon. That is to say, we choose the odd parity state $|\alpha\rangle_b|-\alpha\rangle_d$ and $|-\alpha\rangle_b|\alpha\rangle_d$. However, we should notice that a coherent state $|\alpha\rangle$ also has a probability of no photon. This will cause an erroneous measurement of concurrence. For example, the error probability is $|\langle 0|\pm\sqrt{2}\alpha\rangle|^2$. If $\alpha = 3$, the error probability is about e^{-9}, which can be ignored. In the second and third rounds of PCMs, when the photon in the signal mode interacts with the coherent state in the cross-Kerr nonlinear media, the coherent state will undergo a corresponding phase change, as shown in Eq. (4). In order to select the odd parity state from the input state, the PCM needs to detect this phase change with an X quadrature homodyne measurement, which allows us to distinguish the coherent states $|\alpha\rangle$ and $|\alpha e^{\pm 2i\theta}\rangle$. The error probability of distinction is $P_{\text{error}} = \frac{1}{2}\text{erfc}(\sqrt{2}\alpha\sin^2\theta)$.[63,64] We can see that when $\alpha\sin^2\theta \geq 2.4$, the error probability is $P_{\text{error}} < 10^{-6}$, and thus an approximately precise distinction can be achieved as long as sufficiently strong coherent probe light is used. For example, when the order of the average number of photons for the coherent light reaches 10^{12}, we can obtain $\alpha \sim 10^6$. In this case, only a relatively weak nonlinear phase shift $\theta = 1.6\times10^{-3}$ is needed. Although the effect of natural Kerr nonlinear media is only $\theta \approx 10^{-18}$,[74] a recent experiment reported an observable phase shift due to single post-selected photons.[75] Therefore, as long as sufficient coherent light is selected, the PCM can be implemented approximately deterministically via the weak cross-Kerr nonlinear media and the quadrature homodyne measurement. That is, through the PCM, the parties can correctly select the polarization of state from the input state. This ensures the accuracy of the concurrence measurement for the hybrid entangled state.

In summary, we propose the first protocol to directly measure the concurrence of hybrid entangled states. To complete the measurement, we design a PCM for the polarization qubit and a PCM for the coherent state, respectively. In this protocol, we perform three rounds of PCMs. The results show that we can convert the concurrence into the successful probability of picking up the wanted states from the initial entangled states. This protocol only uses common PBSs, BSs, and weak cross-Kerr nonlinearity, which is feasible for future experiments. This protocol may be useful in future quantum information processing.

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