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Hopf bifurcation control of a Pan-like chaotic system*

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This paper is concerned with the Hopf bifurcation control of a modified Pan-like chaotic system. Based on the Routh–Hurwtiz theory and high-dimensional Hopf bifurcation theory, the existence and stability of the Hopf bifurcation depending on selected values of the system parameters are studied. The region of the stability for the Hopf bifurcation is investigated. By the hybrid control method, a nonlinear controller is designed for changing the Hopf bifurcation point and expanding the range of the stability. Discussions show that with the change of parameters of the controller, the Hopf bifurcation emerges at an expected location with predicted properties and the range of the Hopf bifurcation stability is expanded. Finally, numerical simulation is provided to confirm the analytic results.

Keywords: Hopf bifurcation control, Pan-like chaotic system, Routh–Hurwtiz, hybrid control method, stability control, high-dimensional Hopf bifurcation

PACS: 82.40.Bj **DOI:** 10.1088/1674-1056/27/9/094702

1. Introduction

Since Lorenz found the chaotic attractor by chance during experiments in 1963,[1] many results have been obtained from the investigation of chaotic systems.[2–11] Currently, we are not only concerned on discovering new chaotic systems, but also concerned on the chaotic and bifurcation characteristics of the system itself. Bifurcation control has become one of the important research fields.[12–21] Nguyen *et al.* proposed a novel dynamic state-feedback control law.[21] Analytical schemes to determine the control gains have been derived, according to the conditions for the emergence of Hopf bifurcation. Gan *et al.* analyze the stability and direction of the Hopf bifurcation for a food finite model by the feedback control method.[22] Non-negative equilibrium point of the system is asymptotically stable by analyzing the eigenvalues. The Hopf bifurcation is unstable when the bifurcation parameter exceeds the critical value. Ding *et al.* investigate the problem of bifurcation control for complex networks in a small-world networks model with time delay,[23] presenting a proportional-derivative (PD) feedback controller to control the Hopf bifurcation which inherently occurs due to the networks topology. Liu *et al.* discuss the stable of the wireless network model using the hybrid control.[24] The center manifold theorem and normal form theorem are used to study the stable and direction of the bifurcation periodic solutions. Wu *et al.* carry out the effect of controller parameters on the Hopf bifurcation point, bifurcation type, and periodic solution amplitude by the washout filter for the Rössler system.[25]

At present, there are still little researches on the Pan chaotic system. Pan *et al.* proposed a three-dimensional chaos system,[26] which controls this new attractor by linear feedback functions. The trajectories of the chaotic attractor can be controlled to any periodic target orbits or points. Sundarapandian *et al.* constructed explicit state-feedback control laws for regulating the output of the three-dimensional Pan chaos system so as to track constant reference signals.[27] He also derived new results for the anti-synchronization of identical Pan chaotic systems by active nonlinear control. This control method is effective and convenient, which regards to achieve anti-synchronization of the chaotic systems addressed. AL-Azzawi analyzed the dynamic behaviors of a new three-dimensional Pan chaos system,[28] including some basic dynamic properties, stability, and Hopf bifurcation. The stability and Hopf bifurcation are investigated by the Routh–Hurwitz method and the Cardan method, and the analysis shows that both methods justified the same results.

In this paper, a new five-dimensional Pan-like chaotic system is proposed, in which two new state variables based on the Pan system are set. The system contains many state variables and the dynamic behaviors are very complex. Based on the Routh–Hurwtiz theory and high-dimensional Hopf bifurcation theory, the existence and stability of the Hopf bifurcation are proved. By the hybrid control method, a nonlinear controller is designed to the system for changing the Hopf bifurcation point and expanding the range of the Hopf bifurcation stability. The Hopf bifurcation emerges at an expected location with predicted properties. Meanwhile, the parameters of the nonlinear controller can change the stability of the system. The range

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of stability can be expanded by choosing appropriate parameter values. The purpose of Hopf bifurcation control and the stability control are realized.

2. Dynamic behaviors

The Pan-like chaotic system has the form

$$\begin{cases} \dot{x} = a(y-x)+u, \\ \dot{y} = bx - xz + v, \\ \dot{z} = xy - cz, \\ \dot{u} = x + du, \\ \dot{v} = x + ev, \end{cases} \tag{1}$$

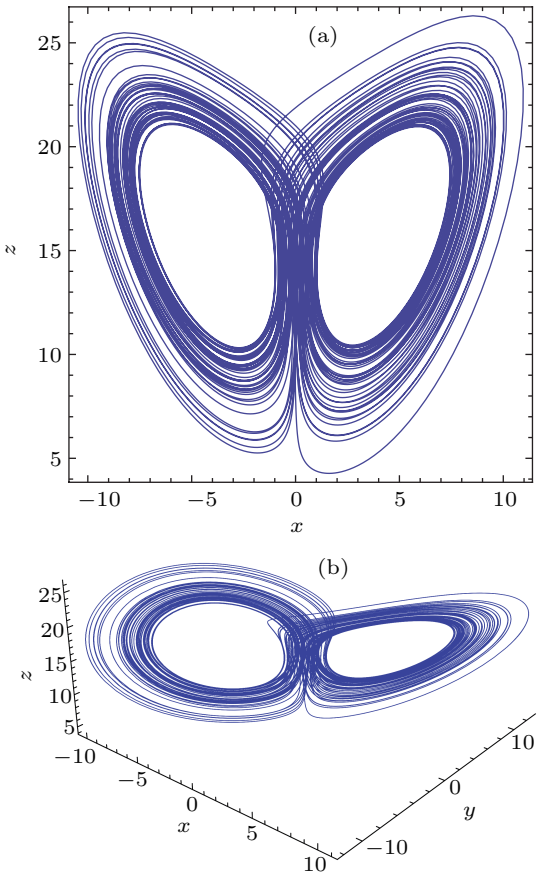


Fig. 1. (color online) Chaotic attractors of the system (1).

where x, y, z, u, v are state variables of the system, and a, b, c, d, e are real parameters. The dynamic behaviors of the system depend on the changes of the real parameters. When the parameters are fixed as $[a,b,c,d,e]^{\mathrm{T}} = \left[7, 16, \frac{2}{3}, -1, -2\right]^{\mathrm{T}}$, the system has a chaotic attractor, which is shown in Fig. 1.

2.1. Hopf bifurcation of equilibrium points

The system has three equilibrium points, given by

$$\begin{aligned} E_0 &= (0,0,0,0,0), \\ E_1 &= \left(\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}, \frac{\sqrt{acde\,(be-1)\,(ad+1)}}{ade}, \right. \\ &\quad \left. b-\frac{1}{e}, -\frac{1}{d}\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}, -\frac{1}{e}\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}\right), \\ E_2 &= \left(-\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}, -\frac{\sqrt{acde\,(be-1)\,(ad+1)}}{ade}, \right. \\ &\quad \left. b-\frac{1}{e}, \frac{1}{d}\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}, \frac{1}{e}\sqrt{\frac{acd\,(be-1)}{e\,(ad+1)}}\right). \end{aligned}$$

Clearly, E_0 is an equilibrium point of the system for all values of the parameters a, b, c, d, e. The equilibrium points E_1 and E_2 of the system exist only when $acde\,(be-1)\,(ad+1) > 0$.

The equilibrium points E_1 and E_2 have symmetrical characteristics, so we only analyze the stability of equilibrium points E_1. The Jacobi matrix of system (1) at the equilibrium points E_1 is given by

$$J_{E_1} = \begin{bmatrix} -a & a & 0 & 1 & 0 \\ b - z_{E_1} & 0 & -x_{E_1} & 0 & 1 \\ y_{E_1} & x_{E_1} & -c & 0 & 0 \\ 1 & 0 & 0 & d & 0 \\ 1 & 0 & 0 & 0 & e \end{bmatrix}. \tag{2}$$

The characteristic equation of system (1) is

$$\lambda^5 + q_1\lambda^4 + q_2\lambda^3 + q_3\lambda^2 + q_4\lambda + q_5 = 0, \tag{3}$$

where

$$\begin{aligned} q_1 &= a + c - d - e, \\ q_2 &= -1 + de - c(d+e) + a\left(-\frac{1+de+e^2}{e} + \frac{c(e+d(ae+be-1))}{e(1+ad)}\right), \\ q_3 &= \frac{1}{e(1+ad)}[e(e+c(de-1)) + a^2d(c(2be-3-de-e^2)+d(1+e^2) \\ &\quad + a(d+2de^2-c(2-be+e^2+de(1+be)+d^2(be-e^2-1)))], \\ q_4 &= \frac{c[e^2+a^2d(-2e(be-1)+d(3-2be+e^2))]+a[e-be^2+d^2e(be-1)+d(3-2be+2e^2)]}{e(1+ad)}, \\ q_5 &= 2acd(be-1). \end{aligned}$$

In the case of $a = 7$, $b = 16$, $d = -1$, $e = -2$, and $c \in (0, 1.9460)$, we have

$$(q_1q_2 - q_3)(q_3q_4 - q_2q_5) - (q_1q_4 - q_5)^2 > 0. \tag{4}$$

Since the above conditions are met, the real parts of characteristic roots of the characteristic equation (3) are all negative. Based on the Routh–Hurwitz theory, the system is asymptotically stable at equilibrium points E_1.

Take c as the Hopf bifurcation parameter. When $c = c_0 = 1.9460$,

$$\begin{aligned} &q_1 = 11.9459 > 0,\\ &q_2 = 82.4176 > 0,\\ &q_3 = 664.458 > 0,\\ &q_4 = 1528.5 > 0,\\ &q_5 = 899.006 > 0,\\ &(q_1q_2 - q_3)(q_3q_4 - q_2q_5) - (q_1q_4 - q_5)^2 = 0. \end{aligned} \tag{5}$$

The matrix $\boldsymbol{J}_{E_1}$ has a pair of pure imaginary roots, that is

$$\lambda_{1,2} = \pm\omega_0 \mathrm{i} = \pm \mathrm{i}\sqrt{\frac{q_5 - q_1q_4}{q_3 - q_1q_2}} = \pm 7.3644\mathrm{i}. \tag{6}$$

The real parts of other characteristic roots are all negative, and

$$\mathrm{Re}(\lambda'(c))|_{c=c_0} = 0.22422 \neq 0.$$

Therefore, based on the Hopf bifurcation theory, the system (1) undergoes Hopf bifurcation at $c = c_0 = 1.9460$.

2.2. Stability of Hopf bifurcation

As shown in Subsection 2.1, the system undergoes Hopf bifurcation at the critical value of bifurcation parameter $c = c_0 = 1.9460$. The characteristic values of the Jacobi matrix are $\lambda_{1,2} = \pm 7.3644\mathrm{i}$, $\lambda_3 = -9.0273$, $\lambda_4 = -2.0008$, and $\lambda_5 = -0.9177$. By making a linear transformation of the system as $(x_1, y_1, z_1, u_z, v_1)^{\mathrm{T}} = \boldsymbol{P}(x_2, y_2, z_2, u_x, v_2)^{\mathrm{T}}$, where

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0.9974 & 1.0711 & -0.2718 & 0.8569 & -0.8673 \\ 1.2127 & -1.2208 & -0.5059 & 193.6125 & -0.0608 \\ 0.0181 & -1.3333 & -0.1246 & -0.9991 & 12.1536 \\ 0.0343 & -0.1265 & -0.1423 & 1162.7907 & 0.9239 \end{bmatrix}, \tag{7}$$

the system has the normal form

$$\begin{cases} \dot{x}_2 = -7.3644y_2 + F_1(x_2, y_2, z_2, u_2, v_2), \\ \dot{y}_2 = 7.3644x_2 + F_2(x_2, y_2, z_2, u_2, v_2), \\ \dot{z}_2 = -9.0273z_2 + F_3(x_2, y_2, z_2, u_2, v_2), \\ \dot{u}_2 = -2.008u_2 + F_4(x_2, y_2, z_2, u_2, v_2), \\ \dot{v}_2 = -0.9177v_2 + F_5(x_2, y_2, z_2, u_2, v_2). \end{cases} \tag{8}$$

Because of the complicated expression of F_i $(i = 1, 2, 3, 4, 5)$, we have omitted them in this paper.

Based on the Hopf bifurcation theory, the indicators

$$\begin{aligned} &g_{11} = -0.0652 - 0.5198\mathrm{i},\\ &g_{02} = -0.1735 - 0.9127\mathrm{i},\\ &g_{20} = 0.0432 - 0.1270\mathrm{i},\\ &G_{21} = 0, g_{21} = 0.0073 + 0.0646\mathrm{i}, \end{aligned}$$

can be obtained by detailed calculation, and the stability coefficient is

$$\begin{aligned} \beta_2 &= 2\mathrm{Re}\left\{\frac{\mathrm{i}}{2\omega_0}\left(g_{20}g_{11} - 2|g_{11}|^2 - \frac{1}{3}|g_{02}|^2\right) + \frac{1}{2}g_{21}\right\} \\ &= 0.0113. \end{aligned} \tag{9}$$

The stability coefficient of system (1) is $\beta_2 > 0$. Therefore, the Hopf bifurcation solution of the system is unstable. The local bifurcation diagram which takes c as the Hopf bifurcation parameter is shown in Fig. 2.

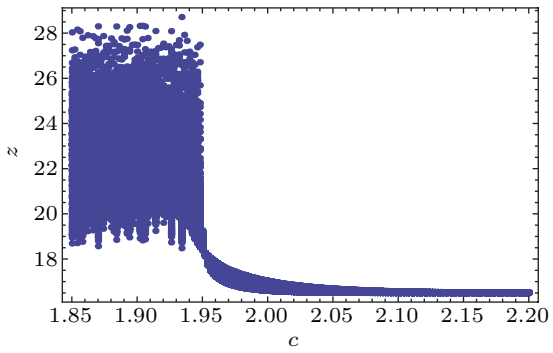


Fig. 2. (color online) Local bifurcation diagram of system (1).

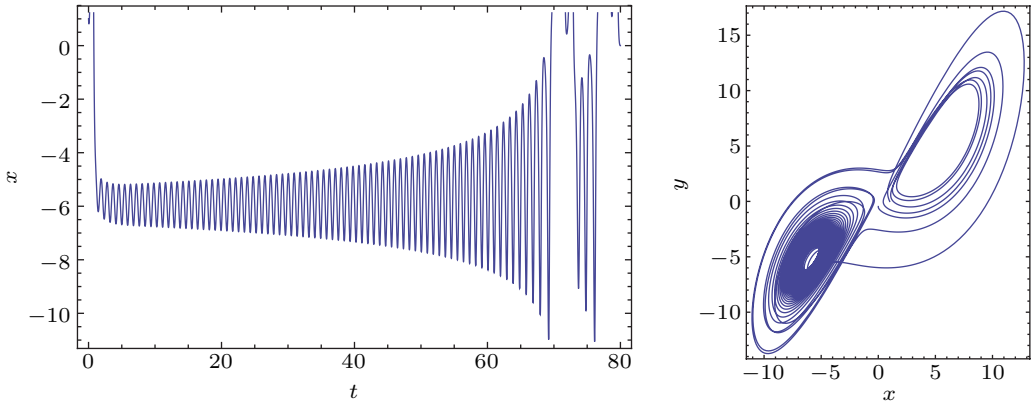


Fig. 3. (color online) Sequence and phase diagram at $c = 1.85$.

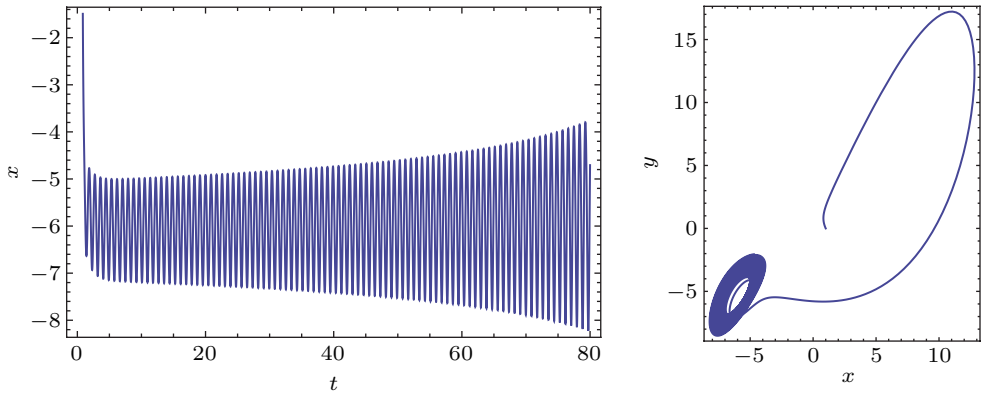


Fig. 4. (color online) Sequence and phase diagram at $c = 1.946$.

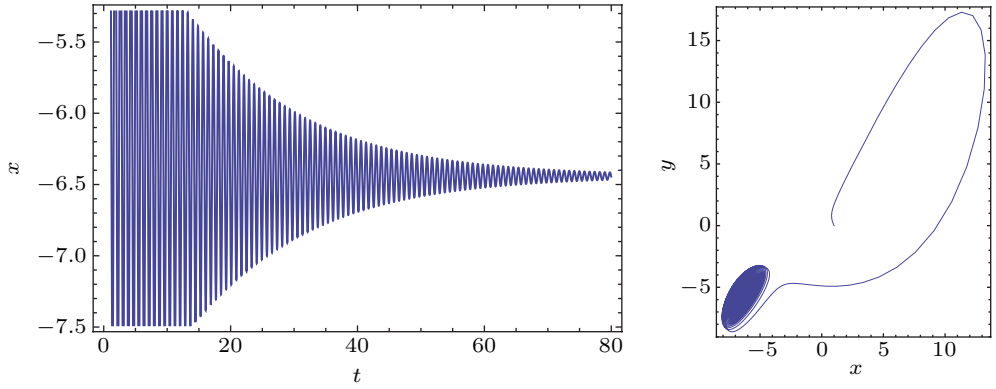


Fig. 5. (color online) Sequence and phase diagram at $c = 2.2$.

When $c > 1.9460$, the system is in a stable state which has a stable limit cycle. When $c < 1.9460$, the Hopf bifurcation appears, which is in an unstable and quasi-periodic state, showing chaotic character finally. The sequence and phase diagram at different values of parameter c are shown in Figs. 3–5.

From the diagram above, when $c = 1.85$, the system is in an unstable state. The system thus has an unstable periodic solution, showing the chaotic phenomenon in the end. When $c = 1.946$, the system is unstable. The bifurcation amplitude is gradually increasing. When $c = 2.2$, the system is in a stable state. This proves that the former theoretical analysis is correct.

3. Control of Hopf bifurcation

3.1. Advance of the Hopf bifurcation

At the equilibrium point E_1, the system (1) is in an unstable state when $c = c_0 = 1.9460$. The hybrid control method is used to control the Hopf bifurcation and stability of the system, so that the critical value of the Hopf bifurcation is changed and the system is stable.

The control system of system (1) is

$$\begin{cases} \dot{x} = a(y-x)+u, \\ \dot{y} = bx - xz + v, \\ \dot{z} = xy - cz, \\ \dot{u} = \alpha_1(x+du) + (1-\alpha_1)(u-u_{E_1})^3, \\ \dot{v} = \alpha_2(x+ev) + (1-\alpha_2)(v-v_{E_1})^3, \end{cases} \tag{10}$$

where α_1 and α_2 are the control parameters, u_{E_1} and v_{E_1} are the values of the equilibrium point E_1. Therefore, the equilibrium point E_1 of system (1) is still the equilibrium of the system (10). After setting up the controller, the structure of the solution of controlled system E_1 is unchanged.

The Jacobi matrix of system (10) at the equilibrium points E_1 is given by

$$\boldsymbol{J}_{E_1} = \begin{bmatrix} -a & a & 0 & 1 & 0 \\ b - z_{E_1} & 0 & -x_{E_1} & 0 & 1 \\ y_{E_1} & x_{E_1} & -c & 0 & 0 \\ \alpha_1 & 0 & 0 & \alpha_1 d & 0 \\ \alpha_2 & 0 & 0 & 0 & \alpha_2 e \end{bmatrix}. \tag{11}$$

The characteristic equation of control system (1) is

$$\lambda^5 + p_1\lambda^4 + p_2\lambda^3 + p_3\lambda^2 + p_4\lambda + p_5 = 0, \tag{12}$$

where p_i $(i = 1,2,3,4,5)$ are the functions of the parameters a, b, c, d, e and the control parameters α_1, α_2. Because they have complicated expressions, we have also omitted them.

When $a=7$, $b=16$, $d=-1$, $e=-2$, and $c>0$, the control parameters α_1 and α_2 of the controlled system meet the conditions based on the Hopf bifurcation theory, that is

$$\begin{cases} (p_1p_2-p_3)(p_3p_4-p_2p_5)-(p_1p_4-p_5)^2=0, \\ p_i>0, \quad i=1,2,3,4,5. \end{cases} \tag{13}$$

The relationship among the control parameters α_1, α_2, and the bifurcation parameter c is shown in Fig. 6.

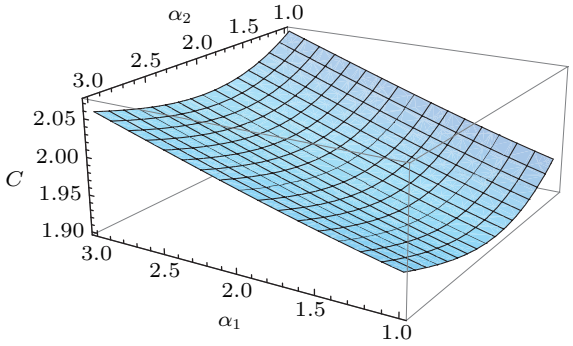


Fig. 6. (color online) Relationship among the parameters α_1, α_2, and c.

The relationship among the control parameters α_1, α_2, and the bifurcation parameter c is nonlinear. It is possible to change the Hopf bifurcation critical values by selecting the appropriate control parameters α_1 and α_2. Similarly, advancement of the bifurcation point can also be achieved.

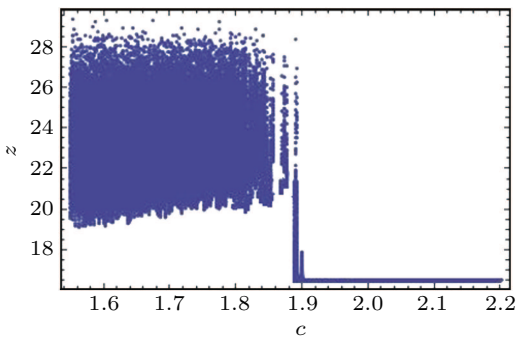


Fig. 7. (color online) Local bifurcation diagram of system (10) when $\alpha_1=1.1$ and $\alpha_2=2$.

When $\alpha_1=1.1$ and $\alpha_2=2$, the critical value of bifurcation parameter $c=c_0=1.9059$ can be obtained according to Eq. (12). The local Hopf bifurcation diagram of system (10) is shown in Fig. 7.

The Hopf bifurcation critical point of the system (10) is advance than the original system (1), and the rationality of the set of controllers is verified.

3.2. Control of the stability

When the Hopf bifurcation of system (1) occurs, the system is in an unstable state. Next, the relationship between the control parameters and the stability of the controlled system is analyzed. When $c=c_0=1.9059$, the system (10) shows Hopf bifurcation. The Jacobi matrix of system (10) has a pair of pure imaginary roots $\lambda'_{1,2}=\pm 7.2618\mathrm{i}$ and three negative roots $\lambda'_3=-8.9125$, $\lambda'_4=-4.0842$, and $\lambda'_5=-1.0092$. The transformation matrix of the system variables can thus be obtained as

$$\boldsymbol{P}=\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0.9968 & -1.0586 & -1.2878 & 0.4692 & -0.8746 \\ -1.2058 & -1.2299 & -2.3830 & -3.6881 & -0.1179 \\ 0.0224 & 0.1481 & -7.1023 & -0.3686 & 12.1132 \\ 0.1164 & 0.2113 & -2.4563 & -23.7558 & 0.6687 \end{bmatrix}. \tag{14}$$

Through transformation, the normal form of controlled system (10) is obtained as

$$\begin{cases} \dot{x}_2=-7.2618y_2+Q_1(x_2,y_2,z_2,u_2,v_2), \\ \dot{y}_2=7.2618x_2+Q_2(x_2,y_2,z_2,u_2,v_2), \\ \dot{z}_2=-8.9125z_2+Q_3(x_2,y_2,z_2,u_2,v_2), \\ \dot{u}_2=-4.0842u_2+Q_4(x_2,y_2,z_2,u_2,v_2), \\ \dot{v}_2=-1.0092v_2+Q_5(x_2,y_2,z_2,u_2,v_2), \end{cases} \tag{15}$$

where Q_i $(i=1,2,3,4,5)$ which determine the bifurcation stability of system (10) consist of nonlinear terms. Through complex calculation, the relationship between the stability parameters and the control parameters is obtained as

$$\begin{aligned} \beta_2 = & -0.0016+0.2332\alpha_1-0.0306\alpha_1^2-0.0814\alpha_2 \\ & +0.0179\alpha_1\alpha_2+0.0291\alpha_2^2. \end{aligned} \tag{16}$$

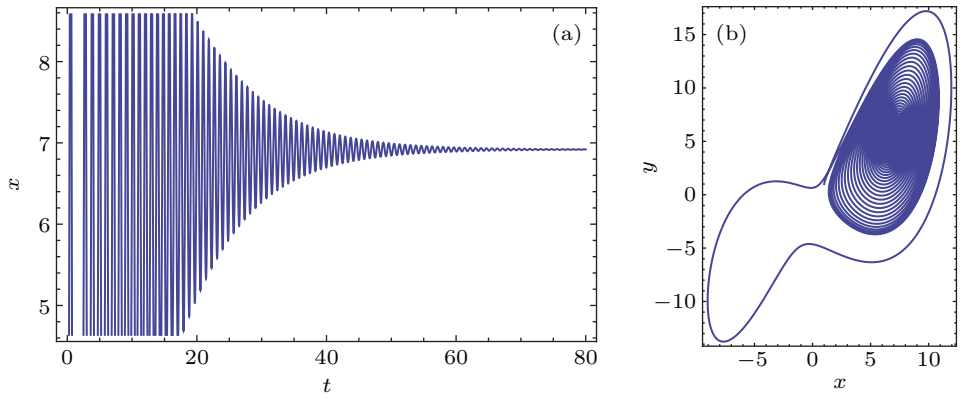


Fig. 8. (color online) The sequence and phase diagram of system (10) when $\alpha_1=10, \alpha_2=1.5$.

In order to support the analytic results, the numerical simulations are carried out. Sequence and phase diagram when $\alpha_1 = 10$, $\alpha_2 = 1.5$, and $c = 1.9059$ are shown Fig. 8.

When $\alpha_1 = 10$, $\alpha_2 = 1.5$, and $\beta_2 = -0.3510 < 0$, the Hopf bifurcation of the control system is stable. Numerical simulation shows that the theoretical analysis and controller settings are correct and reasonable. As long as the appropriate control parameters are selected, the pre-set bifurcation characteristics can be achieved.

4. Conclusion

We analyze the dynamic characteristics of a Pan-like chaotic system based on the Routh–Hurwtiz theory and the high-dimensional Hopf bifurcation theory. The condition under which the Hopf bifurcation emerges is investigated, and the critical value of bifurcation parameters is calculated. The hybrid control method is used to set a nonlinear controller to control the Hopf bifurcation and stability of the system. Through analysis, it shows that the parameters of the linear controller can change the emergence of Hopf bifurcation of the system, as long as parameters are appropriate. At the same time, the nonlinear part of the control parameter can change the stability of the system and expand the system's bifurcation stability range. The accuracy of the theoretical analysis is verified by numerical simulation.

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