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# Effect of elliptical polarizations on nonsequential double ionization in two-color elliptically polarized laser fields*

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Using the classical ensemble model, we investigate the nonsequential double ionization (NSDI) of Ar and Mg in the two-color elliptically polarized laser pulse for different elliptical polarizations. Numerical results show that for Ar atoms the NSDI yield increases as the ellipticity increases, which is different from the case of Mg atoms. Moreover, the correlated behavior in the correlated electron momentum along the *x* direction and ion momentum distributions of Ar atoms are influenced by the ellipticity. By statistical analysis of different times, we can conclude that the ellipticity may be responsible for the NSDI processes. The correlated momenta distributions along the *x* direction at the recollision time are demonstrated and the results show that the travelling time and ellipticity can affect the emitted directions of both electrons.

**Keywords:** nonsequential double ionization, two-color elliptically polarized laser pulse, ellipticity, momentum distributions

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## 1. Introduction

When atoms or molecules are exposed in a strong laser field, numerous physical phenomena might happen,[1,2] such as above-threshold ionization (ATI),[3–5] nonsequential double ionization (NSDI),[6–8] and high-order harmonic generation (HHG).[9] With the development of theoretical studies and experimental technology, a large amount of research has been devoted to looking in greater details at these physical phenomena, such as recollision-induced excitation with subsequent ionization (RESI),[10,11] recollision-induced ionization (RII)[12,13], the multiple recollision of NSDI in the below recollision threshold regime,[14] and the analysis of classical trajectories in the multiple-returning-recollision of NSDI processes.[15–17]

NSDI is an important process in intense laser–atom interactions. In this process, an electron is ionized via over-the-barrier or tunneling ionization, and is driven back to the ion core when the electric field changes its direction, and then another one is ionized by the recollision. This process often occurs in linearly polarized (LP) laser pulses, but for circularly polarized (CP) laser pulses it is prominently suppressed.[18,19] However, Gillen *et al.*[20] demonstrated experimentally a "knee" structure of Mg atoms in CP laser fields. In the past two decades, some works have been devoted to the study of the mechanism of NSDI in elliptically polarized (EP) or CP laser fields.[21–28] It has been illustrated that the recollision is responsible for the NSDI in EP or CP laser fields and the returning electron comes back to the ion core with "elliptical trajectories".

With the advancement of laser technology, a variety of pulses are achievable, such as near-single-cycle laser pulses, EP laser pulses, CP laser pulses, parallel-polarized two-color laser pulses, and two-color circularly polarized (TCCP) laser pulses. Early on, theoretical efforts were devoted to using bichromatic CP pulses to drive HHG.[29] Furthermore, Eichmann *et al.*[30] have realized it in experiment. Subsequently, this pulse has been used to investigate the double ionization of atoms and molecules, to drive high-order above-threshold ionization (HATI),[31,32] and to go into molecular photoelectron momentum distribution.[33] Up to now, it has been reported experimentally that the NSDI events in counter-rotating TCCP laser pulse has been realized and the theoretical analysis of the NSDI events characteristic has been illustrated.[34–38] Furthermore, HHG has been theoretically and experimentally illustrated by a bichromatic elliptically polarized laser field.[39,40] In addition, there are very many variable parameters in the counter-rotating TCCP laser fields, such as elliptical polarization, relative intensity, and phase. Until now, we note that the process of NSDI with two-color elliptically polarized laser pulses for different elliptical polarizations have not been paid much attention.

In this paper, using the three-dimensional classical ensemble model, we investigate NSDI of Ar and Mg atoms in the two-color elliptically polarized laser pulses for different elliptical polarizations. The double ionization yields of Ar and Mg as a function of laser intensity with different ellipticity is

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demonstrated. It is shown that for Ar atoms the NSDI yield increases as the ellipticity increases, which is different from the case of Mg atoms. We also analyze the influence of the ellipticity on correlated behaviors in the correlated electron momentum along the $x$ direction and ion momentum distributions of Ar atoms and Mg atoms, respectively. In addition, we perform the statistical analysis of typical time. It indicates that the ellipticity is responsible for the typical time of the NSDI processes, such as the time of the first ionized and recollision time. The correlated momentum distributions along the $x$ direction at the recollision time are demonstrated and the results show that the travelling time and the ellipticity can affect the emitted directions of both electrons.

## 2. Theoretical model

To solve the time-dependent Schrödinger equation for multi-electron systems in strong laser fields, a huge computational requirement is demanded. The classical ensemble method is a reliable and effective tool to investigate the laser field interaction with atoms,[41,42] and the underlying recollision processes and mechanisms in NSDI can be presented intuitively by tracing the classical trajectories. Here, we use the classical ensemble method to investigate the ionization dynamics of Ar and Mg atoms in the two-color elliptically polarized laser pulses for different elliptical polarizations. In this model, the evolution of the two-electron system is followed by Newton's equation of motion (atomic units are used throughout unless stated otherwise):

$$\frac{\mathrm{d}^2\boldsymbol{r}_i}{\mathrm{d}t^2} = -\nabla[V_{\mathrm{ne}}(\boldsymbol{r}_i) + V_{\mathrm{ee}}(\boldsymbol{r}_1,\boldsymbol{r}_2)] - \boldsymbol{E}(t), \tag{1}$$

where the subscript $i$ is the tag of two electrons, $\boldsymbol{r}_i$ is the position of the $i$th electron, and the $\boldsymbol{E}(t) = \boldsymbol{E}_{\mathrm{r}}(t) + \boldsymbol{E}_{\mathrm{b}}(t)$ is the two-color elliptically polarized laser field, in which the $\boldsymbol{E}_{\mathrm{r}}(t)$ is the fundamental (red) laser pulse and the $\boldsymbol{E}_{\mathrm{b}}(t)$ is the second harmonic (blue) laser pulse. The laser pulses are written as

$$\begin{aligned}\boldsymbol{E}_{\mathrm{r}}(t) = {} & \frac{E_0}{1+\gamma_{\mathrm{E}}} f(t) \left[ \frac{1}{\sqrt{1+{\varepsilon_1}^2}} \cos(\omega_{\mathrm{r}} t + \varphi_0)\hat{\boldsymbol{x}} \right. \\ & \left. + \frac{\varepsilon_1}{\sqrt{1+{\varepsilon_1}^2}} \sin(\omega_{\mathrm{r}} t + \varphi_0)\hat{\boldsymbol{y}} \right], \end{aligned} \tag{2}$$

$$\begin{aligned}\boldsymbol{E}_{\mathrm{b}}(t) = {} & \frac{\gamma_{\mathrm{E}} E_0}{1+\gamma_{\mathrm{E}}} f(t) \left[ \frac{1}{\sqrt{1+{\varepsilon_2}^2}} \cos(\omega_{\mathrm{b}} t + 2\varphi_0)\hat{\boldsymbol{x}} \right. \\ & \left. - \frac{\varepsilon_2}{\sqrt{1+{\varepsilon_2}^2}} \sin(\omega_{\mathrm{b}} t + 2\varphi_0)\hat{\boldsymbol{y}} \right], \end{aligned} \tag{3}$$

with the maximum combined electric field amplitude $E_0$, the electric field amplitude ratio $\gamma_{\mathrm{E}} = 1.4$[38] between the second harmonic laser pulse and the fundamental laser pulse, the fundamental frequency $\omega_{\mathrm{r}} = 0.0576$ a.u. (790 nm in wavelength), and the second harmonic frequency $\omega_{\mathrm{b}} = 0.115$ a.u. (395 nm in wavelength). $f(t)$ is a trapezoidal pulse with a two-cycle turn on, six cycles at full strength, and two-cycle turn off, and $\varphi_0$, $\varepsilon_1$, $\varepsilon_2$ are the random carrier-envelope phase and the degree of ellipticity. We just investigate the effect of the ellipticity ($\varepsilon_2$) on nonsequential double ionization and $\varepsilon_1 = 1.0$ is used throughout until stated otherwise. The potential of nuclear–electron and electron–electron interaction are given by

$$V_{\mathrm{ne}}(\boldsymbol{r}_i) = \frac{-2}{\sqrt{\boldsymbol{r}_i^2 + a^2}}, \tag{4}$$

$$V_{\mathrm{ee}}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{\sqrt{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2 + b^2}}, \tag{5}$$

with the soft core parameters $a = 1.5$ for Ar atoms, $a = 3.0$ for Mg atoms to avoid the autoionization,[17,27] and $b = 0.05$ to remove the numerical singularity.

In our calculation, the initial condition for Eq. (1) is that evolution ensemble starts to form a classically allowed position of the energy. For the Ar and Mg atoms, the energies are $-1.59$ a.u. and $-0.83$ a.u., respectively, which is approximately equal the sum of the first and second ionization energies. We randomly assign the available kinetic energies and the directions of the momentum vectors of two electrons, and then the system evolves in the field-free case to obtain stable momentum and position distributions within the permissible range. We define the double ionization event when the energies of both electrons are greater than zero at the end of the laser field.

## 3. Results and discussion

The top row of Fig. 1 shows the Lissajous curves of two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 0.8, 1.0$), which are the elliptically polarized laser pulses for different elliptical polarizations. The shapes of the electric field are changed from two to three laser lobes. Figures 1(e) and 1(f) depict the double ionization probabilities of Ar and Mg versus laser intensity with the electric field amplitude ratio $\gamma_{\mathrm{E}} = 1.4$ in the two-color elliptically polarized laser pulses for different ellipticities and the pure CP laser field, respectively.

For the two-color elliptically polarized laser fields, the double ionization probability curves of Ar and Mg atoms show the distinct knee structure. For the CP laser field, figure 1(e) shows that the knee structure disappears in the double ionization probability curve of Ar atoms. For Mg atoms, however, figure 1(f) shows that the knee structure exists in the CP laser field. The results are in agreement with the previous theoretical study[26] and experimental work.[20] Furthermore, for Mg atoms, the double ionization probability in two-color elliptically polarized laser pulse is higher than that in CP laser fields. For Ar atoms, the double ionization probability increases as

the ellipticity ($\varepsilon_2 = 0.0, 0.2, 0.4$) increases. When the ellipticity continues to increase, the curves get close. While for Mg atoms, the double ionization probabilities only change a little with different ellipticities. Theoretical research has been devoted to understanding NSDI with elliptical or circular polarization and the apparent species dependency.[23,25] The recollision is possible with elliptical polarization for different trajectories, the chance for which this happens depends critically on the atomic species.

In order to understand the effect of elliptical polarizations on nonsequential double ionization in two-color elliptically polarized laser pulses for different ellipticities, we show the correlated electron momentum distributions along the $x$ direction and the ion momentum distributions of Ar at the end of the laser pulse. The laser intensity is 0.2 PW/cm$^2$. In the top row of Fig. 2, the correlated electron momentum changes as the ellipticity increases. For instance, for the ellipticity $\varepsilon_2 = 0$, the population is mainly distributed in the first quadrant, which indicates that both electrons are most likely to eject to the $+x$ direction and the correlated behavior is correlated. While for the ellipticity ($\varepsilon_2$) from EP to the CP, the population is also distributed in the first quadrant, but much of the population moves to the second and fourth quadrants, showing an obviously anticorrelated behavior.

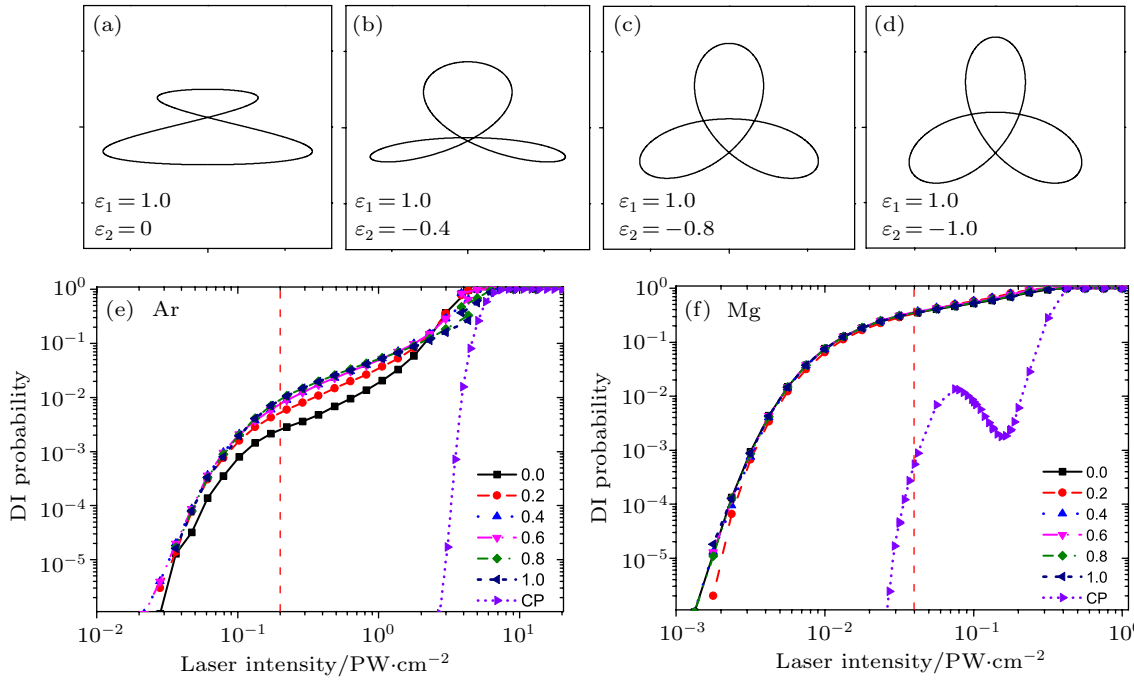



**Fig. 1.** (color online) (a)–(d) Lissajour curves of the two-color elliptically polarized laser pulse for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 0.8, 1.0$). The double ionization (DI) probability of (e) Ar and (f) Mg as a function of laser intensity with $\gamma_E = 1.4$ in the two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$) and the pure CP laser field.

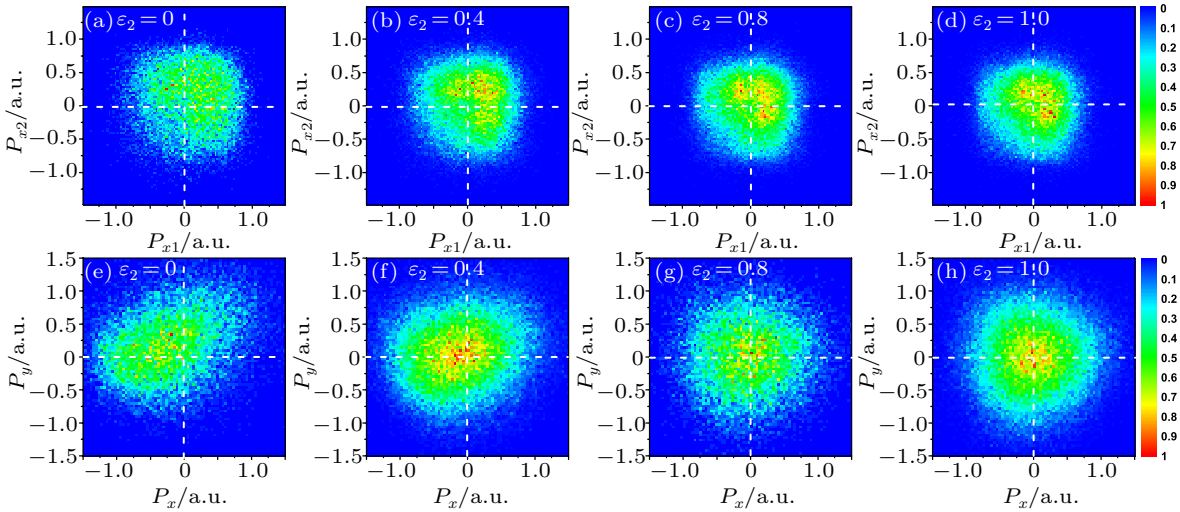



**Fig. 2.** (color online) (a)–(d) The correlated electron momentum distributions along the $x$ direction and (e)–(h) the ion momentum distributions of Ar in two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 0.8, 1.0$). The laser intensity is 0.2 PW/cm$^2$, the electric field amplitude ratio $\gamma_E = 1.4$, the fundamental frequency $\omega_f = 0.0576$ a.u. (790 nm in wavelength), and the second harmonic frequency $\omega_b = 0.115$ a.u. (395 nm in wavelength).

The bottom row of Fig. 2 shows the ion momentum distribution under four different ellipticities. The net momentum of the ion is smaller if the two electrons are emitted in opposite hemispheres than that if the two electrons are emitted in the same hemisphere.[17,43] The population gradually moves to the origin region as the ellipticity increases, which shows a clearly anticorrelated behavior. In addition, the ion momentum distribution presents triangle structure symmetrically (as shown in Fig. 2(h)) due to the shape of the electric fields of the counter-rotating TCCP laser fields, which has been presented in the theoretical study and the experimental observations.[37,38]

Figure 3 shows that the correlated electron momentum distributions along the $x$ direction and the ion momentum distributions of Mg atoms do not change obviously as the ellipticity increases, which is different from Ar atoms as shown in Fig. 2. Furthermore, much of the population moves to the second and fourth quadrants as indicated in the top of Fig. 3, which shows a significant anticorrelated behavior. The origin region of the ion momentum distribution is more populated as indicated in the bottom of Fig. 3, which also shows a significant anticorrelated behavior. Thus the effect of ellipticities on the correlated behaviors of Mg in the correlated electron momentum and the ion momentum distributions is not obvious.

In order to understand the dynamics of the NSDI process more clearly, we show the statistical analysis of the typical time of Ar atoms (upper row) and Mg atoms (bottom row) in Fig. 4 by tracing the NSDI classical trajectories, respectively. The typical time is the single ionization time $t_s$, the recollision time $t_r$, the travelling time $t_{tr}$, and the delay time $t_d$. The single ionization time ($t_s$) is defined as the instant that one electron achieves positive energy before the recollision.[44] The recollision time ($t_r$) is defined as the instant when the distance

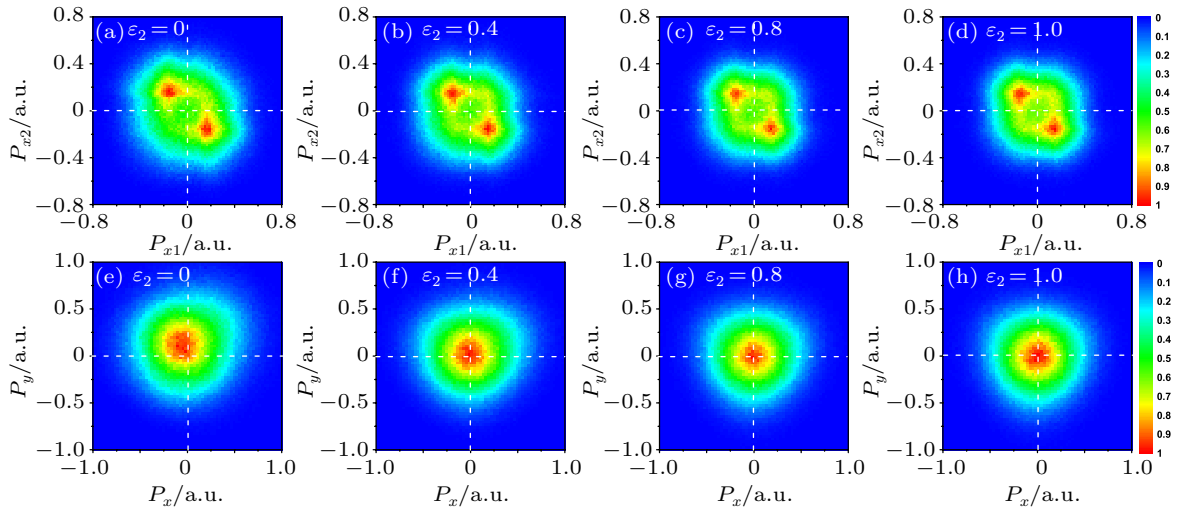



**Fig. 3.** (color online) (a)–(d) The correlated electron momentum distributions along the $x$ direction and (e)–(h) the ion momentum distributions of Mg in the two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 0.8, 1.0$). The laser parameters are the same as those in Fig. 2 except for the laser intensity (0.04 PW/cm$^2$).

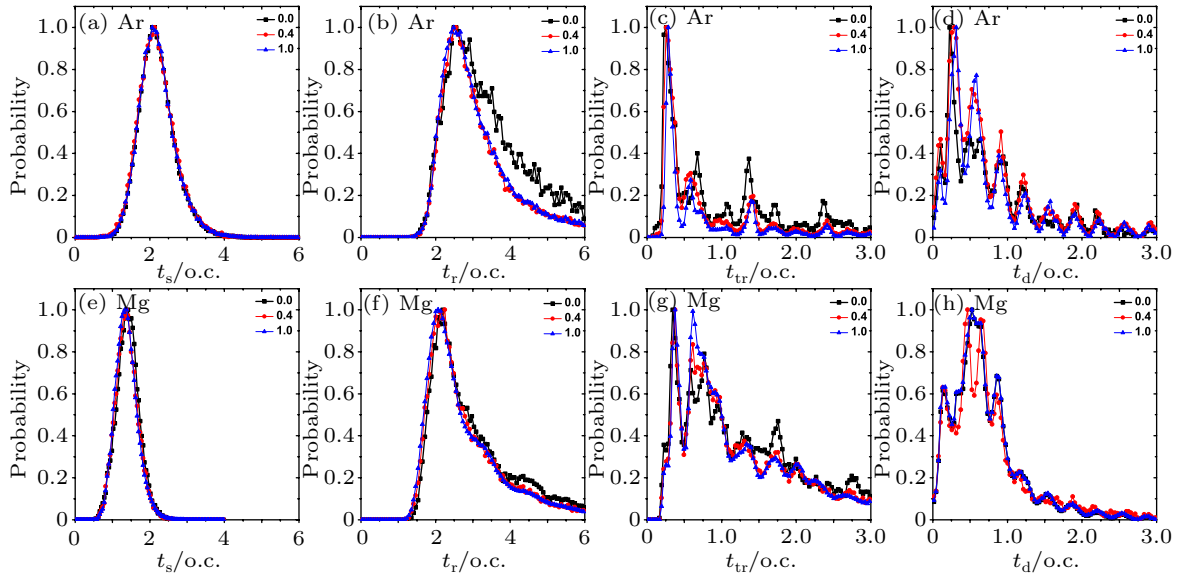



**Fig. 4.** (color online) Statistical distribution of the single ionization time $t_s$, the recollision time $t_r$, travelling time $t_{tr}$, and delay time $t_d$ for Ar atoms (upper row) and Mg atoms (bottom row) with three different ellipticities.

between the two electrons is less than 2.0 a.u. The travelling time ($t_{tr}$) is defined as the interval between the single ionization time and the recollision time. The delay time ($t_d$) is defined as the interval between the recollision time and the double ionization time (the instant when the two electrons both achieve positive energy).

In Figs. 4(a) and 4(e), we show the single ionization time of Ar and Mg atoms with different ellipticities, respectively. The peaks locate at 2 o.c. (optical cycles) and 1.4 o.c., respectively. The first ionization electron is ejected at almost the same time as the ellipticities increase, while for Mg atoms the time changes slightly.

In Figs. 4(b) and 4(f), we show the recollision time of Ar and Mg atoms with different ellipticities, respectively. For Ar atoms, the peaks of recollision time locates around 2.5 o.c. The probabilities of recollision occurring between 3 o.c. and 6 o.c. with the ellipticities $\varepsilon_2 = 0.0$ are higher than the probabilities of the ellipticities $\varepsilon_2 = 0.4, 1.0$ at the same time period, as shown in Fig. 4(b). For Mg atoms, the probabilities of recollision occurring are close with three different ellipticities, as shown in Fig. 4(f).

In Figs. 4(c) and 4(g), we show the travelling time of Ar and Mg atoms with different ellipticities, respectively. For Ar atoms, the main peaks are located around 0.25 o.c. This indicates that the first electron comes back to the parent ion core in a short time ($t_{tr} \leq 0.25$ o.c.) with the "short trajectory" and the typical trajectory was shown in the top row of Fig. 5. The probabilities of travelling time between 0.5 o.c. and 3.0 o.c. with the ellipticities $\varepsilon_2 = 0.0$ are higher than that with the ellipticities $\varepsilon_2 = 0.4, 1.0$. Furthermore, the second peak of travelling time in the ellipticities $\varepsilon_2 = 0.0$ laser field right shifts comparing with that in the ellipticities $\varepsilon_2 = 0.4, 1.0$ laser fields, as shown in Fig. 4(c). This indicates that the recollision process for the ellipticities $\varepsilon_2 = 0.0$ occurs little more slowly than that for the ellipticities $\varepsilon_2 = 0.4, 1.0$ in two-color elliptically polarized laser fields (0.5 o.c.$\leq t_{tr} \leq$ 1.0 o.c.).

However, for Mg atoms, there are two main peaks located around 0.25 o.c. and 0.75 o.c. in Fig. 4(g). The probabilities of the second peak increase as the ellipticity $\varepsilon_2$ increases. The second peaks of Mg atoms are higher than that of Ar, which indicates that more first ionized electron of Mg atoms needs more travelling time to come back to the parent ion core with the "long trajectory" ($t_{tr} > 0.5$ o.c.) than that of Ar atoms, and the typical trajectory was shown in the bottom row of Fig. 5.

In Figs. 4(d) and 4(h), we show the delay time of Ar atoms and Mg atoms with different ellipticities, respectively. For Ar atoms, the main peak locates around 0.25 o.c., as shown in Fig. 4(d), which indicates that RII is the main mechanism. The third peaks locate around 0.5 o.c. Moreover, the probabilities increase as the ellipticity $\varepsilon_2$ increases. It indicates that more RESI occurs as the ellipticity $\varepsilon_2$ increases. In the top row of Fig. 2, the electron momentum mainly distributes in the first quadrant and more electrons are distributed in the second and fourth quadrants as the ellipticity $\varepsilon_2$ increases. This behavior is in accordance with that illustrated in Fig. 4(d), and it is similar to when two electrons are likely to be emitted into opposite directions when the delay time increases, which is illustrated in Ref. [42].

For Mg atoms, the main peak is located around 0.5 o.c., and the first peak (around 0.25 o.c.) is lower than the main peak, as shown in Fig. 4(h). This behavior indicates that mainly the RESI mechanism occurs. This is in accordance with that the electron momentum is mainly distributed in the second and four quadrants in the top row of Fig. 3. Moreover, the probabilities of delay time with three different ellipticities are close. In all of the above, these statistical analysis of the typical time of Ar atoms and Mg atoms well exhibit that ellipticity is responsible for the NSDI processes.

The occurring of recollision processes is dependent on the species of atoms[26,45,46] and the recollision mechanism is responsible for the characteristics of NSDI with elliptical or circular polarization. To understand NSDI of the two-color elliptically polarized laser pulses for different ellipticities $\varepsilon_2 = 0.0$, $\varepsilon_2 = 0.4$, and $\varepsilon_2 = 1.0$, we show the typical trajectories of Ar atoms in Fig. 5. The top row is the "short trajectory" ($t_{tr} \leq 0.5$ o.c.) and the bottom row is the "long trajectory" ($t_{tr} > 0.5$ o.c.). As shown in the top row of Fig. 5 ($t_{tr} \leq 0.5$ o.c.), the first-released electron is pulled back around the ion core with an oblate trajectory only once even if the ellipticity $\varepsilon_2$ increases. The shape of the trajectories is not affected by the ellipticity $\varepsilon_2$ for the "short trajectory". This trajectory is the same as the typical recollision "short trajectory" with LP laser field illustrated in Ref. [25].

In the bottom row of Fig. 5 ($t_{tr} > 0.5$ o.c.), the first-released electron is not "lucky enough" to collide with the other one but it is pulled around the ion core more times with only one recollision. The shape of the trajectories becomes more and more triangular as the ellipticity $\varepsilon_2$ increases due to the counter-rotating TCCP laser fields of the specific dynamical symmetries of the total net electric field, and the same triangle trajectory is shown in Ref. [34]. This exhibits that the ellipticity $\varepsilon_2$ is responsible for the shape of the trajectories when the travelling time is larger than 0.5 o.c., but for the travelling time less than 0.5 o.c. this is not the case.

Figure 6 shows the correlated electron momentum distributions along the $x$ direction of Ar atoms at the recollision time in the two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 1.0$). Even such a small variation of the ellipticities could result in changes in the electron momentum distributions, as shown in Fig. 6. The top row of Fig. 6 shows the electron momentum distributions of the travelling time less than 0.5 o.c. ("short trajectory") at the recollision

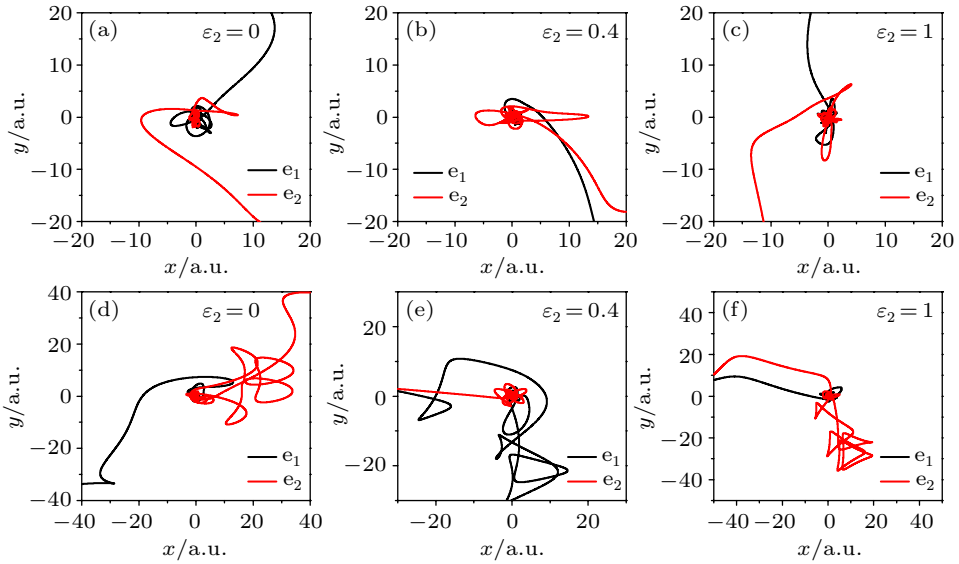



**Fig. 5.** (color online) The typical NSDI trajectory of Ar atoms in the two-color elliptically polarized laser pulses for different ellipticities: (a) and (d) $\varepsilon_2 = 0.0$, (b) and (e) $\varepsilon_2 = 0.4$, (c) and (f) $\varepsilon_2 = 1.0$, at the combined laser intensity of 0.2 PW/cm$^2$. The top row shows trajectories of first ionized electron coming back to the parent ion core with the "short trajectory" ($t_{\rm tr} \leq 0.5$ o.c.). The bottom row shows trajectories of the first ionized electron coming back to the parent ion core with the "long trajectory" ($t_{\rm tr} > 0.5$ o.c.).

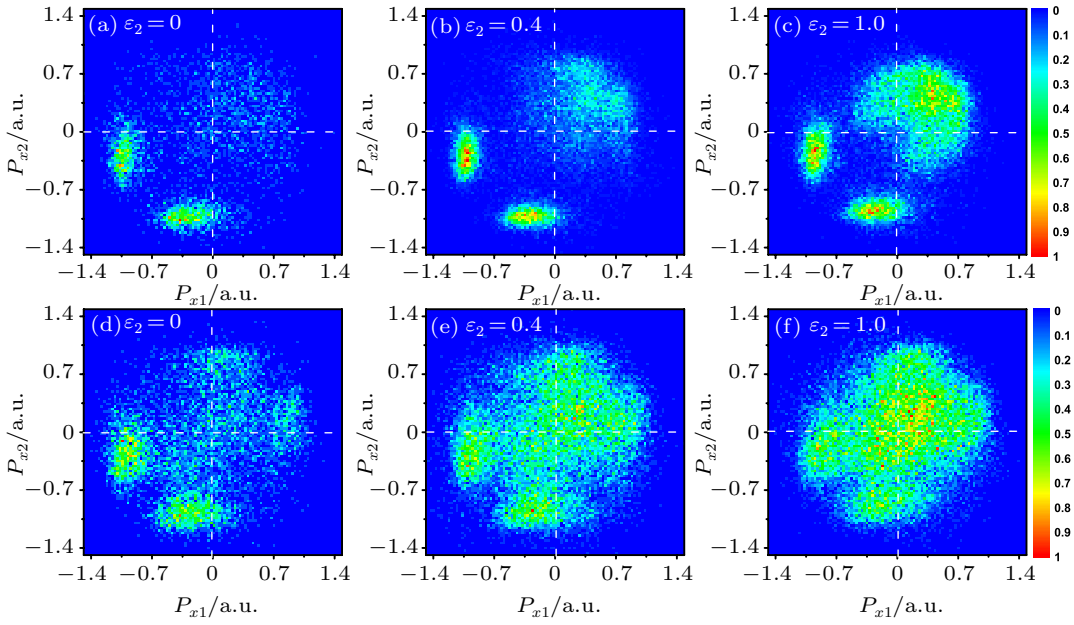



**Fig. 6.** (color online) The correlated electron momentum distributions along the $x$ direction for Ar atoms at the recollision time in the two-color elliptically polarized laser pulses for different ellipticities ($\varepsilon_2 = 0.0, 0.4, 1.0$). The laser parameters are the same as those in Fig. 2. The distributions correspond to the events with travelling time less than 0.5 o.c. (top row) and larger than 0.5 o.c. (bottom row).

time. The population is mainly distributed in the third quadrant for three different ellipticities ($\varepsilon_2$). While for the ellipticity ($\varepsilon_2$) from EP to the CP, much of the population moves to the first, second, and fourth quadrants, as shown in the top of Fig. 6. It indicates that both electrons are not only emitted in the same direction (the $+x$ or $-x$ direction) but also emitted into opposite directions as the ellipticity ($\varepsilon_2$) increases. The bottom row of Fig. 6 shows the electron momentum distribution of the travelling time larger than 0.5 o.c. ("long trajectory") at the recollision time. More of the population moves to the second and fourth quadrants than that of the short trajectories, which means that both electrons are more likely to be emitted into opposite directions with long trajectories. In addition, more of the population moves to the second and fourth quadrants as the ellipticity ($\varepsilon_2$) increases. This well demonstrates that both elliptical polarization and travelling time are responsible for the formation of the electron momentum distribution at the recollision time, meaning that both of them affect

the emitted directions of both electrons.

## 4. Conclusion

We investigate the effect of elliptical polarizations on nonsequential double ionization in the two-color elliptically polarized laser pulses for different ellipticities. The probabilities of the double ionization versus laser intensity show the clear knee structure in these laser fields. The probabilities of Ar atoms increase as the ellipticity increases, which is different from the case of Mg atoms. Our calculations show the correlated electron momentum distributions along the $x$ direction and the ion momentum distributions of Ar atoms and Mg atoms. For Ar atoms, the anticorrelated behavior becomes more and more obvious as the ellipticity increases, but this is not the case for Mg atoms. In addition, we find that travelling time and the ellipticity are responsible for the NSDI processes. The correlated momentums along the $x$ direction distributions at the recollision time are demonstrated and the results show that the travelling time and ellipticity can affect the emitted directions of both electrons.

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