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Deterministic hierarchical joint remote state preparation with six-particle partially entangled state*

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In this paper, we present a novel scheme for hierarchical joint remote state preparation (HJRSP) in a deterministic manner, where two senders can jointly and remotely prepare an arbitrary single-qubit at three receivers' port. A six-particle partially entangled state is pre-shared as the quantum channel. There is a hierarchy among the receivers concerning their powers to reconstruct the target state. Due to various unitary operations and projective measurements, the unit success probability can always be achieved irrespective of the parameters of the pre-shared partially entangled state.

Keywords: joint remote state preparation, hierarchical quantum communication, partially entangled state

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1. Introduction

Over the past few decades, the application of quantum mechanics in the field of computer science and information theory has motivated an emerging research area, quantum computation, and quantum information.[1] Encoding information in quantum states provides a totally new method for information processing beyond the capabilities of its classical counterparts. Transferring a quantum state faithfully and securely between remote communicating peers is of the greatest significance. One of the most remarkable protocols for state transmission is quantum teleportation, put forward by Bennett *et al.* in 1993,[2] allowing the teleportation of an unknown quantum state via an Einstein–Podolsky–Rosen (EPR) pair as the quantum channel with the aid of Bell measurement and two bits of classical communication. Later, a new protocol referred to as remote state preparation (RSP)[3–5] was introduced, in which a known state could be remotely prepared using the same quantum channel as in quantum teleportation but with lower classical communication cost. Afterwards, a special kind of RSP scheme called joint RSP (JRSP)[6,7] was proposed to deal with the situation that the classical information of a target state was shared by two or more senders located in separated places. Some researchers investigated deterministic RSP in noisy environments.[8,9] Up to now, almost all of the studies on multi-party RSP and JRSP have been restricted to symmetric schemes where all receivers have equal power to reconstruct the target state sent by the sender or senders.[10–24] Recently, some hierarchical quantum communication schemes have been proposed, for example, hierarchical quantum information splitting (HQIS),[25–27] hierarchical quantum secret sharing (HQSS),[28] and hierarchical dynamic quantum secret sharing (HDQSS).[29]

In many practical scenarios, joint decision and hierarchial quantum communication are important. We may consider that there exists a code to unlock the bank vault. Because of the security effect it may cause, this particular code cannot be given to a single authorized person. The information (code) is distributed among two authorized persons, so that none of the authorized persons can misuse the code. Now consider that Alice and Amy (two authorized persons) are the president and vice president of a bank. Bob is the bank executive, and Charlie and David are bank managers. If and only if the bank's president and vice president together wish to unlock the bank vault, then they jointly distribute the information (the code which is required to unlock the bank vault) among the bank executive and managers. The bank executive can unlock the bank vault independently. However, if a manager wants to unlock the bank vault, the assistance of the executive and the other manager is required. Thus, the senders, i.e., the bank's president and vice president, cannot issue an individual order that allows the receivers to unlock the bank vault, and the bank executive is more powerful than managers.

Motivated by the practical applications of the hierarchial quantum communication, Shukla *et al.* proposed the first scheme of HJRSP based on a 5-qubit cluster state,[30] where the receivers were graded in accordance to their power to reconstruct the quantum state sent by the senders. In real-world implementations, noisy environments and the decoher-

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ence effect take maximally entangled states to partially entangled states or mix states. Shukla *et al.* investigated HJRSP based on a non-maximally entangled cluster state.[30] However, the success probability is less than 100%. In order to improve the success probability of HJRSP with non-maximally entangled states, we present a novel scheme for deterministic HJRSP using a six-particle partially entangled state as the quantum channel. The senders (and the cooperators if necessary) carry out proper unitary operations and projective measurements. According to these measurement results, the receiver can reestablish the target state with an appropriate unitary operation. Unit success probability can always be achieved irrespective of the parameters of the pre-shared partially entangled state.

The rest of this paper is organized as follows. In Section 2, a particular six-particle partially entangled state is introduced. In Section 3, a novel scheme for deterministic HJRSP is proposed based on the six-particle partially entangled state. In Section 4, performance analysis is presented. Finally, some conclusions are drawn in Section 5.

2. Six-particle partially entangled state

Inspired by some ideas in Ref. [31], the following six-particle partially entangled state is utilized throughout this paper

$$|QC\rangle_{123456} = \frac{1}{\sqrt{2}}(|000000\rangle + \cos\gamma|110111\rangle + \sin\gamma|111111\rangle)_{123456}. \quad (1)$$

This state is characterized by parameter γ. It is apparent from Eq. (1) that when $\gamma = 0$, $|QC\rangle_{123456} = \frac{1}{\sqrt{2}}(|00000\rangle + |11111\rangle)_{12456} \otimes |0\rangle_3$, and this is a separate state with respect to the particles 1, 2, 4, 5, 6, and 3. While for $\gamma = \pi/2$, $|QC\rangle_{123456} = \frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)_{123456}$, and this is a maximally entangled state. In this paper, we are interested in $0 < \gamma < \pi/2$ for which the state $|QC\rangle_{123456}$ is partially entangled with the entanglement degree $C_{1,2,4,5,6|3} = |\sin\gamma|$ quantified by the concurrence.[29]

To obtain $|QC\rangle_{123456}$, a quantum circuit is constructed using a Hadamard gate, control-NOT (CNOT) gates, and rotation gates, as shown in Fig. 1. H denotes the Hadamard operation[1]

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (2)$$

With the dot "•" denoting the control particle, the CNOT gate takes the form of the following 4×4 matrix[1]

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (3)$$

Moreover, the rotation gate $R_y(\gamma)$ can be expressed as[1]

$$R_y(\gamma) = \begin{pmatrix} \cos(\gamma/2) & -\sin(\gamma/2) \\ \sin(\gamma/2) & \cos(\gamma/2) \end{pmatrix}, \quad (4)$$

where γ indicates the tangle of rotation around the y-axis.

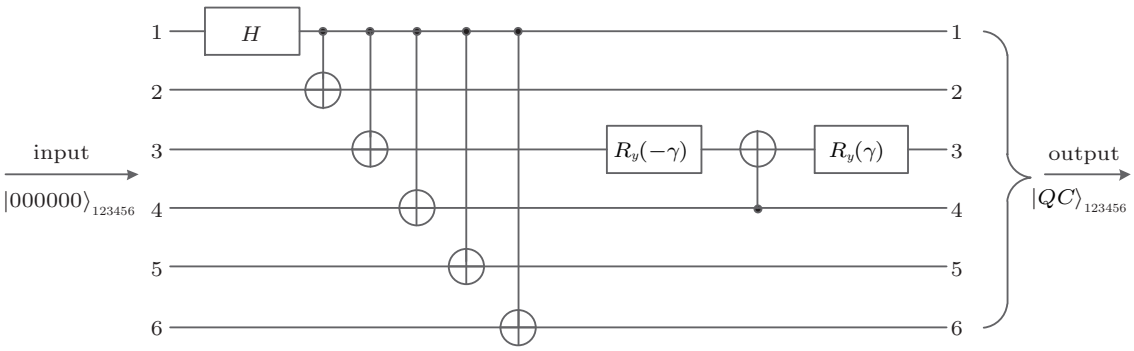


Fig. 1. Quantum circuit to obtain $|QC\rangle_{123456}$.

The input of the circuit is a separate state $|000000\rangle_{123456}$, due to the Hadamard operation and the following five CNOTs, a six-particle GHZ state is obtained

$$|\text{GHZ}\rangle_{\text{six}} = \frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)_{123456}. \quad (5)$$

Then $R_y(-\gamma)$ is performed on particle 3, reads

$$\begin{aligned} |\text{GHZ}\rangle_{\text{six}} &= \frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)_{123456} \xrightarrow[\text{on particle 3}]{R_y(-\gamma)} \\ &\frac{1}{\sqrt{2}}\left\{|00\rangle_{12}\left(\cos\frac{\gamma}{2}|0\rangle - \sin\frac{\gamma}{2}|1\rangle\right)_3|000\rangle_{456}\right. \\ &\left. + |11\rangle_{12}\left(\sin\frac{\gamma}{2}|0\rangle + \cos\frac{\gamma}{2}|1\rangle\right)_3|111\rangle_{456}\right\} \\ &= \frac{1}{\sqrt{2}}\left(\cos\frac{\gamma}{2}|000000\rangle - \sin\frac{\gamma}{2}|001000\rangle\right. \end{aligned}$$

$$+\sin\frac{\gamma}{2}|110111\rangle+\cos\frac{\gamma}{2}|111111\rangle\Big)_{123456}. \quad (6)$$

After the CNOT operation performed on particles 4 and 3, the joint state of six particles evolves to

$$\frac{1}{\sqrt{2}}\Big(\cos\frac{\gamma}{2}|000000\rangle-\sin\frac{\gamma}{2}|001000\rangle+\sin\frac{\gamma}{2}|111111\rangle + \cos\frac{\gamma}{2}|110111\rangle\Big)_{123456}. \quad (7)$$

Subsequently, $R_y(\gamma)$ is performed on particle 3, given by

$$\begin{aligned}&\frac{1}{\sqrt{2}}\Big(\cos\frac{\gamma}{2}|000000\rangle-\sin\frac{\gamma}{2}|001000\rangle+\sin\frac{\gamma}{2}|111111\rangle + \cos\frac{\gamma}{2}|110111\rangle\Big)_{123456}\xrightarrow[\text{on particle 3}]{R_y(\gamma)}\\ &\frac{1}{\sqrt{2}}\Big\{\cos\frac{\gamma}{2}|00\rangle_{12}\Big(\cos\frac{\gamma}{2}|0\rangle+\sin\frac{\gamma}{2}|1\rangle\Big)_3|000\rangle_{456}\\ &-\sin\frac{\gamma}{2}|00\rangle_{12}\Big(-\sin\frac{\gamma}{2}|0\rangle+\cos\frac{\gamma}{2}|1\rangle\Big)_3|000\rangle_{456}\\ &+\sin\frac{\gamma}{2}|11\rangle_{12}\Big(-\sin\frac{\gamma}{2}|0\rangle+\cos\frac{\gamma}{2}|1\rangle\Big)_3|111\rangle_{456}\\ &+\cos\frac{\gamma}{2}|11\rangle_{12}\Big(\cos\frac{\gamma}{2}|0\rangle+\sin\frac{\gamma}{2}|1\rangle\Big)_3|111\rangle_{456}\Big\}\\ =&\frac{1}{\sqrt{2}}(|000000\rangle+\cos\gamma|110111\rangle+\sin\gamma|111111\rangle)_{123456}.\end{aligned} \quad (8)$$

The six-particle partially entangled state $|QC\rangle_{123456}$ is obtained.

3. Deterministic hierarchical joint remote state preparation

In this section, we detail our scheme for deterministic HJRSP based on the six-particle partially entangled state introduced in Section 2. Suppose that two independent senders, Alice and Amy intend to jointly prepare an arbitrary single-qubit (generally referred to as the target state) at the end of receivers, given by

$$|\xi\rangle=a_0|0\rangle+a_1\mathrm{e}^{\mathrm{i}\theta}|1\rangle, \quad (9)$$

where real coefficients $a_0, a_1\geq 0$, and $\theta\in[0,2\pi]$ with the normalization condition $a_0^2+a_1^2=1$. Consistent with some JRSP schemes,[16,21,22] independent of whether the scheme is hierarchical or not, the information characterizing the target state is shared among the senders. Generally, the amplitude information $\{a_0,a_1\}$ is determined by Alice, and the phase parameter θ is chosen by Amy. Further, at least three receivers (generally referred to as Bob, Charlie, and David) are required in a hierarchical scheme for joint remote state preparation. Particularly, Bob is assumed to be the higher-power receiver, who can reconstruct the target state without the help of other receivers. However, the lower-power receiver Charlie (David) can construct the target state if and only if both Bob and David (Charlie) help him.

To accomplish the task of HJRSP, two senders, Alice and Amy previously share the following quantum channel with three receivers, Bob, Charlie, and David.

$$|QC\rangle_{A_1A_2B_1B_2CD}=\frac{1}{\sqrt{2}}\Big(|000000\rangle+\cos\gamma|110111\rangle+\sin\gamma|111111\rangle\Big)_{A_1A_2B_1B_2CD}. \quad (10)$$

Subscripts indicate that particle A_1 belongs to Alice, A_2 belongs to Amy, the higher-power receiver Bob is in possession of particles B_1 and B_2, the lower-power receivers Charlie and David own particles C and D, respectively. Concrete implementation steps are detailed as follows.

Step 1. Alice encodes amplitude information.

Alice encodes the amplitude information of the target state $|\xi\rangle$ on the pre-shared quantum state $|QC\rangle_{A_1A_2B_1B_2CD}$ by performing a projective measurement (denoted as PM_A^1) on her particle with the following basis

$$\begin{aligned}|\psi_1\rangle&=a_0|0\rangle+a_1|1\rangle,\\ |\psi_2\rangle&=a_0|1\rangle-a_1|0\rangle.\end{aligned} \quad (11)$$

Based on the measurement postulate of quantum mechanics, the pre-shared quantum channel state $|QC\rangle_{A_1A_2B_1B_2CD}$ can be rewritten in terms of Alice's measurement basis as

$$|QC\rangle_{A_1A_2B_1B_2CD}=\frac{1}{\sqrt{2}}\Big(|\psi_1\rangle_{A_1}\otimes|\Psi_1\rangle_{A_2B_1B_2CD}-|\psi_2\rangle_{A_1}\otimes|\Psi_2\rangle_{A_2B_1B_2CD}\Big), \quad (12)$$

where

$$\begin{aligned}|\Psi_1\rangle_{A_2B_1B_2CD}&=\Big(a_0|00000\rangle+a_1\cos\gamma|10111\rangle+a_1\sin\gamma|11111\rangle\Big)_{A_2B_1B_2CD},\\ |\Psi_2\rangle_{A_2B_1B_2CD}&=\Big(a_1|00000\rangle-a_0\cos\gamma|10111\rangle-a_0\sin\gamma|11111\rangle\Big)_{A_2B_1B_2CD}.\end{aligned} \quad (13)$$

After the projective measurement PM_A^1, Alice informs her measurement result (denoted as R_A^1) to Amy with 1 classical bit of communication cost. It is clear from Eq. (12) that Alice's PM_A^1 projects the joint state of particles A_2,B_1,B_2,C,D onto one of the two possible states $\{|\Psi_1\rangle_{A_2B_1B_2CD},|\Psi_2\rangle_{A_2B_1B_2CD}\}$ with equal probability of $P_{R_A^1}=1/2$.

Step 2. Amy encodes phase information.

After receiving Alice's result R_A^1, Amy selects suitable unitary operation (denoted as U_A^2) to perform on the particle A_2. Relations between Alice's PM_A^1 results and Amy's unitary operations are listed out in Table 1.

Firstly, let us consider the situation that Alice's PM_A^1 result is $|\psi_1\rangle_{A_1}$. According to Table 1, Amy performs identity operation I on the particle A_2. Subsequently, she implements a projective measurement (denoted as PM_A^2) on particle A_2 under the following basis

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + \mathrm{e}^{-\mathrm{i}\theta}|1\rangle\right),$$
$$|\phi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - \mathrm{e}^{-\mathrm{i}\theta}|1\rangle\right). \quad (14)$$

Table 1. Relations between Alice's PM_A^1 result and Amy's unitary operation. Here I is the identity operator and σ_x is Pauli-X operator.

Alice's PM_A^1 result (R_A^1)	Amy's unitary operation (U_A^2)
$\|\psi_1\rangle_{A_1}$	I
$\|\psi_2\rangle_{A_1}$	σ_x

Amy's operations can be expressed analytically as follows:

$$\begin{aligned}
&|\Psi_1\rangle_{A_2B_1B_2CD}\\
=\ &(a_0|00000\rangle + a_1\cos\gamma|10111\rangle\\
&+ a_1\sin\gamma|11111\rangle)_{A_2B_1B_2CD} \xrightarrow[\text{on particle } A_2]{U_A^2=I}\\
&(a_0|00000\rangle + a_1\cos\gamma|10111\rangle + a_1\sin\gamma|11111\rangle)_{A_2B_1B_2CD}\\
=\ &\frac{1}{\sqrt{2}}\{|\phi_1\rangle_{A_2}\otimes|\Phi_1\rangle_{B_1B_2CD} + |\phi_2\rangle_{A_2}\otimes|\Phi_2\rangle_{B_1B_2CD}\}, \quad (15)
\end{aligned}$$

where

$$\begin{aligned}
|\Phi_1\rangle_{B_1B_2CD} &= (a_0|0000\rangle + a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0111\rangle\\
&\quad + a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1111\rangle)_{B_1B_2CD},\\
|\Phi_2\rangle_{B_1B_2CD} &= (a_0|0000\rangle - a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0111\rangle\\
&\quad - a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1111\rangle)_{B_1B_2CD}. \quad (16)
\end{aligned}$$

After the projective measurement PM_A^2, Amy informs her measurement result (denoted as R_A^2) to Bob with 1 classical bit of communication cost. It is transparent from Eq. (15) that after the projective measurement PM_A^2, Amy will obtain one of the two possible states $\{|\phi_1\rangle_{A_2}, |\phi_2\rangle_{A_2}\}$ with equal conditional probability $P_{R_A^2|R_A^1} = 1/2$.

Step 3. Bob performs unitary operation.

Upon receiving Amy's result (R_A^2), Bob performs suitable local unitary operation (U_B) on his particle B_2, as listed in Table 2.

Table 2. Relations between Amy's PM_A^2 result and Bob's unitary operation. Here I is the identity operator and σ_z is the Pauli-Z operator.

Amy's PM_A^2 result (R_A^2)	Bob's unitary operation (U_B)
$\|\phi_1\rangle_{A_2}$	I
$\|\phi_2\rangle_{A_2}$	σ_z

Corresponding to Amy's PM_A^2 result of $|\phi_1\rangle_{A_2}$ ($|\phi_2\rangle_{A_2}$), Bob performs I (σ_z) on his particle B_2. After that, the joint state of particles B_1, B_2, C, and D evolves to

$$\left(a_0|0000\rangle + a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0111\rangle + a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1111\rangle\right)_{B_1B_2CD}. \quad (17)$$

Step 4. Quantum state reconstruction.

Case 1 The receivers decide that Bob will reconstruct the target state $|\xi\rangle$.

(A) Bob performs CNOT operation, with B_2 as the control particle, and C and D as the target particles. Considering the technological challenges to implement CNOT operation remotely, particles B_2, C, and D are very close. After the CNOT operation, the joint state of particles B_1, B_2, C, and D evolves to

$$\left(a_0|0000\rangle + a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0100\rangle + a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1100\rangle\right)_{B_1B_2CD}. \quad (18)$$

(B) Bob performs $R_y(-\gamma)$ on his particle B_1, followed by a projective measurement (denoted as PM_B) on the same particle with the computational basis $\{|0\rangle, |1\rangle\}$. Bob's $R_y(-\gamma)$ and PM_B operations can be expressed analytically as follows:

$$\begin{aligned}
&\left(a_0|0000\rangle + a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0100\rangle + a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1100\rangle\right)_{B_1B_2CD}\\
&\xrightarrow[PM_B]{R_y(-\gamma)} \cos\frac{\gamma}{2}|0\rangle_{B_1}\otimes\left[a_0|0\rangle + a_1\mathrm{e}^{\mathrm{i}\theta}|1\rangle\right]_{B_2}\otimes|00\rangle_{CD}\\
&- \sin\frac{\gamma}{2}|1\rangle_{B_1}\otimes\left[a_0|0\rangle - a_1\mathrm{e}^{\mathrm{i}\theta}|1\rangle\right]_{B_2}\otimes|00\rangle_{CD}. \quad (19)
\end{aligned}$$

From Eq. (19), it is apparent that after Bob's projective measurement, the state of particle B_2 is perfectly correlated to the target state $|\xi\rangle = a_0|0\rangle + a_1\mathrm{e}^{\mathrm{i}\theta}|1\rangle$.

(C) If Bob's measurement result (denoted as R_B) is $|0\rangle_{B_1}$, he performs identity operator I on particle B_2. As for the result $|1\rangle_{B_1}$, Bob performs Pauli-Z operator σ_z on particle B_2, thus the target state $|\xi\rangle$ can be retrieved.

Case 2 The receivers decide that Charlie will reconstruct the target state $|\xi\rangle$.

(a) Bob performs $R_y(-\gamma)$ on his particle B_1, followed by a projective measurement (PM) on the same particle with the computational basis $\{|0\rangle, |1\rangle\}$. The joint state of particles B_1, B_2, C, and D evolves as

$$\begin{aligned}
&\left(a_0|0000\rangle + a_1\cos\gamma\mathrm{e}^{\mathrm{i}\theta}|0111\rangle + a_1\sin\gamma\mathrm{e}^{\mathrm{i}\theta}|1111\rangle\right)_{B_1B_2CD}\\
&\xrightarrow[\text{on } B_1]{R_y(-\gamma)\&PM} \cos\frac{\gamma}{2}|0\rangle_{B_1}\otimes\left[a_0|000\rangle + a_1\mathrm{e}^{\mathrm{i}\theta}|111\rangle\right]_{B_2CD}\\
&- \sin\frac{\gamma}{2}|1\rangle_{B_1}\otimes\left[a_0|000\rangle - a_1\mathrm{e}^{\mathrm{i}\theta}|111\rangle\right]_{B_2CD}. \quad (20)
\end{aligned}$$

(b) If his measurement result is $|0\rangle_{B_1}$, Bob performs identity operator I on particle B_2, if the measurement result is $|1\rangle_{B_1}$, he performs Pauli-Z operator σ_z on particle B_2. The joint state of particles B_2, C, and D evolves to

$$\left(a_0|000\rangle + a_1\mathrm{e}^{\mathrm{i}\theta}|111\rangle\right)_{B_2CD}. \quad (21)$$

(c) Bob performs projective measurement on particle B_2, with the basis

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{22}$$

The joint state of particles B_2, C, and D is given by

$$\begin{aligned}&\left(a_0|000\rangle + a_1 e^{i\theta}|111\rangle\right)_{B_2CD}\\ =\ &\frac{1}{\sqrt{2}}|+\rangle_{B_2}\left(a_0|00\rangle + a_1 e^{i\theta}|11\rangle\right)_{CD}\\ &+\frac{1}{\sqrt{2}}|-\rangle_{B_2}\left(a_0|00\rangle - a_1 e^{i\theta}|11\rangle\right)_{CD}.\end{aligned} \tag{23}$$

It can be seen from Eq. (23) that Bob will obtain one of the two possible states $\left\{|+\rangle_{B_2}, |-\rangle_{B_2}\right\}$ with equal probability. Bob informs David his measurement result with 1 classical bit of communication cost.

(d) Upon receiving Bob's measurement result, David performs a suitable local unitary operation on his particle D. If Bob's measurement result is $|+\rangle_{B_2}$, David performs identity operator I on his particle D, if Bob's measurement result is $|-\rangle_{B_2}$, David performs Pauli-Z operator σ_z on his particle D, then the joint state of particles C and D is

$$\left(a_0|00\rangle + a_1 e^{i\theta}|11\rangle\right)_{CD}. \tag{24}$$

(e) David performs projective measurement on particle D with the basis $\{|+\rangle, |-\rangle\}$, given by

$$\begin{aligned}&\left(a_0|00\rangle + a_1 e^{i\theta}|11\rangle\right)_{CD}\\ =\ &\frac{1}{\sqrt{2}}\left(a_0|0\rangle + a_1 e^{i\theta}|1\rangle\right)_C \otimes |+\rangle_D\\ &+\frac{1}{\sqrt{2}}\left(a_0|0\rangle - a_1 e^{i\theta}|1\rangle\right)_C \otimes |-\rangle_D.\end{aligned} \tag{25}$$

After his projective measurement, David informs his measurement result to Charlie with 1 classical bit of communication cost.

(f) Upon receiving David's measurement result, Charlie performs the local unitary operation on his particle C. It is clear from Eq. (25) that after David's projective measurement, the state of particle C is perfectly correlated to the target state $|\xi\rangle$. If David's measurement result is $|+\rangle_D$, the state of particle C is the target state $|\xi\rangle$. If David's measurement result is $|-\rangle_D$, Charlie needs to perform Pauli-Z operator σ_z on his particle C, the target state $|\xi\rangle = a_0|0\rangle + a_1 e^{i\theta}|1\rangle$ can be readily recovered.

Case 3 The receivers decide that David will reconstruct the state $|\xi\rangle$.

This is similar to case 2.

As above, the situation concerning Alice's PM_A^1 result $|\psi_1\rangle_{A_1}$ is discussed. Now let us consider the situation where Alice's PM_A^1 result is $|\psi_2\rangle_{A_1}$.

Step 2'. Amy encodes phase information.

After receiving Alice's PM_A^1 result $R_A^1 = |\psi_2\rangle_{A_1}$, Amy performs Pauli-$X$ operation σ_x on particle A_2 according to Table 1. Subsequently, she implements a projective measurement on particle A_2 with the basis expressed as Eq. (14).

Amy's operations can be expressed analytically as follows:

$$\begin{aligned}&|\Psi_2\rangle_{A_2B_1B_2CD}\\ =\ &(a_1|00000\rangle - a_0\cos\gamma|10111\rangle - a_0\sin\gamma|11111\rangle)_{A_2B_1B_2CD}\\ &\xrightarrow[\text{on particle } A_2]{\sigma_x} (a_1|10000\rangle - a_0\cos\gamma|00111\rangle\\ &- a_0\sin\gamma|01111\rangle)_{A_2B_1B_2CD}\\ =\ &\frac{1}{\sqrt{2}}\{|\phi_1\rangle_{A_2}\otimes|\Phi_1'\rangle_{B_1B_2CD} - |\phi_2\rangle_{A_2}\otimes|\Phi_2'\rangle_{B_1B_2CD}\},\end{aligned} \tag{26}$$

where

$$\begin{aligned}|\Phi_1'\rangle_{B_1B_2CD} &= (a_1 e^{i\theta}|0000\rangle - a_0\cos\gamma|0111\rangle\\ &\quad - a_0\sin\gamma|1111\rangle)_{B_1B_2CD},\\ |\Phi_2'\rangle_{B_1B_2CD} &= (a_1 e^{i\theta}|0000\rangle + a_0\cos\gamma|0111\rangle\\ &\quad + a_0\sin\gamma|1111\rangle)_{B_1B_2CD}.\end{aligned} \tag{27}$$

From Eq. (26), it is transparent that after the projective measurement PM_A^2, Amy will obtain one of the two possible states $\{|\phi_1\rangle_{A_2}, |\phi_2\rangle_{A_2}\}$ with equal probability. Subsequently Amy informs this result to Bob with 1 classical bit of communication cost.

Step 3'. Bob performs unitary operation.

Corresponding to Amy's measurement result of $|\phi_1\rangle_{A_2}$ ($|\phi_2\rangle_{A_2}$), Bob performs $\sigma_z(I)$ on his particle B_2. After that, the joint state of particles B_1, B_2, C, and D evolves to

$$(a_1 e^{i\theta}|0000\rangle + a_0\cos\gamma|0111\rangle + a_0\sin\gamma|1111\rangle)_{B_1B_2CD}. \tag{28}$$

Step 4'. Quantum state reconstruction.

Case 4 The receivers decide that Bob will reconstruct the target state $|\xi\rangle$.

(I) Bob performs CNOT operation, with B_2 as the control particle, C and D as the target particles. Considering the technological challenges to implement CNOT operation remotely, particles B_2, C, and D are very close. After the CNOT operation, the joint state of particles B_1, B_2, C, and D evolves to

$$(a_1 e^{i\theta}|0000\rangle + a_0\cos\gamma|0100\rangle + a_0\sin\gamma|1100\rangle)_{B_1B_2CD}. \tag{29}$$

(II) Bob performs $R_y(-\gamma)$ on his particle B_1, followed by a projective measurement (denoted as PM_B') on the same particle with the computational basis $\{|0\rangle, |1\rangle\}$. Bob's $R_y(-\gamma)$ and

PM'_B operations can be expressed analytically as follows:

$$\begin{aligned}&(a_1\,\mathrm{e}^{\mathrm{i}\theta}|0000\rangle + a_0\cos\gamma|0100\rangle + a_0\sin\gamma|1100\rangle)_{B_1B_2CD}\\ &\xrightarrow[PM'_B]{R_y(-\gamma)} \cos\frac{\gamma}{2}|0\rangle_{B_1}\otimes[a_0|1\rangle + a_1\,\mathrm{e}^{\mathrm{i}\theta}|0\rangle]_{B_2}\otimes|00\rangle_{CD}\\ &+\ \sin\frac{\gamma}{2}|1\rangle_{B_1}\otimes[a_0|1\rangle - a_1\,\mathrm{e}^{\mathrm{i}\theta}|0\rangle]_{B_2}\otimes|00\rangle_{CD}. \qquad (30)\end{aligned}$$

(III) If the measurement result of particle B_1 is $|0\rangle_{B_1}$, Bob performs Pauli-X operator σ_x on particle B_2; if the measurement result is $|1\rangle_{B_1}$, Bob performs Pauli-Y operator $i\sigma_y$ on particle B_2, thus retrieving the target state $|\xi\rangle$.

Case 5 The receivers decide that Charlie will reconstruct the target state $|\xi\rangle$.

(i) Bob performs $R_y(-\gamma)$ on his particle B_1, followed by a projective measurement (PM) on the same particle with the computational basis $\{|0\rangle,|1\rangle\}$. The joint state of particles B_1, B_2, C, and D evolves as

$$\begin{aligned}&(a_1\,\mathrm{e}^{\mathrm{i}\theta}|0000\rangle + a_0\cos\gamma|0111\rangle + a_0\sin\gamma|1111\rangle)_{B_1B_2CD}\\ &\xrightarrow[\text{on } B_1]{R_y(-\gamma)\&PM} \cos\frac{\gamma}{2}|0\rangle_{B_1}\otimes[a_1\,\mathrm{e}^{\mathrm{i}\theta}|000\rangle + a_0|111\rangle]_{B_2CD}\\ &-\ \sin\frac{\gamma}{2}|1\rangle_{B_1}\otimes[a_1\,\mathrm{e}^{\mathrm{i}\theta}|000\rangle - a_0|111\rangle]_{B_2CD}. \qquad (31)\end{aligned}$$

(ii) Corresponding to the measurement result $|0\rangle_{B_1}$ ($|1\rangle_{B_1}$), Bob performs identity operator I (σ_z) on particle B_2. The joint state of particles B_2, C, and D evolves to

$$(a_1\,\mathrm{e}^{\mathrm{i}\theta}|000\rangle + a_0|111\rangle)_{B_2CD}. \qquad (32)$$

(iii) Bob performs projective measurement on particle B_2 with the basis $\{|+\rangle,|-\rangle\}$, which can be expressed as

$$\begin{aligned}&(a_1\,\mathrm{e}^{\mathrm{i}\theta}|000\rangle + a_0|111\rangle)_{B_2CD}\\ =\ &\frac{1}{\sqrt{2}}|+\rangle_{B_2}(a_1\,\mathrm{e}^{\mathrm{i}\theta}|00\rangle + a_0|11\rangle)_{CD}\\ &+\frac{1}{\sqrt{2}}|-\rangle_{B_2}(a_1\,\mathrm{e}^{\mathrm{i}\theta}|00\rangle - a_0|11\rangle)_{CD}. \qquad (33)\end{aligned}$$

It is apparent from Eq. (33) that Bob will obtain one of the two possible states $\{|+\rangle_{B_2},|-\rangle_{B_2}\}$ with equal probability. Bob informs David of his measurement result with 1 classical bit of communication cost.

(iv) Upon receiving Bob's measurement result, David performs suitable local unitary operation I/σ_z on his particle D. If Bob's measurement result is $|+\rangle_{B_2}$, David performs identity operator I on his particle D. If Bob's measurement results is $|-\rangle_{B_2}$, David performs Pauli-Z operator σ_z on particle D, then the joint state of particles C and D is

$$(a_1\,\mathrm{e}^{\mathrm{i}\theta}|00\rangle + a_0|11\rangle)_{CD}. \qquad (34)$$

(v) David performs projective measurement on particle D with the basis $\{|+\rangle,|-\rangle\}$, which can be expressed as

$$\begin{aligned}&(a_1\,\mathrm{e}^{\mathrm{i}\theta}|00\rangle + a_0|11\rangle)_{CD}\\ =\ &\frac{1}{\sqrt{2}}(a_1\,\mathrm{e}^{\mathrm{i}\theta}|0\rangle + a_0|1\rangle)_C\otimes|+\rangle_D\\ &+\frac{1}{\sqrt{2}}(a_1\,\mathrm{e}^{\mathrm{i}\theta}|0\rangle - a_0|1\rangle)_C\otimes|-\rangle_D. \qquad (35)\end{aligned}$$

After this projective measurement, David informs his measurement result to Charlie with 1 classical bit of communication cost.

(vi) Upon receiving David's measurement result, Charlie performs local unitary operation on his particle C. It is clear from Eq. (35) that after David's projective measurement, the state of particle C is perfectly correlated to the target state $|\xi\rangle$. If David's measurement result is $|+\rangle_D$, Charlie needs to perform Pauli-X operator σ_x on his particle C. If David's measurement result is $|-\rangle_D$, Charlie needs to perform Pauli-Y operator $i\sigma_y$ on his particle C, the target state $|\xi\rangle = a_0|0\rangle + a_1\,\mathrm{e}^{\mathrm{i}\theta}|1\rangle$ can be readily recovered.

Case 6 The receivers decide that David will reconstruct the state $|\xi\rangle$.

This is similar to case 5.

4. Performance analysis

In Section 3, it can be seen from Case 1 and Case 4 that the higher-power receiver can reconstruct the target state independently. Based on Case 2 and Case 5, the lower-power receiver can reconstruct the target state if and only if all the other receivers help him.

Taking Bob's reconstruction of the target state $|\xi\rangle$ for example, various measurement results and unitary operations are summarized, as shown in Table 3. The success probability of Bob's reconstruction of the target state can be calculated as

$$\begin{aligned}P_{\mathrm{Bob}}^{\mathrm{suc}} &= \sum_{R_A^1}\sum_{R_A^2}\sum_{R_B} P_{R_A^1}P_{R_A^2|R_A^1}P_{R_B|R_A^2R_A^1}\\ &= 4\times\frac{1}{2}\times\frac{1}{2}\times\left[\cos^2\left(\frac{\gamma}{2}\right)+\sin^2\left(\frac{\gamma}{2}\right)\right]\\ &= 1. \qquad (36)\end{aligned}$$

When it comes to Charlie or David's reconstruction of the target state, a similar table can be listed. For the sake of brevity, here we no longer depict it. In conclusion, our HJRSP scheme is deterministic, and the unit success probability can always be achieved independent of the parameters of the pre-shared partially entangled state.

The transmission of classical information plays an important role in the RSP process.[32–35] In our protocol of deterministic HJRSP, 2 bits of classical communication are required for the higher-power receiver to retrieve the target state, and 4 bits of classical communication are required for the lower-power receiver, which can be calculated from Section 3. While comparing Ref. [30] with our present protocol for deterministic HJRSP via partially entangled states, it can be learned that the former scheme takes 3 and 4 bits of classical communication for higher- and lower-power receivers, respectively.

Table 3. Participants' measurement results and unitary operations, where $P_{R_A^1}$ denotes the probability for Alice's measurement result R_A^1. $P_{R_A^2|R_A^1}$ denotes the conditional probability for Amy's measurement result R_A^2, given Alice's R_A^1. $P_{R_B|R_A^2R_A^1}$ denotes the conditional probability for Bob's measurement result R_B, given Alice's R_A^1 and Amy's R_A^2.

Alice encodes amplitude information		Amy encodes phase information		Bob's unitary operation	Bob's reconstruction	
Alice PM_A^1 result $(R_A^1\ \&P_{R_A^1})$	Amy's unitary operation (U_A^2)	Amy's PM_A^2 result $(R_A^2\ \&P_{R_A^2\|R_A^1})$		(U_B)	Bob's PM_B result $(R_B\ \&P_{R_B\|R_A^2R_A^1})$	reconstruction operation (U_B^r)
$\|\psi_1\rangle_{A_1}, \frac{1}{2}$	I	$\|\phi_1\rangle_{A_2}, \frac{1}{2}$	I	$\|0\rangle_{B_1}, \cos^2\left(\frac{\gamma}{2}\right)$	I	
				$\|1\rangle_{B_1}, \sin^2\left(\frac{\gamma}{2}\right)$	σ_z	
		$\|\phi_2\rangle_{A_2}, \frac{1}{2}$	σ_z	$\|0\rangle_{B_1}, \cos^2\left(\frac{\gamma}{2}\right)$	I	
				$\|1\rangle_{B_1}, \sin^2\left(\frac{\gamma}{2}\right)$	σ_z	
$\|\psi_2\rangle_{A_1}, \frac{1}{2}$	σ_x	$\|\phi_1\rangle_{A_2}, \frac{1}{2}$	σ_z	$\|0\rangle_{B_1}, \cos^2\left(\frac{\gamma}{2}\right)$	σ_x	
				$\|1\rangle_{B_1}, \sin^2\left(\frac{\gamma}{2}\right)$	$\mathrm{i}\sigma_y$	
		$\|\phi_2\rangle_{A_2}, \frac{1}{2}$	I	$\|0\rangle_{B_1}, \cos^2\left(\frac{\gamma}{2}\right)$	σ_x	
				$\|1\rangle_{B_1}, \sin^2\left(\frac{\gamma}{2}\right)$	$\mathrm{i}\sigma_y$	

5. Conclusions

In summary, a deterministic HJRSP scheme is proposed based on a pre-shared six-particle partially entangled state. Two senders and three receivers are involved in the proposed scheme. Participants perform various unitary operations and projective measurements on their own particles. The higher-power receiver can reconstruct the target state independently, and the lower-power receiver can reconstruct the target state if and only if all the other receivers help him. For each receiver, the success probability of state reconstruction is always 100%, independent of parameters of the pre-shared partially entangled state. Furthermore, based on our research here, an extension to deterministic HJRSP of a general multi-qubit state is possible.

It is worth pointing out that the pre-shared partially entangled quantum channel is necessary to accomplish the HJRSP tasks, however unit success probability can always be achieved irrespective of the channel's entanglement degree. This point is very interesting and primarily meaningful considering the technological challenges to prepare and maintain maximal entanglement in real-world implementations.

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