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Citation: Chin. Phys. B . 2018, 27(9): 090303. **doi:** 10.1088/1674-1056/27/9/090303

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Entropy of field interacting with two two-qubit atoms*

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(Received 9 February 2018; revised manuscript received 25 June 2018; published online 28 August 2018)

We use quantum field entropy to measure the degree of entanglement for a coherent state light field interacting with two atoms that are initially in an arbitrary two-qubit state. The influence of different mean photon number of the coherent field on the entropy of the field is discussed in detail when the two atoms are initially in one superposition state of the Bell states. The results show that the mean photon number of the light field can regulate the quantum entanglement between the atoms and light field.

Keywords: quantum field entropy, coherent state light field, quantum entanglement

PACS: 03.65.Ud, 42.50.Pq **DOI:** 10.1088/1674-1056/27/9/090303

1. Introduction

In 1935, Einstein *et al.*[1] and Schrödinger[2] proposed the concept of entangled states respectively. Quantum entanglement is a distinctive feature of quantum physics, and it is very useful in quantum information processing, including quantum communication and quantum computation.[3–6] In order to quantitatively describe the degree of entanglement between microsystems of the light field interacting with the atom, Phoenix and Knight[7] studied the entanglement and dynamics between light fields and atoms using von Neumann quantum entropy theory. Thereafter, the coupling entropy characteristic between the light field and the atom has attracted a great deal of attention, the entropy of quantum systems has been studied, and many results have been obtained.[8–18] However, the above literatures are based on the eigenvalue method for calculating the reduced density operator of the atom of the subsystem. In the previous work,[19,20] we formulated the quantum entropy of the interaction between two two-level entangled atoms and a light field of the coherent state and the Schrödinger cat state by using the eigenvalue method to calculate the reduced density operator of the light field of the subsystem. In this paper, we investigate the quantum field entropy in a system of arbitrary two qubit atoms interacting with the coherent state light field. For our purpose, we choose the initial atoms in four different states and examine the evolution characteristics of the quantum field entropy by means of numerical calculations.

2. Theoretical model and its solution

We consider a system composed of two identical two-level atoms resonantly interacting with a single-mode cavity field simultaneously, the Hamiltonian of the system in the rotating wave approximation can be written as ($\hbar = 1$)[21]

$$H = \Omega a^{+}a + \frac{1}{2}\omega\sum_{i=1}^{2}\sigma_i^z + g\sum_{i=1}^{2}(a^{+}\sigma_i^{-} + a\sigma_i^{+}), \tag{1}$$

where a^{+} and a are the creation and annihilation operators of the field mode, ω is the atomic transition frequency, and $\sigma_i^z = |e_i\rangle\langle e_i|$, $\sigma_i^{+} = |e_i\rangle\langle g_i|$, and $\sigma_i^{-} = |g_i\rangle\langle e_i|$ are the spin operators of the i-th atom ($i = 1, 2$). $|e\rangle$ denotes an excited state of the atom, $|g\rangle$ denotes a ground state of the atom, and g is the atom–field coupling constant.

The two atoms are initially prepared in an arbitrary two-qubit state

$$|\Psi_{\rm a}(0)\rangle = a|e_1, e_2\rangle + b|g_1, g_2\rangle + c|e_1, g_2\rangle + d|g_1, e_2\rangle, \tag{2}$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. The radiation field is initially prepared in a single-mode coherent state

$$|\Psi_{\rm f}(0)\rangle = \sum_{n=0}^{\infty} F_n|n\rangle = \sum_{n=0}^{\infty} {\rm e}^{-\frac{1}{2}|\alpha|^2}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{3}$$

where $\alpha = |\alpha|{\rm e}^{{\rm i}\varphi}$, $|\alpha|^2$ is the mean photon number of the coherent light field, and φ is the phase angle of the coherent field.

When two atoms interact with the light field, the system state vector at any time will evolve as

$$\begin{aligned}|\Psi_{\rm s}(t)\rangle \;=\; & |\alpha\rangle|e_1, e_2\rangle + |\beta\rangle|e_1, g_2\rangle + |\gamma\rangle|g_1, e_2\rangle \\ & + |\delta\rangle|g_1, g_2\rangle,\end{aligned} \tag{4}$$

where

$$\begin{aligned}|\alpha\rangle \;&=\; \sum_{n=0}^{\infty} A(n,t)|n\rangle, \;\; |\beta\rangle = \sum_{n=0}^{\infty} C(n,t)|n\rangle, \\ |\gamma\rangle \;&=\; \sum_{n=0}^{\infty} D(n,t)|n\rangle, \;\; |\delta\rangle = \sum_{n=0}^{\infty} B(n,t)|n\rangle.\end{aligned} \tag{5}$$

*Project supported by the National Natural Science Foundation of China (Grant No. 11404108).

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According to the Schrödinger equation, the following results can be obtained:

$$\begin{aligned}
&A(n,t)\\
&= aF_n - \mathrm{i}\sqrt{\frac{n+1}{2(2n+3)}}(c+d)F_{n+1}\sin(\sqrt{2(2n+3)}gt)\\
&\quad -\sqrt{\frac{n+1}{2(2n+3)}}\left[\sqrt{\frac{2(n+1)}{2n+3}}aF_n+\sqrt{\frac{2(n+2)}{2n+3}}bF_{n+2}\right]\\
&\quad \times\left[1-\cos(\sqrt{2(2n+3)}gt)\right],\ \ n=0,1,2,\ldots, \qquad (6)\\
&B(n,t)\\
&= bF_n - \mathrm{i}\sqrt{\frac{n}{2(2n-1)}}(c+d)F_{n-1}\sin(\sqrt{2(2n-1)}gt)\\
&\quad -\sqrt{\frac{n}{2(2n-1)}}\left[\sqrt{\frac{2(n-1)}{2n-1}}aF_{n-2}+\sqrt{\frac{2n}{2n-1}}bF_n\right]\\
&\quad \times\left[1-\cos(\sqrt{2(2n-1)}gt)\right],\ \ n=2,3,4,\ldots,\\
&\quad B(1,t)=bF_1\cos(\sqrt{2}gt)-\mathrm{i}\frac{1}{\sqrt{2}}(c+d)F_0\sin(\sqrt{2}gt),\\
&\quad B(0,t)=bF_0, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
C(n,t) &= \frac{1}{2}(c-d)F_n+\frac{1}{2}(c+d)F_n\cos(\sqrt{2(2n+1)}gt)\\
&\quad -\mathrm{i}\frac{1}{2}\left[\sqrt{\frac{2n}{2n+1}}aF_{n-1}+\sqrt{\frac{2(n+1)}{2n+1}}bF_{n+1}\right]\\
&\quad \times\sin(\sqrt{2(2n+1)}gt),\ \ n=1,2,3,\ldots,\\
C(0,t) &= \frac{1}{2}(c-d)F_0+\frac{1}{2}(c+d)F_0\cos(\sqrt{2}gt)\\
&\quad -\mathrm{i}\frac{1}{2}\sqrt{2}bF_1\sin(\sqrt{2}gt), \qquad (8)\\
D(n,t) &= -\frac{1}{2}(c-d)F_n+\frac{1}{2}(c+d)F_n\cos(\sqrt{2(2n+1)}gt)\\
&\quad -\mathrm{i}\frac{1}{2}\left[\sqrt{\frac{2n}{2n+1}}aF_{n-1}+\sqrt{\frac{2(n+1)}{2n+1}}bF_{n+1}\right]\\
&\quad \times\sin(\sqrt{2(2n+1)}gt),\ \ n=1,2,3,\ldots,\\
D(0,t) &= -\frac{1}{2}(c-d)F_0+\frac{1}{2}(c+d)F_0\cos(\sqrt{2}gt)\\
&\quad -\mathrm{i}\frac{1}{2}\sqrt{2}bF_1\sin(\sqrt{2}gt). \qquad (9)
\end{aligned}$$

3. Field entropy

The reduced density matrix of the light field of the subsystem is given by

$$\begin{aligned}
\hat{\rho}_{\mathrm{field}}(n,t) &= \mathrm{Tr}_{\mathrm{atom}}\hat{\rho}_{\mathrm{system}}(n,t)\\
&= |\alpha\rangle\langle\alpha|+|\beta\rangle\langle\beta|+|\gamma\rangle\langle\gamma|+|\delta\rangle\langle\delta|. \qquad (10)
\end{aligned}$$

The eigenvalues of the reduced density matrix of the light field are

$$\tau_1(t)=-\frac{1}{2}\sqrt{q}-\frac{1}{2}\sqrt{r-\frac{s}{4\sqrt{q}}}-\frac{\vartheta_1}{4}, \qquad (11)$$

$$\tau_2(t)=-\frac{1}{2}\sqrt{q}+\frac{1}{2}\sqrt{r-\frac{s}{4\sqrt{q}}}-\frac{\vartheta_1}{4}, \qquad (12)$$

$$\tau_3(t)=\frac{1}{2}\sqrt{q}-\frac{1}{2}\sqrt{r+\frac{s}{4\sqrt{q}}}-\frac{\vartheta_1}{4}, \qquad (13)$$

$$\tau_4(t)=\frac{1}{2}\sqrt{q}+\frac{1}{2}\sqrt{r+\frac{s}{4\sqrt{q}}}-\frac{\vartheta_1}{4}. \qquad (14)$$

Here

$$\begin{aligned}
&\Delta_1=\vartheta_2^2-3\vartheta_1\vartheta_3+12\vartheta_4,\\
&\Delta_2=2\vartheta_2^3-9\vartheta_1\vartheta_2\vartheta_3+27\vartheta_3^2+27\vartheta_1^2\vartheta_4-72\vartheta_2\vartheta_4,\\
&p=\Delta_2+\sqrt{-4\Delta_1^3+\Delta_2^2},\ \ \Delta=\frac{\sqrt[3]{2}\Delta_1}{3\sqrt[3]{p}}+\frac{\sqrt[3]{p}}{3\sqrt[3]{2}},\\
&q=\frac{\vartheta_1^2}{4}-\frac{2\vartheta_2}{3}+\Delta,\ \ r=\frac{\vartheta_1^2}{2}-\frac{4\vartheta_2}{3}-\Delta,\\
&s=-\vartheta_1^3+4\vartheta_1\vartheta_2-8\vartheta_3, \qquad (15)\\
&\vartheta_1=-(\langle\alpha|\alpha\rangle+\langle\beta|\beta\rangle+\langle\gamma|\gamma\rangle+\langle\delta|\delta\rangle), \qquad (16)
\end{aligned}$$

$$\begin{aligned}
\vartheta_2 &= \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle+\langle\alpha|\alpha\rangle\langle\gamma|\gamma\rangle+\langle\alpha|\alpha\rangle\langle\delta|\delta\rangle+\langle\beta|\beta\rangle\langle\gamma|\gamma\rangle\\
&\quad +\langle\beta|\beta\rangle\langle\delta|\delta\rangle+\langle\gamma|\gamma\rangle\langle\delta|\delta\rangle-|\langle\alpha|\beta\rangle|^2|-\langle\alpha|\gamma\rangle|^2\\
&\quad -|\langle\alpha|\delta\rangle|^2-|\langle\beta|\gamma\rangle|^2-|\langle\beta|\delta\rangle|^2-|\langle\gamma|\delta\rangle|^2, \qquad (17)\\
\vartheta_3 &= -\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle\langle\gamma|\gamma\rangle-\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle\langle\delta|\delta\rangle\\
&\quad -\langle\alpha|\alpha\rangle\langle\gamma|\gamma\rangle\langle\delta|\delta\rangle-\langle\beta|\beta\rangle\langle\gamma|\gamma\rangle\langle\delta|\delta\rangle\\
&\quad -\langle\beta|\gamma\rangle\langle\gamma|\delta\rangle\langle\delta|\beta\rangle-\langle\alpha|\beta\rangle\langle\beta|\gamma\rangle\langle\gamma|\alpha\rangle\\
&\quad -\langle\alpha|\gamma\rangle\langle\beta|\alpha\rangle\langle\gamma|\beta\rangle-\langle\alpha|\delta\rangle\langle\beta|\alpha\rangle\langle\delta|\beta\rangle\\
&\quad -\langle\alpha|\beta\rangle\langle\beta|\delta\rangle\langle\delta|\alpha\rangle-\langle\alpha|\gamma\rangle\langle\gamma|\delta\rangle\langle\delta|\alpha\rangle\\
&\quad -\langle\alpha|\delta\rangle\langle\gamma|\alpha\rangle\langle\delta|\gamma\rangle-\langle\beta|\delta\rangle\langle\gamma|\beta\rangle\langle\delta|\gamma\rangle\\
&\quad +\langle\alpha|\alpha\rangle|\langle\beta|\gamma\rangle|^2+\langle\alpha|\alpha\rangle|\langle\beta|\delta\rangle|^2+\langle\alpha|\alpha\rangle|\langle\gamma|\delta\rangle|^2\\
&\quad +\langle\beta|\beta\rangle|\langle\alpha|\delta\rangle|^2+\langle\beta|\beta\rangle|\langle\alpha|\gamma\rangle|^2+\langle\beta|\beta\rangle|\langle\gamma|\delta\rangle|^2\\
&\quad +\langle\gamma|\gamma\rangle|\langle\alpha|\beta\rangle|^2+\langle\gamma|\gamma\rangle|\langle\beta|\delta\rangle|^2+\langle\gamma|\gamma\rangle|\langle\alpha|\delta\rangle|^2\\
&\quad +\langle\delta|\delta\rangle|\langle\alpha|\beta\rangle|^2+\langle\delta|\delta\rangle|\langle\alpha|\gamma\rangle|^2+\langle\delta|\delta\rangle|\langle\beta|\gamma\rangle|^2, \qquad (18)\\
\vartheta_4 &= \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle\langle\gamma|\gamma\rangle\langle\delta|\delta\rangle+\langle\alpha|\alpha\rangle\langle\beta|\gamma\rangle\langle\gamma|\delta\rangle\langle\delta|\beta\rangle\\
&\quad +\langle\alpha|\alpha\rangle\langle\beta|\delta\rangle\langle\gamma|\beta\rangle\langle\delta|\gamma\rangle+\langle\alpha|\beta\rangle\langle\beta|\gamma\rangle\langle\gamma|\alpha\rangle\langle\delta|\delta\rangle\\
&\quad +\langle\alpha|\beta\rangle\langle\beta|\delta\rangle\langle\gamma|\gamma\rangle\langle\delta|\alpha\rangle+\langle\alpha|\gamma\rangle\langle\beta|\alpha\rangle\langle\gamma|\beta\rangle\langle\delta|\delta\rangle\\
&\quad +\langle\alpha|\gamma\rangle\langle\beta|\beta\rangle\langle\gamma|\delta\rangle\langle\delta|\alpha\rangle+\langle\alpha|\delta\rangle\langle\beta|\alpha\rangle\langle\gamma|\gamma\rangle\langle\delta|\beta\rangle\\
&\quad +\langle\alpha|\delta\rangle\langle\beta|\beta\rangle\langle\gamma|\alpha\rangle\langle\delta|\gamma\rangle+|\langle\alpha|\beta\rangle|^2|\langle\gamma|\delta\rangle|^2\\
&\quad +|\langle\alpha|\gamma\rangle|^2|\langle\beta|\delta\rangle|^2+|\langle\alpha|\delta\rangle|^2|\langle\beta|\gamma\rangle|^2\\
&\quad -\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle|\langle\gamma|\delta\rangle|^2-\langle\alpha|\alpha\rangle\langle\gamma|\gamma\rangle|\langle\beta|\delta\rangle|^2\\
&\quad -\langle\alpha|\alpha\rangle\langle\delta|\delta\rangle|\langle\beta|\gamma\rangle|^2-\langle\beta|\beta\rangle\langle\gamma|\gamma\rangle|\langle\alpha|\delta\rangle|^2\\
&\quad -\langle\beta|\beta\rangle\langle\delta|\delta\rangle|\langle\alpha|\gamma\rangle|^2-\langle\gamma|\gamma\rangle\langle\delta|\delta\rangle|\langle\alpha|\beta\rangle|^2\\
&\quad -\langle\alpha|\beta\rangle\langle\beta|\gamma\rangle\langle\gamma|\delta\rangle\langle\delta|\alpha\rangle-\langle\alpha|\beta\rangle\langle\beta|\delta\rangle\langle\gamma|\alpha\rangle\langle\delta|\gamma\rangle\\
&\quad -\langle\alpha|\gamma\rangle\langle\beta|\alpha\rangle\langle\gamma|\delta\rangle\langle\delta|\beta\rangle-\langle\alpha|\gamma\rangle\langle\beta|\delta\rangle\langle\gamma|\beta\rangle\langle\delta|\alpha\rangle\\
&\quad -\langle\alpha|\delta\rangle\langle\beta|\alpha\rangle\langle\gamma|\beta\rangle\langle\delta|\gamma\rangle\\
&\quad -\langle\alpha|\delta\rangle\langle\beta|\gamma\rangle\langle\gamma|\alpha\rangle\langle\delta|\beta\rangle. \qquad (19)
\end{aligned}$$

The quantum field entropy can be defined as follows:[7]

$$S_{\mathrm{f}}(t)=-\sum_{i=1}^{4}\tau_i(t)\ln[\tau_i(t)]. \qquad (20)$$

4. Numerical results and discussion

For the atomic system of an arbitrarily two-qubit state, in this paper, it is a superposition of different Bell states. In the numerical processing, the evolution of the quantum field entropy $S_{\rm f}(t)$ in four superposition states is prominent for the atoms that are initially in two types of Bell states. Therefore, when the atoms are in the following four types of initial states, the quantum properties of the system are significant.

When the two atoms are initially in a superposition state of two Bell states, namely,

$$|\Psi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|e_1,e_2\rangle+|g_1,g_2\rangle) + \frac{1}{\sqrt{2}}(|e_1,g_2\rangle+|g_1,e_2\rangle)\right], \tag{21}$$

the temporal evolution of the quantum field entropy $S_{\rm f}(t)$ is shown as Fig. 1.

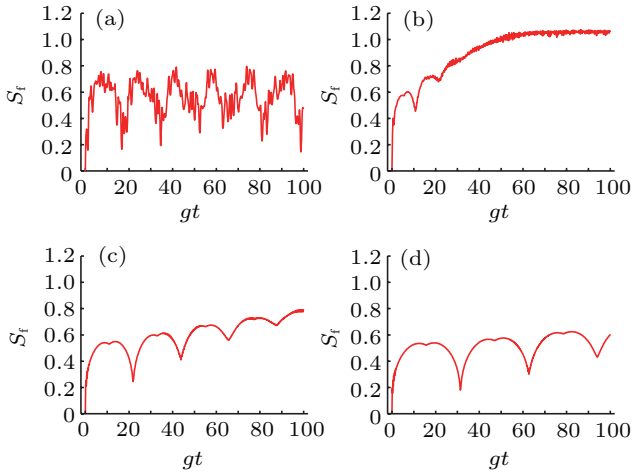


Fig. 1. (color online) Evolution of $S_{\rm f}(t)$ with scaled time gt versus different $|\alpha|$ for $a=b=c=d=1/2$. From (a) to (d), $|\alpha|=0.5, 3.5, 7.0, 10.0$, respectively.

Time evolution characteristics of the $S_{\rm f}(t)$ versus different $|\alpha|$ are shown in Fig. 1 for the atoms initially in

$$|\Psi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|e_1,e_2\rangle+|g_1,g_2\rangle) + \frac{1}{\sqrt{2}}(|e_1,g_2\rangle+|g_1,e_2\rangle)\right].$$

The results obtained by the numerical methods show that the period of the field entropy evolution becomes longer with the increase of the mean photon number of the light field, and meanwhile the field entropy decreases. That is to say, when the light field parameter takes a suitable value, quantum entanglement between the coherent field and the two atoms can be kept for much longer time. Furthermore, the numerical results show that the time evolution characteristics of $S_{\rm f}(t)$ for $a=b=-c=-d=1/2$ are nearly the same as those for $a=b=c=d=1/2$.

When the initial state of the two atoms is

$$|\Psi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|e_1,e_2\rangle-|g_1,g_2\rangle) + \frac{1}{\sqrt{2}}(|e_1,g_2\rangle+|g_1,e_2\rangle)\right]. \tag{22}$$

Similarly, the temporal evolution curves of $S_{\rm f}(t)$ are shown in Fig. 2. The results indicate that the number of oscillations of the field entropy is apparently increased compared to the results in Fig. 1 and the field entropy can be kept as a larger value. Moreover, because the atom is initially in the superposition state, the period becomes irregular. In addition, the light field parameter $|\alpha|$ has a strong effect on the time evolution of $S_{\rm f}(t)$ in the system. It can be observed that the results for $a=-b=-c=-d=1/2$ and $a=-b=c=d=1/2$ are similar in the evolving curve of $S_{\rm f}(t)$.

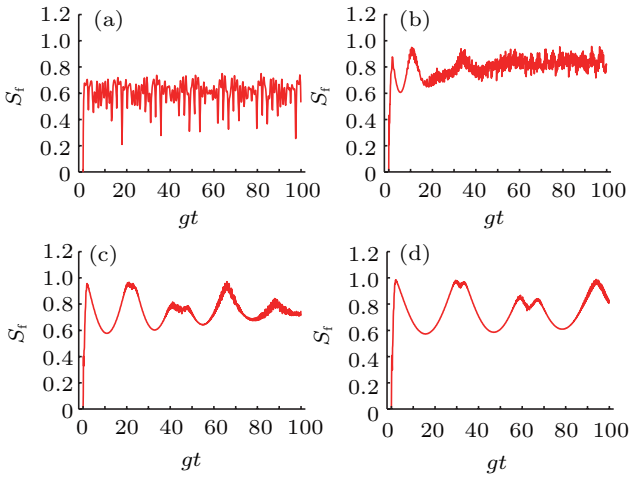


Fig. 2. (color online) Evolution of $S_{\rm f}(t)$ with scaled time gt versus different $|\alpha|$ for $a=-b=c=d=1/2$. From (a) to (d), $|\alpha|=0.5, 3.5, 7.0, 10.0$, respectively.

When the initial state of the two atoms is

$$|\Psi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|e_1,e_2\rangle+|g_1,g_2\rangle) - \frac{1}{\sqrt{2}}(|e_1,g_2\rangle-|g_1,e_2\rangle)\right]. \tag{23}$$

Time evolution of $S_{\rm f}(t)$ is shown as Fig. 3. It is shown that the evolution period of the field entropy changes and the maximal field entropy gradually increases compared to the results in Figs. 1 and 2. The above results correspond with the fact that a different initial state can lead to a different evolution of entropy of the coherent light field. It is also found that the numerical result of field entropy $S_{\rm f}(t)$ is almost the same for $a=b=c=-d=1/2$ and $a=b=-c=d=1/2$.

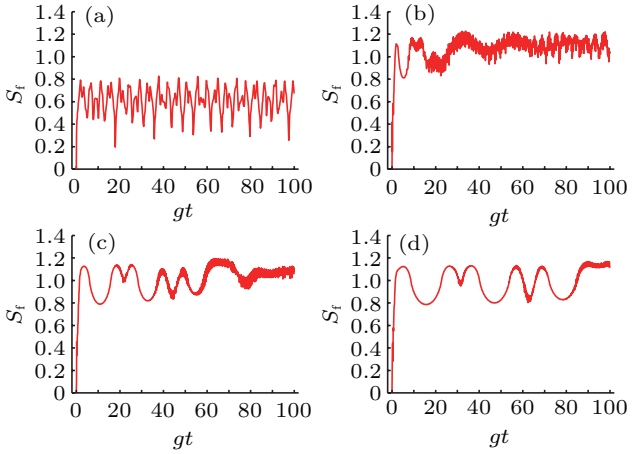


Fig. 3. (color online) Evolution of $S_f(t)$ with scaled time gt versus different $|\alpha|$ for $a = b = -c = d = 1/2$. From (a) to (d), $|\alpha| = 0.5, 3.5, 7.0, 10.0$, respectively.

When the initial state of the two atoms is

$$|\Psi_a(0)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|e_1,e_2\rangle - |g_1,g_2\rangle) + \frac{1}{\sqrt{2}}(|e_1,g_2\rangle - |g_1,e_2\rangle)\right]. \quad (24)$$

Time evolution of $S_f(t)$ is shown as Fig. 4. With the increase of the mean photon number, the maximal field entropy gradually decreases for this initial state. The mean photon number can control the quantum entanglement between the atoms and light field. The numerical results are almost identical for $a = -b = -c = d = 1/2$ and $a = -b = c = -d = 1/2$.

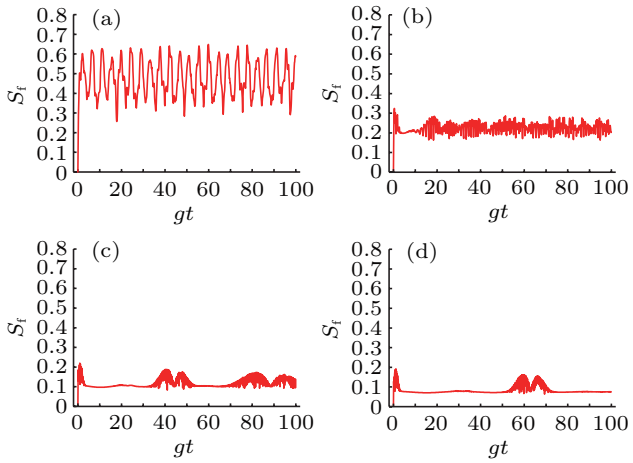


Fig. 4. (color online) Evolution of $S_f(t)$ with scaled time gt versus different $|\alpha|$ for $a = -b = c = -d = 1/2$. From (a) to (d), $|\alpha| = 0.5, 3.5, 7.0, 10.0$, respectively.

Through the entropy theory of atoms and light field presented by Phoenix and Knight, the entropy is a very useful measuring method of quantum entanglement of the quantum state. The time behavior of the field and atomic entropy can reflect the evolution of quantum entanglement between the two atoms and the light field. The higher the entropy, the greater the entanglement. In Figs. 1–4, the results show that the field entropy evolution curve starts at 0, indicating that there is no entangling between the light field and the two atoms at the initial moment. The oscillation of the temporal evolution has larger range and irregularity for smaller $|\alpha|$. With the increase of light field parameter $|\alpha|$, the oscillation frequency of the temporal evolution speeds up, and the phase backward shifts. This phenomenon shows that changing $|\alpha|$ can control the entanglement between the atoms and light field.

5. Conclusion

The quantum field entropy evolution properties of the coherent light field interacting with two two-qubit atoms has been studied. The influences of the mean photon number on the temporal evolution of the quantum field entropy between the light field and two atoms are discussed for the two atoms initially prepared in four specific states. The result shows that the amplitude and phase of the field entropy can be changed by the mean photon number. The mean photon number of the light field and the setting of the initial state of the two two-level atoms play important roles in the evolution of the field entropy and the entanglement between the atom and the field.

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