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Decoherence for a two-qubit system in a spin-chain environment*

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The quantum coherence and correlation dynamics for a two-qubit system in the Ising spin-chain environment are studied. A sudden change of coherence is found near the critical point, which provides us with an effective way to detect the quantum phase transition. By studying the relationship between quantum discord and coherence, we find that coherence displays the behavior of classical correlation for $t < t_0$, and of quantum discord for $t > t_0$, where t_0 is the time-point of a sudden transition between classical and quantum decoherence.

Keywords: quantum coherence, quantum discord, decoherence

PACS: 03.65.Ta, 03.65.Yz, 03.67.Mn **DOI:** 10.1088/1674-1056/27/9/090302

1. Introduction

Quantum coherence, arising from the quantum superposition principle, has played a vital role in quantum information processing, such as entanglement creation,[1,2] nanoscale thermodynamics,[3–5] and quantum biology.[6–8] The rigorous physical conditions for quantifying coherence have been proposed, and the relative entropy coherence and the l_1 norm of coherence have been proved to obey these conditions.[9]

Quantum correlation can be identified as a powerful resource in quantum information processing such as quantum metrology and quantum algorithms.[10–13] Both coherence and correlation reflect the quantum behavior of a physical system, so the relationship between them is essential to understanding the quantum resource. Yao *et al.* proved that basis-free quantum coherence was equivalent to quantum discord.[14] Chitambar and Hsieh unified the resource theories of entanglement and coherence by studying their combined behavior in local incoherent operations and classical communication (LIOCC).[15] The conversion from coherence to quantum discord in multipartite systems was studied in Ref. [16]. The connection among quantum coherence, incompatibility, and quantum correlation defined by the relative quantum coherence was explored in Ref. [17]. The dynamics of correlation and coherence under the incoherent quantum channels was studied,[18] and the equivalency regarding the coherence and discord by quantum measurement was derived.[19]

The interaction of a quantum system with its surrounding environment will destroy the coherence. The dynamics of quantum correlation under the influence of Markovian or non-Markovian environments has been investigated.[20–22] The quantum correlation exhibits many different phenomena, such as sudden transition from quantum correlation to classical correlation, the freezing for quantum discord,[23–27] etc. Recently, the dynamical conditions of frozen coherence have been proposed.[28] Yu *et al.* found that if the relative entropy coherence was frozen for an initial state in an incoherent channel, all measures of coherence were frozen.[29] The topics related to coherence have been widely studied.[30–39]

In addition, quite a few systems coupled to the spin chain bath have been investigated since the spin chains have wide applications in quantum computation. Currently, these investigations only focus either on the correlation or on the coherence alone. The relationship between coherence and correlation in spin baths has not been studied extensively. In this paper, we investigate the dynamics of coherence and correlation of a two-qubit system in the Ising spin-chain baths. We obtain the analytical expression of the dynamical behaviors as quantified by the relative entropy coherence, as well as the l_1 norm of coherence, the quantum discord, and the geometric measure of quantum discord (GMQD). The coherence can also be used to probe the existence of quantum phase transition (QPT), such as entanglement or discord. By comparing and analyzing these behaviors, we find that the quantum coherence displays the behavior of quantum discord and classical correlation at different cases for a class of initial states.

This paper is organized as follows. In Section 2, we review the measures of quantum coherence and quantum correlation. In Section 3, we introduce the system coupled to the spin chain environment. In Section 4, we discuss quantum coherence and the correlation dynamics for a class of initial states. Section 5 is a summary of our results.

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2. Measures of quantum coherence and correlation

2.1. The measures of quantum coherence

In Ref. [9], Baumgratz *et al.* have proposed that any valid measure to quantify quantum coherence should satisfy the following conditions:

(a) Non-negativity, $C(\rho)\geq 0$ for all states ρ, with $C(\delta)=0$ for all incoherent states δ.

(b) Contractivity under incoherent channels Λ, $C(\rho)\geq C(\Lambda(\rho))$.

(c) Contractivity under selective measurements on average, $C(\rho)\geq\sum_j p_j C(\rho_j)$, where $\rho_j=K_j\rho K_j^{\dagger}/p_j$, $p_j=\mathrm{Tr}\left(K_j\rho K_j^{\dagger}\right)$, Tr$\rho$ is the trace of matrix ρ, and K_j is an incoherent Kraus operator with $\sum_j K_j^{\dagger}K_j=I$.

(d) Convexity, $C(q\rho+(1-q)\tau)\leq qC(\rho)+(1-q)C(\tau)$ for any states ρ and τ, $q\in[0,1]$.

It has been proved that the relative entropy coherence and the l_1 norm of coherence satisfy all those conditions.

The relative entropy coherence is defined as

$$C_{\mathrm{r}}(\rho)=S\left(\rho_{\mathrm{diag}}\right)-S(\rho), \tag{1}$$

where ρ_{diag} is the matrix obtained by deleting off diagonal elements of ρ and $S(\rho)=\mathrm{Tr}(\rho\log_2\rho)$.

The l_1 norm of coherence is

$$C_{l_1}(\rho)=\sum_{i\neq j}\left|\rho_{ij}\right|. \tag{2}$$

2.2. The measures of quantum correlation

Quantum discord is defined as the difference generated by two definitions of mutual information[40]

$$D(\rho_{\mathrm{AB}})=I(\rho_{\mathrm{AB}})-C(\rho_{\mathrm{AB}}), \tag{3}$$

where $I(\rho_{\mathrm{AB}})=S(\rho_{\mathrm{A}})+S(\rho_{\mathrm{B}})-S(\rho_{\mathrm{AB}})$ and $C(\rho_{\mathrm{AB}})=\max_{\{B_k\}}\left[S(\rho_{\mathrm{A}})-S(\rho_{\mathrm{AB}}|\{B_k\})\right]$ represent the total correlation and classical correlation, respectively. ρ_{A} or ρ_{B} is the reduced density of ρ_{AB}. By performing the positive-operator valued measurements $\{B_k\}$ on subsystem B, the quantum conditional entropy of the system is obtained, $S(\rho_{\mathrm{AB}}|\{B_k\})=\sum_k p_k S(\rho_k)$, where p_k corresponds to the probability of the measurement outcome k of the B_k, $\rho_k=\mathrm{Tr}_{\mathrm{B}}\left[(I\otimes B_k)\rho_{\mathrm{AB}}(I\otimes B_k)\right]/p_k$, and $p_k=\mathrm{Tr}\left[(I\otimes B_k)\rho_{\mathrm{AB}}(I\otimes B_k)\right]$.

The calculation of quantum discord is difficult because the measurement B_k performed on the subsystems is the optimal basis. Dakić *et al.* introduced GMQD to quantify the quantum correlation in Ref. [41]

$$G(\rho_{\mathrm{AB}})=\min_{\chi\in\Omega_0}\|\rho-\chi\|^2, \tag{4}$$

where $\|X\|^2=\mathrm{Tr}X^2$ and Ω_0 is the set of zero-discord state, i.e., $G(\Omega_0)=0$.

3. Model

In the following, we consider two central spins transversely coupled to their own spin baths modeled by the Ising chain. The total Hamiltonian is $H=H_{\mathrm{E}}+H_{\mathrm{I}}$, where

$$H_{\mathrm{E}}=-\sum_{i=1}^{N}\sum_{l=\mathrm{A}',\mathrm{B}'}J\left(\sigma_{i,l}^{z}\sigma_{i+1,l}^{z}+\lambda\sigma_{i,l}^{x}\right), \tag{5}$$

and

$$H_{\mathrm{I}}=-\sum_{i=1}^{N}J\delta\left(\sigma_{i,\mathrm{A}'}^{x}|e\rangle_{\mathrm{A}}\langle e|+\sigma_{i,\mathrm{B}'}^{x}|e\rangle_{\mathrm{B}}\langle e|\right). \tag{6}$$

H_{E} and H_{I} denote the Hamiltonian of the spin chain and the interaction between the system and the environment, respectively. σ_i^{α} $(\alpha=1,2,3)$ are the Pauli matrices representing the i-th site of the spin chain. N is the number of spins in each chain. J characterizes the strength of the Ising interaction. λ and δ describe the strength of the transverse field and the coupling strength of the central atoms with its spin chain, respectively. The system consists of central spins A and B, where the states $|g\rangle_{\mathrm{A(B)}}$ and $|e\rangle_{\mathrm{A(B)}}$ represent spin up and down, respectively. A$'$ and B$'$ are the baths of qubits.

The initial state of the central qubits is prepared in the Bell-diagonal states,

$$\rho_{\mathrm{AB}}(0)=\left(I+\sum_{\alpha}c_{\alpha}\sigma_{\mathrm{A}}^{\alpha}\otimes\sigma_{\mathrm{B}}^{\alpha}\right)/4,$$

where $\sigma_{\mathrm{A}}^{\alpha}(\sigma_{\mathrm{B}}^{\alpha})(\alpha=1,2,3)$ are the Pauli matrices of A(B), and I is the identity matrix. The initial state of each bath is denoted by $|\varphi(0)\rangle$. The evolved reduced density matrix of the central qubits is derived as

$$\begin{aligned}\rho_{\mathrm{AB}}(t)=&\frac{1}{4}\{[(1+c_3)(|ee\rangle\langle ee|+|gg\rangle\langle gg|)\\&+(1-c_3)(|eg\rangle\langle eg|+|ge\rangle\langle ge|)\\&+[(c_1-c_2)R^2(t)|ee\rangle\langle gg|\\&(c_1+c_2)|R(t)|^2|eg\rangle\langle eg|+\mathrm{H.c.}]\},\end{aligned} \tag{7}$$

where H.c. means Hermitian conjugate, $R(t)=\langle\varphi_-(t)|\varphi_+(t)\rangle$, $|\varphi_{\pm}(t)\rangle=\mathrm{e}^{-\mathrm{i}H_{\pm}t}|\varphi(0)\rangle$, and $H_{\pm}$ is the corresponding Hamiltonian[42,43]

$$H_{\pm}=\sum_k\varepsilon_k^{\pm}\left(\gamma_{k,\pm}^{\dagger}\gamma_{k,\pm}-1/2\right). \tag{8}$$

Here, $\varepsilon_k^{\pm}=2J\sqrt{1+\lambda_{\pm}^2-2\lambda_{\pm}\cos ka}$, $\lambda_+=\lambda+\delta$, $\lambda_-=\lambda$, $k=2n\pi/(Na)$, and $n=-N/2,\ldots,N/2-1$.

Each bath is initially prepared in the ground state of H_-, $|\varphi(0)\rangle=|G\rangle_g$, and $\gamma_{k,-}|G\rangle_g=0$. The ground state $|G\rangle_e$ of H_+ satisfies $\gamma_{k,+}|G\rangle_e=0$, and is related to $|G\rangle_g$ by

$$|G\rangle_g=\prod_{k>0}\left[\cos\alpha_k-\mathrm{i}\sin\alpha_k\gamma_{k,+}^{\dagger}\gamma_{k,-}^{\dagger}\right]|G\rangle_e. \tag{9}$$

Here,

$$\alpha_k = \left(\theta_k^- - \theta_k^+\right)/2, \quad \theta_k^\pm = \arctan\frac{-\sin ka}{\cos ka - \lambda_\pm},$$

and $R(t)$ is derived as

$$R(t) = \exp\left[\mathrm{i}\sum_k \left(\varepsilon_k^- - \varepsilon_k^+\right)/2\right] \times \prod_{k>0}\left(\cos^2\alpha_k + \sin^2\alpha_k \mathrm{e}^{2\mathrm{i}\varepsilon_k^+ t}\right). \quad (10)$$

4. Coherence

We choose parameters $(c_1, c_2) = (1, -c_3)$ in the initial-state $\rho_{\mathrm{AB}}(0)$. By Eq. (3), the analytical expression for quantum discord is given by

$$D(\rho_{\mathrm{AB}}) = \left[(1-c)\log_2(1-c) + (1+c)\log_2(1+c)\right]/2,$$
$$c = \min\left\{|R(t)|^2, c_3\right\}. \quad (11)$$

Using Eq. (4), the GMQD is expressed as

$$G(\rho_{\mathrm{AB}}) = \frac{(1+c_3)^2}{2}|R(t)|^4 + c_3^2 - \max\left\{\frac{(1+c_3^2)}{2}|R(t)|^4, c_3^2\right\}. \quad (12)$$

According to Eq. (1), we can obtain the relative entropy coherence as follows:

$$C_{\mathrm{r}}(\rho_{\mathrm{AB}}) = \sum_{i=1}^{4}\lambda_i\log_2\lambda_i - \frac{1}{2}\left[(1-c_3)\log_2\frac{1-c_3}{4} + (1+c_3)\log_2\frac{1+c_3}{4}\right],$$
$$\lambda_{1,2} = (1+c_3)\left(1\pm|R(t)|^2\right)/4,$$
$$\lambda_{3,4} = (1-c_3)\left(1\pm|R(t)|^2\right)/4. \quad (13)$$

With Eq. (2), the expression for the l_1 norm of coherence is obtained

$$C_{l_1}(\rho_{\mathrm{AB}}) = \sum_{i\neq j}|\rho_{ij}| = |R(t)|^2. \quad (14)$$

Note that the coherence is a basis-dependent quantity, and Eqs. (13) and (14) are obtained in the standard basis with $\{|gg\rangle\langle gg|, |ge\rangle\langle ge|, |eg\rangle\langle eg|, |ee\rangle\langle ee|\}$.

The relative entropy coherence and the l_1 norm of coherence are plotted as a function of the magnetic intensity λ and time t in Fig. 1. The unit of quantity t is $1/J$. We find that both the sudden change of the relative entropy coherence and the l_1 norm of coherence occur at the critical point $\lambda = \lambda_{\mathrm{c}} = 1$. The system has strong quantum fluctuations, which can cause $R(t)$ to change rapidly, and a sudden change may occur. The sudden change of coherence is associated with the QPT. The quantum coherence can also be served as a means to probe the QPT, which is easier to evaluate than quantum discord and entanglement.

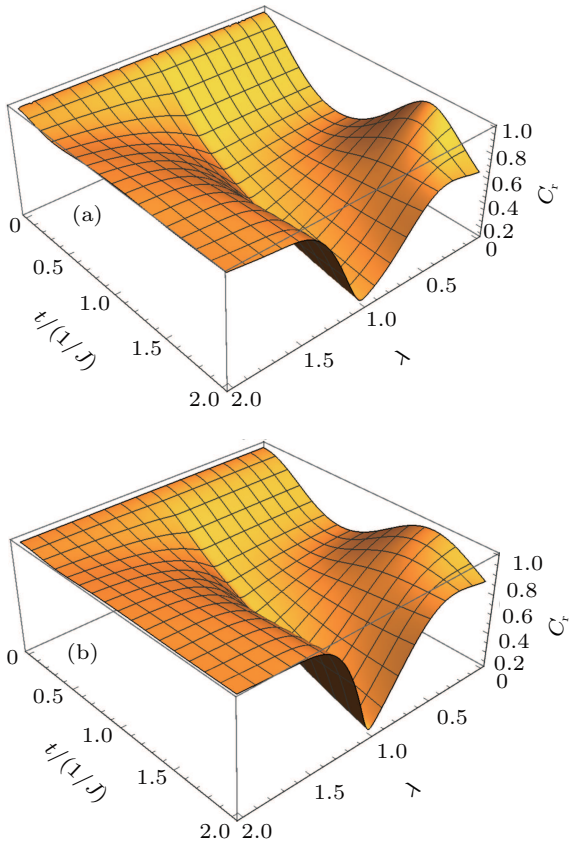


Fig. 1. (color online) (a) The relative entropy coherence C_{r} and (b) the l_1 norm of coherence C_{l_1} as a function of t and λ for $N = 100$, $c_3 = 0.8$, and $\delta = 0.1$.

We plot the evolution of quantum discord, the total correlation, the classical correlation, and the relative entropy coherence for $N = 100$, $c_3 = 0.8$, $\lambda = 0.9$, and $\delta = 0.1$ in Fig. 2. The sudden transition phenomenon between quantum discord and classical correlation occurs at t_0, with $|R(t_0)|^2 = c_3$. The quantum coherence is invariant for $t < t_0$, and the classical correlation is invariant for $t > t_0$. It is interesting to see that the evolution of quantum coherence displays the behavior of quantum discord and classical correlation at different cases. For $t < t_0$, the evolution of quantum coherence is the same as classical correlation, and is the same as quantum correlation for $t > t_0$. It means that the coherence is also part of the total correlation. This result is similar to the dynamics of correlation and coherence under the bit-flip and the phase-flip channel in Ref. [18]. For $t < t_0$, the relative entropy distance between the system quantum state and the nearest classical state is a constant. For $t > t_0$, the parameter c in Eq. (11) varies from c_3 to $|R(t)|^2$. As a result, equation (11) can be re-arranged as Eq. (13). Thus, quantum discord is the same as quantum coherence. The measure of quantum coherence depends on the specific basis. The basis on subsystem B for the relative

entropy measure of Eq. (9) is $\{|g\rangle\langle g|, |e\rangle\langle e|\}$. However, the measure of quantum discord is a basis-independent quantity, and we must select the optimal basis on subsystem B to ensure that classical correlation is maximum. For $t > t_0$, the optimal basis $\{B_k\}$ is $\{|g\rangle\langle g|, |e\rangle\langle e|\}$, which is the same as the measurement basis of the relative entropy coherence. Thus, quantum discord is the same as quantum coherence. The quantum coherence displays the behavior of classical correlation and quantum discord for $t < t_0$ and $t > t_0$, respectively, which is caused by the change of the optimal basis.

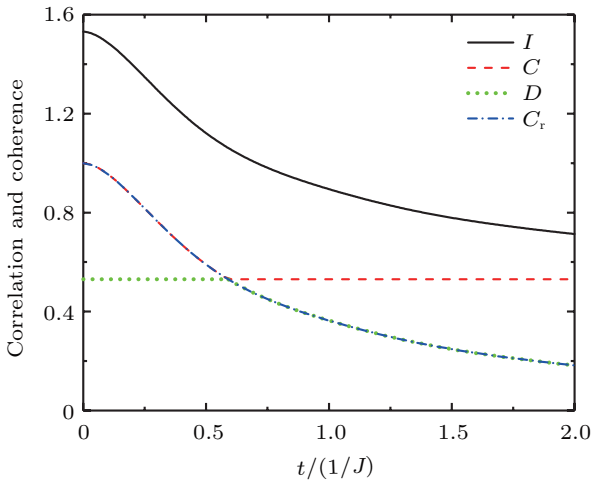


Fig. 2. (color online) Dynamics of total correlation I (black solid line), classical correlation C (red dashed line), quantum discord D (green dotted line), and the relative entropy coherence C_r (blue short dotted line) for $N = 100$, $c_3 = 0.8$, $\lambda = 0.9$, and $\delta = 0.1$.

Figure 3 shows the decay of coherence and correlation for $N = 100$, $c_3 = 0.8$, $\lambda = 0.9$, and $\delta = 0.1$. We find that the variation tendencies of GMQD, the relative entropy coherence, and the l_1 norm coherence are similar.

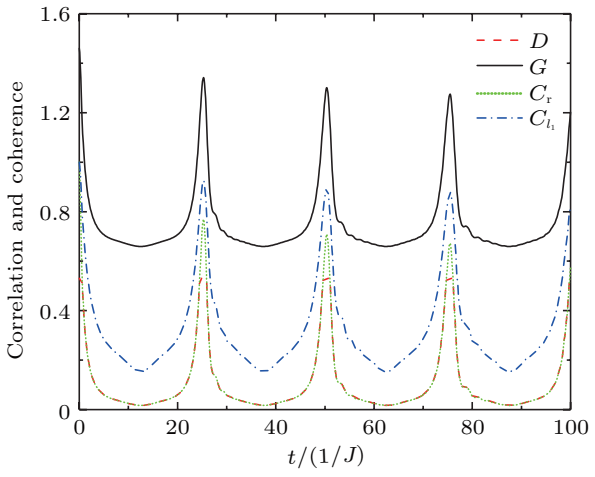


Fig. 3. (color online) Dynamics of quantum discord D (red dashed line), GMQD G (black solid line), the relative entropy coherence C_r (green dotted line), and the l_1 norm coherence C_{l_1} (blue dashed dotted line) for $N = 100$, $c_3 = 0.8$, $\lambda = 0.9$, and $\delta = 0.1$.

5. Conclusions

In this paper, we have investigated the dynamics of two qubits coupled to two separate baths modeled by the Ising spin chain, and derived the analytical expressions of the relative entropy coherence, the l_1 norm of coherence and correlations. Firstly, we have found that quantum coherence can be used to probe the existence of QPT. Then, it has been shown that quantum coherence displays the behavior of classical correlation and quantum discord for $t < t_0$ and $t > t_0$, respectively. At t_0 the sudden transition from quantum discord to classical correlation occurs, and the role of quantum coherence during classical correlation and quantum discord changes. This phenomenon is caused by the changes of the optimal basis for quantum discord. Our results may deepen the understanding of essential relations between coherence and correlation in quantum theory.

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