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Double compound combination synchronization among eight n-dimensional chaotic systems

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Depending on double compound synchronization and compound combination synchronization, a new kind of synchronization is introduced which is the double compound combination synchronization (DCCS) of eight n-dimensional chaotic systems. This kind may be considered as a generalization of many types of synchronization. In the communication, based on many of drive and response systems, the transmitted and received signals will be more secure. Using the Lyapunov stability theory and nonlinear feedback control, analytical formulas of control functions are obtained to insure our results. The corresponding analytical expression and numerical treatment are used to show the validity and feasibility of our proposed synchronization scheme. The eight memristor-based Chua oscillators are considered as an example. Other examples can be similarly investigated. The proposed synchronization technique is supported using the MATLAB simulation outcomes. We obtain the same results of numerical treatment of our synchronization using simulation observations of our example.

Keywords: double compound synchronization, compound combination synchronization, chaotic system, Lyapunov stability

PACS: 05.45.–a, 05.45.Gg, 05.45.Pq, 05.45.Xt **DOI:** 10.1088/1674-1056/27/8/080502

1. Introduction

Chaos synchronization and control theory have a great interest due to their potential applications in several fields during the recent decades.[1–5] Different synchronization types have been introduced and many theoretical results have been achieved, such as complete synchronization,[6–8] anti-synchronization,[9–11] lag synchronization,[12,13] projective synchronization,[13,14] function projective synchronization,[15] etc. Most of the pervious types of synchronization concentrate on synchronization between one drive and one response systems. There exist many techniques of control to achieve synchronization such as sliding mode control, random phase control, adaptive passive control, and others.[16–24]

Synchronization of systems with more than one drive and response systems is very interesting and important, because the transmitted signals may own stronger anti-translated and anti-attack ability. Complexity and the formation of the drive signals are also peppy factors to include communication security. Researchers studied complex schemes to transmit the information signals. Runzi *et al.* investigated combination synchronization of two drive and one response systems,[25,26] while the synchronization of two drive and two response systems (combination–combination synchronization) is studied by Sun *et al.*[27] Mahmoud *et al.* introduced the generalization of combination–combination synchronization of chaotic systems.[28,29] A new kind of compound synchronization (synchronization between three drive systems and one response system) has been illustrated in Ref. [30]. Zhang and Deng studied the double compound synchronization, which is the synchronization of four drive and two response systems.[31] The compound–combination synchronization (synchronization of three drive and two response systems) was presented in Ref. [32].

In this paper, we introduce a new type of synchronization which is a generalization of many pervious types of synchronization and more complexity. This type is called double compound combination synchronization (DCCS) among eight chaotic systems (four drive and other four response chaotic or hyperchaotic systems). In the last years, a new circuit element, namely memristor which is basically a fourth class of electrical circuit, joining the inductor, the capacitor, and the resistor. Memristor produced the circuit systems which have chaotic and hyperchaotic property. In Ref. [33], the authors replaced Chua's diodes with memristors which proposed several nonlinear oscillators from Chua's oscillators. Wen *et al.* regarded the fuzzy modeling and synchronization of memristor-based Lorenz circuits with memristor-based Chua's circuits.[34] Memristor was postulated as the missing fourth passive circuit element.[35] A great breakthrough that a passive two terminal physical implementation was immediately related to memristor theory was obtained by Hewlett–Packard Labs.[36] This new circuit element can be useful for low-power computation and storage to store information and data.[37] Memristor, also, can be used to implement programmable

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analog circuits, leveraging memristor's fine-resolution programmable resistance without causing perturbations due to parasitic components,[38] and soon.

This paper is organized as follows: Section 2, contains the design of DCCS scheme among eight n-dimensional chaotic systems. The definition of this new type of synchronization is given. In Section 3, the DCCS for eight identical memristor-based Chua oscillators as an example, is investigated by using the Lyapunov stability theory and nonlinear feedback control. The analytical expressions of the control functions are derived. In Section 4, the numerical treatments of testing the analytical formula of the control functions to achieve the DCCS for our example are illustrated by using fourth order Runge–Kutta method. The simulation of the chaotic memristor-based Chua oscillator is contracted by using MATLAB/Simulink. We gain a good agreement between numerical treatments and simulation results for our proposed synchronization. The conclusion is finally stated in the last section.

2. Double compound combination synchronization scheme

We will present the technique which is used to achieve the DCCS of four drive and four response n-dimensional chaotic systems. The first scaling drive system is given as follows:

$$\dot{x_1}(t) = f_1(x_1(t)); \tag{1}$$

the second scaling drive system is written as

$$\dot{x_2}(t) = f_2(x_2(t)); \tag{2}$$

the first base drive system is described as

$$\dot{x_3}(t) = f_3(x_3(t)); \tag{3}$$

the second base drive system is drown as:

$$\dot{x_4}(t) = f_4(x_4(t)); \tag{4}$$

and the first response system is written as:

$$\dot{y_1}(t) = g_1(y_1(t)) + u_1; \tag{5}$$

the second response system is defined as:

$$\dot{y_2}(t) = g_2(y_2(t)) + u_2; \tag{6}$$

the third response system is given by:

$$\dot{y_3}(t) = g_3(y_3(t)) + u_3; \tag{7}$$

the fourth response system is:

$$\dot{y_4}(t) = g_4(y_4(t)) + u_4, \tag{8}$$

where

$$x_i = \mathrm{diag}\,(x_{i1}, x_{i2}, \ldots, x_{in}),$$
$$y_i = \mathrm{diag}\,(y_{i1}, y_{i2}, \ldots, y_{in})$$

are the state variables of systems (1)–(8),

$$f_i(x_i) = \mathrm{diag}\,(f_{i1}(x_i), f_{i2}(x_i), \ldots, f_{in}(x_i)),$$
$$g_i(y_i) = \mathrm{diag}\,(g_{i1}(y_i), g_{i2}(y_i), \ldots, g_{in}(y_i))$$

are eight diagonal matrices of continuous functions and

$$u_i(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) = \mathrm{diag}\,(u_{i1}, u_{i2}, \ldots, u_{in}), \quad i = 1, 2, 3, 4$$

are four control functions of the response systems (5)–(8). The topological structure diagram of the drive systems and the response systems is shown in Fig. 1. Now, we will state the new definition of the DCCS as

Definition 1 For an eight constant diagonal matrices $A_i, B_i \in (\mathbb{R}^n \times \mathbb{R}^n)$ and at least one of $B_i \neq 0$ $(i = 1, 2, 3, 4)$, such that

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \|(A_1x_1 + A_2x_2)(A_3x_3 + A_4x_4) - B_1y_1 - B_2y_2 - B_3y_3 - B_4y_4\| = 0, \tag{9}$$

then the response systems (5)–(8) are synchronized as DCCS with the drive systems (1)–(4), where $\|\cdot\|$ means the norm of a matrix and $e = \mathrm{diag}\,(e_1, e_2, \ldots, e_n)$ is a diagonal error matrix.

Remark 1 If two of $B_i = 0$, $(i = 1, 2, 3, 4)$, this kind of synchronization turned into a double compound synchronization.

Remark 2 The compound combination synchronization is a special case of our kind, if one of $A_i = 0$ and two of $B_i = 0$, $(i = 1, 2, 3, 4)$.

Remark 3 The synchronization of (i) one drive and one response chaotic systems, (ii) two drive and one response systems, (iii) two drive and two response systems, and so on are special cases of our eight chaotic systems.

Remark 4 For the choice, $A_i = 0$ and three of $B_i = 0$, $(i = 1, 2, 3, 4)$, chaos control problem can be obtained.

Remark 5 The error $e = (A_1x_1 + A_2x_2)(A_3x_3 + A_4x_4) - B_1y_1 - B_2y_2 - B_3y_3 - B_4y_4$ can be written for example as $e = [A_1x_1(A_3x_3 + A_4x_4) - B_1y_1 - B_2y_2] + [A_2x_2(A_3x_3 + A_4x_4) - B_3y_3 - B_4y_4]$, which is considered as the sum of two errors of the synchronization of compound combination. So, the new Definition 1 can be considered as a DCCS.

Let $A_i = \mathrm{diag}\,(a_{i1}, a_{i2}, \ldots, a_{in})$, $B_i = \mathrm{diag}\,(b_{i1}, b_{i2}, \ldots, b_{in})$, $i = 1, 2, 3, 4$, therefore, the error system can be described as:

$$\begin{aligned}
e_1 &= (a_{11}x_{11} + a_{21}x_{21})(a_{31}x_{31} + a_{41}x_{41}) - (b_{11}y_{11} + b_{21}x_{21} + b_{31}x_{31} + b_{41}x_{41}),\\
e_2 &= (a_{12}x_{12} + a_{22}x_{22})(a_{32}x_{32} + a_{42}x_{42}) - (b_{12}y_{12} + b_{22}x_{22} + b_{32}x_{32} + b_{42}x_{42}),\\
&\cdots,\\
e_n &= (a_{1n}x_{1n} + a_{2n}x_{2n})(a_{3n}x_{3n} + a_{4n}x_{4n}) - (b_{1n}y_{1n} + b_{2n}x_{2n} + b_{3n}x_{3n} + b_{4n}x_{4n}).
\end{aligned} \tag{10}$$

From systems (10) and (1)–(4), the error dynamical system can be given as:

$$\begin{aligned}
\dot{e}_1 &= (a_{11}f_{11} + a_{21}f_{21})(a_{31}x_{31} + a_{41}x_{41}) + (a_{11}x_{11} + a_{21}x_{21})(a_{31}f_{31} + a_{41}f_{41}) - (b_{11}g_{11} + b_{21}g_{21}\\
&\quad + b_{31}g_{31} + b_{41}g_{41}) - (b_{11}u_{11} + b_{21}u_{21} + b_{31}u_{31} + b_{41}u_{41}),\\
\dot{e}_2 &= (a_{12}f_{12} + a_{22}f_{22})(a_{32}x_{32} + a_{42}x_{42}) + (a_{12}x_{12} + a_{22}x_{22})(a_{32}f_{32} + a_{42}f_{42}) - (b_{12}g_{12} + b_{22}g_{22}\\
&\quad + b_{32}g_{32} + b_{42}g_{42}) - (b_{12}u_{12} + b_{22}u_{22} + b_{32}u_{32} + b_{42}u_{42}),\\
&\cdots,\\
\dot{e}_n &= (a_{1n}f_{1n} + a_{2n}f_{2n})(a_{3n}x_{3n} + a_{4n}x_{4n}) + (a_{1n}x_{1n} + a_{2n}x_{2n})(a_{3n}f_{3n} + a_{4n}f_{4n}) - (b_{1n}g_{1n} + b_{2n}g_{2n}\\
&\quad + b_{3n}g_{3n} + b_{4n}g_{4n}) - (b_{1n}u_{1n} + b_{2n}u_{2n} + b_{3n}u_{3n} + b_{4n}u_{4n}).
\end{aligned} \tag{11}$$

Denote

$$\begin{aligned}
&U_1 = b_{11}u_{11} + b_{21}u_{21} + b_{31}u_{31} + b_{41}u_{41},\ \ U_2 = b_{12}u_{12} + b_{22}u_{22} + b_{32}u_{32} + b_{42}u_{42},\\
&\quad \cdots,\ U_n = b_{1n}u_{1n} + b_{2n}u_{2n} + b_{3n}u_{3n} + b_{4n}u_{4n}.
\end{aligned} \tag{12}$$

We will introduce a theorem for the analytical expression of the control functions $U = \mathrm{diag}\,(U_1, U_2, \ldots, U_n)$.

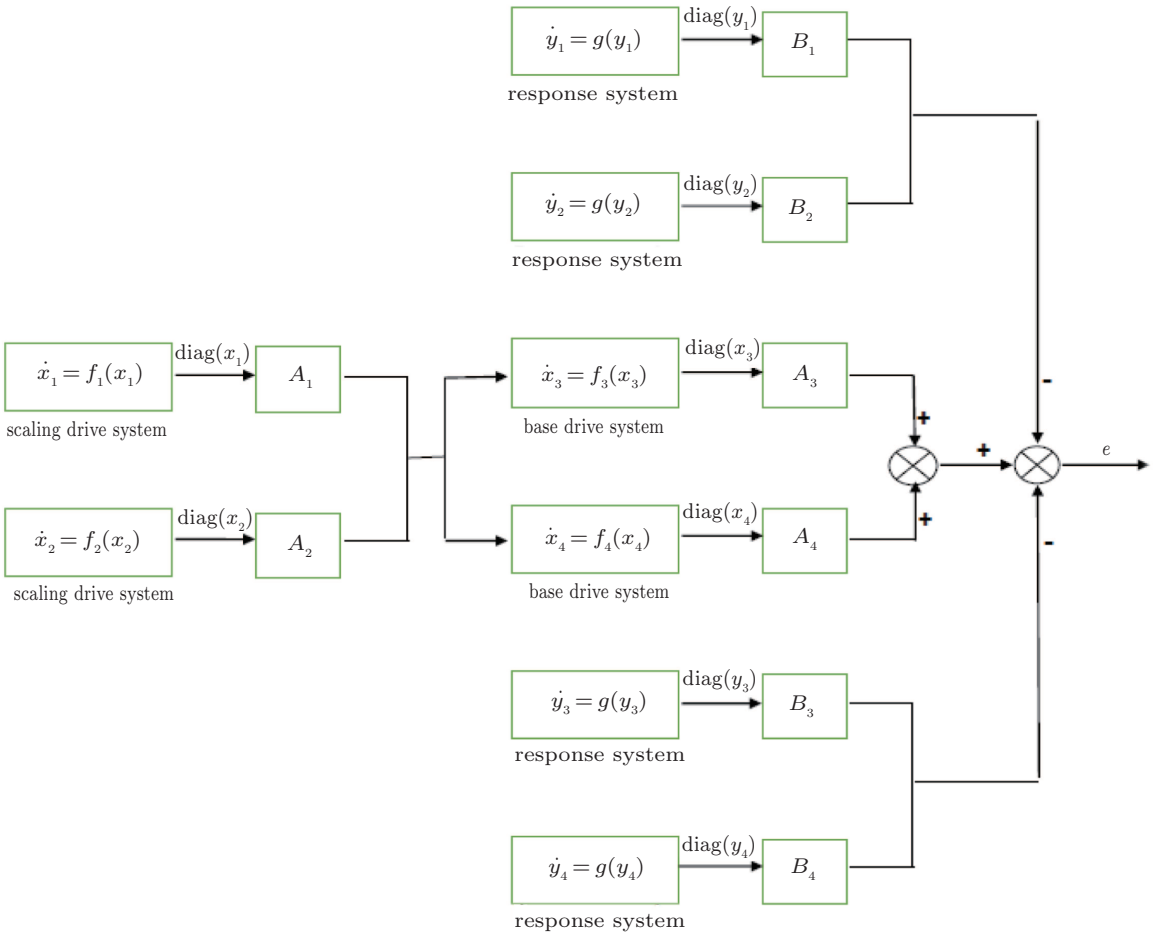


Fig. 1. (color online) The topological structure diagram of the drive systems (1)–(4) and the response systems (5)–(8).

Theorem 1 If the control functions are selected as follows:

$$\begin{aligned}
U_1 &= (a_{11}f_{11}+a_{21}f_{21})(a_{31}x_{31}+a_{41}x_{41})+(a_{11}x_{11}+a_{21}x_{21})(a_{31}f_{31}+a_{41}f_{41})-(b_{11}g_{11}+b_{21}g_{21}\\
&\quad +b_{31}g_{31}+b_{41}g_{41})+k_1e_1,\\
U_2 &= (a_{12}f_{12}+a_{22}f_{22})(a_{32}x_{32}+a_{42}x_{42})+(a_{12}x_{12}+a_{22}x_{22})(a_{32}f_{32}+a_{42}f_{42})-(b_{12}g_{12}+b_{22}g_{22}\\
&\quad +b_{32}g_{32}+b_{42}g_{42})+k_2e_2,\\
&\quad \cdots,\\
U_n &= (a_{1n}f_{1n}+a_{2n}f_{2n})(a_{3n}x_{3n}+a_{4n}x_{4n})+(a_{1n}x_{1n}+a_{2n}x_{2n})(a_{3n}f_{3n}+a_{4n}f_{4n})-(b_{1n}g_{1n}+b_{2n}g_{2n}\\
&\quad +b_{3n}g_{3n}+b_{4n}g_{4n})+k_ne_n.
\end{aligned} \tag{13}$$

then the drive systems (1)–(4) and the response systems (5)–(8) achieved DCCS, where $k_1,k_2,\ldots,k_n$ are real positive numbers s.t. $K=\mathrm{diag}\,(k_1,k_2,\ldots,k_n)$ is a positive-definite diagonal gain matrix.

Proof We may define a Lyapunov function as:

$$V(e_1,e_2,\ldots,e_n)=\frac{1}{2}\sum_{i=1}^{n}e_i^2, \tag{14}$$

and its total time derivative is:

$$\dot{V}=\sum_{i=1}^{n}e_i\dot{e}_i, \tag{15}$$

from Eq. (11), we have

$$\begin{aligned}
\dot{V}=\sum_{i=1}^{n}e_i[&(a_{1i}f_{1i}+a_{2i}f_{2i})(a_{3i}x_{3i}+a_{4i}x_{4i})\\
&+(a_{1i}x_{1i}+a_{2i}x_{2i})(a_{3i}f_{3i}+a_{4i}f_{4i})\\
&-(b_{1i}g_{1i}+b_{2i}g_{2i}+b_{3i}g_{3i}+b_{4i}g_{4i})-U_i].
\end{aligned} \tag{16}$$

Substituting system (13) into Eq. (16), we obtain

$$\dot{V}=-\sum_{i=1}^{n}k_ie_i^2, \tag{17}$$

since $\dot{V}$ is negative-definite and $V(t)$ is positive-definite, the Lyapunov direct method implies that $\lim\limits_{t\to\infty}\|e\|=0$. Therefore, the drive systems (1)–(4) and the response systems (5)–(8) are synchronized.

Corollary 1 The double compound synchronization can be achieved between the drive systems (1)–(4) and the response systems (5)–(8) if one takes $B_3=B_4=0$. So the control functions take the form:

$$\begin{aligned}
U_1 &= (a_{11}f_{11}+a_{21}f_{21})(a_{31}x_{31}+a_{41}x_{41})+(a_{11}x_{11}+a_{21}x_{21})(a_{31}f_{31}+a_{41}f_{41})-(b_{11}g_{11}+b_{21}g_{21})+k_1e_1,\\
U_2 &= (a_{12}f_{12}+a_{22}f_{22})(a_{32}x_{32}+a_{42}x_{42})+(a_{12}x_{12}+a_{22}x_{22})(a_{32}f_{32}+a_{42}f_{42})-(b_{12}g_{12}+b_{22}g_{22})+k_2e_2,\\
&\quad \cdots,\\
U_n &= (a_{1n}f_{1n}+a_{2n}f_{2n})(a_{3n}x_{3n}+a_{4n}x_{4n})+(a_{1n}x_{1n}+a_{2n}x_{2n})(a_{3n}f_{3n}+a_{4n}f_{4n})-(b_{1n}g_{1n}+b_{2n}g_{2n})+k_ne_n.
\end{aligned} \tag{18}$$

Corollary 2 For the choice $A_4=0$, $B_3=B_4=0$, then the compound combination synchronization between the drive systems (1)–(4) and the response systems (5)–(8) is achieved under the following control functions:

$$\begin{aligned}
U_1 &= a_{31}x_{31}(a_{11}f_{11}+a_{21}f_{21})+a_{31}f_{31}(a_{11}x_{11}+a_{21}x_{21})-(b_{11}g_{11}+b_{21}g_{21})+k_1e_1,\\
U_2 &= a_{32}x_{32}(a_{12}f_{12}+a_{22}f_{22})+a_{32}f_{32}(a_{12}x_{12}+a_{22}x_{22})-(b_{12}g_{12}+b_{22}g_{22})+k_2e_2,\\
&\quad \cdots,\\
U_n &= a_{3n}x_{3n}(a_{1n}f_{1n}+a_{2n}f_{2n})+a_{3n}f_{3n}(a_{1n}x_{1n}+a_{2n}x_{2n})-(b_{1n}g_{1n}+b_{2n}g_{2n})+k_ne_n.
\end{aligned} \tag{19}$$

Corollary 3 If we choose $A_1=A_2=A_3=A_4=0$, $B_2=B_3=B_4=0$, then the response system (5) will be considered as a chaos control problem, under the following control functions:

$$\begin{aligned}
U_1 &= b_{11}g_{11}+k_1e_1,\\
U_2 &= b_{12}g_{12}+k_2e_2,\\
&\quad \cdots,\\
U_n &= b_{1n}g_{1n}+k_ne_n.
\end{aligned} \tag{20}$$

3. Illustrative example

In this section, we apply the synchronization scheme of Section 2 to study the DCCS for eight identical memristor-based Chua oscillators as an example. The first scaling drive memristor-based Chua oscillator is given by

$$\begin{aligned}
\dot{x}_{11} &= c_{11}x_{12}+c_{12}x_{11}-c_{13}x_{11}x_{14}^2,\\
\dot{x}_{12} &= x_{11}-x_{12}+x_{13},\\
\dot{x}_{13} &= -c_{14}x_{12}-c_{15}x_{13},
\end{aligned}$$

$$\dot{x}_{14} = x_{11}, \tag{21}$$

where c_{1i}, $(i = 1,2,\ldots,5)$ are constant parameters. We calculated the Lyapunov exponents of system (21) and they are: $\lambda_1 = 0.2522$, $\lambda_2 = -0.0296$, $\lambda_3 = -0.0419$, $\lambda_4 = -7.5153$, for $c_{11} = 15.6$, $c_{12} = 3$, $c_{13} = 21$, $c_{14} = 15$, $c_{15} = 0.5$ and $x_{10} = \mathrm{diag}\,(0.2, 0.2, 0.2, 0.2)$. It is clear that one of them is positive which means that its solution is chaotic as shown in Fig. 2.

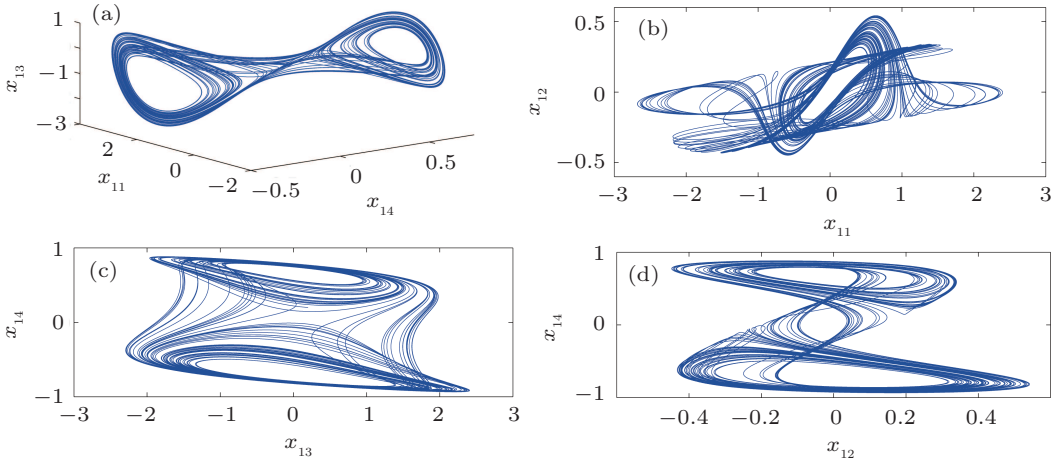


Fig. 2. (color online) Chaotic attractor of system (21) when $c_{11} = 15.6$, $c_{12} = 3$, $c_{13} = 21$, $c_{14} = 15$, $c_{15} = 0.5$ and $x_{10} = \mathrm{diag}\,(0.2, 0.2, 0.2, 0.2)$: (a) x_{14}, x_{11}, x_{13} space, (b) x_{11}, x_{12} space, (c) x_{13}, x_{14} space, (d) x_{12}, x_{14} space.

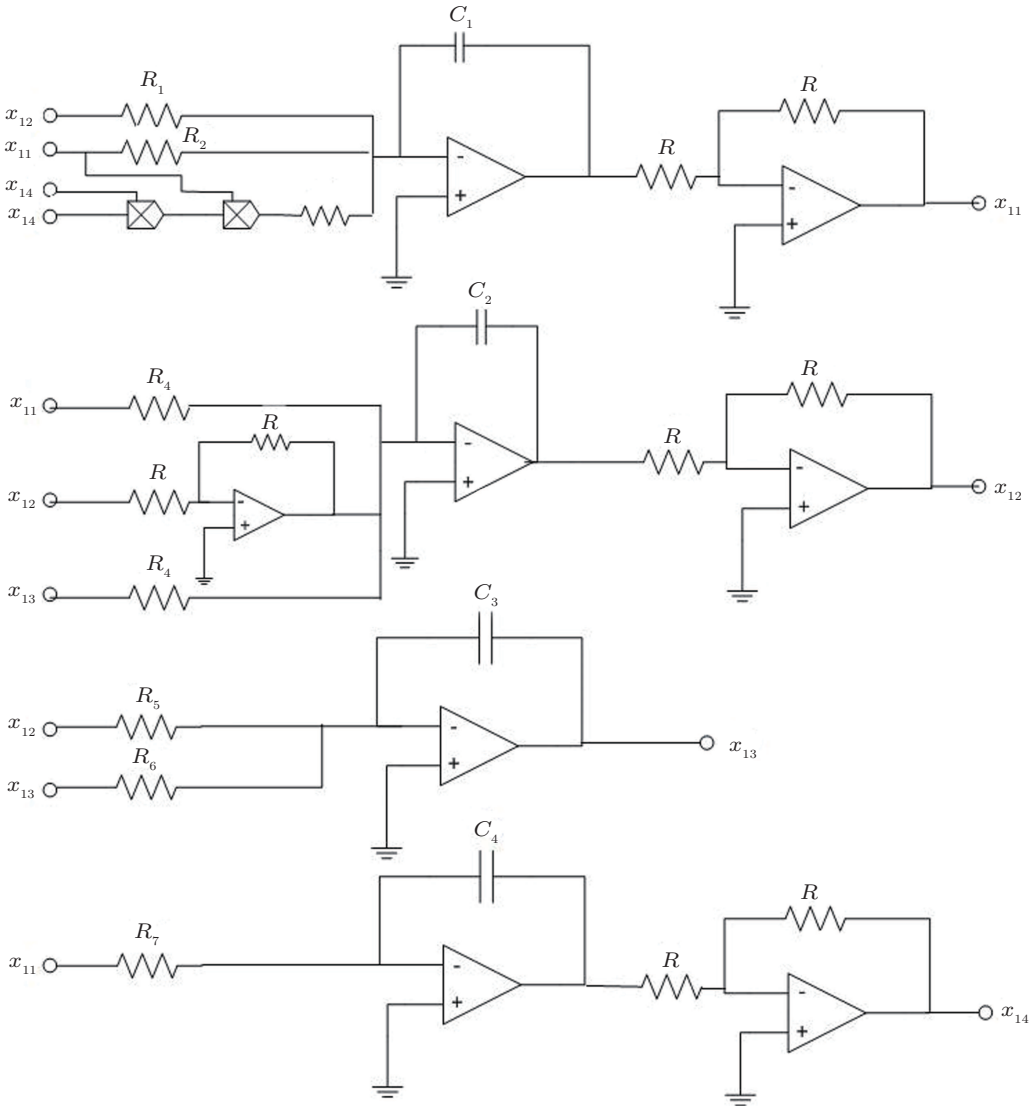


Fig. 3. Circuit implementation of system (21).

We designed circuit implementation for system (21) as shown in Fig. 3. The circuit equations in terms of the circuit parameters are:

$$\begin{aligned}
\dot{x}_{11} &= \frac{1}{R_2C_1}x_{12} + \frac{1}{R_1C_1}x_{11} - \frac{1}{R_3C_1}x_{11}x_{14}^2,\\
\dot{x}_{12} &= \frac{1}{R_4C_2}x_{11} - \frac{1}{R_4C_2}x_{12} + \frac{1}{R_4C_2}x_{13},\\
\dot{x}_{13} &= -\frac{1}{R_5C_3}x_{12} - \frac{1}{R_6C_3}x_{13},\\
\dot{x}_{14} &= \frac{1}{R_7C_4}x_{11}.
\end{aligned} \tag{22}$$

The circuit consists of four channels to realize the integration, addition, and subtraction of the state variables x_{11}, x_{12}, x_{13}, and x_{14}, respectively. These variables are corresponding to the state voltages of the four channels, respectively. For the same parameters values of system (21) in Fig. 2, the corresponding values of circuit elements are $R_1 = 16.67$ kΩ, $R_2 = 2.18$ kΩ, $R_3 = 1.59$ kΩ, $R_4 = 40$ kΩ, $R_5 = 3.33$ kΩ, $R_6 = 100$ kΩ, $R_7 = 100$ kΩ, $R = 10$ kΩ, and the capacitors are $C_1 = 30$ μF, $C_2 = 25$ μF, $C_3 = 20$ μF, and $C_4 = 10$ μF. The circuit implementation of system (22) is drown in Fig. 3. The corresponding simulation observations of the chaotic memristor-based Chua oscillator (21) appear in Fig. 4 which agree with numerical simulation in Fig. 2.

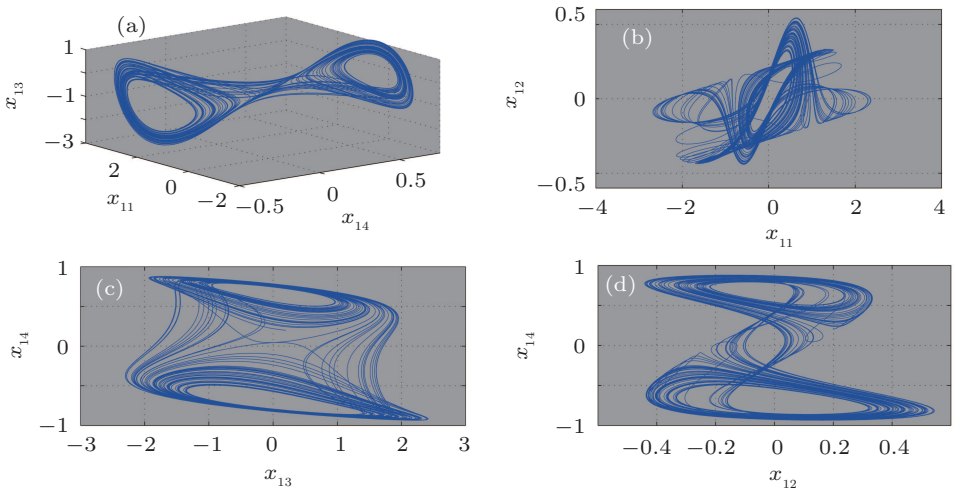


Fig. 4. (color online) Simulation observations of system (21) for the same parameters in Fig. 2: (a) x_{14},x_{11},x_{13} space; (b) x_{11},x_{12} space; (c) x_{13},x_{14} space; (d) x_{12},x_{14} space.

The second scaling drive memristor-based Chua oscillator is given by

$$\begin{aligned}
\dot{x}_{21} &= c_{21}x_{22} + c_{22}x_{21} - c_{23}x_{21}x_{24}^2,\\
\dot{x}_{22} &= x_{21} - x_{22} + x_{23},\\
\dot{x}_{23} &= -c_{24}x_{22} - c_{25}x_{23},\\
\dot{x}_{24} &= x_{21},
\end{aligned} \tag{23}$$

and the two base drive memristor-based Chua oscillators are shown below

$$\begin{aligned}
\dot{x}_{31} &= c_{31}x_{32} + c_{32}x_{31} - c_{33}x_{31}x_{34}^2,\\
\dot{x}_{32} &= x_{31} - x_{32} + x_{33},\\
\dot{x}_{33} &= -c_{34}x_{32} - c_{35}x_{33},\\
\dot{x}_{34} &= x_{31},
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{x}_{41} &= c_{41}x_{42} + c_{42}x_{41} - c_{43}x_{41}x_{44}^2,\\
\dot{x}_{42} &= x_{41} - x_{42} + x_{43},\\
\dot{x}_{43} &= -c_{44}x_{42} - c_{45}x_{43},\\
\dot{x}_{44} &= x_{41},
\end{aligned} \tag{25}$$

where c_{ij}, $(i = 2,3,4; j = 1,2,\ldots,5)$ are constant parameters. The four response memristor-based Chua oscillators are given, respectively, as

$$\begin{aligned}
\dot{y}_{11} &= d_{11}y_{12} + d_{12}y_{11} - d_{13}y_{11}y_{14}^2 + u_{11},\\
\dot{y}_{12} &= y_{11} - y_{12} + y_{13} + u_{12},\\
\dot{y}_{13} &= -d_{14}y_{12} - d_{15}y_{13} + u_{13},\\
\dot{y}_{14} &= y_{11} + u_{14},
\end{aligned} \tag{26}$$

$$\begin{aligned}
\dot{y}_{21} &= d_{21}y_{22} + d_{22}y_{21} - d_{23}y_{21}y_{24}^2 + u_{21},\\
\dot{y}_{22} &= y_{21} - y_{22} + y_{23} + u_{22},\\
\dot{y}_{23} &= -d_{24}y_{22} - d_{25}y_{23} + u_{23},\\
\dot{y}_{24} &= y_{21} + u_{24},
\end{aligned} \tag{27}$$

$$\begin{aligned}
\dot{y}_{31} &= d_{31}y_{32} + d_{32}y_{31} - d_{33}y_{31}y_{34}^2 + u_{31},\\
\dot{y}_{32} &= y_{31} - y_{32} + y_{33} + u_{32},\\
\dot{y}_{33} &= -d_{34}y_{32} - d_{35}y_{33} + u_{33},\\
\dot{y}_{34} &= y_{31} + u_{34},
\end{aligned} \tag{28}$$

$$\begin{aligned}
\dot{y}_{41} &= d_{41}y_{42} + d_{42}y_{41} - d_{43}y_{41}y_{44}^2 + u_{41},\\
\dot{y}_{42} &= y_{41} - y_{42} + y_{43} + u_{42},\\
\dot{y}_{43} &= -d_{44}y_{42} - d_{45}y_{43} + u_{43},\\
\dot{y}_{44} &= y_{41} + u_{44},
\end{aligned} \tag{29}$$

where d_{kj}, $(k = 1,2,\ldots,4; j = 1,2,\ldots,5)$ are constant parameters and u_{kl}, $(k = 1,2,\ldots,4; l = 1,2,\ldots,4)$ are the control functions.

According to Theorem 1, we obtain the control functions as follows:

$$
\begin{aligned}
U_1 &= (a_{31}x_{31}+a_{41}x_{41})[a_{11}(c_{11}x_{12}+c_{12}x_{12}-c_{13}x_{11}{x_{14}}^2)+a_{21}(c_{21}x_{22}+c_{22}x_{21}-c_{23}x_{21}{x_{24}}^2)]+(a_{11}x_{11}\\
&\quad +a_{21}x_{21})[a_{31}(c_{31}x_{32}+c_{32}x_{31}-c_{33}x_{31}{x_{34}}^2)+a_{41}(c_{41}x_{42}+c_{42}x_{41}-c_{43}x_{41}{x_{44}}^2)]-b_{11}(d_{11}y_{12}+d_{12}y_{11}\\
&\quad -d_{13}y_{11}{y_{14}}^2)-b_{21}(d_{21}y_{22}+d_{22}y_{21}-d_{23}y_{21}{y_{24}}^2)-b_{31}(d_{31}y_{32}+d_{32}y_{31}-d_{33}y_{31}{y_{34}}^2)-b_{41}(d_{41}y_{42}\\
&\quad +d_{42}y_{41}-d_{43}y_{41}{y_{44}}^2)+k_1e_1,\\
U_2 &= (a_{32}x_{32}+a_{42}x_{42})[a_{12}(x_{11}-x_{12}+x_{13})+a_{22}(x_{21}-x_{22}+x_{23})]+(a_{12}x_{12}+a_{22}x_{22})[a_{32}(x_{31}-x_{32}+x_{33})\\
&\quad +a_{42}(x_{41}-x_{42}+x_{43})]-b_{12}(y_{11}-y_{12}+y_{13})-b_{22}(y_{21}-y_{22}+y_{23})-b_{32}(y_{31}-y_{32}+y_{33})-b_{42}(y_{41}\\
&\quad -y_{42}+y_{43})+k_2e_2,\\
U_3 &= (a_{33}x_{33}+a_{43}x_{43})[a_{13}(-c_{14}x_{12}-c_{15}x_{13})+a_{23}(-c_{24}x_{22}-c_{25}x_{23})]+(a_{13}x_{13}+a_{23}x_{23})[a_{33}(-c_{34}x_{32}\\
&\quad -c_{35}x_{33})+a_{43}(-c_{44}x_{42}-c_{45}x_{43})]-b_{13}(-d_{14}y_{12}-d_{15}y_{13})-b_{23}(-d_{24}y_{22}-d_{25}y_{23})-b_{33}(-d_{34}y_{32}\\
&\quad -d_{35}y_{33})-b_{43}(-d_{34}y_{32}-d_{35}y_{33})+k_3e_3,\\
U_4 &= (a_{34}x_{34}+a_{44}x_{44})[a_{14}x_{11}+a_{24}x_{21}]+(a_{14}x_{14}+a_{24}x_{24})[a_{34}x_{31}+a_{44}x_{41}]-b_{14}y_{11}-b_{24}y_{21}-b_{34}y_{31}\\
&\quad -b_{44}y_{41}+k_4e_4,
\end{aligned}
\tag{30}
$$

From Eq. (15) the Lyapunov function for this example is: $V=\sum_{i=1}^{4}e_i$, which is positive and its derivative is negative using Eq. (30).

4. Numerical simulation

In this section, numerical treatments are illustrated to state the effectiveness of the suggested scheme of DCCS of eight identical memristor-based Chua oscillators. If we take, $c_{i1}=d_{i1}=15.6$, $c_{i2}=d_{i2}=3$, $c_{i3}=d_{i3}=21$, $c_{i4}=d_{i4}=15$, $c_{i5}=d_{i5}=0.5$, $(i=1,2,3,4)$,

$$
\begin{aligned}
A_1 &= \mathrm{diag}\,(a_{11},a_{12},a_{13},a_{14})=\mathrm{diag}\,(1,-1,2,1),\\
A_2 &= \mathrm{diag}\,(a_{21},a_{22},a_{23},a_{24})=\mathrm{diag}\,(2,2,2,2),\\
A_3 &= \mathrm{diag}\,(a_{31},a_{32},a_{33},a_{34})=\mathrm{diag}\,(1,1,-1,2),\\
A_4 &= \mathrm{diag}\,(a_{41},a_{42},a_{43},a_{44})=\mathrm{diag}\,(5,1,-1,1),\\
B_1 &= \mathrm{diag}\,(b_{11},b_{12},b_{13},b_{14})=\mathrm{diag}\,(2,3,2,2),\\
B_2 &= \mathrm{diag}\,(b_{21},b_{22},b_{23},b_{24})=\mathrm{diag}\,(4,1,3,-2),\\
B_3 &= \mathrm{diag}\,(b_{31},b_{32},b_{33},b_{34})=\mathrm{diag}\,(1,-1,2,3),\\
B_4 &= \mathrm{diag}\,(b_{41},b_{42},b_{43},b_{44})=\mathrm{diag}\,(2,3,-1,1),\\
K &= \mathrm{diag}\,(k_1,k_2,k_3,k_4)=\mathrm{diag}\,(2,1,3,2),
\end{aligned}
$$

and the initial conditions for the four drive systems (21)–(25) and the four response systems (26)–(29), are respectively,

$$
\begin{aligned}
x_{10} &= \mathrm{diag}\,(0.2,0.2,0.2,0.2),\\
x_{20} &= \mathrm{diag}\,(-0.4386,0.0531,1.2655,0.4452),\\
x_{30} &= \mathrm{diag}\,(-0.4386,0.0531,1.26550.4452),\\
x_{40} &= \mathrm{diag}\,(-0.4386,0.0531,1.2655,0.4452),\\
y_{10} &= \mathrm{diag}\,(1,1.2,1.4,0.8),\\
y_{20} &= \mathrm{diag}\,(1,-1,1.65,0.355),\\
y_{30} &= \mathrm{diag}\,(-4.21,-5.52,2.11,-1.14),\\
y_{40} &= \mathrm{diag}\,(0.5,1,1.2,-0.54).
\end{aligned}
$$

The corresponding numerical results are shown in Figs. 5 and 6. Figure 5 presents the state variables of the four drive systems and the four response systems. The synchronization errors converge to zero as shown in Fig. 6.

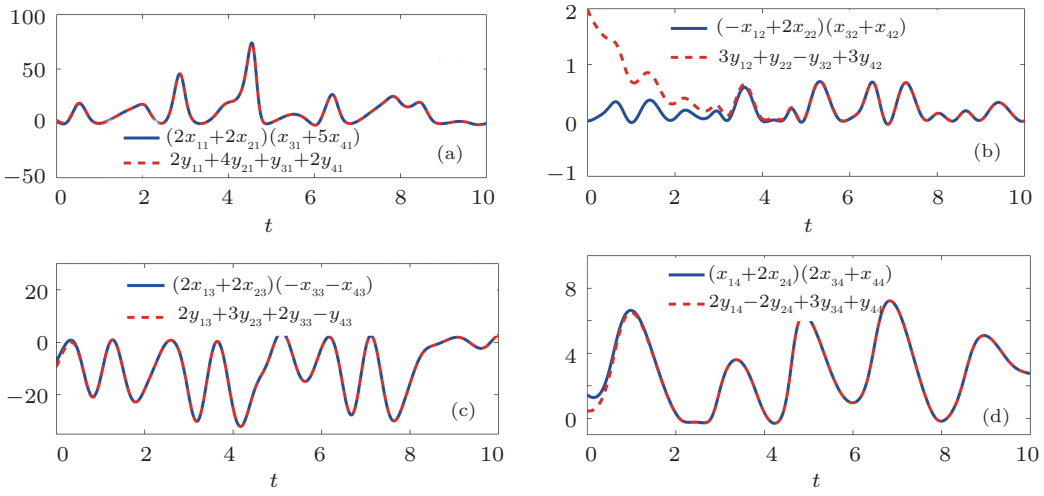


Fig. 5. (color online) State variables synchronization for the four drive systems (21)–(25) (solid curves) and the four response system (26)–(29) (dashed curves): (a) $(x_{11}+2x_{21})(x_{31}+5x_{41})$ and $(2y_{11}+4y_{21}+y_{31}+2y_{41})$ versus t; (b) $(-x_{12}+2x_{22})(x_{32}+x_{42})$ and $(3y_{12}+y_{22}-y_{32}+3y_{42})$ versus t; (c) $(2x_{13}+2x_{23})(-x_{33}-x_{43})$ and $(2y_{13}+3y_{23}+2y_{33}-y_{43})$ versus t; (d) $(x_{14}+2x_{24})(2x_{34}+x_{44})$ and $(2y_{14}-2y_{24}+3y_{34}+y_{44})$ versus t.

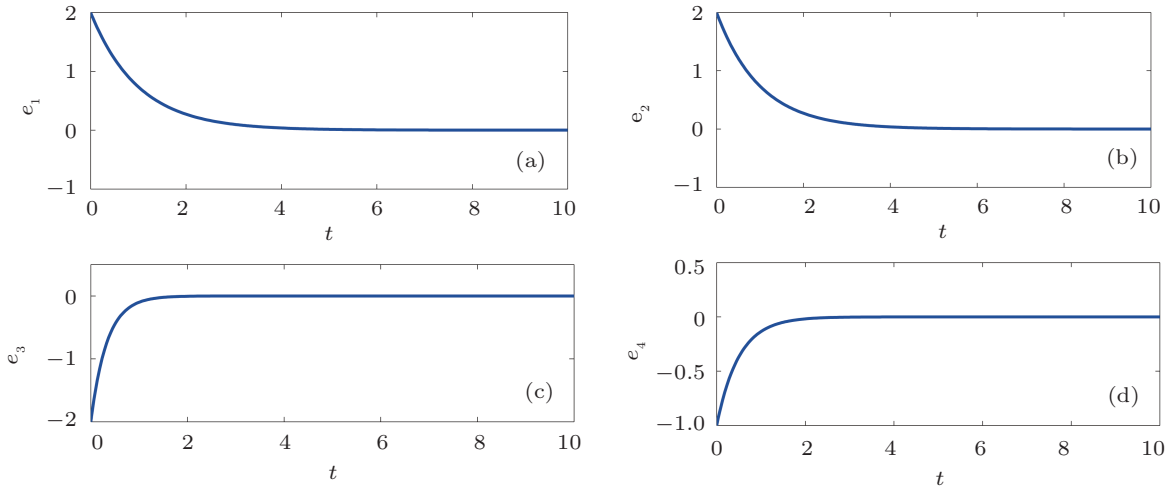


Fig. 6. (color online) Synchronization errors for the four drive systems (21)–(25) and the four response systems (26)–(29): (a) (t, e_1) diagram; (b) (t, e_2) diagram; (c) (t, e_3) diagram; (d) (t, e_4) diagram.

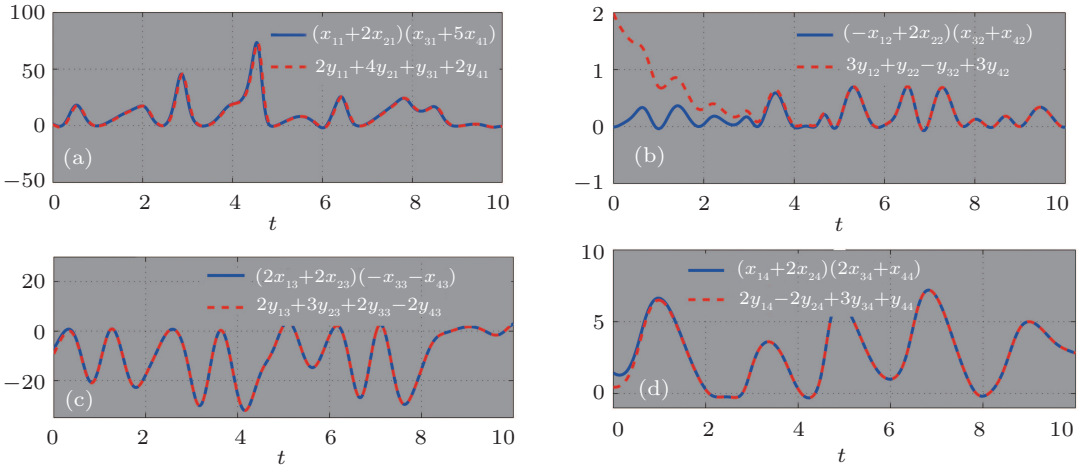


Fig. 7. (color online) Synchronization results from simulation observation of four drive (21)–(25) and four response (26)–(29) memristor-based Chua chaotic systems: (a) $(x_{11}+2x_{21})(x_{31}+5x_{41})$ and $(2y_{11}+4y_{21}+y_{31}+2y_{41})$ versus t; (b) $(-x_{12}+2x_{22})(x_{32}+x_{42})$ and $(3y_{12}+y_{22}-y_{32}+3y_{42})$ versus t; (c) $(2x_{13}+2x_{23})(-x_{33}-x_{43})$ and $(2y_{13}+3y_{23}+2y_{33}-y_{43})$ versus t; (d) $(x_{14}+2x_{24})(2x_{34}+x_{44})$ and $(2y_{14}-2y_{24}+3y_{34}+y_{44})$ versus t.

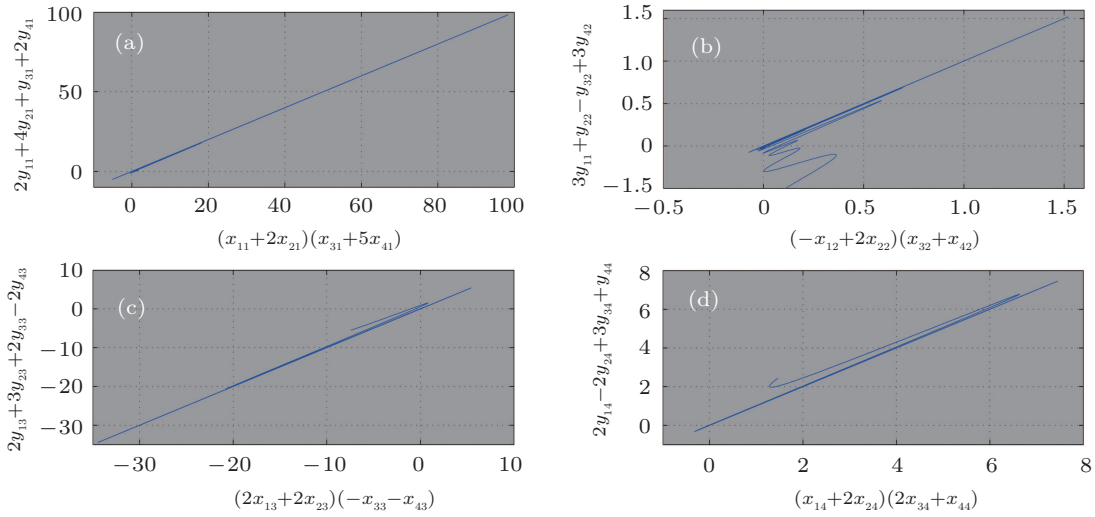


Fig. 8. (color online) Synchronization results from simulation observation of drive (21)–(25) and response (26)–(29) memristor-based Chua chaotic systems: (a) $(x_{11}+2x_{21})(x_{31}+5x_{41})$, $(2y_{11}+4y_{21}+y_{31}+2y_{41})$ space; (b) $(-x_{12}+2x_{22})(x_{32}+x_{42})$, $(3y_{12}+y_{22}-y_{32}+3y_{42})$ space; (c) $(2x_{13}+2x_{23})(-x_{33}-x_{43})$, $(2y_{13}+3y_{23}+2y_{33}-y_{43})$ space; (d) $(x_{14}+2x_{24})(2x_{34}+x_{44})$, $(2y_{14}-2y_{24}+3y_{34}+y_{44})$ space.

Synchronization results also have been calculated from MATLAB/Simulink and these results are shown in Figs. 7 and 8. Figure 7 shows the same results of Fig. 5, while figure 8 explains the relationship between the drive and response systems which

clearly appears as the form of a line $y = x$, this means that there is an agreement between the results of Figs. 8 and 6. The synchronization setup time[39] with different values of k_1, k_2, k_3, and k_4 are calculated as shown in Fig. 9. According to Fig. 9, the synchronization setup time (e) approaches to zero fast for reasonable large values of k_i.

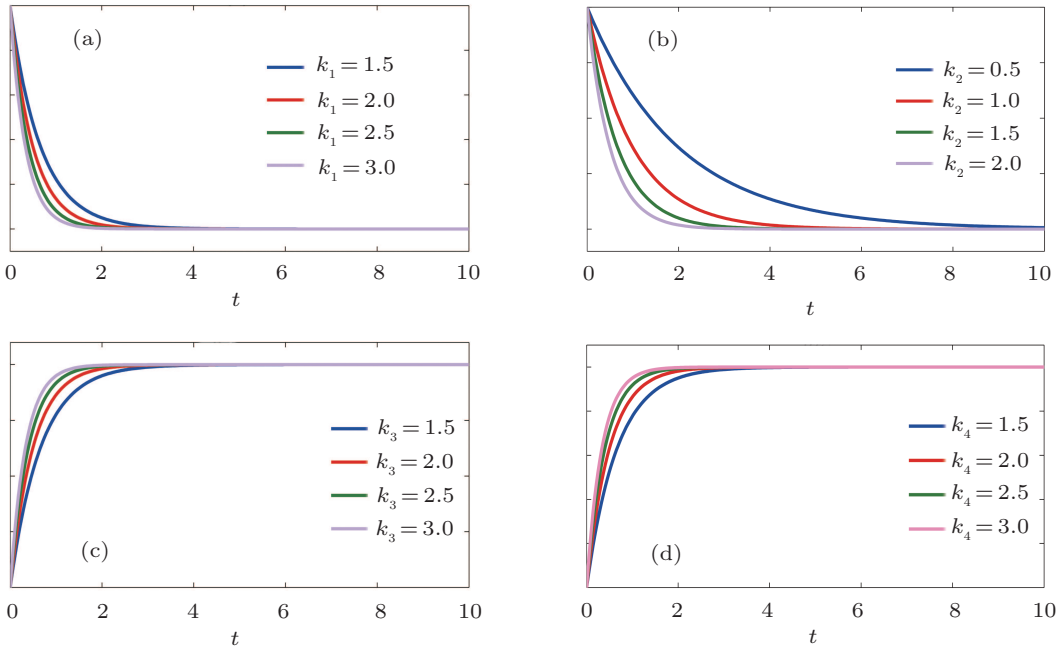


Fig. 9. (color online) Synchronization setup time with different parameter: (a) k_1, (b) k_2, (c) k_3, (d) k_4.

5. Conclusion

The object of this paper is to propose a new type of synchronization among eight n-dimensional chaotic systems. The proposed scheme is used to achieve the DCCS between four drive and four response n-dimensional chaotic systems. Using the Lyapunov stability theory and nonlinear feedback control, analytical expressions are obtained for achieving DCCS. As an example, we consider the eight identical memristor-based Chua chaotic systems. Numerical results have been calculated to illustrate the validity of our scheme. Based on complexity of DCCS, our results make more secure and interesting to transmit and receive signals in applications of communications. The numerical results of DCCS are shown in Figs. 5 and 6, and agree with the MATLAB/Simulink results as shown in Figs. 7 and 8.

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