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Robustness of coherence between two quantum dots mediated by Majorana fermions*

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We study three important measurements used to identify the quantum correlations between two quantum dots (QDs) mediated by a pair of Majorana fermions (MFs) in a superconducting quantum wire. We find that, in addition to the quantum discord, the robustness of coherence (ROC) can also be considered as a quantity to measure the quantum correlation for the special case where the quantum entanglement is vanishing. For comparison, we study the quantum correlation between two QDs mediated by other fermions, i.e., regular fermions and superconducting fermions. We find that, when the quantum entanglement is not vanishing, i.e., the concurrence is finite, the detailed difference between the concurrence and ROC can be considered as an important implication for the existence of MFs.

Keywords: quantum coherence, Majorana fermion, non-locality

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1. Introduction

Majorana fermion (MF) refers to a kind of exotic particle that is identical to its own antiparticle. MF has been proposed[1] in particle physics as a possible elementary building block. In the past few years, due to possibly being realized in condensed matter systems using quasiparticle excitations[2] and the potential applications in topological quantum computations,[3,4] MFs have been widely investigated from both theoretical and experimental sides. These quasiparticles have been proposed to exist in different solid-state systems, e.g., the 5/2 fractional quantum Hall systems,[5] the chiral p-wave superconductors,[6–9] and the surface state of topological insulators with superconducting proximity effect.[10] Experimentally,[11–14] the end states of a semiconductor nanowire[15,16] with strong spin–orbit coupling[17] are demonstrated to be a pair of MFs under proper experimental conditions, i.e., the nanowire has been put on the top of an s-wave superconductor and subject to a proper Zeeman field. As a quasiparticle, MF in solid-state systems is a superposition of particle and hole states. The MF can not emerge separately in solid-state systems, a proper combination of MF-pair can form a nonlocal electronic level. This unique property makes the non-locality to be an intrinsic feature of MFs even if the two MFs are separated far away from each other and the quantum entanglement[18,19] between these two MFs is vanishing.

Several approaches[20–25] have been proposed to detect the quantum non-locality of MFs.[26–31] Wang *et al.* proposed[20] an experiment to detect the MFs by measuring the quantum entanglement between two QDs. However, it is difficult to discriminate the MFs and the other intermediaries, i.e., the topological trivial superconductors and regular fermions, in their proposal. In Ref. [21], Wang *et al.* studied the spectral density of the cross-correlation of two QDs mediated by a pair of MFs. They found that the spectral density mediated by MFs is antisymmetric with respect to the simultaneous modulation of the QD energy levels, while the spectrum is symmetric if the current is mediated by the regular fermion. Li *et al.* studied[22] the quantum correlation between two QDs mediated by a pair of MFs. As an evidence of the non-locality of MFs, they have shown that, being different from the quantum entanglement, the quantum discord is not vanishing when the MFs are completely separated from each other.

Recently, quantum coherence has been proposed as a physical resource to measure the quantum correlation.[32] Napoli *et al.* proposed[33] an operational and observable measure of the quantum coherence, the robustness of coherence (ROC), under the general abstract notion of robustness of asymmetry.[34] The quantum coherence as a resource to identify the non-locality of MFs has not been investigated rigorously as far as we know. In this paper, we study the ROC between two QDs mediated by a pair of MFs.

2. Model Hamiltonian

The model Hamiltonian is set up as follows (see Fig. 1 for schematic setup). A quantum wire with strong spin–orbit coupling is put on the top of an s-wave superconductor, the proximity effect drives the quantum wire into a superconducting phase. In the regime of topological superconductor phase, a pair of zero-energy MFs are associated to the two ends of the quantum wire. The model Hamiltonian used to describe

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the MFs is given by

$$H_{\rm MF} = {\rm i}\varepsilon_{\rm MF}\gamma_1\gamma_2/2, \tag{1}$$

where $\varepsilon_{\rm MF} \sim {\rm e}^{-L/\xi}$ and L is the length of the quantum wire, ξ is the superconducting coherent length. γ_1 and γ_2 are the two Majorana operators which satisfy the anticommutation relation $\{\gamma_i,\gamma_j\} = \delta_{ij}$ and the Majorana condition $\gamma_j^\dagger = \gamma_j$ $(i,j=1,2)$. Two QDs are put close to the ends of the quantum wire, i.e., $\rm QD_1$ and $\rm QD_2$. We use the following Hamiltonian to describe the QDs:

$$H_{\rm QD} = \varepsilon_{d1}d_1^\dagger d_1 + \varepsilon_{d2}d_2^\dagger d_2, \tag{2}$$

where ε_{d1} and ε_{d2} are the on-site energies of the two QDs, and $d_j^\dagger$ and d_j are the creation and annihilation operators on $\rm QD_{\it j}$. The tunneling coupling between the QDs and MFs can be written as[9,15,21,35]

$$H_{\rm T} = \sum_{j=1,2}\lambda_j\left(d_j^\dagger - d_j\right)\gamma_j, \tag{3}$$

where λ_j is the tunneling coupling constant between $\rm QD_{\it j}$ and MFs. The total Hamiltonian

$$H_{\rm QD}^{\rm MF} = H_{\rm MF} + H_{\rm QD} + H_{\rm T}. \tag{4}$$

Under the following transformation:

$$\gamma_1 = {\rm i}\left(f - f^\dagger\right), \quad \gamma_2 = f + f^\dagger, \quad d_1 \to {\rm i}d_1, \tag{5}$$

we find that the total Hamiltonian can be rewritten as

$$\begin{aligned} H_{\rm QD}^{\rm MF} &= \varepsilon_{\rm M}\left(n_f - \frac{1}{2}\right) + \sum_{j=1,2}\left[\varepsilon_{dj}n_{dj} + \lambda_j\left(d_j^\dagger f + {\rm h.c.}\right)\right] \\ &\quad - \lambda_1\left(d_1^\dagger f^\dagger + f d_1\right) + \lambda_2\left(d_2^\dagger f^\dagger + f d_2\right), \end{aligned} \tag{6}$$

where $n_f = f^\dagger f$ and $n_{dj} = d_j^\dagger d_j$ are the electron occupation number operators of the paired MFs and the on-site states at $\rm QD_{\it j}$, and h.c. refers to the Hermitian conjugate. On the electron occupation number basis, the model Hamiltonian can be written as an 8×8 matrix.

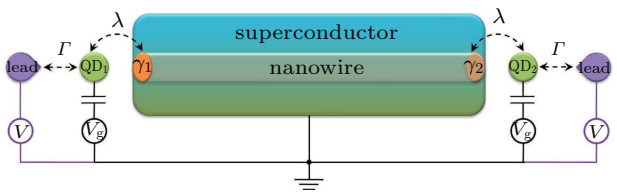


Fig. 1. (color online) Schematic set up for detecting the quantum correlations between two QDs mediated by a pair of MFs. The MFs (γ_1 and γ_2) are located at the two ends of the semiconductor nanowire with strong spin–orbit coupling and placed in proximity to an s-wave superconductor. λ refers to the coupling between the MFs and QDs. Two electrode leads are coupled to the QDs to investigate the decoherence effects on the quantum correlations.

In order to find the distinguishable features of MFs, here we consider two other setups for comparison. The first case is QDs mediated by regular fermions as shown in Ref. [22], where the Hamiltonian is written as

$$H_{\rm QD}^{\rm RF1} = \varepsilon_c c^\dagger c + \sum_{j=1,2}\left[\varepsilon_j n_{dj} + \lambda_j\left(d_j^\dagger c + {\rm h.c.}\right)\right], \tag{7}$$

$$H_{\rm QD}^{\rm RF2} = \sum_{j=1,2}\left[\varepsilon_{dj}n_{dj} + \varepsilon_{cj}c_j^\dagger c_j + \lambda_j\left(d_j^\dagger c + {\rm h.c.}\right)\right], \tag{8}$$

for one and two non-interacting regular fermions between the QDs, respectively. Here ε_c and ε_{cj} are the energies of the regular fermions, $c^\dagger$ (c) and $c_j^\dagger$ (c_j) are the creation (annihilation) operators of the regular fermions, λ_j $(j=1,2)$ is the tunneling coupling constant between $\rm QD_{\it j}$ and regular fermions. The second case is QDs mediated by a pair of superconducting electrons, the Hamiltonian can be written as

$$\begin{aligned} H_{\rm QD}^{\rm SC} &= \sum_{j=1,2}\left[\varepsilon_{dj}n_{dj} + \lambda_j\left(d_j^\dagger c + {\rm h.c.}\right) + \varepsilon_{cj}c_j^\dagger c_j\right) \\ &\quad + \Delta(c_1^\dagger c_2^\dagger + c_2 c_1). \end{aligned} \tag{9}$$

The time evaluation of the density matrix for these model Hamiltonians (6)–(9) can be solved analytically for given initial states when the decoherence effect is not considered.

Moreover, in order to investigate the decoherence effect on quantum correlations, we assume that the two electrode leads are weakly coupled to the two QDs. This coupling is described by

$$H_{\rm TL} = \sum_{j=1,2}\sum_k\left(t_j d_j^\dagger c_{kj} + {\rm h.c.}\right),$$

where t_j is the coupling strength between the j-th electrode lead and the corresponding QD, c_{kj} and $c_{kj}^\dagger$ are annihilation and creation operators of a quantum state with wave vector k on the j-th electrode lead. The two electrode leads are described as $H_{\rm leads} = \sum_{j=1,2}\sum_k \varepsilon_{kj}c_{kj}^\dagger c_{kj}$, where ε_{kj} is the energy level of the quantum state with wave vector k on the j-th electrode lead. The chemical potential μ of the electrode leads is assumed to be much lower than the QD on-site energy, $\mu \ll \varepsilon_1, \varepsilon_2$, such that the electrons can only jump outwards from the QDs to the leads. The leads are regarded as an environment and the quantum state in the central regime is described by a reduced density matrix $\rho(t)$. The time evaluation of this reduced density matrix can be described by the standard Born–Markov master equation[36] in the weak coupling limit

$$\frac{{\rm d}\rho(t)}{{\rm d}t} = {\rm i}\left[\rho, H_{\rm tot}\right] + \sum_{j=1,2}\mathscr{D}^{(j)}[\rho(t)], \tag{10}$$

where $H_{\rm tot}$ is the model Hamiltonian of the whole system as shown in Eqs. (6)–(9)for different situations. $\mathscr{D}^{(j)}[\rho] = \Gamma_j\left(d_j\rho d_j^\dagger - \frac{1}{2}\left\{d_j^\dagger d_j, \rho\right\}\right)$ is the dissipator of the quantum master equation at $\rm QD_{\it j}$, Γ_j is the corresponding tunneling rate, it is associated with both the coupling strength t_j and the density of states on the j-th lead. For simplicity, we assume $\Gamma_1 = \Gamma_2 = \Gamma$. Generally, one can express this reduced density matrix $\rho(t)$ as

a 8×8 or 16×16 matrix for different situations in the electron occupation number representation. By tracing over the degrees of freedom of the intermediate MFs, regular fermions, or superconducting states in $\rho(t)$, we can obtain the further reduced 4×4 density matrix of the two QDs, $\rho_d(t)$, and study the quantum correlation between these QDs.

In this work, we focus on the ROC as a resource to measure the quantum correlation between the two QDs mediated by the MFs. Here we give a brief introduction to ROC. The ROC of a state ρ is given by the minimum weight of another state τ such that its convex mixture with ρ yields an incoherent state δ.[33] More specifically, let $\mathfrak{D}(\mathbb{C}^d)$ be the convex set of density operators acting on a d-dimensional Hilbert space, $\mathfrak{I}$ be the subset of incoherent states in $\mathfrak{D}(\mathbb{C}^d)$. The ROC of a state $\rho\in\mathfrak{D}(\mathbb{C}^d)$ is given by

$$\mathscr{C}_{\mathscr{R}}=\min_{\tau\in\boldsymbol{D}(\mathbb{C}^d)}\left\{s\geq 0\left|\frac{\rho+s\tau}{1+s}=:\delta\in\mathfrak{I}\right.\right\},\tag{11}$$

where τ is another state in $\mathfrak{D}(\mathbb{C}^d)$ and δ is an incoherent state. Numerically, it is proved that the ROC can be recast via the semidefinite program as[37]

$$\text{maximize}\quad -\mathrm{Tr}[\rho W],\tag{12}$$

$$\text{subject to}\quad \Pi(W)\geq 0\quad\text{and}\quad W\leq 1,\tag{13}$$

for any given witness operator W. The witness operator W is defined as the Hermitian operator which satisfies $\Pi(W)\geq 0$ if and only if $\mathrm{Tr}(\delta W)=\mathrm{Tr}(\delta\Pi(W))\geq 0$ for all incoherent states $\delta\in\mathfrak{I}$. Here Π is the full dephasing operator, which maps the witness operator W into its diagonal part $\Pi(W)=\sum_{j=0}^{d-1}|j\rangle\langle j|W|j\rangle\langle j|$ in a computational reference basis $\{|j\rangle\}_{j=0}^{d-1}$. The MATLAB[38] codes used to evaluate the semidefinite program of ROC can be found in the supplemental materials of Refs. [33] and [34]. The open-source modeling system for convex optimization, CVX,[39] is required in running the MATLAB codes.

Two other measurements, the concurrence[40,41] and the quantum discord,[42,43] which are used to identify the quantum correlations between the two QDs, are calculated as well, they are widely studied in different systems in quantum information field. For the sake of completeness, here we give a brief introduction to the definitions of these measurements. We refer to Ref. [22] and references therein for the detailed discussion of these quantities for the model Hamiltonian investigated in this work. The concurrence, as a measure of the quantum entanglement, can be obtained as follows for the two-qubit states:

$$\mathscr{C}(\rho)=\max\left\{0,\sqrt{\Lambda_1}-\sqrt{\Lambda_2}-\sqrt{\Lambda_3}-\sqrt{\Lambda_4}\right\},\tag{14}$$

where Λ_1–Λ_4 are the four eigenvalues of the following matrix in decreasing order:

$$\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y),\tag{15}$$

where ρ^* is the complex conjugate of the density matrix ρ.

The definition of quantum discord is accomplished by using two other important quantities in quantum information theory, the quantum mutual information and the classical correlation. The quantum mutual information $\mathscr{M}(\rho)$ is a quantum substitution of the mutual information in classical information theory, $\mathscr{M}(\rho)=S(\rho_{\mathscr{A}})+S(\rho_{\mathscr{B}})-S(\rho)$, where $\rho_{\mathscr{A}}$ ($\rho_{\mathscr{B}}$) is the reduced density matrix for the subsystem $\mathscr{A}$ ($\mathscr{B}$) of the composite bipartite system $\mathscr{A}\mathscr{B}$, and ρ refers to the total density matrix of the $\mathscr{A}\mathscr{B}$ system. $S(\rho)=-\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy. The other quantity, classical correlation $\mathscr{C}\mathscr{C}(\rho)$, is defined by using the von Neumann-type measurements, $\mathscr{C}\mathscr{C}(\rho)=S(\rho_{\mathscr{A}})-\min_{\{\mathscr{B}_j\}}\sum_j p_j S(\rho_j)$. The inferior limit is taken over different sets of orthogonal projection operators, $\{\mathscr{B}_j\}s$, acting on the subsystem $\mathscr{B}$ only. For a given set of orthogonal projection operators $\{\mathscr{B}_j\}$, one can calculate the conditional density matrix ρ_j associated with the measurement result j, and the corresponding probability p_j to obtain this measurement result j. Their expressions are given by $\rho_j=(\mathbb{1}\otimes\mathscr{B}_j)\rho(\mathbb{1}\otimes\mathscr{B}_j)$, where $\mathbb{1}$ is the identity matrix acting on the subsystem $\mathscr{A}$, $p_j=\mathrm{tr}(\rho_j)$. Finally, the quantum discord is the difference of these two quantities

$$\mathscr{D}(\rho)=\mathscr{M}(\rho)-\mathscr{C}\mathscr{C}(\rho).\tag{16}$$

3. Results

Firstly, we consider the quantum correlations for the model Hamiltonian without decoherence, Eq. (6). In the electron occupation number representation $|n_{d1},n_{d2},n_f\rangle$, equation (6) can be reexpressed as a 8×8 matrix, it is easy to check that the parity of the electron number is conserved, which makes the model Hamiltonian be expressed as $H_{\mathrm{QD}}^{\mathrm{MF}}=H_{\mathrm{u}}+H_{\mathrm{g}}$, where H_{u} and H_{g} are both 4×4 matrices in the parity-even and parity-odd subspaces of the Hilbert space, respectively. Here we consider the product state $|\psi(0)\rangle=|1,1,0\rangle$ in the parity-even subspace as an initial state, the corresponding Hamiltonian H_{u} can be written as

$$H_{\mathrm{u}}=\begin{pmatrix}-\frac{\varepsilon_{\mathrm{M}}}{2} & \lambda & \lambda & 0\\ \lambda & \varepsilon_{d2}+\frac{\varepsilon_{\mathrm{M}}}{2} & 0 & \lambda\\ \lambda & 0 & \varepsilon_{d1}+\frac{\varepsilon_{\mathrm{M}}}{2} & \lambda\\ 0 & \lambda & \lambda & \varepsilon_{d1}+\varepsilon_{d2}-\frac{\varepsilon_{\mathrm{M}}}{2}\end{pmatrix},\tag{17}$$

on the bases $|0,0,0\rangle$, $|0,1,1\rangle$, $|1,0,1\rangle$, and $|1,1,0\rangle$. The time evaluation of the initial state is solved to be

$$\begin{aligned}|\psi(t)\rangle &= C_1(t)|0,0,0\rangle+C_2(t)|0,1,1\rangle\\ &\quad+C_3(t)|1,0,1\rangle+C_4(t)|1,1,0\rangle,\end{aligned}\tag{18}$$

where the time-dependent coefficients are given by

$$\begin{aligned}C_1(t) &= \frac{1}{2}\left[-\mathrm{e}^{\mathrm{i}\omega t}+\cos(\Omega t)\right]+\frac{\mathrm{i}\omega}{2\Omega}\sin(\Omega t),\\ C_2(t) &= C_3(t)=-\frac{\mathrm{i}\lambda}{\Omega}\sin(\Omega t),\end{aligned}$$

$$C_4(t) = \frac{1}{2}\left[e^{i\omega t} + \cos(\Omega t)\right] + \frac{i\omega}{2\Omega}\sin(\Omega t), \qquad (19)$$

with $\omega = \varepsilon_M/2$ and $\Omega = \sqrt{\varepsilon_M^2 + 16\lambda^2}/2$. By tracing over the MF states in the density matrix $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$, one can obtain the reduced density matrix $\rho_d(t)$. Expressed in the electron occupation number representation of the QDs, $|n_{d1}, n_{d2}\rangle = |0,0\rangle$, $|0,1\rangle$, $|1,0\rangle$, and $|1,1\rangle$, $\rho_d(t)$ has a X-type shape. All of the 8 non-zero elements in $\rho_d(t)$ can be obtained by the joint measurement experiment proposed in Ref. [22].

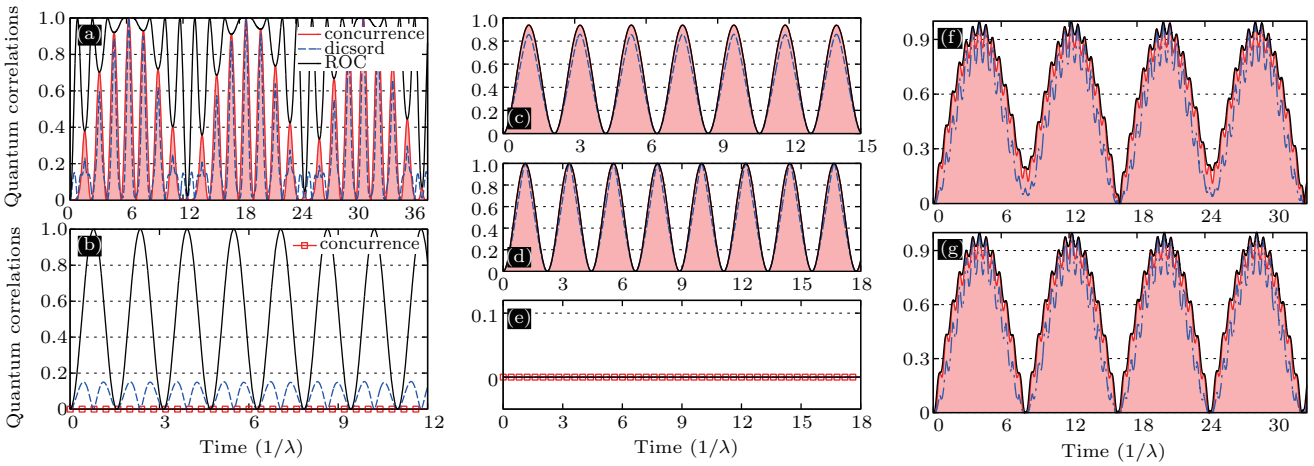


Fig. 2. (color online) Time evaluation of the quantum correlations between two QDs mediated by different kinds of fermions. The parameters are set as follows: tunneling coupling $\lambda_1 = \lambda_2 = \lambda$, tunneling rate between the QDs and the leads $\Gamma = 0$, (a) QD energies $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and inter-MF coupling $\varepsilon_M = 0.5\lambda$ for two QDs mediated by a pair of MFs, (b) $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and $\varepsilon_M = 0$ for two QDs mediated by a pair of MFs, (c) $\varepsilon_{d1} = \varepsilon_{d2} = 0.25\lambda$ and energy $\varepsilon_c = 0.5\lambda$ for two QDs mediated by a regular fermion, (d) $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and $\varepsilon_c = 0$ for two QDs mediated by a regular fermion, (e) $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and $\varepsilon_{c1} = \varepsilon_{c2} = 0$ for two QDs mediated by a pair of regular fermions, (f) $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and $\varepsilon_{c1} = \varepsilon_{c2} = 0.5\lambda$ for two QDs mediated by a pair of superconducting fermions with pairing potential $\Delta = 5\lambda$, (g) $\varepsilon_{d1} = \varepsilon_{d2} = 0$ and $\varepsilon_{c1} = \varepsilon_{c2} = 0$ for two QDs mediated by a pair of superconducting fermions with pairing potential $\Delta = 5\lambda$.

Figures 2(a) and 2(b) show the time evaluation of the quantum correlations calculated from $\rho_d(t)$. When the direct coupling between the two MFs, ε_M, is not vanishing, there are two frequencies $\omega/2\pi$ and $\Omega/2\pi$ (see Fig. 2(a) for $\varepsilon_M = 0.5\lambda$). Both quantum entanglement (red solid line with the area bellow the line filled) and quantum discord (dashed blue line) have the similar tendency, i.e., the position of the fast oscillation peak of both quantum entanglement and quantum discord is located at $t = 2N\pi/\Omega$, the position of the slow oscillation peak is located at $t = 2N\pi/\omega$, both quantum entanglement and quantum discord achieve their maxima at these points, concurrence = discord = 1. Near the nodes of the slow oscillation, quantum entanglement and quantum discord are a bit different from each other. However, when we consider the quantum coherence (the solid black line), it is quite different from the quantum entanglement and quantum discord. Firstly, the ROC is greater than (at least equals to) the quantum entanglement in the whole time domain. Secondly, the maximum quantum coherence in each fast oscillation period can approach one. Thirdly, the location of the fast oscillation peak (valley) of the quantum coherence is associated to the oscillation valley (peak) of the quantum entanglement. Their tendencies are opposite to each other.

Figure 2(b) shows the quantum correlations of two QDs mediated by a pair of MFs without direct coupling between the two MFs, $\varepsilon_M = 0$. One can find that, similar to the quantum discord, the quantum coherence can also be used to identify the quantum correlation between two qubits without quantum entanglement. For comparison, we show the quantum correlations between two QDs mediated by a regular fermion, Figs. 2(c) and 2(d) for different parameters. Both figures 2(c) and 2(d) show that the quantum entanglement and quantum coherence are exact the same. Figure 2(e) shows that the quantum correlation between the two QDs is vanishing if these two QDs are mediated by a pair of regular fermions without interaction. Furthermore, figures 2(f) and 2(g) show the quantum correlations between two QDs mediated by a pair of normal superconducting fermions with a superconducting energy gap $\Delta = 5\lambda$ and different parameters given in the caption. The trembles on the lines are induced by the finite superconducting energy gap. We find that the quantum entanglement, quantum discord, and quantum coherence have some differences, they are not coincide to each other exactly, however, the general trends are the same. These results indicate that the differences between the quantum entanglement and quantum coherence can be considered as an important identity for MFs in the one-dimensional quantum wire system.

Now we consider the decoherence induced by the leads. By solving the Born–Markov master equation in Eq. (10), we can obtain the time-dependent density matrices. Figure 3 shows the quantum correlations between the two QDs mediated by a pair of MFs. In Fig. 3(a), the direct coupling between the two MFs, $\varepsilon_M = 0.5\lambda$. There are three different time scales, the fast oscillation, the slow oscillation, and the deco-

herence time induced by the environment. Similar to the case without decoherence, one can find that the ROC is great than the quantum entanglement in the whole time domain. More importantly, the oscillation peak of concurrence and quantum discord is smaller than the valley of ROC in each fast oscillation period, there is a gap between the ROC and the other two measurements, which demonstrates that the quantum coherence is more robust against decoherence from the environment. Figure 3(b) shows the quantum correlations between the two QDs mediated by a pair of MFs without direct coupling between the two MFs, $\varepsilon_{\rm M}=0$. One can find that, similar to the quantum discord, the ROC can also be used to identify the quantum correlations between two QDs without quantum entanglement, and numerically, the ROC is more significant than the quantum discord. The insert in Fig. 3(b) shows the details of quantum discord in the regime $t\in[0,8]$ in units of $1/\lambda$, where one can find that the maximum quantum discord is about 0.1.

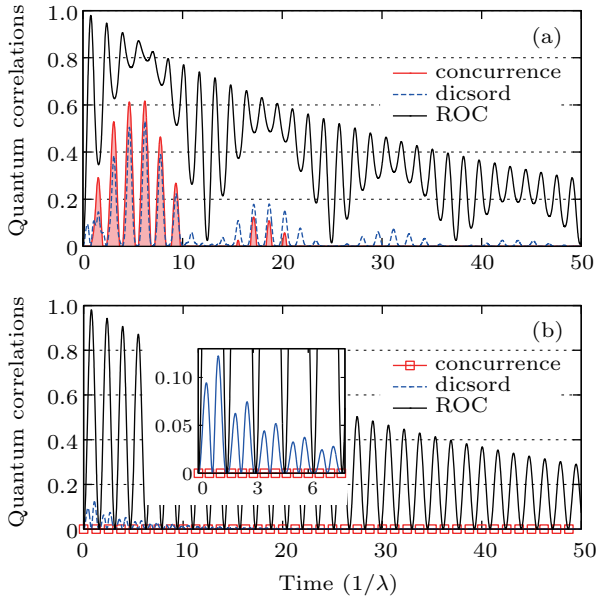


Fig. 3. (color online) Time evaluation of the quantum correlations between two QDs mediated by MFs with quantum decoherence. The parameters are set as follows: tunneling coupling constant $\lambda_1=\lambda_2=\lambda$, QD energies $\varepsilon_{d1}=\varepsilon_{d2}=0$, tunneling rate between the QDs and the leads $\Gamma=0.05$, (a) $\varepsilon_{\rm M}=0.5\lambda$, (b) $\varepsilon_{\rm M}=0$. The insert show the details of the quantum discord in the interval $t\in[0,8]$ in units of $1/\lambda$.

4. Summary

We calculate three important measurements used to identify the quantum correlations between two QDs mediated by a pair of MFs. We find that, in addition to the quantum discord, the ROC can also be considered as a quantity to measure the quantum correlations when the quantum entanglement is vanishing. When the quantum entanglement is not vanishing, i.e., the concurrence is finite, comparing with other cases like the QDs mediated by regular fermions and superconducting fermions, we find that the detailed difference between the concurrence and ROC can be considered as an important implication of the existence of MFs.

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