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Citation: Chin. Phys. B . 2018, 27(7): 076502. **doi:** 10.1088/1674-1056/27/7/076502

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Thermal conductivity of systems with a gap in the phonon spectrum*

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(Received 7 November 2017; revised manuscript received 17 April 2018; published online 25 June 2018)

An original theoretical model for describing the low-temperature thermal conductivity in systems with a region of forbidden values (a gap) in the phonon spectrum is proposed. The model is based on new experimental results on the temperature dependence of the phonon diffusion coefficient in nanoceramics and dielectric glasses which showed a similar anomalous behavior of the diffusion coefficient in these systems that may be described under the assumption of a gap in the phonon spectrum. In this paper, the role of the gap in low-temperature behavior of the thermal conductivity, $\kappa(T)$, is analyzed. The plateau in the temperature dependence of the thermal conductivity is shown to correlate with the position and the width of the gap. The temperature dependence of thermal conductivity of such systems when changing the scattering parameters related to various mechanisms is studied. It is found that the umklapp process (U-processes) involving low-frequency short-wavelength phonons below the gap forms the behavior of the temperature dependence of thermal conductivity in the plateau region. A comparison of the calculated and experimental results shows considerable possibilities of the model in describing the low-temperature thermal conductivity in glass-like systems.

Keywords: phononic band gap, phonon diffusion coefficient, thermal conductivity, glassy crystals

PACS: 65.60.+a, 63.50.–x, 61.43.Fs **DOI:** 10.1088/1674-1056/27/7/076502

1. Introduction

In recent years, considerable attention of researchers has been attracted to new complex systems which, though not being amorphous materials, exhibit glass-like thermal properties (the so-called glassy crystals).[1] Such materials include solid alcohols,[2,3] clathrates,[4,5] ferroelectric relaxors (single crystals and ceramics),[6,7] cryocrystal nanocomposite,[8,9] and thermoelectrcs.[10–12]

Manifestation of glass-like thermal properties in systems of different nature calls for new theoretical models able to explain both their similarities and differences in a unified approach. Not all models describing the low-temperature properties of glasses seem to be suitable also for these new complex systems.

It is well known that to explain the anomalous behavior of the thermal properties of the amorphous state (excessive heat capacity, the plateau in the thermal conductivity,[13,14] etc.), various phenomenological models were developed, which introduced into consideration a number of low-energy elementary excitations: tunneling two-level systems (TLS)[15,16] responsible for the thermal properties at $T < 1$ K, relaxation systems (RS),[17] and low-frequency quasi-local vibrational modes (LFM)[18] forming the plateau ($T < 10$ K). The soft-potential model describes TLS, RS, and LFM within a unified approach.[19] The majority of works devoted to this problem use the time-relaxation approximation (τ-approximation), according to which all the information about the scattering can be described by the temperature and frequency dependence of the relaxation time (or the average path length) of phonons. In this case, the group velocity of phonons is assumed to be constant and their density of states always corresponds to the density of states of the Debye model ($g(\omega) \sim \omega^2$). The most experimental studies analyze the temperature dependence only of the thermal conductivity depending on many independent variables responsible for different scattering mechanisms. Combining these parameters makes it possible to achieve a good agreement with the experimental results even in the time-relaxation approximation, at least up to the temperatures of the plateau region. In new glass-like systems, the behavior of $\kappa(T)$ in the plateau region is more variable. At these temperatures, one can observe both an increase in $\kappa(T)$ and the formation of a local minimum (see Fig. 7), which further complicates the description of this dependence in τ-approximation. Unfortunately, we are unaware of any theoretical work in which the increase of the thermal conductivity above the plateau and, especially, the non-trivial behavior of $\kappa(T)$ in the plateau region observed in glassy-crystals (for example, Fig. 1 in Ref. [3]) would be explained.

Impetus to writing this work has been given by recent studies of the phonon diffusion coefficient $D(T)$ in glasses. It turned out that this formally simpler quantity entering in the integrand of the thermal conductivity coefficient demonstrates an anomalous temperature dependence close to T^{-5}[7,20] for all the systems considered. In explaining such a behavior of this characteristic, the authors of the known theoretical studies restricted their consideration to the effective diffusion coeffi-

*Project supported by the Ural Branch of the Russian Academy of Sciences, Russia (Grant No. 18-2-2-12), the Russian Foundation for Basic Research, Russia (Grant Nos. 16-07-00529 and 18-07-00191), and the Financing Program, Russia (Grant No. AAAA-A16-116021010082-8).

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cient $D_{\text{eff}} = \kappa/C$, where κ is the thermal conductivity coefficient and C is the heat capacity.[21,22] Assuming that in the plateau region $\kappa =$ const and putting $C \sim T^5$, they obtained the necessary temperature dependence. This explanation does not seem convincing, because the dependence $D(T) \sim T^{-5}$ does not coincide with the plateau region.

To describe the anomalous dependence $D(T)$, in Ref. [7] the renormalization of the phonon spectrum was taken into account. It turned out that the phonon scattering at structural defects of the amorphous state may have a resonance character leading to the splitting of the acoustic branch and the appearance of a region of forbidden values (a gap) in the spectrum. The observed temperature dependence of the phonon diffusion coefficient is universal for all investigated glasses, and takes place when the temperature only approaches the plateau region, which clearly points to the association of this anomaly with the plateau in the thermal conductivity. In this connection, the study of the effect of the phonon spectrum renormalization seems to be necessary in theoretical description of the thermal conductivity in glasses and glassy crystals.

We are aware of only one study in which the temperature dependence of the thermal conductivity of a real system was described using a phonon spectrum different from the Debye one.[23–26] In those papers, the low-temperature thermal conductivity of quasi-one-dimensional systems was studied. The anomalies in the behavior of $\kappa(T)$ were ascribed to strong phonon–phonon scattering by low-lying transverse acoustic phonons propagating along a preferred direction (bending phonons[27]), with a very weak dispersion over a large part of the Brillouin zone. The flattening of the dispersion curve for transverse phonons is due to the formation of a gap in the bending phonons spectrum. In Ref. [28], the thermal conductivity of systems with a complete gap in the phonon spectrum was considered qualitatively and it was shown that the gap may cause the formation of a plateau in $\kappa(T)$. Unfortunately, the lack of experimental data on the frequency dependence of the phonon diffusion coefficient interfered with further elaboration of this idea.

In this paper, the effect of the model parameters (both conventional and new ones) on the fine structure of low-temperature heat conductivity of systems with a gap in the phonon spectrum is studied, and the possibilities of the model proposed for describing the thermal conductivity of the glass-like systems are demonstrated.

2. The phonon spectrum and diffusion coefficient at resonance scattering

In this section, we consider some examples of the gap formation in the phonon density of states and the phonon diffusion coefficient appearing in the expression for the heat conductivity. The formation of a gap (pseudogap for quasi-one-dimensional crystals) in the phonon spectrum may be due to both the peculiarities of the phonon spectrum of a perfect crystal, e.g., for the bending phonons of quasi-one-dimensional crystals, and the acoustic branch splitting at resonance scattering by defects.

The mechanism of resonance scattering of phonons on defects was used more than once to explain the anomalies in the temperature dependence of the thermal conductivity coefficient.[29] Unfortunately, in the great majority of works, only the correction to the phonon relaxation time due to this mechanism was taken into account, though it has long been known[30] that with such a scattering, the phonon dispersion law can vary drastically, leading to a change in the phonon spectrum of the system and a non-trivial behavior of the frequency dependence of the phonon group velocity. It is natural to assume that the gap formation in the phonon spectrum of glasses is due to the processes of resonance scattering of phonons on defects.

Detailed theoretical studies of the phonon spectrum in the presence of defects which scatter phonons in a resonant way were carried out in Refs. [31] and [32], where the scalar model was used to investigate the phonon spectrum of a crystal with a simple cubic lattice containing substitution impurity atoms with a mass M and a force constant γ differing from those in the matrix (M_0, γ_0). The calculations were performed by the Green function method. The Green function of a crystal with defects has the form

$$G(\omega^2, \boldsymbol{q}) = \frac{1}{\omega^2 - \omega^2(\boldsymbol{q}) - \Sigma(\omega, \boldsymbol{q})}, \tag{1}$$

where $\omega^2(\boldsymbol{q})$ is the dispersion law of a perfect crystal which is assumed to be the Debye one $\omega_{\text{D}} = v_0 q_{\max}$ (v_0 is the speed of sound averaged over polarization and q is the wave vector),

$$\Sigma(\omega, \boldsymbol{q}) = \text{Re}(\Sigma(\omega, \boldsymbol{q})) + \text{i}\frac{2\omega}{\tau(\omega)} \tag{2}$$

is the self-energy part determined by the phonon elastic scattering at the defects, and $\tau(\omega)$ is the phonon lifetime.

In this paper, we restrict our consideration to an approximation linear in the concentration of scattering centers, in which $\Sigma(\omega, \boldsymbol{q}) = ct(\omega, \boldsymbol{q})$, c is the relative concentration of impurity atoms, and $t(\omega, \boldsymbol{q})$ is the one-site scattering matrix.[32] It was shown that the inclusion of terms of higher-order in concentration is responsible for the fine structure of the phonon spectrum, without changes in the gap parameters. Here we consider only the resonance scattering, assuming the impurity atom to be heavy ($M \gg M_0$) and weakly bound to the atoms of the matrix ($\gamma < \gamma_0$). This case is considered in detail in Refs. [31] and [32], where in the limit of small ω and $\boldsymbol{q}$ for the matrix $t(\omega, \boldsymbol{q})$ we obtained[32]

$$t(\omega, \boldsymbol{q}) \approx t(\omega) = \frac{\varepsilon \omega_{\text{r}}^2 \omega^2}{\omega^2 - \omega_{\text{r}}^2 - \text{i}\omega^3 \Gamma/\omega_{\text{D}}}, \tag{3}$$

where $\Gamma \ll 1$ is attenuation at resonance. The resonance frequency is defined by dimensionless parameters $\eta = \gamma/(2\gamma_0 + \gamma)$ and $\varepsilon = M/M_0 - 1 : \omega_r^2 = \omega_D^2 \eta/\varepsilon \ll \omega_D^2$. The parameter η only depends on γ and reaches its maximum $\eta = 1/3$ at $\gamma = \gamma_0$ in the case of isotopic impurity atoms, i.e. the diagonal disorder. The renormalized phonon dispersion law is defined by the poles of the Green function (1). At a sufficient concentration of defects, resonance scattering may result in a splitting of the host-phonon dispersion curve at frequency ω_r, which leads to the appearance of two branches. The approximate expression for the phonon dispersion law for the lower (1) and upper (2) branches in this case has the form[32]

$$\tilde{\omega}_{1,2}^2(q) \simeq \frac{1}{2}\{\omega_r^2(1+c\varepsilon) + (vq)^2 \mp \sqrt{(\omega_r^2(1+c\varepsilon) + (vq)^2)^2 - 4\omega_r^2(vq)^2\varepsilon}\}, \quad (4)$$

from which it follows that between the branches there is an area of forbidden values (a gap) of width $\Delta\omega = \omega_2 - \omega_1$. The lower boundary of the gap coincides with the upper boundary of the first first zone and is found from Eq. (4) in the limit $q \to \infty$: $\omega_1 = \omega_r$. The upper boundary of the gap follows from Eq. (4) at $q = 0$:

$$\omega_2 = \sqrt{\omega_r^2 + c\eta\omega_D^2} = \omega_r\sqrt{1+c\varepsilon}. \quad (5)$$

Figure 1 shows the dispersion curves of a crystal with isotopic impurities calculated by Eq. (4) for the values of parameters c and ε, for which the gap edges (in dimensionless units $z = \omega/\omega_D$) are equal to (1) $z_1 \simeq 0.05$ and $z_2 \simeq 0.25$, (2) $z_1 \simeq 0.075$ and $z_2 \simeq 0.17$, and (3) $z_1 \simeq 0.10$ and $z_2 \simeq 0.2$. It should be noted that the low-frequency branch with a very weak dispersion over a large part of the Brillouin zone is similar to the dispersion curve of bending phonons in quasi-one-dimensional crystals.[33] This points to the same nature of the phenomenon: approaching the gap in the spectrum of bending phonons in quasi-one-dimensional crystals, and the complete gap in our model.

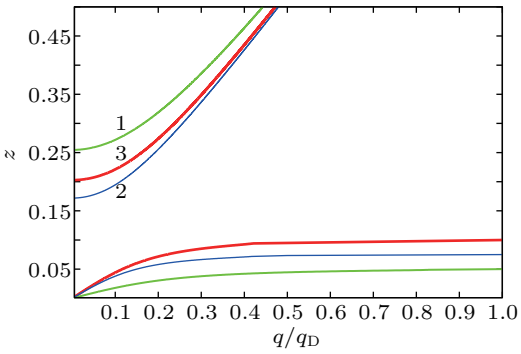


Fig. 1. (color online) Phonon dispersion for the parameter values of (1) $c = 0.2$ and $\varepsilon = 133$, (2) $c = 0.75$ and $\varepsilon = 60$, and (3) $c = 0.1$ and $\varepsilon = 33$, which correspond the gap edges (in dimensionless units $z = \omega/\omega_D$) of (1) $z_1 \simeq 0.05$ and $z_2 \simeq 0.25$, (2) $z_1 \simeq 0.075$ and $z_2 \simeq 0.17$, and (3) $z_1 \simeq 0.10$ and $z_2 \simeq 0.2$.

The phonon density of states $g_{ph}(\omega) = 2\omega\sum_q \mathrm{Im}G(\omega, q)/\pi$ in the case of diagonal disorder is presented in Fig. 2 for the same parameter values as that in Fig. 1. It should be noted that the total density of vibrational states of the system which determines its heat capacity includes, along with phonons, the contribution from the vibrational states localized on defects which are responsible for the maximum of function $C(T)/T^3$.[28]

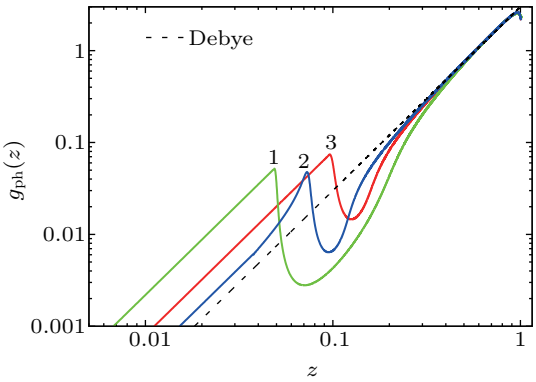


Fig. 2. (color online) The phonon density of states for the same parameter values as that in Fig. 1. The dashed line shows the phonon density of states in the Debye model.

The expression for the phonon diffusion coefficient with the elastic scattering on defects obtained in Ref. [32] can be written more generally as

$$D(\omega) = v_0(\omega)\tau(\omega)/3 = \frac{v_0^2}{3}\frac{2\omega}{c\Phi\mathrm{Im}t(\omega)}, \quad (6)$$

where $\Phi = \partial\log(\omega^2 + c\mathrm{Re}t(\omega))/\partial\log\omega^2$. The appearance of this term in the expression for the diffusion coefficient is due to the renormalization of the phonon dispersion law in the presence of scattering centers. The frequency dependence of the phonon diffusion coefficient in the case of resonance scattering was considered in detail in Ref. [34], in which the temperature dependence of the phonon diffusion coefficient in nanoceramics was investigated. It was found experimentally that this characteristic varies greatly with a slight change in the technology of sample preparation (i.e., the structure), so that the exponent in the dependence $D \sim T^n$ can range from minus 5 to plus 6 in a narrow temperature range. As it is difficult to assume that small changes in the sample structure alter the mechanism of phonon scattering, the observed anomaly cannot be described within the τ-approximation in which $D_{ph} \sim v^2\tau$ at $v = \mathrm{const}$.

To explain the variability of this quantity, the phonon spectrum renormalization was accounted for in Ref. [34], where an elastic medium of density ρ_0, the properties of which are described by one elastic modulus K_0 (scalar model) with the dispersion law $\omega = v_0 q$ ($v_0^2 = K_0/\rho_0$) for phonons of any polarization, was considered as the theoretical model. The phonon scattering centers were randomly space-distributed

spherical shells with outer radius R, thickness d, and elastic parameters v, ρ ($K = v^2/\rho$). These shells modeled the grain boundaries, and the material inside and outside them the ceramic grains. The standard problem of elastic plane wave scattering on a spherical inclusion was solved exactly. It was found that if the material of the shells is acoustically soft ($K \ll K_0$), the one-site scattering matrix may have a resonance character, and the expression for the matrix in the long-wavelength limit has the form similar to Eq. (3), with a resonance frequency $\omega_r = (v_0/R)x_r$, $x_r = \sqrt{KR/K_0 d} < 1$. Here $x = qR = \omega R/v_0$ is a natural variable in problems of the elasticity theory. With a sufficient concentration of shells in the system, like in the case of the heavy impurity, there occurs a splitting of the acoustic branch with the formation of two bands, separated by a gap with parameters

$$x_1 = x_r, \quad x_2 = \sqrt{x_r^2 + 6c_v}, \tag{7}$$

where $c_v = n(4\pi R^3/3)/v$ is the share of the system volume occupied by the spheres.

Figure 3 shows the dependences $D_0(x)$ calculated using the approximate expressions for low frequencies at different values of the two main parameters: x_r and c_v. For all curves, Rayleigh scattering ($D_0(x) \simeq 1/x^4$) is observed at small x. At large x, geometric scattering is observed in both the presence and the absence of resonance. In the intermediate region ($x \simeq 1$), at small x_r (large c_v) there arises a gap whose edges are defined by Eq. (7). This gap becomes a minimum with increasing x_r (decreasing c_v). The upper curve in Fig. 3 is related to a system containing absolutely hard spheres (well-stabilized grain boundaries). In this case, the gap is lacking upon transition from the Rayleigh to the geometric scattering. This figure illustrates the diversity in the frequency dependence of the phonon diffusion coefficient which should affect the calculation of heat conductivity.

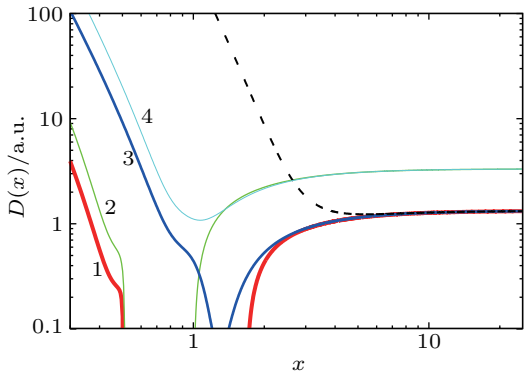


Fig. 3. (color online) Phonon diffusion coefficient $D(x)$ calculated at different values parameters of (1) $x_r = 0.5$ and $c_v = 0.5$, (2) $x_r = 0.5$ and $c_v = 0.2$, (3) $x_r = 1.0$ and $c_v = 0.5$, and (4) $x_r = 1.0$ and $c_v = 0.2$. The dashed curve corresponds to the case of absolutely hard spheres.

From the data presented in this section, it follows that in the case of resonance scattering of phonons at structural defects, the phonon density of states and the phonon diffusion coefficient are very sensitive to the resonance scattering parameters x_r and c_v.

3. Thermal conductivity of systems with a gap in the phonon spectrum

For the thermal conductivity, we use the standard expression[29]

$$\kappa(T) = \beta^2 k_B \int d\omega g_{ph}(\omega)\omega^2 D(T,\omega)\cosh(\beta\omega/2)/4, \tag{8}$$

where $\beta = \hbar/k_B T$ and k_B is the Boltzmann constant. To analyze the effect of the gap in the phonon spectrum, we take into account, one after another, additional contributions to the diffusion coefficient $D(T,\omega)$, assuming

$$D^{-1}(T,\omega) = \sum D^{-1}(T,\omega_i), \tag{9}$$

where $i =$ r, B, tls, and U denotes the resonance scattering, boundary scattering, scattering at two-level systems (TLS), and phonon–phonon scattering, respectively.

The dependences $\kappa(T)$ presented in Fig. 4 were calculated with scattering matrix (3) at $\eta = 1/3$, and to eliminate the integral divergence, only the boundary scattering for the lower zone phonons was additionally taken into account, i.e. only two terms D_r and D_B in Eq. (9). The curves presented in this figure demonstrate an important result of the model proposed. The thermal conductivity at low temperatures is formed by the first-zone phonons whose contribution to the thermal conductivity at the plateau temperature attains saturation, and subsequent increase in the thermal conductivity is due to the second-zone phonons involved in the process. The contributions from the zones 1 and 2 to the overall thermal conductivity are shown for the bottom curve in Fig. 4 by dashed lines. The temperature of the plateau attainment is $T_{pl}/T_D \simeq 0.3\omega_r/\omega_D$. We start from the model of crystalline structure of the amorphous state,[35] which implies that the dynamic properties of an amorphous material correspond qualitatively to the behavior of a crystal lattice comprising bulk structural defects (clusters having a size of tens of nanometers).

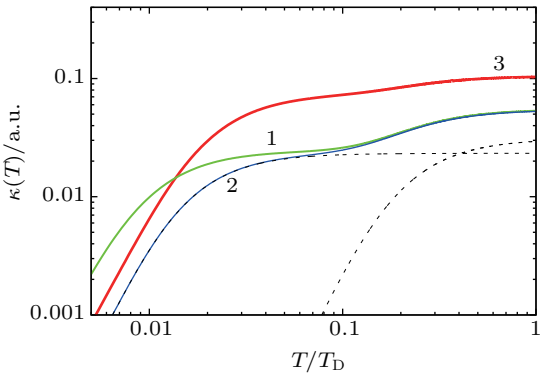


Fig. 4. (color online) The thermal conductivity for different values of the position and the width of the gap. The line numbers correspond to calculations with the same parameters as that in Fig. 1.

For the long-wave thermal phonons, clusters of this size can be considered, to a first approximation, as point defects, which allows us to use the results of Refs. [31] and [32]. Part of the generalized degrees of freedom of the atoms in such clusters correspond to multiwell potentials with tunneling transitions between their minima, and the energy distribution function of TLS, $\rho(\Delta)$, can be determined from the low-temperature behavior of the specific heat capacity.[19] At the same time, other degrees of freedom may correspond to a sufficiently large effective mass, about the sum of the masses of all atoms in the cluster (i.e., $\varepsilon \gtrsim 100$) and a weak coupling with the matrix, in order to ensure resonance scattering of long-wavelength phonons at such defects. The calculation of the thermal conductivity was performed with parameters close to this value, for which the plateau position is close to universal.[14] However, one cannot judge the gap and its parameters in the spectrum of the systems considered only by the behavior of $\kappa(T)$, without invoking the results of other independent experiments, for example, on the phonon diffusion coefficient.[7]

Consider the effect of low-energy two-level systems on the thermal conductivity in the presence of a gap in the phonon spectrum, adding to Eq. (9) a third term with the inverse relaxation rate proportional to the frequency, i.e., assuming $\rho(\Delta) = \text{const}$:

$$\tau_{\text{tls}}^{-1} = A_{\text{tls}}\omega \tanh(\omega/2\beta) \equiv A_{\text{tls}}\omega c_{\text{tls}}(T), \qquad (10)$$

where A_{tls} is the constant of the phonon interaction with TLS and $c_{\text{tls}}(T)$ is the effective concentration of TLS which decreases with increasing temperature because of equalization of the level population.

Figure 5 shows the dependence $\kappa(T)$ calculated at different ratios between D_{B} and D_{tls}, with the same gap edges in the phonon spectrum as curve 1 in Fig. 1: $z_1 = 0.05$ and $z_2 = 0.25$. The top curve refers to a system without TLS ($D_{\text{tls}}^{-1} = 0$), the bottom one to the case of dominant TLS scattering ($D_{\text{B}}^{-1} = 0$), and the rest to the case when the two terms are comparable in magnitude. It should be noted that for all the curves one can distinguish three areas characterized by different temperature dependences. In the low-temperature limit, the exponent in $\kappa(T) \sim T^n$ ranges from 2 for the bottom curve to 3 for the top one. In the second area (plateau region), the plateau is observed only for the systems without TLS, while the other exhibit a slow growth of $\kappa(T) \sim a + bT^n$, $n < 2$ due to the equalization of the level population (a decrease of $c_{\text{tls}}(T)$ from (12)), and saturation of the phonon contribution to the first zone occurs at much higher temperatures. Inclusion of the second-zone phonons in the heat transfer (third area) is characterized by a rapid increase in thermal conductivity. Thus the behavior of $\kappa(T)$ at temperatures of the plateau and below is determined by the ratios between the inverse relaxation rates for various scattering processes, and when selecting the appropriate parameters additional experimental data are required, in particular, the specific heat capacity in the low-temperature limit.

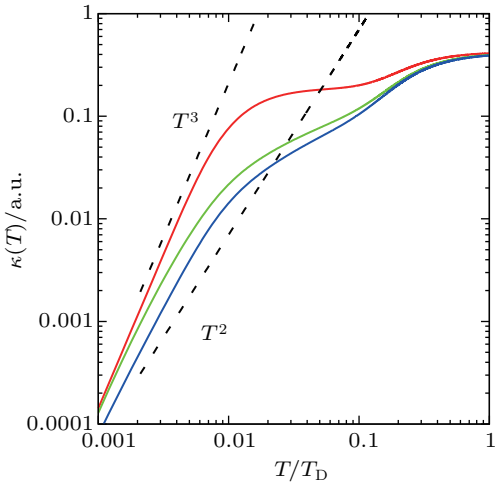


Fig. 5. (color online) The thermal conductivity for different values of the position and the width of the gap. The line numbers correspond to calculations with the same parameters as that in Fig. 1.

We introduce an additional term describing the phonon–phonon interaction in Eq. (9). An important consequence of the phonon spectrum splitting is the fact that in this case in the system there arises an additional group of short-wavelength phonons which may be involved in the U-processes. These are low-frequency phonons of the first branch with a very weak dispersion over a large part of the Brillouin zone (see Fig. 1). The phonon–phonon interaction involving such phonons was studied in detail in Ref. [26] to describe the anomalies of the low-temperature thermal conductivity in quasi-one-dimensional systems. As in the model presented there forms a complete gap in the phonon spectrum, in contrast to the quasi-one-dimensional systems, in which a gap arises only for the bending phonons, the expression obtained in Ref. [26] for the inverse relaxation rate of U-processes would not do, and the standard expression is used for the first-zone phonons

$$\tau_{\text{U}}^{-1} = A_{\text{U}}\omega^2 \exp(-E_{\text{U}}/k_{\text{B}}T), \qquad (11)$$

with the assumption that $E_{\text{U}} \lesssim \hbar\omega_{\text{r}}$.

From Fig. 6, in the presence of a gap in the phonon spectrum, an important role in the $\kappa(T)$ behavior in the plateau region is played by the U-processes involving short-wavelength low-energy phonons of the lower zone which suppress the growth of $\kappa(T)$ caused by the equalization of the TLS level population. First, the U-processes can lead to a flattening of the curve, and then with an increase in the intensity of the phonon–phonon interaction, to the formation of a local minimum.

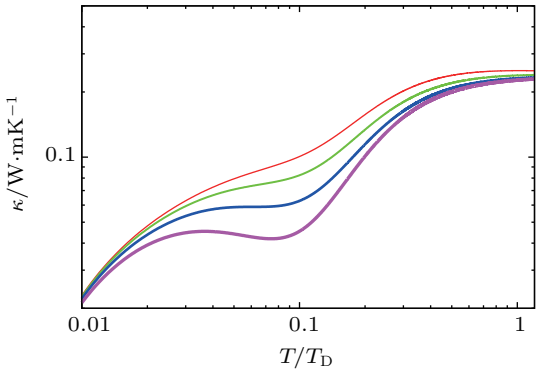


Fig. 6. (color online) The change in the behavior of the thermal conductivity with increasing intensity (from top to down) of U-processes involving phonons of the lower zone.

The calculation results presented in this section show that the proposed model based on a small number of scattering mechanisms can qualitatively describe any anomalous behavior of $\kappa(T)$ ever observed in the experiment.

At the same time, these results suggest that the temperature dependence behavior in particular temperature intervals may be obtained with various sets of parameters, so the plateau itself is obtained in the absence of TLS (Fig. 4), or with TLS when taking account of the U-processes involving phonons of the first zone (Fig. 5), and the temperature dependence $\kappa(T)$ when approaching the plateau region can be obtained as both the TLS concentration (Fig. 5) and the intensity of U-processes (Fig. 6). Thus in theoretical description of such complex integral characteristics of the thermal conductivity, the results of independent experiments are needed to define the absolute values of the model parameters. As no additional experimental data for new glass-like systems are available, we cannot give preference to any one set of parameters. Hence, figure 7 shows the characteristic features of the thermal conductivity (the growth in the plateau region, the plateau itself, and the local minimum) for primary alcohols, and the theoretical curves calculated within the model proposed are presented without discussing the parameters used in the calculation.

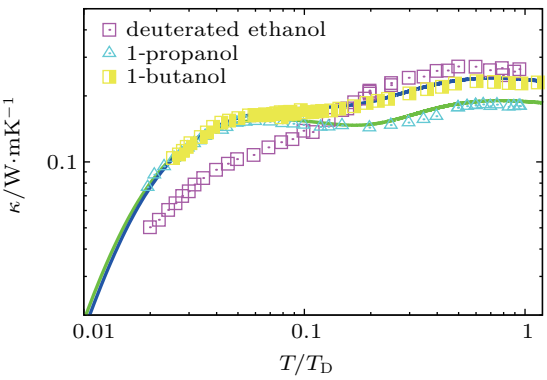


Fig. 7. (color online) Experimental results obtained for primary alcohols in Refs. [2] (1-propanol), [3] (1-butanol), and [36] (deuterated ethanol), and the curves calculated within the model proposed.

4. Conclusion

The theoretical study of the low-temperature thermal conductivity of disordered systems with a gap in the phonon spectrum using different values of scattering parameters shows the opportunities offered by the model in describing the thermal conductivity of the glass-like systems. The case where a complete gap appears as a result of resonance scattering of phonons at defects is considered. The conditions for the spectrum splitting and the formation of a low-frequency acoustic branch with a very weak dispersion over a large part of the Brillouin zone, separated from the remaining states by a gap, are found.

The position and the width of the gap correlate with those of the plateau region in the temperature dependence of the thermal conductivity coefficient. It is shown that the thermal conductivity at temperatures of the plateau region and below is determined only by the phonons of the lower zone, whereas the high-frequency phonons of the second zone remain frozen. The increase of the thermal conductivity above the plateau is due to the inclusion of these phonons into the heat transfer process. Another important consequence of the model used is the appearance of an additional group of short-wavelength phonons with a frequency much lower than the Debye one, which can take part in the U-processes at low temperatures. It is shown that such processes may be responsible for the temperature dependence of thermal conductivity in the plateau region observed in some glass-like systems.

Unfortunately, additional experimental data on low-temperature properties of glassy crystals are unavailable. Nonetheless, a qualitative comparison with the experiments on the thermal conductivity shows that the approach is suggested to be quite reasonable.

Acknowledgment

The author is grateful to Ostanin S for useful comments and Krivchikov A I for providing the experimental data in numerical form.

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