http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics B

Interaction between many parallel screw dislocations and a semi-infinite crack in a magnetoelectroelastic solid

Xin Lv(吕鑫), Guan-Ting Liu(刘官厅)

Citation: Chin. Phys. B . 2018, 27(7): 074601. **doi:** 10.1088/1674-1056/27/7/074601

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Interaction between infinitely many dislocations and a semi-infinite crack in one-dimensional hexagonal quasicrystal

Guan-Ting Liu(刘官厅), Li-Ying Yang(杨丽英)

Chin. Phys. B . 2017, 26(9): 094601. **doi:** 10.1088/1674-1056/26/9/094601

The interaction between a screw dislocation and a wedge-shaped crack in one-dimensional hexagonal piezoelectric quasicrystals

Li-Juan Jiang(姜丽娟), Guan-Ting Liu(刘官厅)

Chin. Phys. B . 2017, 26(4): 044601. **doi:** 10.1088/1674-1056/26/4/044601

Mechanics of high-capacity electrodes in lithium-ion batteries

Ting Zhu

Chin. Phys. B . 2016, 25(1): 014601. **doi:** 10.1088/1674-1056/25/1/014601

The influence of defects on the effective Young's modulus of a defective solid

Shen Wei, Fan Qun-Bo, Wang Fu-Chi, Ma Zhuang

Chin. Phys. B . 2013, 22(4): 044601. **doi:** 10.1088/1674-1056/22/4/044601

Screw dislocations interacting with two asymmetrical interfacial cracks emanating from an elliptical hole

Zeng Xin, Fang Qi-Hong, Liu You-Wen, P. H. Wen

Chin. Phys. B . 2013, 22(1): 014601. **doi:** 10.1088/1674-1056/22/1/014601

Interaction between many parallel screw dislocations and a semi-infinite crack in a magnetoelectroelastic solid*

Xin Lv(吕鑫)† and Guan-Ting Liu(刘官厅)‡

College of Mathematics Science, Inner Mongolia Normal University, Hohhot 010022, *China*

(Received 18 November 2017; revised manuscript received 23 February 2018; published online 25 June 2018)

Based on the fundamental equations of magnetoelectroelastic material and the analytic theory, and using the Muskhelishvili-introduced well-known elastic techniques combined with the superposition principle, the closed form solution of the generalized stress field of the interaction between many parallel screw dislocations and a semi-infinite crack in an infinite magnetoelectroelastic solid is obtained, on the assumption that the surface of the crack is impermeable electrically and magnetically. Besides, the Peach–Koehler formula of n parallel screw dislocations is given. Numerical examples show that the generalized stress varies with the position of point z and is related to the material constants. The results indicate that the stress concentration occurs at the dislocation core and the tip of the crack. The result of interaction makes the system stay in a lower energy state.

Keywords: parallel screw dislocations, semi-infinite crack, magnetoelectroelastic solid, interaction

PACS: 46.05.+b, 46.50.+a, 61.72.Lk **DOI:** 10.1088/1674-1056/27/7/074601

1. Introduction

Magnetoelectroelastic (MEE) material is a new type of intelligent material which can achieve the conversion of magnetic energy, electric energy, and mechanical energy, and has wide applications in lots of multi-functional equipment. So far, great progress has been made regarding the interaction among multi-defects in magnetoelectric composites.[1–11] Hao *et al.*[12] studied the interaction of a screw dislocation with a semi-infinite interfacial crack in an MEE bi-material. Fang *et al.*[13,14] derived the solutions of the interaction between a screw dislocation and other defects such as rigid lines, interfacial cracks, and inhomogeneity in MEE material. Hu and Li[15] presented the general solutions of the singular stress, electric and magnetic field in a piezoelectromagnetic strip containing a Griffith crack. Xiao *et al.*[16] discussed the generalized screw dislocation interacting with a wedge-shaped MEE bi-material interface. Liu and Guo[17] dealt with the problem of the interaction between a screw dislocation and an oblique edge crack in a half-infinite MEE solid. As mentioned above, few studies on the interaction between many dislocations and cracks have been done up to now. In this paper, we will study the interaction between many parallel dislocations and a semi-infinite crack in an MEE solid. The analytic solutions of the dislocations and the crack are obtained according to the method in Ref. [18]. This study expands the research scope of fracture mechanics of the MEE solid, and provides the theoretical basis for the application of this material in the fields of engineering and technology.

2. Basic equations

For an MEE solid, supposing that its polarized directions of the electric and magnetic fields are along the x_3 axis in the three-dimensional (3D) space coordinate system with an isotropic x_1–x_2 plane, we consider the mechanical–electric–magnetic coupling anti-plane deformation problem. It means that all the quantities can be determined by anti-plane displacement $u_3(x_1,x_2)$, in-plane electric potential $\varphi(x_1,x_2)$, and magnetic potential $\psi(x_1,x_2)$. The basic equations can be written as follows.[19]

Constitutive equations are given by

$$\begin{cases} \sigma_{32} = 2C_{44}\varepsilon_{32} - e_{15}E_2 - q_{15}H_2, \\ \sigma_{31} = 2C_{44}\varepsilon_{31} - e_{15}E_1 - q_{15}H_1, \\ D_2 = 2e_{15}\varepsilon_{32} + \kappa_{11}E_2 + d_{11}E_2, \\ D_1 = 2e_{15}\varepsilon_{31} + \kappa_{11}E_1 + d_{11}E_1, \\ B_2 = 2q_{15}\varepsilon_{32} + d_{11}E_2 + \mu_{11}E_2, \\ B_1 = 2q_{15}\varepsilon_{31} + d_{11}E_1 + \mu_{11}E_1. \end{cases} \tag{1}$$

Generalized strain-displacement relations are given by

$$\begin{cases} \varepsilon_{32} = \dfrac{1}{2}\dfrac{\partial u_3}{\partial x_2}, \quad \varepsilon_{31} = \dfrac{1}{2}\dfrac{\partial u_3}{\partial x_1}, \\ E_2 = -\dfrac{\partial \varphi}{\partial x_2}, \quad E_1 = -\dfrac{\partial \varphi}{\partial x_1}, \\ H_2 = -\dfrac{\partial \psi}{\partial x_2}, \quad H_1 = -\dfrac{\partial \psi}{\partial x_2}. \end{cases} \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11262017, 11262012, and 11462020), the Natural Science Foundation of Inner Mongolia Autonomous Region, China (Grant No. 2015MS0129), the Key Project of Inner Mongolia Normal University, China (Grant No. 2014ZD03), and the Graduate Research Innovation Project of Inner Mongolia Autonomous Region, China (Grant No. S20171013502).

†Corresponding author. E-mail: lvxinnmg@qq.com

‡Corresponding author. E-mail: guantingliu@imnu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

Equilibrium equations are given by

$$\begin{cases} \dfrac{\partial \sigma_{32}}{\partial x_2} + \dfrac{\partial \sigma_{31}}{\partial x_1} = 0, \\ \dfrac{\partial D_2}{\partial x_2} + \dfrac{\partial D_1}{\partial x_1} = 0, \\ \dfrac{\partial B_2}{\partial x_2} + \dfrac{\partial B_1}{\partial x_1} = 0, \end{cases} \tag{3}$$

where none of the body force, body volume charge density, and body volume current are considered; σ_{3k}, ε_{3k}, and u_3 are the stress, the strain, and the displacement, respectively; D_k, E_k, and φ are the electric displacement, the electric field, and the electric potential, respectively, with $k = 1, 2$; B_k, H_k, and ψ are the magnetic induction, the magnetic field, and the magnetic potential, respectively; C_{44}, κ_{11}, and μ_{11} are the elastic coefficient, the dielectric coefficient, and the magnetic permeability coefficient, respectively; e_{15}, q_{15}, and d_{11} are piezoelectric, piezomagnetic, and magnetoelectric coupling coefficient, respectively.

Substituting Eqs. (1) and (2) into Eq. (3), we obtain

$$\boldsymbol{C}\nabla^2 \boldsymbol{u} = 0, \tag{4}$$

in which

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \tag{5}$$

$$\boldsymbol{C} = \begin{bmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -d_{11} \\ q_{15} & -d_{11} & -\mu_{11} \end{bmatrix}, \tag{6}$$

$$\boldsymbol{u} = [u_3, \varphi, \psi]^{\mathrm{T}}. \tag{7}$$

Because $\boldsymbol{C}$ is nonsingular, equation (4) is equivalent to

$$\nabla^2 \boldsymbol{u} = 0. \tag{8}$$

3. Analytic solution of *n* parallel dislocations in coupling field

The physical model considered in this paper is shown in Fig. 1. There are n screw dislocations in the vicinity of a semi-infinite crack in an MEE solid that is infinitely large.

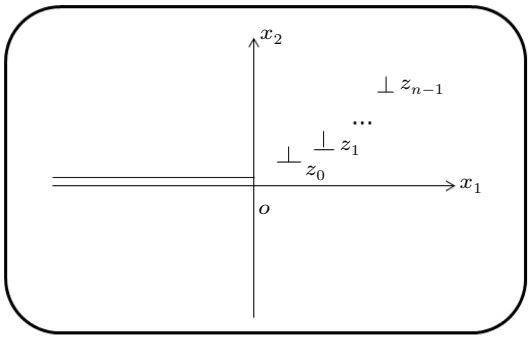


Fig. 1. Many parallel dislocations around a semi-infinite crack in MEE solid.

We first consider the analytic solution of a dislocation. Let one of the dislocations be located at the z_0 point in the x_1–x_2 plane and its Burgers vector is $(0, 0, b_3, b_\varphi, b_\psi)$ in the problem under consideration. The dislocation conditions are given by

$$\oint_\lambda \mathrm{d}u_3 = b_3, \quad \oint_\lambda \mathrm{d}\varphi = b_\varphi, \quad \oint_\lambda \mathrm{d}\psi = b_\psi, \tag{9}$$

where λ denotes the Burgers contour surrounding the dislocation z; b_3 is the displacement jump value along x_3; b_φ is the potential jump value; b_ψ is the magnetic potential jump value. Note that the three integrals are conducted in physical space.

By means of properties of the analytic function, from Eq. (8), we define

$$\boldsymbol{u} = \mathrm{Re}\left[\boldsymbol{f}(z)\right], \tag{10}$$

where $\boldsymbol{f}(z)$ represents the analytic function vector and Re denotes its real part.

We introduce the following representations of symbols:

$$[\sigma_{31}, D_1, B_1]^{\mathrm{T}} = \boldsymbol{\Sigma}_1, \quad [\sigma_{32}, D_2, B_2]^{\mathrm{T}} = \boldsymbol{\Sigma}_2, \tag{11}$$

$$[\varepsilon_{31}, -E_1, -H_1]^{\mathrm{T}} = \boldsymbol{Z}_1, \quad [\varepsilon_{32}, -E_2, -H_2]^{\mathrm{T}} = \boldsymbol{Z}_2. \tag{12}$$

According to the force conditions of the dislocation, the form of analytic function $\boldsymbol{f}(z)$ can be taken as

$$\boldsymbol{f}(z) = \boldsymbol{A}\ln(z - z_0), \tag{13}$$

where $z = x_1 + \mathrm{i}x_2$.

Moreover,

$$\begin{aligned} \mathrm{Re}\left[\int_{z_0} \boldsymbol{f}'(z)\,\mathrm{d}z\right] &= \mathrm{Re}\left[\int_{z_0} \frac{\boldsymbol{A}}{z - z_0}\,\mathrm{d}z\right] \\ &= \mathrm{Re}\left[2\pi\mathrm{i}\boldsymbol{A}\right] \\ &= [b_3, b_\varphi, b_\psi]^{\mathrm{T}}. \end{aligned} \tag{14}$$

From Eq. (14), we have

$$\boldsymbol{A} = \frac{1}{2\pi\mathrm{i}}\left[b_3, b_\varphi, b_\psi\right]^{\mathrm{T}}. \tag{15}$$

From Eqs. (10), (13), and (15), we can obtain the complex representation of the generalized displacement.

Based on Eq. (1), equation (11) can be written as

$$\boldsymbol{\Sigma}_1 - \mathrm{i}\boldsymbol{\Sigma}_2 = \boldsymbol{C}f'(z) = \frac{\boldsymbol{C}\cdot\boldsymbol{A}}{z - z_0}. \tag{16}$$

Equation (16) is the generalized stress in the form of the matrix.

According to Eqs. (2) and (12), the generalized strain can be obtained as

$$\boldsymbol{Z}_1 - \mathrm{i}\boldsymbol{Z}_2 = \frac{\boldsymbol{A}}{z - z_0}. \tag{17}$$

From Eq. (16), the generalized stress induced by dislocation z_i acting on z_1 is

$$\boldsymbol{F}_{i1} = \frac{\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}}{z_1 - z_i} \quad (i = 2, 3, \ldots, n). \tag{18}$$

Supposing that there are n parallel screw dislocations located at points $z_1, z_2, \ldots, z_n$, respectively, and their Burgers vectors are $(0, 0, b_3^{(i)}, b_\varphi^{(i)}, b_\psi^{(i)})$. Hence, the form of the corresponding

matrix is taken as $\boldsymbol{A}_{(i)}$. Using the superposition principle, the field force of the dislocation z_i, acting on an arbitrary point z in the x_1–x_2 plane is

$$\boldsymbol{F}(z)=\sum_{i=1}^{n}\frac{\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}}{z-z_i}. \tag{19}$$

Equation (19) is the analytic solution of n parallel dislocations in the coupling field.

4. Interaction among n parallel dislocations

In this section, we consider the interactions among n parallel dislocations. According to Ref. [20] , using the superposition principle, the force induced by dislocations $z_2, z_3, \ldots, z_n$, acting on z_1, is given by

$$\overline{g}=\pi\mathrm{Res}\left[\sum_{i=2}^{n}\frac{\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}}{z_1-z_i}+\frac{\boldsymbol{C}\cdot\boldsymbol{A}_{(1)}}{z-z_1}\right]^2, \tag{20}$$

where Res denotes the residue on variable z, and $\overline{()}$ refers to the complex conjugate of variable in the bracket. According to residue theory, we have

$$g=2\pi\boldsymbol{A}_{(1)}\sum_{i=2}^{n}\frac{\boldsymbol{C}\boldsymbol{A}_{(i)}}{\overline{z_1}-\overline{z_i}}. \tag{21}$$

Equation (21) is the Peach–Koehler formula of n parallel dislocations in an MEE solid. It is indicated that the force induced by dislocations $z_2, z_3, \ldots, z_n$, acting on z_1 is inversely proportional to the distance from it to the dislocations, and it is proportional to the Burgers vectors in an MEE solid.

5. Interaction between n parallel dislocations and a semi-infinite crack

The interaction between n parallel dislocations and a semi-infinite crack has received more attention.

From Eq. (19), the generalized distribution force in the crack surface, produced by n parallel dislocations is

$$\boldsymbol{F}(t)=\sum_{i=1}^{n}\frac{\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}}{t-z_i},\quad t\in(-\infty,0). \tag{22}$$

Because the crack is free of traction on its surface, the generalized stress produced by the dislocations in the crack surface is counteracted by the equivalent and reverse additional distribution force produced by the crack surface. The boundary conditions of the crack are expressed as follows:

$$\begin{cases} z\to\infty:\boldsymbol{\Sigma}_1-\mathrm{i}\boldsymbol{\Sigma}_2=0, \\ z=t\in(-\infty,0):\boldsymbol{\Sigma}_1-\mathrm{i}\boldsymbol{\Sigma}_2=-\boldsymbol{F}(t). \end{cases} \tag{23}$$

The three sets of equations for the MEE solid together with Eq. (23) constitute the Riemann–Hilbert boundary problem.[21,22]

According to Muskhelishvili's method,[21] we obtain the generalized stress field induced by additional distribution force, namely,

$$\begin{aligned}\tilde{F}(z) &= -\frac{z^{-\frac{1}{2}}}{\pi}\int_{-\infty}^{0}\frac{(-t)^{\frac{1}{2}}}{t-z}\boldsymbol{F}(t)\,\mathrm{d}t \\ &= -\sum_{i=1}^{n}\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}\frac{1}{\sqrt{z}(\sqrt{z_i}+\sqrt{z})}.\end{aligned} \tag{24}$$

Submitting Eq. (19) to Eq. (24), the generalized stress field of the interaction between n parallel dislocations and a semi-infinite crack is derived and given by

$$\boldsymbol{F}_{\mathrm{total}}(z)=\sum_{i=1}^{n}\boldsymbol{C}\cdot\boldsymbol{A}_{(i)}\left(\frac{1}{z-z_i}-\frac{1}{\sqrt{z}(\sqrt{z_i}+\sqrt{z})}\right). \tag{25}$$

6. Numerical examples

To discuss the interaction between many parallel dislocations and a semi-infinite crack in an MEE solid, the material constants we choose are listed in Table 1.[23]

Table 1. Material constants in MEE solid.

$C_{44}/\mathrm{N\cdot m^{-2}}$	$e_{15}/\mathrm{C\cdot m^{-2}}$	$\kappa_{11}/\mathrm{C^2\cdot N^{-1}\cdot m^{-2}}$	$q_{15}/\mathrm{N\cdot(A\cdot m)^{-1}}$	$\mu_{11}/\mathrm{N\cdot s^2\cdot C^{-2}}$	$d_{11}/\mathrm{N\cdot s\cdot V^{-1}\cdot C^{-1}}$
4.4×10^{10}	5.8	56.4×10^{-10}	275	297×10^{-6}	5.2×10^{-12}

All the Burgers vectors of the dislocations are taken to be 10^{-9} in magnitude. In addition, $\boldsymbol{F}_\sigma$ represents the stress of the elastic field, $\boldsymbol{F}_\mathrm{D}$ the stress of the electric field, and $\boldsymbol{F}_\mathrm{B}$ the stress of the magnetic field in Fig. 2.

It is found from Fig. 2 that the generalized stress increases with the decrease of the distance from the points on the crack to the dislocation. The singularity of the stress in the dislocation core is of the first order in MEE materials.

Letting $n=5$, and $z_1=1$, $z_2=2$, $z_3=3$, $z_4=4$, and $z_5=5$, figure 3 shows that the stress concentrations occur, respectively, at the dislocation cores and the tip of the crack. The stress field, electric field, and magnetic field from the crack tip to the dislocation z_1 first decrease and then increase. From Figs. 3(a), 3(b), and 3(c), we can see that the interaction between dislocations and the crack has a greater influence on the elastic field than the electric field and the magnetic field. This is because the material constants relating to the elastic field are far greater than those relating to the electric field and magnetic field.

Taking the stress of the elastic field for example, figure 4 illustrates that the crack weakens the stress field of n parallel dislocations. The resultant interaction makes the system stay in a lower energy state. Besides, the interaction among dislocations plays a major role in the generalized stress field as the position of point z goes away from the crack tip.

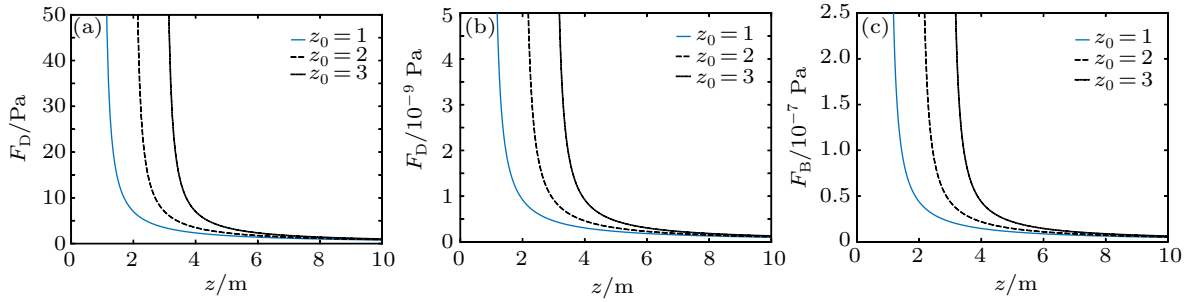


Fig. 2. (color online) (a) Stress produced by elastic field, $\boldsymbol{F}_\sigma$, (b) stress produced by electric field, $\boldsymbol{F}_D$, and (c) stress of magnetic field, $\boldsymbol{F}_B$, varying with the position of point z.

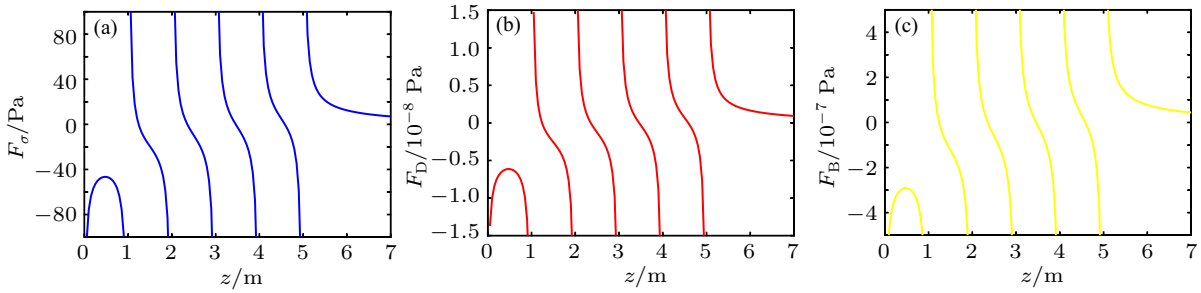


Fig. 3. (color online) Total stresses of (a) elastic field, (b) electric field, and (c) magnetic field varying position of point z.

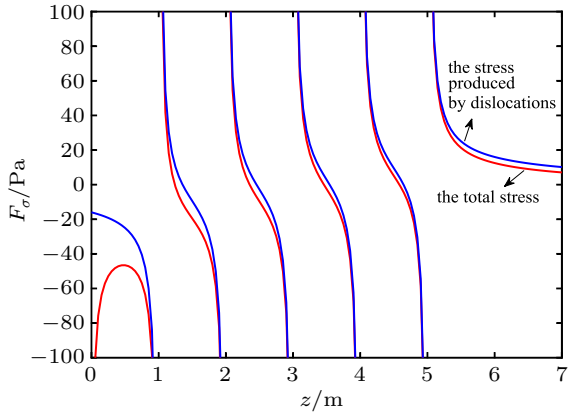


Fig. 4. (color online) Comparison between the total stress and stress produced by dislocations of elastic field.

7. Conclusions

In this paper, a mathematical model of the interaction between n parallel screw dislocations and a semi-infinite crack in an MEE solid is established, and the analytic solutions of the stress field, electric field, and magnetic field are obtained. Finally, the interaction among n parallel screw dislocations and the interaction between n parallel dislocations and a semi-infinite crack are studied further by numerical examples.

(i) Equation (21) gives the Peach–Koehler formula of n parallel screw dislocations in an MEE solid. The results indicate that the Burgers vectors and the distance between dislocations affect the interaction among n parallel screw dislocations.

(ii) From Eq. (25), we can see that the generalized stress in an MEE solid is related to the Burgers vectors and the material constants. In addition, the interaction in the elastic field is more obvious than that in the electric field or magnetic field.

(iii) The generalized stress first decreases and then increases from the dislocation z_1 and the crack tip. At the position far away from the crack, the generalized stress is mainly affected by the dislocations.

(iv) The stress accumulates at the dislocation core and the tip of the crack. The resultant interaction makes the system stay in a lower energy state.

References

[1] Nan C W 1994 *Phys. Rev. B* **50** 6082
[2] Sih G C, Jones R and Song Z F 2003 *Theor. Appl. Fract. Mec.* **40** 161
[3] Gao C F, Kessler H and Balke H 2003 *Int. J. Eng. Sci.* **41** 969
[4] Gao C F, Kessler H and Balke H 2003 *Int. J. Eng. Sci.* **41** 983
[5] Wang B L and Mai Y W 2003 *Eur. J. Mech.* **22** 591
[6] Wang B L and Mai Y W 2004 *Mech. Res. Commun.* **31** 65
[7] Zhao M H, Wang H, Yang F and Liu T 2006 *Theor. Appl. Fract. Mec.* **45** 227
[8] Wang B L and Mai Y W 2007 *Int. J. Solid Struct.* **44** 387
[9] Zhong X C 2008 *Study on Fracture of Magnetoelectroelastic Solid* (Ph.D. Dissertation) (Hunan: Central South University) (in Chinese)
[10] Li G, Wang B L and Han J C 2010 *Int. J. Solid Struct.* **47** 419
[11] Guo J H, Lu Z X and SIRI G L 2013 *Acta Mater. Compos. Sin.* **30** 192 (in Chinese)
[12] Hao R J and Liu J X 2006 *Mech. Res. Commun.* **33** 415
[13] Fang Q H, Liu Y W and Jiang C P 2005 *Int. J. Eng. Sci.* **43** 1011
[14] Yu M, Fang Q H, Liu Y W and Zeng X 2012 *Theor. Appl. Fract. Mech.* **59** 34
[15] Hu K Q and Li G Q 2005 *Mech. Mater.* **37** 925
[16] Xiao Z M, Chen B J and Luo J 2010 *Acta Mech.* **214** 261
[17] Liu X and Guo J H 2016 *Theor. Appl. Fract. Mec.* **86** 225
[18] Liu G T and Yang L Y 2017 *Chin. Phys. B* **26** 094601
[19] Qin Q H and Yang Q S 2006 *Macro-Micro Theory on Multifield Coupling Behavior of Heterogeneous Materials* (Vol. 1) (Beijing: Higher Education Press) p. 101 (in Chinese)
[20] Lin I H and Thomson R 1986 *Acta Metall.* **34** 187
[21] Muskhelishvili N I 1953 *Singular Integral Equations* (Noordhoff: Groningen)
[22] Liu G T, Guo R P and Fan T Y 2003 *Chin. Phys. B* **12** 1149
[23] Sih G C and Chen E P 2003 *Theor. Appl. Fract. Mech.* **40** 1