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Entangling two oscillating mirrors in an optomechanical system via a flying atom*

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We propose a novel scheme for generating the entanglement of two oscillating mirrors in an optomechanical system via a flying atom. In this scheme, a two-level atom, in an arbitrary superposition state, passes through an optomechanical system with two oscillating cavity-mirrors, and then its states are detected. In this way, we can generate the entangled states of the two oscillating mirrors. We derive the analytical expressions of the entangled states and make numerical calculations. We find that the entanglement of the two oscillating mirrors can be controlled by the initial state of the atom, the optomechanical coupling strength, and the coupling strength between the atom and the cavity field. We investigate the dynamics of the system with dissipations and discuss the experimental feasibility.

Keywords: quantum entanglement, optomechanical system, dissipative dynamics

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1. Introduction

Quantum entanglement,[1] as a cornerstone of quantum physics, has potential applications in quantum information science, such as quantum computing,[2] quantum teleportation,[3,4] quantum dense coding,[5] quantum cryptography,[6,7] and so on. So far, quantum entanglement has been realized in various physical systems, such as photonic systems,[8–10] cavity quantum electrodynamics,[11–15] superconductor circuits,[16,17] and ion traps.[18,19] Quantum entanglement has been studied theoretically and experimentally, such as based on photons,[20,21] atoms,[22–24] ions,[25,26] and so on.

Cavity optomechanics, studying the interaction between optical fields and mechanical resonators via the radiation pressure, has attracted massive theoretical and experimental researches in recent years. It provides a platform for realizing novel quantum effects in an extremely wide range from microscale to macroscale. Recently, theoretical and experimental efforts have demonstrated many quantum effects in optomechanical systems, for example, the optomechanically induced transparency[27–29] similar to electromagnetically induced transparency (EIT),[30] the optomechanical storage,[31] the normal mode splitting,[32,33] the preparation of nonclassiacl states,[34,35] the mechanical oscillator squeezing,[36,37] the ground state cooling of the mechanical resonators,[38,39] and so on.[40,41] The quantum entanglement has also been proposed in various optomechanical systems. Vitali *et al*. theoretically investigated stationary entanglement between an optical cavity-mode and a macroscopic vibrating mirror in an optomechanical system.[42] Later, Palomaki *et al*. experimentally demonstrated this kind of entanglement.[43] Genes *et al*. proposed the tripartite and the bipartite continuous variable entanglement by placing a two-level atomic ensemble inside the optomechanical system.[44]

In this paper, we propose a scheme for generating the entanglement of two oscillating mirrors. The system considered consists of an optical cavity with two oscillating cavity-mirrors, an atomic source, a Ramsey zone, and a detector of atomic states. A two-level atom, coming from the atomic source, enters into the optomechanical cavity and interacts with the cavity-field. Meanwhile, the cavity-field also interacts with the two oscillating cavity-mirrors. After the atom exits from the cavity, it goes through a Ramsey zone, and then its states are detected. In this way, we can generate the entangled states of the two oscillating mirrors. Our scheme is different from the previous ones. For example, Zhou *et al*.[45] investigated entanglement of two optomechanical oscillators and two-mode fields induced by atomic coherence. The two modes of the cavity field interact via the coherent three-level atoms and also interact with one movable mirror, respectively, resulting in entanglement between the two movable mirrors. Liao *et al*.[46] investigated the entanglement between two macroscopic mechanical resonators in a two-cavity optomechanical system. Two cavity fields interact with each other via a single photon exchange, and each cavity field interacts with a mechanical resonator, and finally the entanglement of the two mechanical resonators is generated.

The remainder of this paper is organized as follows. In

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Section 2, we give a description of our theoretic model. In Section 3, we describe the process of the entanglement generation and derive the analytical expressions for the generated entangled states. In Section 4, we discuss the measure of the generated entangled states between the two mechanical mirrors, and analyze the effects of dissipations on the generated entangled states. Finally, we discuss the experimental feasibility of our proposal and summarize our results in Section 5.

2. Model and theory

Our proposed scheme is shown in Fig. 1. A two-level atom, coming from the atomic source, enters into the optomechanical cavity and interacts with the cavity-field. Meanwhile, the cavity-field also interacts with the two oscillating cavity-mirrors. After the atom exits from the cavity, it goes through a Ramsey zone, and then its states are detected. The total Hamiltonian of the system can be written as follows:[47,48]

$$H = \hbar\omega_{\rm c}c^{\dagger}c+\frac{\hbar}{2}\omega_{\rm a}\sigma_z-{\rm i}\hbar g_{\rm ca}\left(c\sigma_+-c^{\dagger}\sigma_-\right) + \sum_{j=1,2}\hbar\omega_{\rm m}b_j^{\dagger}b_j+\hbar g_{\rm cb}c^{\dagger}c\left[\left(b_2^{\dagger}+b_2\right)-\left(b_1^{\dagger}+b_1\right)\right], \quad (1)$$

where c ($c^{\dagger}$) and b_j ($b_j^{\dagger}$) are the annihilation (creation) operators of the cavity mode with frequency $\omega_{\rm c}$ and the mechanical modes with frequencies $\omega_{\rm m}$, respectively. $\sigma_z = |e\rangle\langle e|-|g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, and $\sigma_- = |g\rangle\langle e|$ are the Pauli operators of the two-level atom with transition frequency $\omega_{\rm a}$. $g_{\rm cb} = (\omega_{\rm c}/L)\sqrt{\hbar/2m\omega_{\rm m}}$ is the single-photon optomechanical coupling strength between the cavity field and the mechanical modes, with m the mass of the oscillating cavity-mirror, and L the distance between the two cavity-mirrors at equilibrium. $g_{\rm ca}$ is the Jaynes–Cummings coupling strength between the atom and the cavity field. In the interaction picture with respect to $H_0 = \hbar\omega_{\rm c}c^{\dagger}c+\sum_{j=1,2}\hbar\omega_{\rm m}b_j^{\dagger}b_j+\frac{\hbar}{2}\omega_{\rm a}\sigma_z$, the interaction Hamiltonian is given in the form

$$H_{\rm I} = \hbar g_{\rm cb}c^{\dagger}c\left[\left(b_2^{\dagger}-b_1^{\dagger}\right){\rm e}^{{\rm i}\omega_{\rm m}t}+\left(b_2-b_1\right){\rm e}^{-{\rm i}\omega_{\rm m}t}\right] - {\rm i}\hbar g_{\rm ca}\left(c\sigma_+{\rm e}^{-{\rm i}(\omega_{\rm c}-\omega_{\rm a})t}-c^{\dagger}\sigma_-{\rm e}^{{\rm i}(\omega_{\rm c}-\omega_{\rm a})t}\right). \quad (2)$$

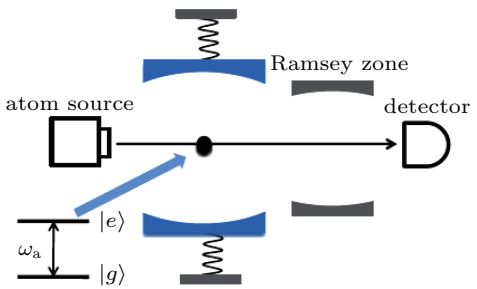


Fig. 1. (color online) Sketch of the system. A two-level atom successively passes through a cavity optomechanical system with two oscillating cavity-mirrors and a Ramsey zone, and then its states are detected.

We consider the resonance condition $\omega_{\rm c} = \omega_{\rm m}+\omega_{\rm a}$ in the following. When $\omega_{\rm m} \gg \{g_{\rm cb}, g_{\rm ca}\}$, the rapidly oscillating terms can be neglected, and an effective Hamiltonian can be derived by using the coarse-grained method[49]

$$H_{\rm eff} = \frac{{\rm i}\hbar g_{\rm cb}g_{\rm ca}}{\omega_{\rm m}}\left[c\left(b_2^{\dagger}-b_1^{\dagger}\right)\sigma_+-c^{\dagger}\left(b_2-b_1\right)\sigma_-\right] - \frac{2\hbar g_{\rm cb}^2}{\omega_{\rm m}}\left(c^{\dagger}c\right)^2-\frac{\hbar g_{\rm ca}^2}{\omega_{\rm m}}\left(c^{\dagger}c\sigma_z+\sigma_+\sigma_-\right), \quad (3)$$

where the first term describes the three-part interactions, which play a crucial role in generating the entangled states. The second term leads to a Kerr-like effect of the cavity-field, and the last term describes the photon-number dependent Stark shifts.

3. Entanglement generation

We consider that initially the cavity is in a single-photon state $|1\rangle_c$ and the mirrors are cooled to their ground states $|0\rangle_{m_j}$. The atomic source sends a two-level atom in the superposition state $(\cos\theta|e\rangle+\sin\theta|g\rangle)$ into the cavity. Then the initial state of the system can be written as

$$|\psi(0)\rangle = (\cos\theta|e\rangle+\sin\theta|g\rangle)|1\rangle_c|0\rangle_{m_1}|0\rangle_{m_2}. \quad (4)$$

When the atom is in the cavity, the state of the system satisfies the Schrödinger's equation

$${\rm i}\hbar\frac{{\rm d}}{{\rm d}t}|\psi(t)\rangle = H_{\rm eff}|\psi(t)\rangle. \quad (5)$$

Assuming that the atom leaves the cavity at time t_1, we find

$$\begin{aligned}|\psi(t_1)\rangle = {} & M_1(t_1)|e\rangle|1\rangle_c|0\rangle_{m_1}|0\rangle_{m_2} \\ & +M_2(t_1)|g\rangle|1\rangle_c|0\rangle_{m_1}|0\rangle_{m_2} \\ & +M_3(t_1)|e\rangle|0\rangle_c|1\rangle_{m_1}|0\rangle_{m_2} \\ & +M_4(t_1)|e\rangle|0\rangle_c|0\rangle_{m_1}|1\rangle_{m_2}, \end{aligned} \quad (6)$$

where

$$M_1(t_1) = \cos\theta\,{\rm e}^{2{\rm i}L_+t_1}, \quad (7)$$

$$M_2(t_1) = \frac{\sin\theta}{R}\left[R\cos(Rt_1)+{\rm i}L_-\sin(Rt_1)\right]{\rm e}^{{\rm i}\frac{g_{\rm cb}^2}{\omega_{\rm m}}t_1}, \quad (8)$$

$$M_3(t_1) = -\frac{\sin\theta}{R}K\sin(Rt_1)\,{\rm e}^{{\rm i}\frac{g_{\rm cb}^2}{\omega_{\rm m}}t_1}, \quad (9)$$

$$M_4(t_1) = \frac{\sin\theta}{R}K\sin(Rt_1)\,{\rm e}^{{\rm i}\frac{g_{\rm cb}^2}{\omega_{\rm m}}t_1}, \quad (10)$$

with $R^2 = \left(g_{\rm cb}^4+g_{\rm ca}^4\right)/\omega_{\rm m}^2$, $L_{\pm} = \left(g_{\rm cb}^2\pm g_{\rm ca}^2\right)/\omega_{\rm m}$, and $K = g_{\rm cb}g_{\rm ca}/\omega_{\rm m}$. From Eq. (6), we can see that the four parts, i.e., the cavity, the atom, and the two mechanical mirrors are in an entangled state.

When the atom is not in the cavity, there is no interaction between the atom and the optomechanical system ($g_{\rm ca}=0$), and the effective Hamiltonian of the system becomes

$$H'_{\rm eff} = -\frac{2\hbar g_{\rm cb}^2}{\omega_{\rm m}}\left(c^{\dagger}c\right)^2. \quad (11)$$

After the atom leaves the optomechanical system at time t_1 and before it arrives at the detector at time t_2, on one hand, the state of the system evolves according to $U(t) = \exp\left[-\frac{\mathrm{i}}{\hbar}H'_{\mathrm{eff}}t\right]$, and on the other hand, we let the atom pass through a Ramsey zone with a classical $\pi/2$ pulse, which transforms the atomic states as $|e\rangle \to (|e\rangle + |g\rangle)/\sqrt{2}$ and $|g\rangle \to (-|e\rangle + |g\rangle)/\sqrt{2}$, then the state of the system at time t_2 can be found as

$$|\psi(t_2)\rangle = \frac{1}{\sqrt{2}N_-}|e\rangle|\Psi_-(t_2)\rangle + \frac{1}{\sqrt{2}N_+}|g\rangle|\Psi_+(t_2)\rangle, \quad (12)$$

where

$$\begin{aligned}|\Psi_\pm(t_2)\rangle = {} & N_\pm[J_\pm(t_1,\tau)|1\rangle_c|0\rangle_{m_1}|0\rangle_{m_2} \\ & + M_3(t_1)|0\rangle_c|1\rangle_{m_1}|0\rangle_{m_2} \\ & + M_4(t_1)|0\rangle_c|0\rangle_{m_1}|1\rangle_{m_2}],\end{aligned} \quad (13)$$

and

$$N_\pm = [1 \pm (M_1(t_1)M_2^*(t_1) + M_2(t_1)M_1^*(t_1))]^{-1/2},$$

$J_\pm(t_1,\tau) \equiv [M_1(t_1) \pm M_2(t_1)]\,\mathrm{e}^{\mathrm{i}\frac{2g_{\mathrm{cb}}^2}{\omega_{\mathrm{m}}}\tau}$, $\tau = t_2 - t_1$. When the atomic states are detected, the other three parts, i.e., the cavity and the two mechanical mirrors will collapse to the entangled states $|\Psi_\pm(t_2)\rangle$, here the $+$ and $-$ signs correspond to the detected atomic states $|g\rangle$ and $|e\rangle$, respectively. The tripartite entangled states $|\Psi_\pm(t_2)\rangle$ are generated with the probabilities $P_\pm = \left[2N_\pm^2\right]^{-1}$, respectively. It can be seen that the generated tripartite entangled states are analogous to the W states.

The reduced density matrix of the two mechanical modes is $\rho^{(\mathrm{m})}(t_2) = \mathrm{Tr_c}\left[|\Psi_\pm(t_2)\rangle\langle\Psi_\pm(t_2)|\right]$, and in the basis $\{|0_{m_1}0_{m_2}\rangle, |0_{m_1}1_{m_2}\rangle, |1_{m_1}0_{m_2}\rangle, |1_{m_1}1_{m_2}\rangle\}$, it can be expressed as

$$\rho^{(\mathrm{m})}(t_2) = \begin{pmatrix} \rho_{11}^{(\mathrm{m})} & 0 & 0 & 0 \\ 0 & \rho_{22}^{(\mathrm{m})} & \rho_{23}^{(\mathrm{m})} & 0 \\ 0 & \rho_{32}^{(\mathrm{m})} & \rho_{33}^{(\mathrm{m})} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (14)$$

where $\rho_{11}^{(\mathrm{m})} = N_\pm^2|M_1(t_1) \pm M_2(t_1)|^2$, $\rho_{22}^{(\mathrm{m})} = N_\pm^2|M_4(t_1)|^2$, $\rho_{33}^{(\mathrm{m})} = N_\pm^2|M_3(t_1)|^2$, $\rho_{23}^{(\mathrm{m})} = N_\pm^2 M_4(t_1)M_3^*(t_1)$, and $\rho_{32}^{(\mathrm{m})} = N_\pm^2 M_3(t_1)M_4^*(t_1)$. It can be seen that this reduced density matrix is only dependent on t_1, the time the atom inside the cavity, which is determined by the atomic velocity and the cavity length. It does not depend on $\tau = t_2 - t_1$, the time from the atom leaving the cavity to the atom arriving at the detector.

4. Entanglement measure and dissipative dynamics

In the previous section, we have obtained the entangled states between the two mechanical mirrors. In this section, we give a proper description and quantification of the entanglement. The concurrence $C_{jk}(t)$ introduced by Wooters is defined as[50]

$$C_{jk}(t) = \max\{0, \lambda_1(t) - \lambda_2(t) - \lambda_3(t) - \lambda_4(t)\}, \quad (15)$$

where $\lambda_i(t)$ are the eigenvalues (in decreasing order) of the Hermitian matrix $\left[\sqrt{\rho_{jk}}\widetilde{\rho_{jk}}\sqrt{\rho_{jk}}\right]^{1/2}$, $\widetilde{\rho_{jk}} = (\sigma_y \otimes \sigma_y)\rho_{jk}^*(\sigma_y \otimes \sigma_y)$, ρ_{jk}^* is the complex conjugate of ρ_{jk}, and σ_y is the Pauli matrix $\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$.

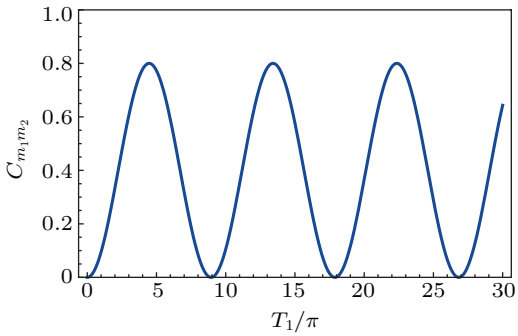


Fig. 2. (color online) The time evolution of the concurrence $C_{m_1m_2}$. Here, $T_1 = g_{\mathrm{cb}}t_1$ is the scaled time. The parameters $\omega_{\mathrm{m}} = 2\pi \times 10^7$ Hz, $g_{\mathrm{cb}} = 0.05\omega_{\mathrm{m}}$, $g_{\mathrm{ca}} = \sqrt{2}g_{\mathrm{cb}}$, $\theta = \pi/2$.

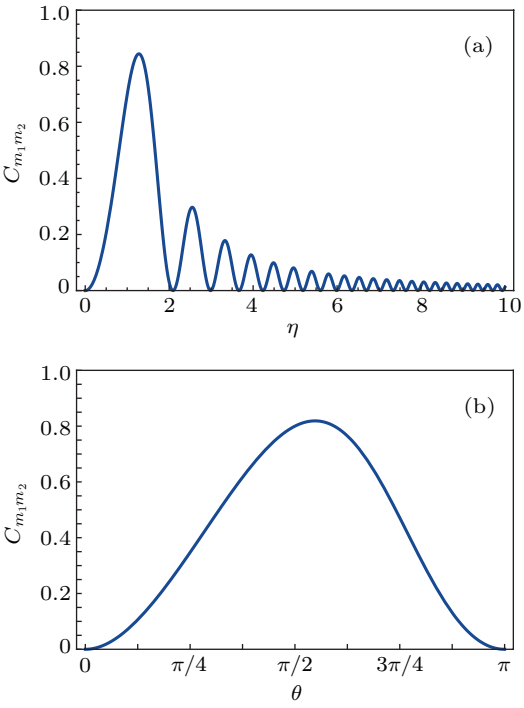


Fig. 3. (color online) (a) The variation of the concurrence $C_{m_1m_2}$ with η ($g_{\mathrm{ca}}/g_{\mathrm{cb}}$). The parameter $\theta = \pi/2$. (b) The variation of the concurrence $C_{m_1m_2}$ with θ. The parameter $g_{\mathrm{ca}} = \sqrt{2}g_{\mathrm{cb}}$. Other common parameters $\omega_{\mathrm{m}} = 2\pi \times 10^7$ Hz, $g_{\mathrm{cb}} = 0.05\omega_{\mathrm{m}}$, $T_1 = 14$.

As we have noted from Eq. (14) that the reduced state of the two mechanical modes does not depend on τ, it only depends on t_1, which is determined by the atomic velocity and the cavity length. So the concurrence of this state will also

have these properties. From Fig. 2, we can see that the concurrence $C_{m_1m_2}$ changes with time t_1 periodically. We can also examine the effects of the parameters $\eta = g_{ca}/g_{cb}$ (the ratio of the atom–cavity coupling strength and the single-photon optomechanical coupling strength) and θ (the initial atomic state) on the degree of entanglement. Figure 3(a) shows the variation of the concurrence $C_{m_1m_2}$ versus η. With the increase of η, the oscillation amplitude of the concurrence $C_{m_1m_2}$ decreases. Figure 3(b) describes the influence of θ on the concurrence $C_{m_1m_2}$. The maximum concurrence occurs at about $\theta = \pi/2$, i.e., the atom is initially in the ground state $|g\rangle$. From Eq. (3), we know that the first term of the Hamiltonian describes the interaction of three parts, which is very important to the generation of the entangled states. Remember that in this paper we consider the resonance condition $\omega_c = \omega_m + \omega_a$. When a photon of the cavity field annihilates, the atom makes a transition from the ground state $|g\rangle$ to the excited state $|e\rangle$, at the same time, either of the two mechanical modes creates a phonon, and the two phonons from the two mechanical modes are in an entangled state. When the atom is initially in the excited state $|e\rangle$, for it to make a transition to the ground state $|g\rangle$, one needs to annihilate a phonon from one of the two mechanical modes [see the term $c^\dagger (b_2 - b_1)\sigma_-$ in Eq. (3)]. However, since the initial states of the two mechanical modes are ground states $|0\rangle_{m_1}|0\rangle_{m_2}$, this process cannot occur, and in turn, the entanglement cannot be generated.

In the above discussion, we do not take into account the dissipation in the processes of the state-generation. Now we discuss the effects of the dissipation on the dynamics of the system after the tripartite entangled states $|\Psi_\pm\rangle$ have been generated. For simplicity, we assume that the cavity and the two mechanical mirrors are coupled to their baths at zero temperature, then the dynamics can be described by the following master equation:[51]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\left[H'_{\mathrm{eff}}, \rho\right] + \frac{\gamma_c}{2}\left(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c\right) + \sum_{j=1,2}\frac{\gamma_{m_j}}{2}\left(2b_j\rho b_j^\dagger - b_j^\dagger b_j\rho - \rho b_j^\dagger b_j\right), \quad (16)$$

where γ_c and γ_{m_j} are the decay rates of the cavity and the mechanical mirrors, respectively. We analyze the dissipation process beginning from the state $\rho(0) = |\Psi_+\rangle\langle\Psi_+|$. The density matrix $\rho(t)$ in the basis $\{|1\rangle \equiv |000\rangle, |2\rangle \equiv |001\rangle, |3\rangle \equiv |010\rangle, |4\rangle \equiv |100\rangle\}$ (here l,m,n denote the photon-number of the cavity mode and the phonon-number of the mechanical modes, respectively) can be written as

$$\rho(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & \rho_{24} \\ 0 & \rho_{32} & \rho_{33} & \rho_{34} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}. \quad (17)$$

The time evolution of the matrix elements can be expressed as

$$\begin{aligned}
&\frac{\mathrm{d}\rho_{11}}{\mathrm{d}t} = \gamma_c\rho_{44} + \gamma_{m_1}\rho_{33} + \gamma_{m_2}\rho_{22},\\
&\frac{\mathrm{d}\rho_{22}}{\mathrm{d}t} = -\gamma_{m_2}\rho_{22}, \quad \frac{\mathrm{d}\rho_{33}}{\mathrm{d}t} = -\gamma_{m_1}\rho_{33},\\
&\frac{\mathrm{d}\rho_{44}}{\mathrm{d}t} = -\gamma_c\rho_{44}, \quad \frac{\mathrm{d}\rho_{23}}{\mathrm{d}t} = -\Gamma_3\rho_{23},\\
&\frac{\mathrm{d}\rho_{24}}{\mathrm{d}t} = -(\mathrm{i}\Lambda + \Gamma_2)\rho_{24}, \quad \frac{\mathrm{d}\rho_{32}}{\mathrm{d}t} = -\Gamma_3\rho_{32},\\
&\frac{\mathrm{d}\rho_{34}}{\mathrm{d}t} = -(\mathrm{i}\Lambda + \Gamma_1)\rho_{34}, \quad \frac{\mathrm{d}\rho_{42}}{\mathrm{d}t} = (\mathrm{i}\Lambda - \Gamma_2)\rho_{42},\\
&\frac{\mathrm{d}\rho_{43}}{\mathrm{d}t} = (\mathrm{i}\Lambda - \Gamma_1)\rho_{43}, \qquad (18)
\end{aligned}$$

with $\Lambda = 2g_{cb}^2/\omega_m$, $\Gamma_1 = (\gamma_c + \gamma_{m_1})/2$, $\Gamma_2 = (\gamma_c + \gamma_{m_2})/2$, $\Gamma_3 = (\gamma_{m_1} + \gamma_{m_2})/2$. The solution of Eq. (18) can be found as

$$\begin{aligned}
&\rho_{11}(t) = |M_1 + M_2|^2 f_c + |M_3|^2 f_{m_1} + |M_4|^2 f_{m_2},\\
&\rho_{22}(t) = |M_4|^2 \mathrm{e}^{-\gamma_{m_2}t}, \quad \rho_{33}(t) = |M_3|^2 \mathrm{e}^{-\gamma_{m_1}t},\\
&\rho_{44}(t) = |M_1 + M_2|^2 \mathrm{e}^{-\gamma_c t}, \quad \rho_{23}(t) = M_4 M_3^* \mathrm{e}^{-\Gamma_3 t},\\
&\rho_{24}(t) = (M_1 + M_2)^* M_4 \mathrm{e}^{-(\mathrm{i}\Lambda + \Gamma_2)t}, \quad \rho_{32}(t) = M_3 M_4^* \mathrm{e}^{-\Gamma_3 t},\\
&\rho_{34}(t) = (M_1 + M_2)^* M_3 \mathrm{e}^{-(\mathrm{i}\Lambda + \Gamma_1)t},\\
&\rho_{42}(t) = (M_1 + M_2) M_4^* \mathrm{e}^{(\mathrm{i}\Lambda - \Gamma_2)t},\\
&\rho_{43}(t) = (M_1 + M_2) M_3^* \mathrm{e}^{(\mathrm{i}\Lambda - \Gamma_1)t}, \qquad (19)
\end{aligned}$$

with $f_c(t) = 1 - \mathrm{e}^{-\gamma_c t}$, $f_{m_1}(t) = 1 - \mathrm{e}^{-\gamma_{m_1}t}$, $f_{m_2}(t) = 1 - \mathrm{e}^{-\gamma_{m_2}t}$.

The reduced density matrix for the two mechanical modes is $\rho^{(\mathrm{m})}(t) = \mathrm{Tr}_c[\rho(t)]$. It can be expressed as

$$\begin{aligned}
\rho^{(\mathrm{m})}(t) = {} & N_+^2\{[\rho_{11}(t) + \rho_{44}(t)]|00\rangle\langle 00| \\
& + \rho_{22}(t)|01\rangle\langle 01| + \rho_{33}(t)|10\rangle\langle 10| \\
& + \rho_{23}(t)|01\rangle\langle 10| + \rho_{32}(t)|10\rangle\langle 01|\}. \qquad (20)
\end{aligned}$$

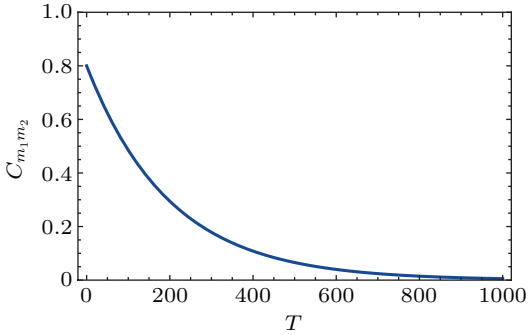


Fig. 4. (color online) The time evolution of the concurrence $C_{m_1m_2}(t)$. Here $T = g_{cb}t$ is the scaled time. The parameters $\omega_m = 2\pi \times 10^7$ Hz, $g_{cb} = 0.05\omega_m$, $g_{ca} = \sqrt{2}g_{cb}$, $\theta = \pi/2$, $\gamma_c = 0.05g_{cb}$, $\gamma_{m_1} = \gamma_{m_2} = 0.005g_{cb}$, $T_1 = 14$.

Once again, we use the concurrence $C_{m_1m_2}(t)$ to measures the entanglement of the two mechanical oscillators. By using

the reduced density matrix (20), we can calculate the concurrence $C_{m_1m_2}(t)$, and the time evolution of $C_{m_1m_2}(t)$ is shown in Fig. 4. It can be seen that the degree of entanglement decreases nearly exponentially with the increase of time and becomes zero eventually.

5. Discussion and conclusion

We now discuss the experimental feasibility of our scheme. We consider an ideal case for the entanglement generation process, in which the loss of the system is neglected . In our scheme, we use the condition $\omega_{\rm m} \gg \{g_{\rm cb}, g_{\rm ca}\}$. Our system needs to work in the strong coupling regime $\{g_{\rm cb}, g_{\rm ca}\} \gg \{\gamma_c, \gamma_m\}$.[52]

The cavity is initially prepared in a single photon state, which can be realized by passing a two-level atom in the excited state through the cavity with a π-pause duration.[48] The two mechanical mirrors are prepared in their ground states, which can be realized by the ground state cooling method.[39]

In summary, we have proposed a novel scheme for generating the entanglement of two oscillating mechanical mirrors. The system considered consists of an optical cavity with two oscillating cavity-mirrors, an atomic source, a Ramsey zone, and a detector for atomic states. The degree of entanglement is measured by the concurrence. We find that the concurrence can be controlled by the atomic velocity, the cavity length, the initial states of the atom, the optomechanical coupling strength, and the atom–cavity coupling strength. We investigate the influences of the dissipations on the dynamics of the generated entangled states, and find that the degree of entanglement nearly exponentially decreases with the increase of time.

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