

Slater determinant and exact eigenstates of the two-dimensional Fermi-Hubbard model

Jun-Hang Ren(任军航), Ming-Yong Ye(叶明勇), Xiu-Min Lin(林秀敏)

Citation: Chin. Phys. B . 2018, 27(7): 073102. **doi:** 10.1088/1674-1056/27/7/073102

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Energy states of the Hulthen plus Coulomb-like potential with position-dependent mass function in external magnetic fields

M Eshghi, R Sever, S M Ikhdair

Chin. Phys. B . 2018, 27(2): 020301. **doi:** 10.1088/1674-1056/27/2/020301

Approximate energies and thermal properties of a position-dependent mass charged particle under external magnetic fields

M Eshghi, H Mehraban, S M Ikhdair

Chin. Phys. B . 2017, 26(6): 060302. **doi:** 10.1088/1674-1056/26/6/060302

Binding energy of the donor impurities in GaAs-$Ga_{1-x}Al_xAs$ quantum well wires with Morse potential in the presence of electric and magnetic fields

Esra Aciksoz, Orhan Bayrak, Asim Soylu

Chin. Phys. B . 2016, 25(10): 100302. **doi:** 10.1088/1674-1056/25/10/100302

Effects of effective attractive multi-body interaction on quantum phase and transport dynamics of a strongly correlated bosonic gas across the superfluid to Mott insulator transition

Sun Jian-Fang, Cui Guo-Dong, Jiang Bo-Nan, Qian Jun, Wang Yu-Zhu

Chin. Phys. B . 2013, 22(11): 110307. **doi:** 10.1088/1674-1056/22/11/110307

Alternative routes to equivalent classical models of a quantum system

M. Radonjić, Slobodan Prvanović, Nikola Burić

Chin. Phys. B . 2012, 21(12): 120301. **doi:** 10.1088/1674-1056/21/12/120301

Slater determinant and exact eigenstates of the two-dimensional Fermi–Hubbard model*

Jun-Hang Ren(任军航)[1,2], Ming-Yong Ye(叶明勇)[1,2,†], and Xiu-Min Lin(林秀敏)[1,2]

[1] Fujian Provincial Key Laboratory of Quantum Manipulation and New Energy Materials, College of Physics and Energy, Fujian Normal University, Fuzhou 350117, China

[2] Fujian Provincial Collaborative Innovation Center for Optoelectronic Semiconductors and Efficient Devices, Xiamen 361005, China

(Received 19 March 2018; revised manuscript received 18 April 2018; published online 25 June 2018)

We consider the construction of exact eigenstates of the two-dimensional Fermi–Hubbard model defined on an $L \times L$ lattice with a periodic condition. Based on the characteristics of Slater determinants, several methods are introduced to construct exact eigenstates of the model. The eigenstates constructed are independent of the on-site electron interaction and some of them can also represent exact eigenstates of the two-dimensional Bose–Hubbard model.

Keywords: Hubbard model, exact eigenstate, Slater determinant

PACS: 31.15.aq, 03.65.Ge **DOI:** 10.1088/1674-1056/27/7/073102

1. Introduction

The two-dimensional Fermi–Hubbard model is important when attempting to understand strongly correlated electronic systems.[1–5] In the model, the electrons can hop from one site to its nearest neighbors and there will be an interaction between two electrons when they are on the same site. The competition between the electron hopping and the on-site electron interaction makes it hard to find exact eigenstates of the model.[6–8] In 1990, Yang and Zhang showed that many new exact eigenstates of the model could be constructed from the known ferromagnetic eigenstates based on the SO_4 symmetry of the model.[9] There are also other methods to construct exact eigenstates of the model.[10] Finding out more exact eigenstates of this model not only is an intellectual challenge, but also can increase our understanding of the model especially when the SO_4 symmetry is considered.[9,11] In this paper, we will present some new methods to construct more exact eigenstates of the model by connecting the problem with the Slater determinant. The eigenstates we constructed are independent of the on-site electron interaction, i.e., they are eigenstates for any value of the on-site electron interaction strength, which were ever believed to have been all figured out by Yang and Zhang and no new such eigenstates could be constructed.[9]

The two-dimensional Fermi–Hubbard model we considered is defined on a periodic two-dimensional $L \times L$ lattice with the Hamiltonian

$$H = T + V, \tag{1}$$

$$T = -t \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} (a^{\dagger}_{\boldsymbol{r}} a_{\boldsymbol{r}'} + b^{\dagger}_{\boldsymbol{r}} b_{\boldsymbol{r}'} + \text{H.c.}), \tag{2}$$

$$V = U \sum_{\boldsymbol{r}} a^{\dagger}_{\boldsymbol{r}} a_{\boldsymbol{r}} b^{\dagger}_{\boldsymbol{r}} b_{\boldsymbol{r}}, \tag{3}$$

where the lattice sites are described by $\boldsymbol{r} = (x, y)$ with x and y being integers $0, 1, \ldots, L-1$. The operators $a_{\boldsymbol{r}}$ and $b_{\boldsymbol{r}}$ are the annihilation operators in coordinate space for the spin-up and spin-down electrons, respectively, and they obey the usual commutation relations for Fermi operators. The notation $\langle \boldsymbol{r}, \boldsymbol{r}' \rangle$ indicates that the sum is over the nearest neighbors and the parameter U describes the on-site electron interaction strength.

When the on-site electron interaction strength U is zero, it is easy to find exact eigenstates of the model Hamiltonian H. By introducing the following annihilation operators in momentum space:

$$a_{\boldsymbol{k}} = \frac{1}{L} \sum_{\boldsymbol{r}} a_{\boldsymbol{r}} \, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \quad b_{\boldsymbol{k}} = \frac{1}{L} \sum_{\boldsymbol{r}} b_{\boldsymbol{r}} \, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{4}$$

where $\boldsymbol{k} = (k_x, k_y)$, k_x and k_y are limited to the range $(-\pi, \pi]$ and are $2\pi/L$ multiplied by some integers, the operator T can be rewritten in the diagonal form

$$T = \sum_{\boldsymbol{k}} E(\boldsymbol{k}) (a^{\dagger}_{\boldsymbol{k}} a_{\boldsymbol{k}} + b^{\dagger}_{\boldsymbol{k}} b_{\boldsymbol{k}}), \tag{5}$$

where

$$E(\boldsymbol{k}) = -2t(\cos k_x + \cos k_y). \tag{6}$$

However, it is usually hard to find exact eigenstates of H for nonzero U.

2. Slater determinant and exact eigenstates

In the following, we will give some methods to construct exact eigenstates of the Hamiltonian H for nonzero U. To

*Project supported by the National Natural Science Foundation of China (Grant No. 11674059) and Natural Science Foundation of Fujian Province, China (Grant Nos. 2016J01008 and 2016J01009).

†Corresponding author. E-mail: myye@fjnu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

reach this goal, we consider the following quantum state with j spin-up electrons and $(n-j)$ spin-down electrons:

$$|\Psi\rangle = A^{\dagger}|0\rangle, \tag{7}$$

$$A^{\dagger} = \frac{1}{L^n}\sum_{\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n} a^{\dagger}_{\boldsymbol{r}_1}\ldots a^{\dagger}_{\boldsymbol{r}_j} b^{\dagger}_{\boldsymbol{r}_{j+1}}\ldots b^{\dagger}_{\boldsymbol{r}_n} f(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n), \tag{8}$$

where $|0\rangle$ is the vacuum state and the function f is required to satisfy two conditions: (i) when $\boldsymbol{r}_\alpha = \boldsymbol{r}_\beta$ for any $\alpha \neq \beta$, there is $f = 0$; (ii) the state $|\Psi\rangle$ is an exact eigenstate of the operator T. The first condition guarantees that there will not be two electrons in the same lattice site in coordinate space so that we can neglect the operator V in the Hamiltonian H. The second condition ensures that the state $|\Psi\rangle$ is an exact eigenstate of the Hamiltonian H. To satisfy the first condition, we assume that the function f is in the following form:

$$f = \begin{vmatrix} \mathrm{e}^{\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{r}_1} & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{r}_1} & \ldots & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_n\cdot\boldsymbol{r}_1} \\ \mathrm{e}^{\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{r}_2} & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{r}_2} & \ldots & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_n\cdot\boldsymbol{r}_2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{e}^{\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{r}_n} & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{r}_n} & \ldots & \mathrm{e}^{\mathrm{i}\boldsymbol{k}_n\cdot\boldsymbol{r}_n} \end{vmatrix} g(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots\boldsymbol{r}_n), \tag{9}$$

where $\boldsymbol{k}_i$ $(i = 1,\ldots,n)$ are different vectors in momentum space. Since the function f is proportional to a Slater determinant, the first condition will be satisfied due to the characteristics of the determinant. The problem is now reduced to find a group of $\boldsymbol{k}_i$ and a function $g(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n)$ that make the function f satisfy the second condition.

2.1. The case $g(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n) = 1$

The simplest case may be $g(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n) = 1$. In this case, from Eq. (9) there is

$$f = \sum_{p\in S_n}(-1)^{\sigma(p)}\prod_{\alpha=1}^{n}\mathrm{e}^{\mathrm{i}\boldsymbol{k}_{p(\alpha)}\cdot\boldsymbol{r}_\alpha}, \tag{10}$$

where S_n denotes the set of all permutations of the set $\{1,2,3,\ldots,n\}$, $p(i)$ denotes the value in the i-th position after the permutation p, and $\sigma(p)$ is the inverse number of the permutation p. Substituting Eq. (10) into the state $|\Psi\rangle$ in Eq. (7), we can obtain

$$|\Psi\rangle = \sum_{p\in S_n}(-1)^{\sigma(p)} a^{\dagger}_{\boldsymbol{k}_{p(1)}}\ldots a^{\dagger}_{\boldsymbol{k}_{p(j)}} b^{\dagger}_{\boldsymbol{k}_{p(j+1)}}\ldots b^{\dagger}_{\boldsymbol{k}_{p(n)}}|0\rangle. \tag{11}$$

Now we find that $|\Psi\rangle$ is obviously an eigenstate of the operator T with the eigenvalue $E(\Psi) = \sum_{i=1}^{n} E(\boldsymbol{k}_i)$, because each term behind the sum of Eq. (11) has the same kinetic energy $E(\Psi)$ independent of the permutation p. As no two electrons of the state $|\Psi\rangle$ can be on the same lattice site in coordinate space, $|\Psi\rangle$ as an eigenstate of T is also an eigenstate of the Hamiltonian H.

An enormous number of eigenstates of H can be constructed through Eq. (11), as long as we choose a group of different $\boldsymbol{k}_i$ $(i = 1,\ldots,n)$ in momentum space. However, it is a pity that $|\Psi\rangle$ in Eq. (11) is the same as the following seemingly different state $|\Psi_y\rangle$ with the definition

$$|\Psi_y\rangle = (S_+)^j|\Psi_0\rangle, \tag{12}$$

$$|\Psi_0\rangle = b^{\dagger}_{\boldsymbol{k}_1} b^{\dagger}_{\boldsymbol{k}_2}\ldots b^{\dagger}_{\boldsymbol{k}_n}|0\rangle, \tag{13}$$

$$S_+ = \sum_{\boldsymbol{k}} S_{\boldsymbol{k}}, \quad S_{\boldsymbol{k}} = a^{\dagger}_{\boldsymbol{k}} b_{\boldsymbol{k}}, \tag{14}$$

where the sum is over all vector $\boldsymbol{k}$ in momentum space. The state $|\Psi_y\rangle$ is obtained from the ferromagnetic state $|\Psi_0\rangle$ by flipping the spin of j electrons and it is already known to be an eigenstate of the Hamiltonian H.[9] To show the relation between the state $|\Psi\rangle$ in Eq. (11) and $|\Psi_y\rangle$, we note that there are

$$|\Psi_y\rangle = \left(\sum_{i=1}^{n} S_{\boldsymbol{k}_i}\right)^j|\Psi_0\rangle, \tag{15}$$

$$(-1)^{\sigma(p)}\prod_{i=1}^{n} b^{\dagger}_{\boldsymbol{k}_{p(i)}}|0\rangle = |\Psi_0\rangle \tag{16}$$

for each permutation $p \in S_n$. Therefore, from Eq. (11), we have

$$\begin{aligned} |\Psi\rangle &= \sum_{p\in S_n} S_{\boldsymbol{k}_{p(1)}}\ldots S_{\boldsymbol{k}_{p(j)}}(-1)^{\sigma(p)}\prod_{i=1}^{n} b^{\dagger}_{\boldsymbol{k}_{p(i)}}|0\rangle \\ &= \sum_{p\in S_n} S_{\boldsymbol{k}_{p(1)}}\ldots S_{\boldsymbol{k}_{p(j)}}|\Psi_0\rangle \\ &= (n-j)!\left(\sum_{i=1}^{n} S_{\boldsymbol{k}_i}\right)^j|\Psi_0\rangle \\ &= (n-j)!|\Psi_y\rangle, \end{aligned} \tag{17}$$

which shows that $|\Psi\rangle$ is the same as $|\Psi_y\rangle$ in physics. In the above, we have used the equation

$$\sum_{p\in S_n} S_{\boldsymbol{k}_{p(1)}} S_{\boldsymbol{k}_{p(2)}}\ldots S_{\boldsymbol{k}_{p(j)}} = (n-j)!\left(\sum_{i=1}^{n} S_{\boldsymbol{k}_i}\right)^j, \tag{18}$$

where $\boldsymbol{k}_i$ $(i = 1,\ldots,n)$ are different vectors in momentum space. Although our state $|\Psi\rangle$ in Eq. (11) is not a new eigenstate, it shows an interesting relation between the Slater determinant and the eigenstates of the Fermi–Hubbard model Hamiltonian H.

2.2. The case $g(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n) \neq 1$

To give some new eigenstates of H, in this case we assume

$$g(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_n) = \prod_{\alpha=1}^{n}\mathrm{e}^{\mathrm{i}\boldsymbol{q}_\alpha\cdot\boldsymbol{r}_\alpha}, \tag{19}$$

where $\boldsymbol{q}_\alpha$ $(\alpha = 1,\ldots,n)$ are vectors in momentum space and they are required not to be all the same, otherwise the function f in the following Eq. (20) will have the same form as Eq. (10). Substituting Eq. (19) into Eq. (9), we obtain

$$f = \sum_{p\in S_n}(-1)^{\sigma(p)}\prod_{\alpha=1}^{n}\mathrm{e}^{\mathrm{i}(\boldsymbol{q}_\alpha+\boldsymbol{k}_{p(\alpha)})\cdot\boldsymbol{r}_\alpha}, \tag{20}$$

which leads to

$$|\Psi\rangle = \sum_{p\in S_n} |\Psi_p\rangle, \tag{21}$$

with

$$|\Psi_p\rangle = (-1)^{\sigma(p)} \prod_{i=1}^{j} a^{\dagger}_{\boldsymbol{q}_i+\boldsymbol{k}_{p(i)}} \prod_{i'=j+1}^{n} b^{\dagger}_{\boldsymbol{q}_{i'}+\boldsymbol{k}_{p(i')}} |0\rangle \tag{22}$$

according to Eq. (7). It can be seen that for each permutation p, the state $|\Psi_p\rangle$ is an eigenstate of the operator T, and the corresponding eigenvalue is

$$E_p = \sum_{\alpha=1}^{n} E(\boldsymbol{q}_\alpha + \boldsymbol{k}_{p(\alpha)}). \tag{23}$$

Usually the eigenvalue E_p is dependent on p, and the state $|\Psi\rangle$ in Eq. (21) as a sum of $|\Psi_p\rangle$ is not an eigenstate of the operator T. To make the state $|\Psi\rangle$ in Eq. (21) an eigenstate of the operator T (or H), we can choose proper vectors $\boldsymbol{k}_\alpha$ and $\boldsymbol{q}_\alpha$ ($\alpha = 1,\ldots,n$) in momentum space such that the eigenvalue E_p is not a function of p. This can be done at least in the following two situations.

(i) Assume $\boldsymbol{k}_\alpha = (k_\alpha, 0)$ and $\boldsymbol{q}_\alpha = (0, q_\alpha)$ for $\alpha = 1,2,\ldots,n$, which are illustrated in Fig. 1. Now there is

$$E(\boldsymbol{q}_\alpha + \boldsymbol{k}_{p(\alpha)}) = E(\boldsymbol{q}_\alpha) + E(\boldsymbol{k}_{p(\alpha)}) \tag{24}$$

according to Eq. (23), which leads to

$$E_p = \sum_{\alpha=1}^{n} E(\boldsymbol{q}_\alpha) + \sum_{\alpha=1}^{n} E(\boldsymbol{k}_\alpha) \tag{25}$$

according to Eq. (23). Since the eigenvalue E_p does not depend on the permutation p, the state $|\Psi\rangle$ in Eq. (21) is an eigenstate of the Hamiltonian H under the above assumptions. In other words, the state

$$|\Psi\rangle = \sum_{p\in S_n} (-1)^{\sigma(p)} \prod_{i=1}^{j} a^{\dagger}_{(k_{p(i)},q_i)} \prod_{i'=j+1}^{n} b^{\dagger}_{(k_{p(i')},q_{i'})} |0\rangle \tag{26}$$

with different k_i is an eigenstate of the Hamiltonian H, and the corresponding eigenvalue is

$$E_p = -2t \sum_{i=1}^{n} [\cos k_i + \cos q_i]. \tag{27}$$

Since $\boldsymbol{k}_\alpha = (k_\alpha, 0)$ for $\alpha = 1,2,\ldots,n$ are assumed to be different from each other, the number of electrons n of the eigenstate $|\Psi\rangle$ in Eq. (26) cannot be larger than L as we are considering an $L\times L$ lattice. When the eigenstate $|\Psi\rangle$ in Eq. (26) has L electrons, the corresponding eigenvalue will be

$$E_p(L) = -2t \sum_{i=1}^{L} [\cos k_i + \cos q_i] = -2t \sum_{i=1}^{L} \cos q_i, \tag{28}$$

where L is assumed to be an even number. As $\boldsymbol{q}_\alpha = (0, q_\alpha)$ for $\alpha = 1,2,\ldots,L$ cannot all be the same, the minimal $E_p(L)$ will be

$$E_p(L)_{\min} = -2t\left(L - 1 + \cos\frac{2\pi}{L}\right), \tag{29}$$

which can be achieved when $q_1 = 2\pi/L$ and $q_\alpha = 0$ for $\alpha = 2,3,\ldots,L$. The average energy $E_p(L)_{\min}/L$ for each electron will be $-2t$ for large L, which is also true when L is an odd number.

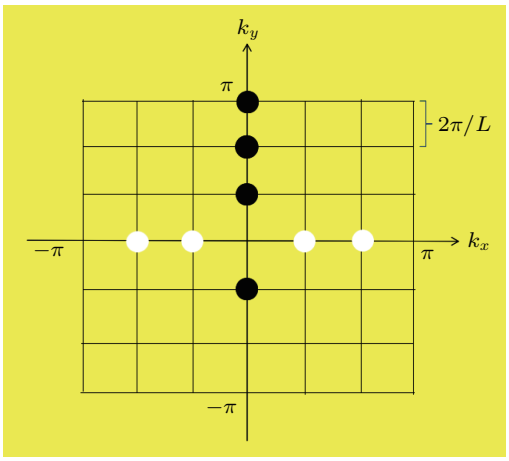


Fig. 1. (color online) Lattice sites in momentum space. The black dots represent $\boldsymbol{q}_\alpha = (0, q_\alpha)$ and the white dots represent $\boldsymbol{k}_\alpha = (k_\alpha, 0)$, where $L = 6$ is chosen as an example.

(ii) Assume $\boldsymbol{k}_\alpha = (k_\alpha, k_\alpha + \lambda)$ and $\boldsymbol{q}_\alpha = (0,0)$ or $(\lambda, -\lambda)$ for some nonzero λ ($\alpha = 1,2,\ldots n$). Now there is

$$E(\boldsymbol{q}_\alpha + \boldsymbol{k}_{p(\alpha)}) = E(\boldsymbol{k}_{p(\alpha)}) \tag{30}$$

for any permutation p according to Eq. (6), which leads to

$$E_p = \sum_{\alpha=1}^{n} E(\boldsymbol{k}_{p(\alpha)}) = \sum_{\alpha=1}^{n} E(\boldsymbol{k}_\alpha) \tag{31}$$

according to Eq. (23). Note that the eigenvalue E_p does not depend on the permutation p again, therefore the state $|\Psi\rangle$ in Eq. (21) is an eigenstate of the Hamiltonian H under the above assumptions.

In the above two situations, we have given two methods to construct the eigenstate of the Hamiltonian H by choosing proper vectors $\boldsymbol{k}_\alpha$ and $\boldsymbol{q}_\alpha$ ($\alpha = 1,\ldots,n$) in momentum space. These eigenstates do not depend on the electron interaction strength U. We need to show that our eigenstates have never been constructed, because Yang and Zhang have claimed that they have constructed all U-independent eigenstates of H.[9] The eigenstates without double occupation in coordinate space constructed by Yang and Zhang are described by $|\Psi_y\rangle$ in Eq. (12), which has a total spin $s = n/2$ for any j (the state has n electrons).[9] However our eigenstates with n electrons such as $|\Psi\rangle$ in Eq. (26) do not have a total spin, so that they are different from the eigenstates constructed by Yang and

Zhang. To demonstrate it, we assume $n = 2$ and $j = 1$ for our eigenstate $|\Psi\rangle$ in Eq. (26), which can now be written as

$$\begin{aligned}|\Psi\rangle &= \sum_{p\in S_2}(-1)^{\sigma(p)}\prod_{i=1}^{1}a^{\dagger}_{(k_{p(i)},q_i)}\prod_{i'=2}^{2}b^{\dagger}_{(k_{p(i')},q_{i'})}|0\rangle \\ &= [a^{\dagger}_{(k_1,q_1)}b^{\dagger}_{(k_2,q_2)} - a^{\dagger}_{(k_2,q_1)}b^{\dagger}_{(k_1,q_2)}]|0\rangle. \end{aligned} \tag{32}$$

When $k_1 \neq k_2$ and $q_1 \neq q_2$, the vectors (k_1,q_1), (k_2,q_2), (k_2,q_1), and (k_1,q_2) are four different vectors in momentum space, so that the state $|\Psi\rangle$ in Eq. (32) cannot have a total spin, i.e., it is the sum of a state with the total spin $s = 0$ and a state with the total spin $s = 1$.

3. Discussion and summary

We have considered the construction of exact eigenstates of the two-dimensional Fermi–Hubbard model. One may wonder whether our methods are also applicable for constructing exact eigenstates of the two-dimensional Bose–Hubbard model, where the operators obey the Bose commutation relations and many particles can be on the same site. Since our constructions prevent double occupation on the same site and do not use the Fermi commutation relations, the eigenstates we constructed for the Fermi–Hubbard model will also be eigenstates for the Bose–Hubbard model when the Bose commutation relations do not make the states zero. For example, the state $|\Psi\rangle$ in Eq. (26) will also be an exact eigenstate of the two-dimensional Bose–Hubbard model, where $a_{\boldsymbol{k}}$ and $b_{\boldsymbol{k}}$ describe two kinds of Boson particles, under the condition that $\boldsymbol{k}_\alpha = (k_\alpha, 0)$ for $\alpha = 1,2,\ldots,n$ are different from each other and $\boldsymbol{q}_\alpha = (q_\alpha, 0)$ for $\alpha = 1,2,\ldots,n$ are also different from each other. This is because each term behind the sum of $|\Psi\rangle$ in Eq. (26) is orthogonal to the other terms under the above condition, so that $|\Psi\rangle$ is not zero.

The number of electrons in our constructed eigenstates cannot reach or be close to the Hall filling, i.e., L^2 electrons in the $L \times L$ lattice, which is believed to have a rich physics.[12,13] However, Yang has shown that applying the $\eta^{\dagger}$ pair operator on a known eigenstate can obtain a new eigenstate with two more electrons.[14] Therefore, we can apply the $\eta^{\dagger}$ pair operator many times on our constructed eigenstates to obtain new eigenstates with the number of electrons reaching or being close to the Hall filling. The eigenstates obtained in this way are not the ground state, but their corresponding eigenvalues can give some upper bound for the ground state energy and the knowing of these excited states can reduce the size of the space to numerically search the ground state, as the ground state is orthogonal to all excited states.

In summary, we have given some methods to construct exact eigenstates of the two-dimensional Fermi–Hubbard model. The basic idea is to prevent two electrons from being on the same site through the Slater determinant, therefore the eigenstates constructed are independent of the on-site interaction strength U. The finding that there are many U-independent eigenstates itself is of interest, and the connection with the Slater determinant can give some inspiration for understanding the two-dimensional Fermi–Hubbard model.

Acknowledgment

M. Y. Ye would like to thank Z. J. Yao for helpful discussions about the Hubbard model.

References

[1] Hubbard J 1963 *Proc. R. Soc. A* **276** 238
[2] Hayes B 2009 *Am. Sci.* **97** 438
[3] Yun S J, Dong T K and Zhu S N 2017 *Chin. Phys. Lett.* **34** 080201
[4] Cheuk L W, Nichols M A, Lawrence K R, Okan M, Zhang H, Khatami E, Trivedi N, Paiva T, Rigol M and Zwierlein M W 2016 *Science* **353** 1260
[5] Wang Y L, Huang L, Du L and Dai X 2016 *Chin. Phys. B* **25** 037103
[6] Lieb E H and Wu F Y 2003 *Physica A* **321** 1
[7] Essler F H L, Frahm H, Göhmann F, Klümper A and Korepin V E 2005 *The One-dimensional Hubbard Model* (Cambridge: Cambridge University Press) p. 5
[8] Li Y Y, Cao J P, Yang W L, Shi K J and Wang Y P 2014 *Nucl. Phys. B* **879** 98
[9] Yang C N and Zhang S C 1990 *Mod. Phys. Lett. B* **4** 759
[10] Ye M Y and Lin X M 2018 *Phys. Status Solidi B* **255** 1700321
[11] Su G and Ge M L 1992 *Commun. Theor. Phys.* **17** 1
[12] Shen S Q, Qiu Z M and Tian G S 1994 *Phys. Rev. Lett.* **72** 1280
[13] Chen Y H, Tao H S, Yao D X and Liu W M 2012 *Phys. Rev. Lett.* **108** 246402
[14] Yang C N 1989 *Phys. Rev. Lett.* **63** 2144