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Fixed time integral sliding mode controller and its application to the suppression of chaotic oscillation in power system*

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Chattering phenomenon and singularity are still the main problems that hinder the practical application of sliding mode control. In this paper, a fixed time integral sliding mode controller is designed based on fixed time stability theory, which ensures precise convergence of the state variables of controlled system, and overcomes the drawback of convergence time growing unboundedly as the initial value increases in finite time controller. It makes the controlled system converge to the control objective within a fixed time bounded by a constant as the initial value grows, and convergence time can be changed by adjusting parameters of controllers properly. Compared with other fixed time controllers, the fixed time integral sliding mode controller proposed in this paper achieves chattering-free control, and integral expression is used to avoid singularity generated by derivation. Finally, the controller is used to stabilize four-order chaotic power system. The results demonstrate that the controller realizes the non-singular chattering-free control of chaotic oscillation in the power system and guarantees the fixed time convergence of state variables, which shows its higher superiority than other finite time controllers.

Keywords: fixed time stability, integral sliding mode control, four-order power system, chaotic oscillation, non-singular chattering-free

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1. Introduction

In recent years, power system, one of the typical nonlinear dynamical systems, has shown more and more obvious nonlinear dynamic behaviors, and the resulting chaotic oscillation of power system has been extensively studied.[1–5] Chaotic oscillation influences the stable operation of power system, which will cause voltage collapse, angle instability and even large-scale blackout accident in the system.[6,7] Therefore, it is necessary to control chaotic oscillation of power system. Many control strategies have been suggested to suppress the chaotic oscillations of power systems, such as inverse system control,[8] adaptive optimal control,[9] state feedback control,[10] adaptive compensation control,[11] least square support vector machine control,[12] output delay feedback control,[13] adaptive backstepping control,[14] adaptive fuzzy integral sliding mode control,[15] etc. The proposed control strategies are useful in exploring the suppression of chaotic oscillations in power systems. However, these methods can only achieve the asymptotic convergence of state variables, but cannot make them converge accurately within finite time. In addition, most of these methods are mainly for studying two-order power system, while more complex model of power system is rarely controlled. In fact, the two-order system is simplified from the four-order power system, and the chaos phenomenon in a four-order power system has been fully studied.[16–23] Therefore, it is necessary to directly control the four-order power system.

Comparing with the asymptotic convergence of state variables, finite time stability theory forces state variable accurately to converge to the control objective within finite time,[24,25] and has advantages in the suppression of chaotic oscillation in power system. In Ref. [26], the finite time stability theory is used to design an equivalent fast terminal fuzzy sliding mode controller for the two-order power system. However, the parameters of the controller are fuzzed and eventually prolong the whole control process. In addition, although finite time stability theory can force state variables precisely to converge to their objective, it cannot guarantee the system convergence within bounded time independent of the initial value, which prevents it from being applied to practical systems. Also, if the initial value is unknown in advance, its convergence time cannot be determined. Compared with finite time stability theory, fixed time stability theory overcomes the drawback of initial value determination of finite time stability theory,[27–30] and make state variables converge accurately within finite time upper bounded by a constant independent of the initial value. Therefore, it is of great practical value to use fixed time stability theory to design a controller.

Among many control methods of chaotic power systems, sliding mode control has been widely applied to the suppression of chaotic power systems because of its advantages, such as fast response, insensitivity to parameter change and distur-

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bance, no need for the on-line identification of the system, and simple physical realization. However, the problems of chattering and singularity are still the basic problems of sliding mode control. In Ref. [31], the fixed time control theory is used to design a fractional order fixed time terminal sliding mode controller for the two-order power system. However, it will cause chattering phenomenon because of symbolic function in its controller. In addition, there are too many controllers, which reduces the control reliability. In Ref. [32], a fixed time sliding mode controller is designed for a four-order power system to suppress chaos in power system, but the controller is not continuous which will also cause chattering. Also, a saturation function is used to solve the singularity problem, which increases control complexity and prolongs convergence time of state variables.[31]

In order to solve the above problems, a fixed time integral sliding mode control method is proposed in the paper. The method avoids singularity problem by using the integral expression, and utilize the continuity of fixed time expression to make the controller continuous. Therefore, it also avoids chattering problem. The control scheme not only avoids the singularity and chattering problem, but also realizes fixed time convergence of state variables, which proves its higher superiority than many other finite time controllers.

The rest of the paper is organized as follows. The fixed time integral mode controller is shown in Section 2. In Section 3, the controller proposed in the paper is used to suppress the chaotic oscillation in a power system model. Section 4 gives the comparison of the proposed controller with previous controllers.

2. Design of fixed time integral sliding mode controller

2.1. Basic concepts and mathematical foundations

Definition[27,28] Consider differential inequality

$$\dot{x} \le f(x), \quad x(0) = x_0. \tag{1}$$

The time for state variable x of Eq. (1) to converge to the origin is set to be $T(x_0)$, for $\forall x_0 \in \mathbb{R}$, $\exists T_{\max} > 0$, such that $\lim_{x_0 \to \infty} [T(x_0)] \le T_{\max}$ holds true, then the origin of Eq. (1) is said to be fixed-time stable.

Lemma 1 (Fixed-time stability theory)[27,28] Consider differential inequality

$$\begin{cases} \dot{x} \le -\alpha \operatorname{sign}(x) |x|^m - \beta \operatorname{sign}(x) |x|^n, \\ x(0) = x_0, \end{cases} \tag{2}$$

where α, β, m, and n are all real numbers, and satisfy $\alpha > 0$, $\beta > 0$, $m > 1$, and $0 < n < 1$.

The time for the state variable of Eq. (2) to converge to the origin is set to be $T(x_0)$, then the state variable x will converge to the origin in a fixed time upper bound by $T_{\max}$, that is $\lim_{x_0 \to \infty} [T(x_0)] \le T_{\max}$, and

$$T_{\max} = \frac{1}{\alpha(m-1)} + \frac{1}{\beta(1-n)}. \tag{3}$$

Lemma 2[29] If real number p satisfies $p > 1$, then for any real numbers $\xi_1 \ge 0$, $\xi_2 \ge 0, \ldots, \xi_N \ge 0$, the following inequality holds:

$$\sum_{i=1}^{N} \xi_i^p \ge N^{1-p} \left(\sum_{i=1}^{N} \xi_i \right)^p. \tag{4}$$

Lemma 3[29] If real number p satisfies $0 < p \le 1$, then for any real numbers $\xi_1 \ge 0$, $\xi_2 \ge 0, \ldots, \xi_N \ge 0$, the following inequality holds:

$$\sum_{i=1}^{N} \xi_i^p \ge \left(\sum_{i=1}^{N} \xi_i \right)^p. \tag{5}$$

2.2. Controller design

Consider the controlled system as follows:

$$\dot{x}_i = f_i(x_i, t) + u_i, \tag{6}$$

where x_i represents the state variable of the system, $f_i(x_i, t)$ denotes an arbitrary nonlinear function, $i = 1, 2, 3, \ldots, N$, and u_i is the control input.

The control objective of state variable x_i in controlled system (6) is set to be $x_{i\mathrm{d}}$, then the control error is $e_i = x_i - x_{i\mathrm{d}}$. The equation of error dynamics is

$$\dot{e}_i = \dot{x}_i - \dot{x}_{i\mathrm{d}} = f_i(x_i, t) + u_i - \dot{x}_{i\mathrm{d}}. \tag{7}$$

Then the integral sliding mode function is designed as follows:

$$s_i = e_i + \alpha \int_0^t \operatorname{sign}(e_i) |e_i|^m \, \mathrm{d}t + \beta \int_0^t \operatorname{sign}(e_i) |e_i|^n \, \mathrm{d}t. \tag{8}$$

In order to make system (7) reach the sliding surface in the approaching motion process within a fixed time, let

$$\begin{aligned} \dot{s}_i &= \dot{e}_i + \alpha \operatorname{sign}(e_i) |e_i|^m + \beta \operatorname{sign}(e_i) |e_i|^n \\ &= f_i(x_i, t) + u_i - \dot{x}_{i\mathrm{d}} + \alpha \operatorname{sign}(e_i) |e_i|^m + \beta \operatorname{sign}(e_i) |e_i|^n \\ &= -\alpha \operatorname{sign}(s_i) |s_i|^m - \beta \operatorname{sign}(s_i) |s_i|^n. \end{aligned} \tag{9}$$

When the approaching motion process of sliding mode control is completed, $s_i = 0$, then from Eq. (9) we can obtain $\dot{s}_i = 0$, that is,

$$\dot{e}_i = -\alpha \operatorname{sign}(e_i) |e_i|^m - \beta \operatorname{sign}(e_i) |e_i|^n. \tag{10}$$

Then the system starts the sliding mode motion process, and Lemma 1 ensures that the system error e_i converges to the origin within a fixed time, so that state variable x_i of the controlled system (6) converges to the control objective $x_{i\mathrm{d}}$.

According to Eq. (9), the designed controller can be solved as follows:

$$\begin{aligned} u_i = & -f_i(x_i,t)+\dot{x_{id}}-\alpha\text{sign}(e_i)\,|e_i|^m-\beta\text{sign}(e_i)\,|e_i|^n \\ & -\alpha\text{sign}(s_i)\,|s_i|^m-\beta\text{sign}(s_i)\,|s_i|^n. \end{aligned} \tag{11}$$

Equation (11) is a fixed time integral sliding mode controller designed for the controlled system (6). Since $\alpha\text{sign}(e_i)\,|e_i|^m$, $\beta\text{sign}(e_i)\,|e_i|^n$, $\alpha\text{sign}(s_i)\,|s_i|^m$, and $\beta\text{sign}(s_i)\,|s_i|^n$ are continuous, chattering will be avoided.

From Eqs. (9) and (10), one obtains that in the approaching motion process and in the sliding mode motion process the sliding mode function s_i and error function e_i satisfy, respectively,

$$\begin{cases} \dot{s}_i = -\alpha\text{sign}(s_i)\,|s_i|^m-\beta\text{sign}(s_i)\,|s_i|^n, \\ \dot{e}_i = -\alpha\text{sign}(e_i)\,|e_i|^m-\beta\text{sign}(e_i)\,|e_i|^n. \end{cases} \tag{12}$$

It is known by Lemma 1 that system (7) reaches the sliding surface and errors converge to the origin in a fixed time from Eq. (12).

2.3. Convergence time analysis

(i) In order to analyze the time of the system (7) reaching the sliding surface, the Lyapunov function is constructed as

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} s_i^2.$$

Then the time derivative of V_1 can be obtained as

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{N} s_i\dot{s}_i \\ &= \sum_{i=1}^{N} s_i[-\alpha\text{sign}(s_i)\,|s_i|^m-\beta\text{sign}(s_i)\,|s_i|^n] \\ &= \sum_{i=1}^{N}[-\alpha\,|s_i|^{m+1}-\beta\,|s_i|^{n+1}] \\ &= -\alpha\sum_{i=1}^{N}|s_i|^{m+1}-\beta\sum_{i=1}^{n}|s_i|^{n+1} \\ &= -\alpha\sum_{i=1}^{N}(s_i^2)^{\frac{m+1}{2}}-\beta\sum_{i=1}^{n}(s_i^2)^{\frac{n+1}{2}}. \end{aligned} \tag{13}$$

Since $(m+1)/2 > 1$ and $(n+1)/2 < 1$, according to Lemma 2 and Lemma 3, one obtains

$$\begin{cases} -\alpha\sum\limits_{i=1}^{N}(s_i^2)^{\frac{m+1}{2}} \le -AV_1^{m_A}, \\ -\beta\sum\limits_{i=1}^{n}(s_i^2)^{\frac{n+1}{2}} \le -BV_1^{n_B}, \end{cases} \tag{14}$$

where

$$\begin{cases} A = \alpha N^{1-m_A}2^{m_A}, \\ B = \beta 2^{m_B}, \\ m_A = (m+1)/2, \\ m_B = (n+1)/2. \end{cases}$$

From Eqs. (13) and (14), one obtains

$$\dot{V}_1 \le -AV_1^{m_A}-BV_1^{n_B}. \tag{15}$$

The time used for approaching motion process of Eq. (9) is set as $T(s_i(0))$, then according to Eqs. (3) and (15), the time for the system (7) to reach sliding surface satisfies:

$$\lim_{s_i(0)\to\infty}[T(s_i(0))] \le \frac{1}{A(m_A-1)}+\frac{1}{B(1-n_B)} = T'_{\max}. \tag{16}$$

Combing Eq. (16) and the definition, one obtains that the system (7) reaches sliding surface within a fixed time upper bounded by $T'_{\max}$.

(ii) In order to analyze the time of error function converging to the origin, the Lyapunov function is similarly constructed as

$$V_2 = \frac{1}{2}\sum_{i=1}^{N} e_i^2.$$

One can obtain

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^{N} e_i\dot{e}_i = -\alpha\sum_{i=1}^{N}|e_i|^{m+1}-\beta\sum_{i=1}^{n}|e_i|^{n+1} \\ &\le -AV_2^{m_A}-BV_2^{n_B}. \end{aligned} \tag{17}$$

The time used for the sliding mode motion process in Eq. (10) is set to be $T(e_i(0))$, then according to Eqs. (3) and (17), the time for error function to converge to the origin in the sliding mode motion process satisfies

$$\lim_{e_i(0)\to\infty}[T(e_i(0))] \le \frac{1}{A(m_A-1)}+\frac{1}{B(1-n_B)} = T'_{\max}. \tag{18}$$

According to Eq. (18) and the definition, one obtains that error function e_i converges to the origin within a fixed time upper bounded by $T'_{\max}$.

The total time required for system stability is composed of approaching motion time and sliding mode motion time. Then according to Eqs. (16) and (18), the total time T_Σ required for system stability satisfies

$$\begin{aligned} \lim_{\substack{s_i(0)\to\infty \\ e_i(0)\to\infty}}(T_\Sigma) &= \lim_{\substack{s_i(0)\to\infty \\ e_i(0)\to\infty}}[T(s_i(0))+T(e_i(0))] \\ &\le \frac{2}{A(m_A-1)}+\frac{2}{B(1-n_B)} = 2T'_{\max}. \end{aligned} \tag{19}$$

Equation (19) shows that the state variables of system (6) converging to the origin within a fixed time upper bounded by $2T'_{\max}$. Parameters A, B, m_A, and m_B can be adjusted by appropriately selecting α, β, m, and n, and thus changing $T'_{\max}$ in Eq. (19). In this way, the convergence time of state variables can be adjusted artificially. That is, the convergence rate of state variables can be adjusted according to the actual requirements.

3. Fixed time integral sliding mode control for chaotic power system

3.1. Dynamic analysis

The proposed controller is used to control the three-bus power system model discussed in Refs. [16]–[23], which is a classic model of four-order power system. Dynamic equations of power system are described as[16–23]

$$\begin{cases} \dot{\delta}_m = \omega, \\ \dot{\omega} = 16.6667\sin(\delta-\delta_m+0.0873)V \\ \quad -0.1667\omega+1.8807, \\ \dot{\delta} = 496.8718V^2-166.6667\cos(\delta-\delta_m-0.0873)V \\ \quad -666.6667\cos(\delta-0.2094)V-93.3333V \\ \quad +33.3333Q_1+43.333, \\ \dot{V} = -78.7638V^2+26.2172\cos(\delta-\delta_m-0.0124)V \\ \quad +104.8689\cos(\delta-0.1346)V+14.5229V \\ \quad -5.2288Q_1-7.0327, \end{cases} \tag{20}$$

where δ_m represents the generator power angle; ω denotes the generator frequency deviation; δ and V refer to the phase angle and amplitude of the load bus in three-bus power system, respectively; and Q_1 is the reactive power of load.

For system (20), when system parameter $Q_1 = 11.377$, and the initial value is selected to be $(0.38, 0.2, 0.1, 0.95)$, four Lyapunov exponents of the system can be calculated to be $\sigma_1 = 0.289$, $\sigma_2 = -0.002$, $\sigma_3 = -4.045$, and $\sigma_3 = -66.48$. One of the Lyapunov exponents is positive, indicating the chaotic oscillation of the system. Phase portrait of the chaotic attractor is shown in Fig. 1, and time response of state variables in the chaotic power system is shown in Fig. 2.

As can be seen from Figs. 1 and 2, the system is in a state of chaotic oscillations and state variables of the system behave like irregular and non-periodic chaotic oscillations. Non-periodic chaotic oscillation is accompanied by voltage instability and frequency oscillation, which will result in voltage collapse, and thus causing a large blackout accident.[6,7] Therefore, it is urgent to stabilize the chaotic power system to ensure its safe and stable operation.

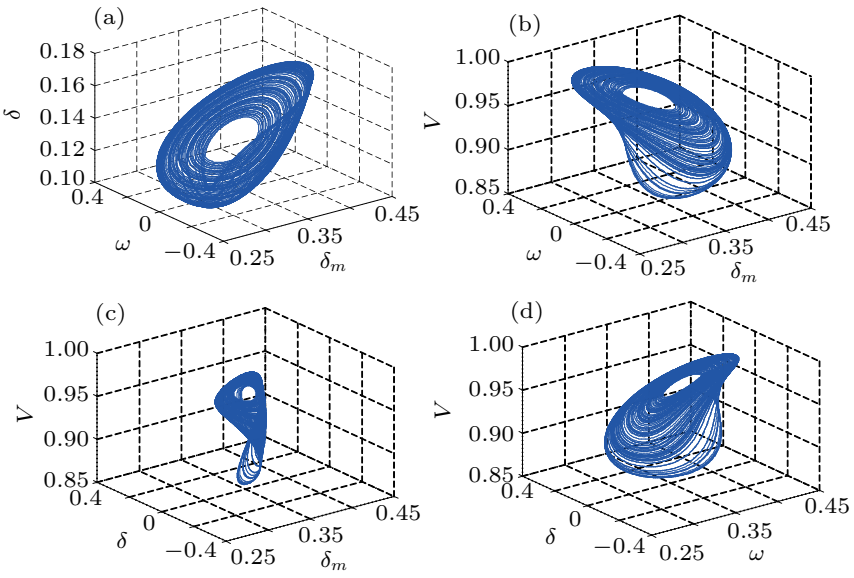


Fig. 1. (color online) Three-dimensional phase portraits in chaotic power system composed of (a) δ_m–ω–δ, (b) δ_m–ω–V, (c) δ_m–δ–V, and (d) ω–δ–V.

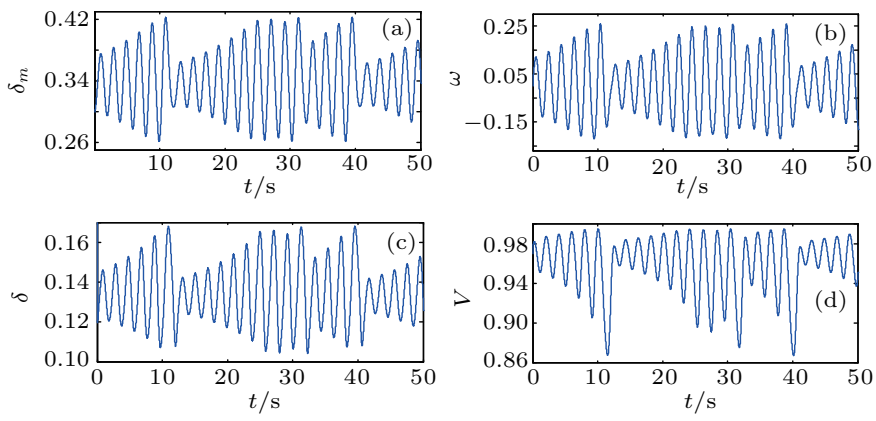


Fig. 2. (color online) Time responses of state variables of (a) δ_m, (b) ω, (c) δ, and (d) V in chaotic power system.

3.2. Design of chaotic oscillating controller

In order to transform the system (20) from chaotic state to equilibrium state and make the system return to its original state, only voltage and frequency are required to be controlled.[32,33] Hence, the control objectives of power system are set as $\omega_{\rm d}=0$ and $V_{\rm d}=1$, then frequency and voltage control errors are $e_2=\omega$ and $e_4=V-1$, and dynamic equations of controlled system are as follows:

$$\begin{cases} \dot{\delta}_m = f_1, \\ \dot{e}_2 = f_2 + u_2, \\ \dot{\delta} = f_3, \\ \dot{e}_4 = f_4 + u_4, \end{cases} \tag{21}$$

where

$$\begin{cases} f_1 = e_2, \\ f_2 = 16.6667\sin(\delta-\delta_m+0.0873)(e_4+1) \\ \quad -0.1667e_2+1.8807, \\ f_3 = 496.8718(e_4+1)^2 \\ \quad -166.6667\cos(\delta-\delta_m-0.0873)(e_4+1) \\ \quad -666.6667\cos(\delta-0.2094)(e_4+1) \\ \quad -93.3333(e_4+1)+33.3333Q_1+43.333, \\ f_4 = -78.7638(e_4+1)^2 \\ \quad +26.2172\cos(\delta-\delta_m-0.0124)(e_4+1) \\ \quad +104.8689\cos(\delta-0.1346)(e_4+1) \\ \quad +14.5229(e_4+1)-5.2288Q_1-7.0327. \end{cases}$$

That is, the whole power system can be stabilized by controlling state variable e_2 and e_4 of system (21) to the origin through using controllers u_2 and u_4.

According to Eq. (11), the controllers can be designed as follows:

$$\begin{aligned} u_i = & -f_i-\alpha\,\mathrm{sign}(e_i)\,|e_i|^m-\beta\,\mathrm{sign}(e_i)\,|e_i|^n \\ & -\alpha\,\mathrm{sign}(s_i)\,|s_i|^m-\beta\,\mathrm{sign}(s_i)\,|s_i|^n. \end{aligned} \tag{22}$$

where $i=2,4$, and s_i is given by Eq. (8).

3.3. Simulation

In this subsection, we illustrate the control effect of the proposed controller. The control parameters of Eq. (22) are selected as $\alpha=\beta=10$, $m=3/2$, and $n=2/3$. Under these conditions, one obtains that the times for the system (21) to reach the sliding surface and for the errors to converge to the origin under any initial values satisfy $\lim\limits_{s_i(0)\to\infty}[T(s_i(0))]\le 0.54$ s and $\lim\limits_{e_i(0)\to\infty}[T(e_i(0))]\le 0.54$ s from Eqs. (16) and (18), respectively. According to Eq. (19), the total time required to obtain stability of the system satisfies $\lim\limits_{s_i(0)\to\infty}[T(s_i(0))]+\lim\limits_{e_i(0)\to\infty}[T(e_i(0))]\le 1.08$ s. The curves of sliding mode function, error function, state variables, and control input changing with time are shown in Figs. 3–6.

As shown in Figs. 3 and 4, system (21) reaches the sliding surface and the errors converge to the origin within 0.2 s, which shows fast convergence of approaching motion and sliding mode motion.

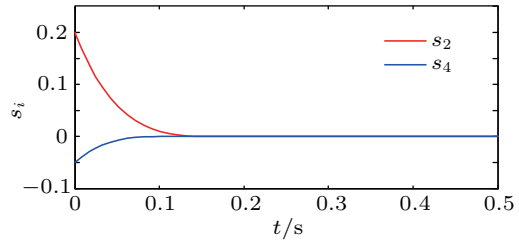


Fig. 3. (color online) Sliding mode functions under control.

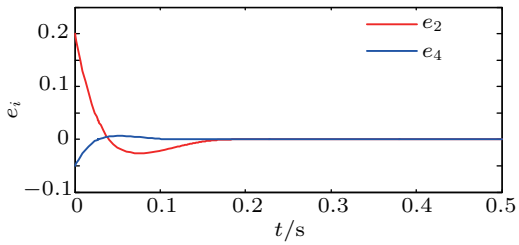


Fig. 4. (color online) Error functions under control.

Figure 5 clearly shows that voltage and frequency are stabilized, the controlled system quickly recovers from chaotic state to equilibrium state within 0.2 s, and the purpose of stabilizing power system is well achieved.

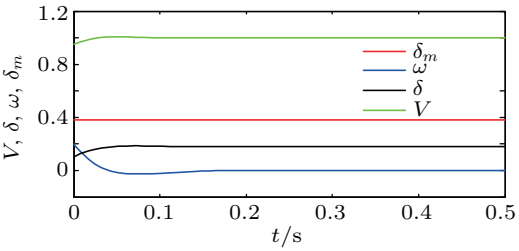


Fig. 5. (color online) State variables of controlled power system.

As shown in Fig. 6, control inputs behave like neither chattering in Refs. [25] and [32] nor singularity in Ref. [32], and can guarantee fixed time stability of the system.

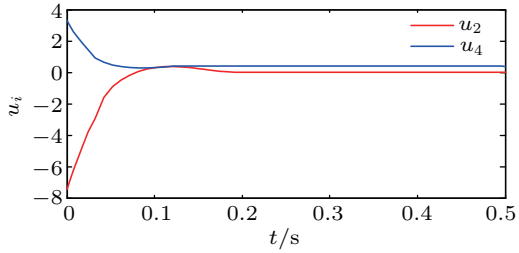


Fig. 6. (color online) Control inputs in the proposed method.

4. Comparison

4.1. Example 1

In 2014, a finite time integral sliding mode controller was designed by Ni.[25] In order to illustrate the superiority of the controller in dealing with chattering problem, the fixed time controller designed in the paper is compared with Ni's finite time controller. The finite time integral sliding mode controller given by Ni is designed as

$$u_i = -f_i - \alpha \mathrm{sign}(e_i) - \beta e_i - \alpha \mathrm{sign}(s_i) - \beta s_i, \tag{23}$$

where $i = 2, 4$, and

$$s_i = e_i + \alpha \int_0^t \mathrm{sign}(e_i)\mathrm{d}t + \beta \int_0^t e_i \mathrm{d}t.$$

Obviously, equation (23) shows a symbolic function, which will cause chattering in control process. The parameters are selected as $\alpha = \beta = 10$, two control inputs are shown in Figs. 7 and 8.

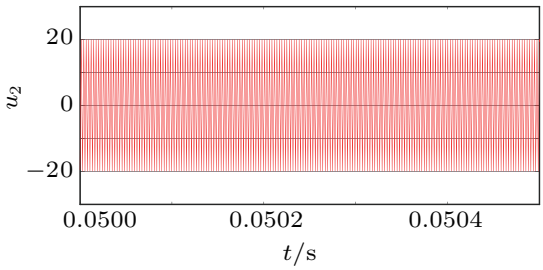


Fig. 7. (color online) Control input u_2 designed by Ni.

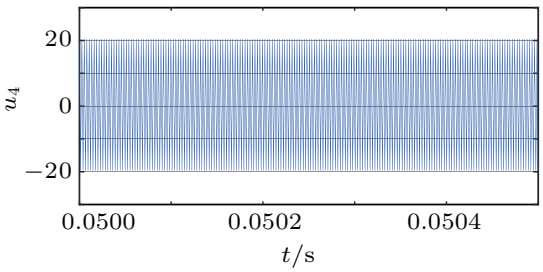


Fig. 8. (color online) Control input u_4 designed by Ni.

As clearly shown in Figs. 7 and 8, the control inputs have serious high-frequency chattering, which means that the system states behave like high-frequency vibration along the specified state trajectory. Chattering not only affects control accuracy and increases the energy consumption of the controlled system, but also destroys the components of a controller. In order to suppress chattering, Ni used the relay characteristics to improve the controller as follows:

$$u_i = -f_i - \frac{\alpha e_i}{|e_i| + 0.001} - \beta e_i - \frac{\alpha s_i}{|s_i| + 0.001} - \beta s_i. \tag{24}$$

In Eq. (24), the parameters are selected as $\alpha = \beta = 10$, two control inputs can be obtained as indicated in Fig. 9. The improved controller does not completely eliminate chattering as shown in Fig. 9. In addition, the improvement method has a disadvantage of inconsistence between theoretical derivation and simulation results.

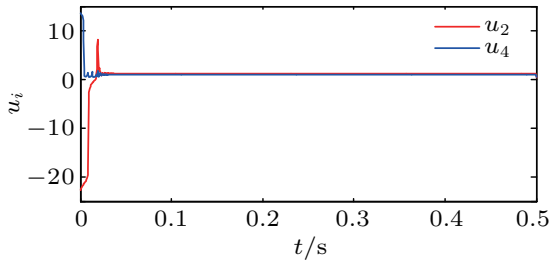


Fig. 9. (color online) Control inputs after improvement by Ni.

4.2. Example 2

In 2018, a non-singular chattering-free terminal sliding mode finite time controller was proposed by Aghababa.[34] According to the method, terminal sliding mode function can be designed as

$$s_i = e_i + \beta \int_0^t \mathrm{sign}(e_i)\,|e_i|^n \,\mathrm{d}t, \tag{25}$$

where $i = 2, 4$, and the parameter n is any real number satisfying $0 < n < 1$.

Like Eq. (9), let

$$\begin{aligned} \dot{s}_i &= \dot{e}_i + \beta \mathrm{sign}(e_i)\,|e_i|^n = f_i + u_i + \beta \mathrm{sign}(e_i)\,|e_i|^n \\ &= -\beta \mathrm{sign}(s_i)\,|s_i|^n. \end{aligned} \tag{26}$$

From Eq. (26), controller can be solved as

$$u_i = -f_i - \beta \mathrm{sign}(e_i)\,|e_i|^n - \beta \mathrm{sign}(s_i)\,|s_i|^n. \tag{27}$$

The controller can also eliminate chattering and singularity problems. However, the combination of Eq. (26) and Eq. (27) shows that in the approaching movement process and the sliding mode movement process, s_i and e_i satisfy the following equations:

$$\begin{cases} \dot{s}_i = -\beta \mathrm{sign}(s_i)\,|s_i|^n, \\ \dot{e}_i = -\beta \mathrm{sign}(e_i)\,|e_i|^n. \end{cases} \tag{28}$$

From Eq. (28), one obtains the times for s_i and e_i to converge to zero from any initial values, to be[35]

$$\begin{cases} T(s_i(0)) = \dfrac{1}{\beta(1-n)}\,|s_i(0)|^{1-n}, \\ T(e_i(0)) = \dfrac{1}{\beta(1-n)}\,|e_i(0)|^{1-n}. \end{cases} \tag{29}$$

Since $1 - n > 0$ in Eq. (29), as the values $|s_i(0)|$ and $|e_i(0)|$ grow, the times $T(s_i(0))$ and $T(e_i(0))$ will be larger and larger. In the limit condition, one has $\lim\limits_{s_i(0)\to\infty} [T(s_i(0))] \to \infty$, $\lim\limits_{e_i(0)\to\infty} [T(e_i(0))] \to \infty$. In other words, the convergence time grows unboundedly with the increase in initial value. This is the inherent drawback of finite time controller.

In order to illustrate the superiority of the controller in reducing the convergence time of sliding mode function and error function, the fixed time controller designed in the paper is also compared with the finite time controller proposed by Aghababa. Letting the initial value of the state variable of power system given in Eq. (20) be $(5,8,4,16)$, sliding mode functions and error functions are shown in Figs. 10 and 11, respectively.

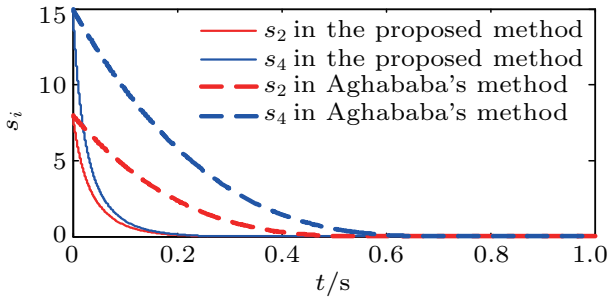


Fig. 10. (color online) Sliding mode functions under controller proposed in the paper (solid curve) and given by Aghababa (dashed curve).

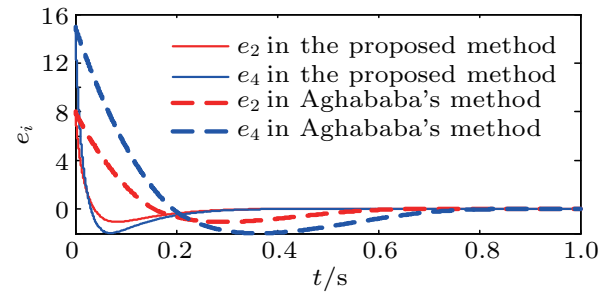


Fig. 11. (color online) Error functions under controller proposed in the paper (solid curve) and given by Aghababa (dashed curve).

It is obvious that the convergence times of sliding mode functions and error functions in the proposed method converge to the origin within 0.54 s, while the convergence times of sliding mode functions and error functions in Aghababa's method increase with increasing initial value and prolongs the convergence time.

4.3. Example 3

In fact, like Eq. (28), one can also easily design a global fast terminal sliding function to make s_i and e_i satisfy the following equations:

$$\begin{cases} \dot{s}_i = -\alpha s_i - \beta \operatorname{sign}(s_i) |s_i|^n, \\ \dot{e}_i = -\alpha e_i - \beta \operatorname{sign}(e_i) |e_i|^n, \end{cases} \tag{30}$$

where $i = 2,4$, and

$$s_i = e_i + \alpha \int_0^t e_i \mathrm{d}t + \beta \int_0^t \operatorname{sign}(e_i) |e_i|^n \mathrm{d}t.$$

That is, the times for s_i and e_i to converge to zero from any initial value are respectively,[35]

$$\begin{cases} T(s_i(0)) = \dfrac{1}{\beta(1-n)} \ln \dfrac{\alpha |s_i(0)|^{1-n} + \beta}{\beta}, \\ T(e_i(0)) = \dfrac{1}{\beta(1-n)} \ln \dfrac{\alpha |e_i(0)|^{1-n} + \beta}{\beta}. \end{cases} \tag{31}$$

When $|s_i(0)| \to \infty$ and $|e_i(0)| \to \infty$ in Eq. (31), then $\lim\limits_{s_i(0)\to\infty} [T(s_i(0))] \to \infty$ and $\lim\limits_{e_i(0)\to\infty} [T(e_i(0))] \to \infty$. Such a design process also has the drawback as described in example 2.

5. Conclusions

In this work, the main conclusions are as follows.

(i) The fixed time integral sliding mode controller designed in this paper can realize the non-singular chattering-free control of the controlled system, and ensure state variables converging to the control objective in fixed time upper bounded by a constant independent of the initial values.

(ii) The controller is used to stabilize chaotic oscillation in the four-order power system, and results show that the controller has a good control effect. In addition, the controller designed in the paper is compared with the controller proposed in other references, which shows that the controller proposed in the paper is better.

(iii) The controller can be used for synchronizing and controlling other complex dynamical systems.

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