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Phase precision of Mach–Zehnder interferometer in PM2.5 air pollution*

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This paper theoretically explores the effect of PM2.5 air pollution on the phase precision of a Mach–Zehnder interferometer. With the increasing of PM2.5 concentration, phase precision for inputs of coherent state & vacuum state and inputs of coherent state & squeezed vacuum state will gradually decrease and be lower than the standard quantum limit. When the value of relative humidity is increasing, the precision of two input cases is decreasing much faster. We also find that the precision for inputs of coherent state & squeezed state is better than that of coherent state & vacuum state when PM2.5 concentration is lower. As PM2.5 concentration increases, the precision for inputs of coherent state & squeezed state decreases faster, and then the two precisions tend to be the same while the concentration is higher.

Keywords: Mach–Zehnder interferometer, PM2.5, phase precision

PACS: 03.67.–a, 03.65.Ta **DOI:** 10.1088/1674-1056/27/7/070304

1. Introduction

Optical interferometry is becoming increasingly important in modern physics. In a classical optical system, the phase precision can in principle be proportional to $1/\sqrt{N}$, commonly referred to as the standard quantum limit (SQL) or the shot-noise limit. Compared with its classical counterpart, quantum-enhanced interferometry is used to estimate phases beyond the shot-noise (or standard quantum) limit.[1–3] Quantum-enhanced interferometry was initiated by Caves, who first suggested how to reach a sub-shot-noise sensitivity by using coherent and squeezed-vacuum light as the input of a Mach–Zehnder interferometer.[4] Using Caves' scheme, the phase sensitivity is bounded by $\Delta\varphi \sim 1/N^{3/4}$.[5] Subsequently, some experiments proved that the precision could exceed the standard limit by using Caves' scheme.[6–8] In addition to squeezed vacuum states, some studies found that by using other non-classical quantum states, such as a spin squeezing state,[9,10] a NOON state,[11,12] and an entangled coherent state,[13,14] the phase precision could achieve the Heisenberg limit.

With the development of industry and the growth of populations, air pollution is becoming more and more serious. In July 1997, the United States Environmental Protection Agency (EPA) promulgated new primary and secondary national ambient air quality standards for PM2.5 (particulate matter less than 2.5 μm). The diameter of PM2.5 particles are very small and have long residence time in the atmosphere. In free space, photon loss comes from the light scattering or absorption by the abundant fine particles of atmospheric aerosol, which will reduce the visibility.[15,16] The ability of light scattering is strongest when the diameter of fine particles is close to the light wavelength.[17] In quantum communication, the light wavelength is about 0.8 μm. So PM2.5 particles are mainly responsible for photon attenuation. In optical interferometry, photonic loss is the main obstacle to the practical implementation of quantum enhanced protocols, which will reduce the estimated precision and degrade the performance of non-classical quantum states.[18–20] However, research into the relationship between PM2.5 air pollution and the phase precision of optical interferometry has not yet been started. In this paper, we will study the effect of PM2.5 air pollution on the phase precision of the Mach–Zehnder interferometer.

We focus on the phase precision of the two input cases under the influence of PM2.5 air pollution. In the first input case, one input port of the Mach–Zehnder interferometer is not excited, that is to say, the port is fed with a vacuum state. The other input port is fed with an important classical state, a coherent state. In the other input case, one input state is a coherent state, and the other input state is a non-classical state, a squeezed vacuum state. This kind of strategy is relatively easy to realize in current technology. Other non-classical states, such as the NOON state and spin squeezing state with a definite photon number, are notoriously hard to be realized. For example, researchers can only prepare the NOON state with a photon number N of no more than 10 to this day.[21,22] So we are not concerned with those non-classical states in this paper.

2. Phase precision of two input cases in noiseless case

In this section, we will calculate the precision of two input cases without photon loss. As shown in Fig. 1, BS1 and BS2 are two 50/50 beam splitters in the Mach–Zehnder inter-

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ferometer. Two incoming light beams are injected from both ports of BS1 and then recombined by a second beam splitter BS2. The phase difference between two arms is φ. We assume that the two beams acquire $-\pi/2$ and $\pi/2$ phase when passing through BS1 and BS2, respectively.

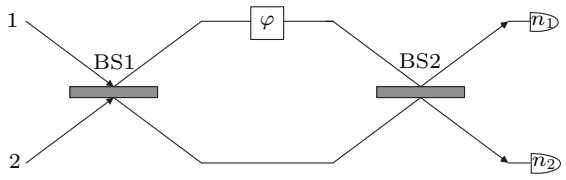


Fig. 1. Schematic diagram of the Mach–Zehnder interferometer.

Let $|\text{in}\rangle$ be the initial state entering the interferometer and $|\text{out}\rangle$ be the final state leaving the interferometer. Then the relationship between two states is[23]

$$|\text{out}\rangle = \mathrm{e}^{\mathrm{i}(\pi/2)J_x}\,\mathrm{e}^{-\mathrm{i}\varphi J_z}\,\mathrm{e}^{-\mathrm{i}(\pi/2)J_x}\,|\text{in}\rangle\,, \tag{1}$$

where J_x and J_z denote x and y components of the total angular momentum J, $J_x = \frac{1}{2}(a^\dagger b + b^\dagger a)$ and $J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$. The photon number difference between the two out ports is $n_1 - n_2 = 2J_z$. The average of the J_z operator is

$$\begin{aligned}\langle J_z\rangle &= \langle\text{out}|J_z|\text{out}\rangle = \cos\varphi\,\langle\text{in}|J_z|\rangle - \sin\varphi\,\langle\text{in}|J_x|\text{in}\rangle \\ &= \cos\varphi\,\langle J_z\rangle_{\text{in}} - \sin\varphi\,\langle J_x\rangle_{\text{in}}\,.\end{aligned} \tag{2}$$

It was found that

$$\begin{aligned}\langle J_z^2\rangle &= \langle\text{out}|J_z^2|\text{out}\rangle = \cos^2\varphi\,\langle J_z^2\rangle_{\text{in}} + \sin^2\varphi\,\langle J_x^2\rangle_{\text{in}} \\ &\quad - \sin\varphi\cos\varphi\,\langle J_xJ_z + J_zJ_x\rangle_{\text{in}}\,.\end{aligned} \tag{3}$$

Hence the variance of J_z is

$$\begin{aligned}(\Delta J_z)^2 &= \langle J_z^2\rangle - \langle J_z\rangle^2 = \cos^2\varphi(\Delta J_z)^2_{\text{in}} + \sin^2\varphi(\Delta J_x)^2_{\text{in}} \\ &\quad - 2\sin\varphi\cos\varphi\,\text{cov}(J_x,J_z)_{\text{in}},\end{aligned} \tag{4}$$

where

$$\text{cov}(J_x,J_z)_{\text{in}} = \frac{1}{2}\langle J_xJ_z + J_zJ_x\rangle_{\text{in}} - \langle J_x\rangle_{\text{in}}\langle J_z\rangle_{\text{in}}\,.$$

The phase precision can be calculated through

$$\Delta\varphi = \frac{\Delta J_z}{\left|\dfrac{\mathrm{d}\langle J_z\rangle}{\mathrm{d}\varphi}\right|}. \tag{5}$$

If the two input ports of the Mach–Zehnder interferometer are fed with a coherent state $|\alpha\rangle$ and a vacuum state $|0\rangle$, respectively, in other words, the input state is $|\text{in}\rangle = |\alpha\rangle\,|0\rangle$, we find

$$\begin{aligned}&\langle J_z\rangle_{\text{in}} = \frac{1}{2}|\alpha|^2\,, \quad \langle J_x\rangle_{\text{in}} = 0, \\ &(\Delta J_z)^2_{\text{in}} = (\Delta J_x)^2_{\text{in}} = \frac{1}{4}|\alpha|^2\,, \quad \text{cov}(J_x,J_z)_{\text{in}} = 0.\end{aligned} \tag{6}$$

Thus the precision is

$$\Delta\varphi^{|\alpha\rangle|0\rangle} = \frac{1}{|\alpha\sin\varphi|} = \frac{1}{\sqrt{N}\,|\sin\varphi|}. \tag{7}$$

Since the mean photon number of input states is $N = |\alpha|^2$, we acquire the last term of Eq. (7). When $\varphi = \pi/2$ or $3\pi/2$, the optimal precision is achieved, which is

$$\Delta\varphi^{|\alpha\rangle|0\rangle} = \frac{1}{\sqrt{N}}. \tag{8}$$

The input state $|\text{in}\rangle = |\alpha\rangle\,|r\rangle$ represents a coherent state entering one port and a squeezed vacuum state entering the other. Thus

$$\begin{aligned}&\langle J_z\rangle_{\text{in}} = \frac{1}{2}(|\alpha|^2 - \sinh^2 r), \quad \langle J_x\rangle_{\text{in}} = 0, \\ &(\Delta J_z)^2_{\text{in}} = \frac{1}{4}|\alpha|^2 + \frac{1}{8}\sinh^2 2r, \\ &(\Delta J_x)^2_{\text{in}} = \frac{1}{4}(|\alpha|^2\cosh 2r - \text{Re}(\alpha^2)\sinh 2r + \sinh^2 r), \\ &\text{cov}(J_x,J_z)_{\text{in}} = 0.\end{aligned} \tag{9}$$

The phase precision is

$$\Delta\varphi^{|\alpha\rangle|r\rangle} = \frac{\sqrt{\cot^2\varphi(|\alpha|^2 + \frac{1}{2}\sinh^2 2r) + |\alpha|^2\,\mathrm{e}^{-2r} + \sinh^2 r}}{\left||\alpha|^2 - \sinh^2 r\right|}. \tag{10}$$

When $\varphi = \pi/2$ or $3\pi/2$, the phase precision is expressed as

$$\Delta\varphi^{|\alpha\rangle|r\rangle} = \frac{\sqrt{|\alpha|^2\,\mathrm{e}^{-2r} + \sinh^2 r}}{\left||\alpha|^2 - \sinh^2 r\right|}. \tag{11}$$

The optimal result can be obtained numerically. When the photon number of squeezed vacuum states is smaller than mean photon number N and approaches $\sqrt{N}/2$, then[5]

$$\Delta\varphi^{|\alpha\rangle|r\rangle} = \frac{1.14}{N^{0.76}} \approx \frac{1}{N^{3/4}}, \tag{12}$$

where the mean photon number is given by $N = |\alpha|^2 + \sinh^2 r$. Mixing a coherent state with a squeezed state provides a precision higher than the standard quantum limit.

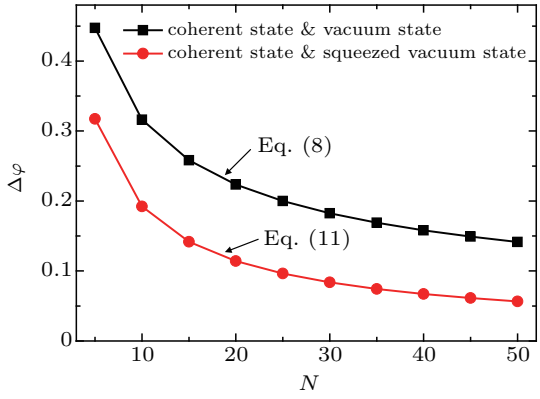


Fig. 2. (color online) Phase precision of the two input cases in noiseless case.

Figure 2 indicates the precision of two input cases. When a small amount of the squeezed vacuum state is mixed with the coherent state, the precision is better than the precision for inputs of coherent state & vacuum state.

3. Quantum state in the presence of noise

That PM2.5 fine particles scatter or absorb photons can be regarded as that the quantum system suffers from unwanted interactions with the environment. These unwanted interactions show up as noise in quantum information processing systems. Supposing the input ports and the output ports are in a clean indoor environment, PM2.5 particles always exist in the free space during the transmission. So we focus on the precision influenced by photon loss in the stage of transmission. Figure 3 indicates where photon loss may appear (gray area).

Mathematically, the interaction of a quantum system with its environment can be expressed as some set of $\{E_k\}$.[24] The output state due to the interaction can be described as

$$\rho_{\text{out}} = \xi(\rho) = \sum_k E_k \rho E_k^{\dagger}. \tag{13}$$

Most quantum states are not single qubit states. However, these states possess symmetry under particle exchange, so

$$c_{\{n\}} = c_{P\{n\}}, \tag{14}$$

where P is an arbitrary permutation of N indices. Taking into account the symmetry property, an l-mode Fock state $|n\rangle = |n_1\rangle|n_2\rangle\ldots|n_l\rangle$ with $N = n_1 + n_2 + \cdots + n_l$ indistinguishable particles can be expressed as the particle description[25,26]

$$\begin{aligned} &|\{n\}\rangle \\ &= \sqrt{\frac{n_1!n_2!\ldots n_l!}{N!}} \\ &\quad \times \sum_P \left|P(1_1, 1_2, \ldots, 1_{n_1}, 2_1, 2_2, \ldots, 2_{n_2}, \ldots, l_1, l_2, \ldots, l_{n_l})\right\rangle, \end{aligned} \tag{15}$$

where $l_1, l_2, \ldots, l_{n_l}$ indicates n_l indices (or particles) in mode l and $\sum_P$ is the summation over all permutations of the indices (or particles) inside the brackets. Because any quantum state can be constructed by a Fock state $|\{n\}\rangle$, we can translate all quantum states into a particle description.

In the particle description of quantum state ρ_{in} with definite N photon number, the photons it brings can be treated as distinguishable particles. The Kraus operators can act independently on each particle. So the initial state ρ_{in} under the influence of quantum noise is written as

$$\begin{aligned} \rho_{\text{out}} &= \xi^{\otimes N}(\rho_{\text{in}}) = \sum_{k1\ldots kN} E_{k1} \otimes \cdots \otimes E_{kN} \rho_{\text{in}} E_{k1}^{\dagger} \otimes \cdots \otimes E_{kN}^{\dagger} \\ &= \sum_{k1\ldots kN} \left(\bigotimes_{i=1}^{N} E_{ki} \right) \rho_{\text{in}} \left(\bigotimes_{i=1}^{N} E_{ki}^{\dagger} \right), \end{aligned} \tag{16}$$

where E_{ki} denotes the Kraus operator E_k for the i-th particle (photon).

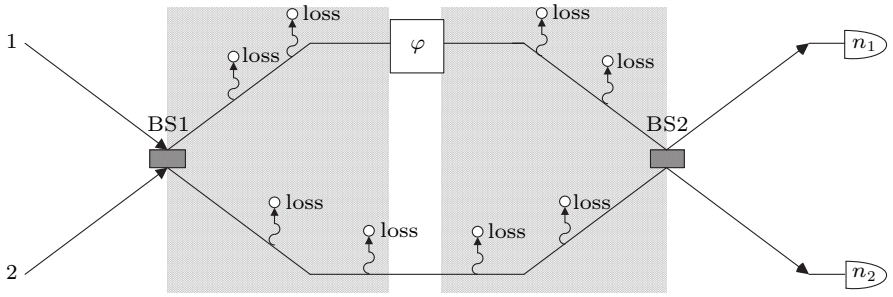


Fig. 3. Possible positions where PM2.5 fine particles can appear during the transmission.

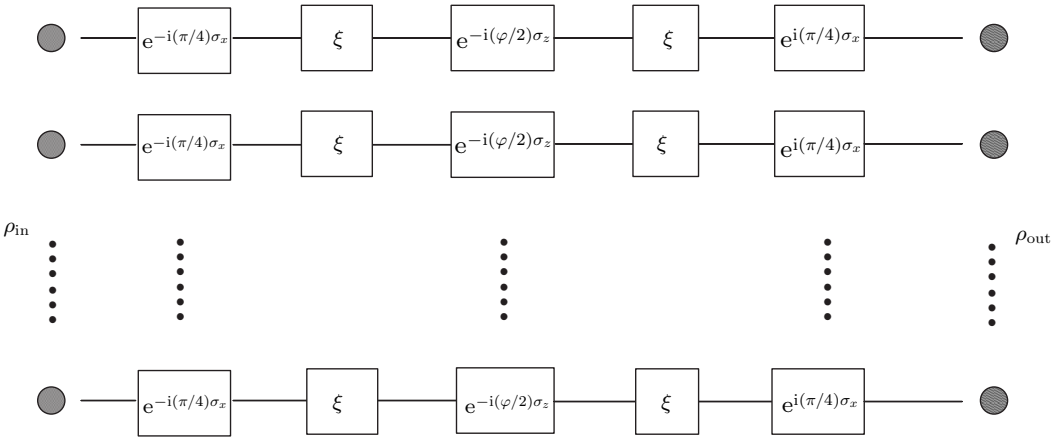


Fig. 4. Process of each photons in noisy Mach–Zehnder interferometer during the transmission.

Considering the quantum noise, figure 4 indicates the process of the initial state $\rho_{\rm in}$ traveling through a Mach–Zehnder interferometer. The transformation $U^i_{\rm BS1} = {\rm e}^{-{\rm i}(\pi/4)\sigma^i_x}$ means the i-th photon of the initial state $\rho_{\rm in}$ traveling through BS1, $U(\varphi)^i = {\rm e}^{-{\rm i}(\varphi/2)\sigma^i_z}$ denotes the phase shift, $U^i_{\rm BS2} = {\rm e}^{{\rm i}(\pi/4)\sigma^i_x}$ represents the i-th photon traveling through BS2, and ξ is the quantum noise acting on the photon during the transmission. Symbols σ^i_x and σ^i_z are the Pauli x and the Pauli z matrices acting on the i-th qubit. The combined operation of all photons is a tensor product

$$\rho_{\rm out} = \zeta(\rho_{\rm in}) = U^{\otimes N}_{\rm BS2}\xi^{\otimes N}\left\{U(\varphi)^{\otimes N}\xi^{\otimes N}\left[U^{\otimes N}_{\rm BS1}\rho_{\rm in}U^{\dagger\otimes N}_{\rm BS1}\right]U(\varphi)^{\dagger\otimes N}\right\}U^{\dagger\otimes N}_{\rm BS2}, \tag{17}$$

where $U(\varphi)^{\otimes N} = {\rm e}^{-{\rm i}(\varphi/2)\sum\limits_i\sigma^i_z}$, $U^{\otimes N}_{\rm BS1} = {\rm e}^{-{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}$, and $U^{\otimes N}_{\rm BS2} = {\rm e}^{{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}$. So

$$\begin{aligned}\rho_{\rm out} = {\rm e}^{{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}\sum_{k1\cdots kN}\left\{\left(\bigotimes_{i=1}^{N}E_{ki}\right){\rm e}^{-{\rm i}(\varphi/2)\sum\limits_i\sigma^i_z}\sum_{k1\cdots kN}\left[\left(\bigotimes_{i=1}^{N}E_{ki}\right){\rm e}^{-{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}\rho_{\rm in}\,{\rm e}^{{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}\left(\bigotimes_{i=1}^{N}E^\dagger_{ki}\right)\right]\right.\\ \left.\times\,{\rm e}^{{\rm i}(\varphi/2)\sum\limits_i\sigma^i_z}\left(\bigotimes_{i=1}^{N}E^\dagger_{ki}\right)\right\}{\rm e}^{-{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}.\end{aligned} \tag{18}$$

Because $J_x = \frac{1}{2}\sum\limits_{i=1}^{N}\sigma^i_x$ and $J_z = \frac{1}{2}\sum\limits_{i=1}^{N}\sigma^i_z$, the tensor product of the phase shift operation can be written as

$$U(\varphi)^{\otimes N} = {\rm e}^{-{\rm i}\varphi J_z}, \tag{19}$$

and the transformations of BS1 and BS2 are

$$U^{\otimes N}_{\rm BS1} = {\rm e}^{-{\rm i}(\pi/2)J_x}, \tag{20}$$

$$U^{\otimes N}_{\rm BS2} = {\rm e}^{{\rm i}(\pi/2)J_x}. \tag{21}$$

Inserting Eqs. (19)–(21) into Eq. (18) and getting rid of the operators $\{E_k\}$ in Eq. (18), we find Eq. (18) to be similar to Eq. (1) in the noiseless case.

The average value of an observable O can be expressed by

$$\langle O\rangle = {\rm Tr}\left[O\zeta(\rho_{\rm in})\right] = {\rm Tr}\left[\zeta^\dagger(O)\rho_{\rm in}\right], \tag{22}$$

where

$$\begin{aligned}\zeta^\dagger(O) = {\rm e}^{{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}\sum_k\left\{\left(\bigotimes_{i=1}^{N}E^\dagger_{ki}\right){\rm e}^{{\rm i}(\varphi/2)\sum\limits_i\sigma^i_z}\right.\\ \times\sum_k\left[\left(\bigotimes_{i=1}^{N}E^\dagger_{ki}\right){\rm e}^{-{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}O\,{\rm e}^{{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}\left(\bigotimes_{i=1}^{N}E_{ki}\right)\right]\\ \left.\times\,{\rm e}^{-{\rm i}(\varphi/2)\sum\limits_i\sigma^i_z}\left(\bigotimes_{i=1}^{N}E_{ki}\right)\right\}{\rm e}^{-{\rm i}(\pi/4)\sum\limits_i\sigma^i_x}.\end{aligned} \tag{23}$$

If the observable $O = J_z$, we can derive the phase precision of definite N photon number states in a noisy interferometer.

In Eqs. (8) and (12), we find the precision scale as $1/N^{1/2}$ or $1/N^{3/4}$, where N denotes the mean photon number for the input of coherent state & vacuum state or that of coherent state & squeezed vacuum state. Obviously, the phase bound is related with mean photon number. While calculating the phase precision, we consider those indefinite photon number states with mean photon number N as a quantum state with a definite photon number N. Thus, the above-mentioned calculation for definite photon number state can apply to indefinite photon number states, such as inputs of coherent state & vacuum state, or to inputs of coherent state & squeezed vacuum state.

4. Effects of PM2.5 on the phase precision

The IMPROVE project established empirical formulas[27] to evaluate the contribution of bulk PM2.5 chemical species to the total light extinction

$$\begin{aligned}b_{\rm ext} = {}& 3f({\rm RH})\left[(NH_4)_2SO_4 + NH_4NO_3\right] + 4\left[{\rm OMC}\right]\\ & + 10[{\rm LAC}] + 1[{\rm SOIL}] + 0.6[{\rm CM}] + 10,\end{aligned} \tag{24}$$

where $b_{\rm ext}$ is the light extinction coefficient, expressed in inverse megameters (Mm^{-1}). Species concentrations shown in brackets are in microgram per meter cubed ($\mu g/m^3$). OMC, LAC, SOIL, and CM represent organic carbon mass, light-absorbing carbon, fine soil, and coarse mass. 10 is the extinction from gas molecules. $f({\rm RH})$ is the water growth factor as a function of relative humidity (RH). Most aerosol species, such as gas molecules, $(NH_4)_2SO_4$, and NH_4NO_3, can contribute to light scattering. While other species, such as LAC, can cause light absorption. Some research has found that CM has a very small contribution to $b_{\rm ext}$.[28,29] So the equation (24) can be rewritten as

$$b_{\rm ext} = [3f({\rm RH})P_1 + 4P_2 + 10P_3 + 1P_4]C_{\rm tot} + 10, \tag{25}$$

where P_1 to P_4 indicate the proportion of $(NH_4)_2SO_4$ and NH_4NO_3, OMC, LAC, and SOIL in the total concentration of PM2.5.

We can evaluate the average value of P_i from data samples from November 2013 to November 2014 in Hangzhou, China.[30] The P_i values are shown in Table 1. The value $f({\rm RH})$ varies with RH, as shown in Table 2.[31]

Table 1. The average proportion of each species in the total concentration of PM2.5.

P_1	P_2	P_3	P_4
0.41	0.29	0.05	0.24

Table 2. Statistical summary of mean f(RH) values in selected relative humidity ranges.

RH	(40,45]	(45,50]	(50,55]	(55,60]	(60,65]	(65,70]	(70,75]	(75,80]	(80,85]	(85,90]
f(RH)	1.22	1.27	1.33	1.38	1.45	1.55	1.65	1.83	2.10	2.46

Due to light scattering or absorption by the abundant fine particles in PM2.5, there is photon loss when the quantum state is transmitted in free space. The attenuation of light propagation through free space according to Beer's law can be described as[32]

$$I = I_0 \exp(-b_{\text{ext}} L), \tag{26}$$

where I_0 is the initial light intensity and I is light intensity after it passes through the distance L. Influenced by PM2.5 air pollution, the probability η of losing a photon is

$$\eta = \frac{I_0 - I}{I_0} = 1 - \exp(-b_{\text{ext}} L). \tag{27}$$

Amplitude damping describes the phenomenon of energy dissipation. The operation elements are

$$\boldsymbol{E}_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta} \end{bmatrix}, \tag{28}$$

$$\boldsymbol{E}_1 = \begin{bmatrix} 0 & \sqrt{\eta} \\ 0 & 0 \end{bmatrix}. \tag{29}$$

$\boldsymbol{E}_1$ operation changes state $|1\rangle$ into state $|0\rangle$ with probability η, corresponding to the physical process of losing a photon to the environment. $\boldsymbol{E}_0$ leaves $|1\rangle$ unchanged with probability $1-\eta$, which means that the photon is not lost.

For the J_z observable, there is

$$J_z = \frac{1}{2}\sum_{i=1}^{N} \sigma_z^i, \tag{30}$$

so

$$J_z^2 = \frac{1}{4}\sum_{i=1}^{N} I^i + \frac{1}{4}\sum_{i\neq j=1}^{N} \sigma_z^i \sigma_z^j, \tag{31}$$

where I is the identity matrix. Using Eq. (23) and Eqs. (28)–(31), we can conclude that

$$\begin{aligned} \langle J_z \rangle_{\text{amp}} &= \text{Tr}\left[\zeta^{\dagger}(J_z)\rho_{\text{in}}\right] \\ &= (1-\eta)\left[\cos\varphi \langle J_z \rangle_{\text{in}} - \sin\varphi \langle J_x \rangle_{\text{in}}\right], \end{aligned} \tag{32}$$

$$\begin{aligned} (\Delta J_z)^2_{\text{amp}} &= \left\langle J_z^2 \right\rangle_{\text{amp}} - \langle J_z \rangle^2_{\text{amp}} \\ &= \frac{N}{4}\eta(2-\eta) + (1-\eta)^2 \cos^2\varphi (\Delta J_z)^2_{\text{in}} \\ &\quad + (1-\eta)^2 \sin^2\varphi (\Delta J_x)^2_{\text{in}} \\ &\quad - 2(1-\eta)^2 \sin\varphi \cos\varphi \text{cov}(J_x, J_z)_{\text{in}}. \end{aligned} \tag{33}$$

If the two input ports are fed into a coherent state and a vacuum state,

$$\Delta\varphi^{|\alpha\rangle|0\rangle}_{\text{amp}} = \frac{(\Delta J_z)_{\text{amp}}}{\left|\dfrac{\text{d}\langle J_z \rangle_{\text{amp}}}{\text{d}\varphi}\right|} = \frac{1}{|(1-\eta)\sqrt{N}\sin\varphi|}. \tag{34}$$

When $\varphi = \pi/2$ or $3\pi/2$, the optimal phase precision is

$$\Delta\varphi^{|\alpha\rangle|0\rangle}_{\text{amp}} = \frac{1}{(1-\eta)\sqrt{N}}. \tag{35}$$

Similarly, if the input states are $|\text{in}\rangle = |\alpha\rangle|r\rangle$, we obtain

$$\Delta\varphi^{|\alpha\rangle|r\rangle}_{\text{amp}} = \frac{\sqrt{\cot^2\varphi(|\alpha|^2 + \frac{1}{2}\sinh^2 2r) + |\alpha|^2 \text{e}^{-2r} + \sinh^2 r + \dfrac{N}{\sin^2\varphi}\dfrac{\eta(2-\eta)}{(1-\eta)^2}}}{\left||\alpha|^2 - \sinh^2 r\right|}. \tag{36}$$

Obviously, the numerator is the smallest when $\varphi = \pi/2$ or $3\pi/2$, and equation (36) becomes

$$\Delta\varphi^{|\alpha\rangle|r\rangle}_{\text{amp}} = \frac{\sqrt{|\alpha|^2 \text{e}^{-2r} + \sinh^2 r + \dfrac{N\eta(2-\eta)}{(1-\eta)^2}}}{\left||\alpha|^2 - \sinh^2 r\right|}. \tag{37}$$

Substituting Eq. (27) into Eq. (35) and Eq. (37), we can find the relationship between phase precision and PM2.5 air pollution.

5. Comparison and discussion

As shown in Fig. 5, the precisions of two input cases in the presence of PM2.5 get worse as the PM2.5 concentration gets bigger. When the PM2.5 concentration is close to zero, the photon loss mainly comes from the scattering of gas molecules. Because the scattering effect of gas molecules is relatively weak, the phase precision is close to the precision of the non-loss case. With PM2.5 concentration continuously increasing, the fine particles, such as organic carbon mass, light-absorbing carbon, fine soil, and coarse mass increase. Due to the light scattering or absorbing from the abundant fine particles, the precision gets worse as PM2.5 concentration increases. As seen in Eq. (26), long distance transmission will accelerate light attenuation. The precision via longer transmission distance $L = 1$ km is worse than precision through transmission distance $L = 0.1$ km.

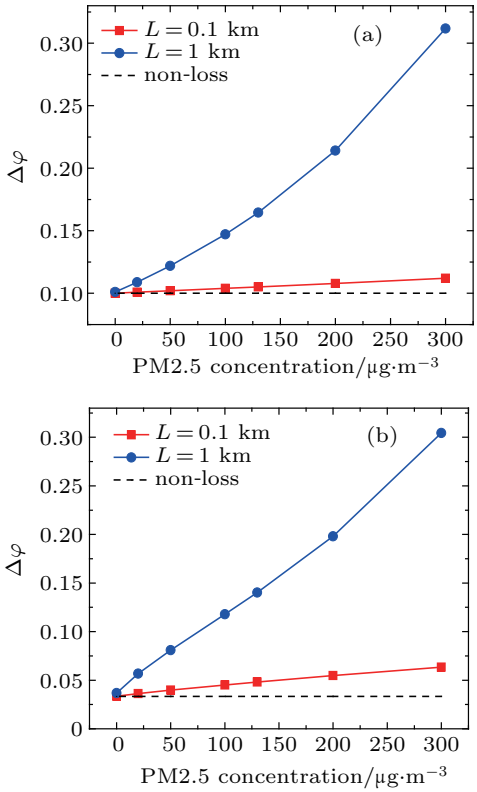


Fig. 5. (color online) Phase precisions influenced by PM2.5 for inputs of (a) coherent state & vacuum state ($N = 100$) and (b) coherent state & squeezed state ($N = 100$).

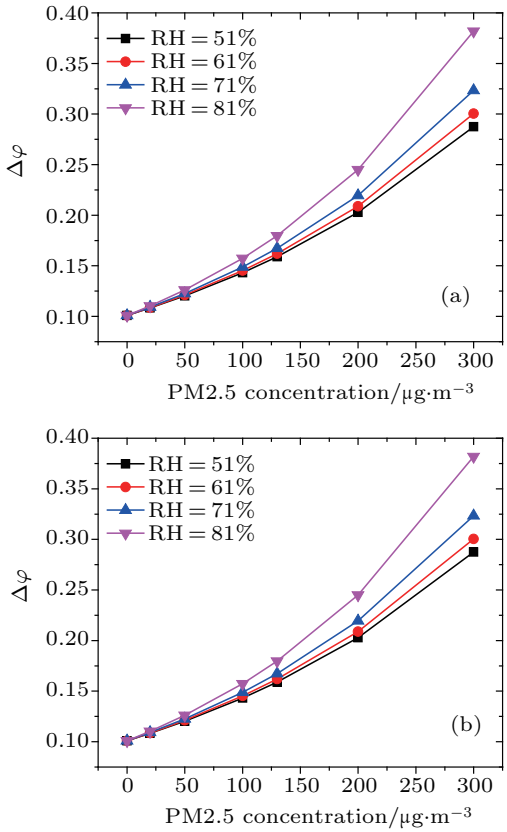


Fig. 6. (color online) Phase precisions in the presence of different relative humidity for inputs of (a) coherent state & vacuum state ($N = 100$) and (b) coherent state & squeezed state ($N = 100$).

Except for transmission distance and PM2.5 concentration, relative humidity can also impact on the phase precision. Some species, such as $(NH_4)_2SO_4$ and NH_4NO_3, have the property of moisture absorption. With RH value increasing, the particle sizes of $(NH_4)_2SO_4$ and NH_4NO_3 increase by aerosol hygroscopic growth. Some research has found that the hygroscopic growth speed of aerosol particles with size 0.01–1 μm is faster and will finally increase to size 0.5–1.5 μm.[33] In quantum communication, the photon wavelength is about 0.8 μm. As the diameters of most particles are near photon wavelength, the scattering ability of PM2.5 was enhanced. Figure 6 shows that the phase precision decreases faster when the relative humidity is higher.

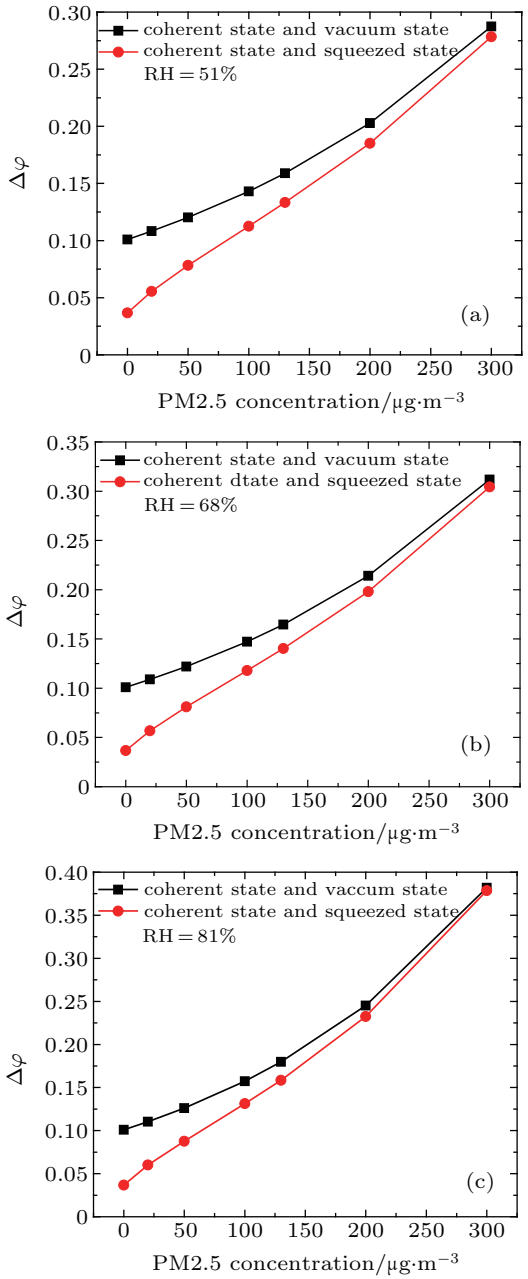


Fig. 7. (color online) Comparison of phase precisions of two input cases ($N = 100$) in the presence of three relative humidities (a) RH = 51%, (b) RH = 68%, and (c) RH = 81%.

In Hangzhou 2014, the lowest, highest, and annual average relative humidities are 51%, 81%, and 68%, respec-

tively. As seen in Fig. 7, the precision changing trends of the above three cases of relative humidity are the same. At first, when PM2.5 concentration is not big, the precisions for inputs of coherent state & squeezed state are better than those for inputs of coherent state & vacuum state. Finally, when the PM2.5 concentration is larger, the precisions of the two input cases tend to be the same. The optimal phase precision for inputs of coherent state & squeezed state is obtained when the coherent beam carries most of the photons, which means that $|\alpha|^2 \gg \sinh^2 r$. So the mean photon number $N \approx |\alpha|^2 \approx |\alpha|^2 - \sinh^2 r$ and

$$\Delta\varphi_{\mathrm{amp}}^{|\alpha\rangle|r\rangle} \approx \sqrt{\left[\mathrm{e}^{-2r} + \frac{\eta(2-\eta)}{(1-\eta)^2}\right] \Big/ N}.$$

Even when the amount of squeezed photon is smaller, the squeezing strength r is still big. Thus the term e^{-2r} can be neglected when η is not close to zero. If $\eta \to 1$, $\frac{\eta(2-\eta)}{(1-\eta)^2} \to \frac{1}{(1-\eta)^2}$. In this case, the approximate optimal phase precision approaches $\frac{1}{(1-\eta)\sqrt{N}}$ and is the same as the precision for inputs of coherent state & vacuum state. Because η will increase along with the increasing of PM2.5 concentration, the two precisions are the same when PM2.5 concentration is higher.

6. Conclusion

We analyzed the phase precision of a Mach–Zehnder interferometer in the presence of PM2.5 air pollution. With the increasing of PM2.5 concentration, the phase precision of two input cases will steadily decrease and be less than the standard quantum limit. Compared with the precision of coherent state & vacuum state, the precision of coherent state & squeezed state decreases faster as PM2.5 concentration increases. The precisions of two input cases tend to be the same when the fine particles concentration approaches 300 $\mu g/m^3$. Except for PM2.5 concentration, transmission distance and relative humidity can also impact on phase precision. Because of the scattering or absorbing from the abundant fine particles, the long transmission distance will attenuate the intensity of light. The longer the distance is, the lower the precision is. Due to ammonium salt having the property of moisture absorption, high relative humidity improves the scattering ability of PM2.5. The precisions of two input cases decrease much faster when the value of relative humidity is higher.

Phase precision for inputs of coherent state & squeezed state is better than that of the other input case when PM2.5 concentration is smaller than 200 $\mu g/m^3$. Coherent state & squeezed state will lose its advantage in heavy air pollution. High relative humidity is harmful to the Mach–Zehnder interferometer. As shown in Fig. 6, phase precisions of two input cases get close to the same when PM2.5 concentration is not bigger than 50 $\mu g/m^3$, which means that high relative humidity has little effect on phase precision when air quality is good. If we want to obtain high precision in free space, the atmospheric parameters need to be monitored and we should ensure the experiment is carried out in a dry and clean environment.

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