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Chin. Phys. B . 2016, 25(7): 077802. **doi:** 10.1088/1674-1056/25/7/077802

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# Transition intensity calculation of Yb:YAG*

Hong-Bo Zhang(张洪波)[1], Qing-Li Zhang(张庆礼)[2,†], Xing Wang(王星)[1,‡], Gui-Hua Sun(孙贵花)[2], Xiao-Fei Wang(王小飞)[2], De-Ming Zhang(张德明)[2], and Dun-Lu Sun(孙敦陆)[2]

[1] *School of Aeronautics and Astronautics Engineering, Air Force Engineering University, Xi'an* 710038, *China*
[2] *The Key Laboratory of Photonic Devices and Materials of Anhui Province, Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei* 230031, *China*

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The Yb:YAG is an excellent high-average power and ultra-short pulse laser crystal. Transition intensity parameters $A_{tp}^{k}$ and Huang–Rhys factors are fitted to its emission spectrum by the full-profile fitting method. Calculated results indicate that the emission spectrum of Yb:YAG at cryogenic temperature consists of three pure electron state transitions and two phonon-assisted transitions, one vibronic transition releases one-phonon of 3 $cm^{-1}$, and the other vibronic transition absorbs one-phonon of 22 $cm^{-1}$. At 300 K, the phonon assisted transition of 3 $cm^{-1}$ turns into two- or more-phonon assisted transitions. The procedure absorbing phonon can reduce the thermal load of Yb:YAG and improve the laser efficiency, which may be one of the reasons why Yb:YAG has excellent performance. The emission bands of Yb:YAG are broadened thermally, and the peak values decrease by several times. The emission cross sections of Yb:YAG determined by Fuchtbauer–Ladenburg (F–L) formula are remarkably different from those calculated with $A_{tp}^{k}$, which indicates that it is necessary for a laser material to determine its transition intensity parameters $A_{tp}^{k}$ in order to reasonably evaluate the laser performance.

**Keywords:** transition intensity parameters, rare earth, photoluminescence, $Yb^{3+}$:YAG

**PACS:** 78.55.–m, 78.60.Lc, 75.10.Dg **DOI:** 10.1088/1674-1056/27/6/067801

## 1. Introduction

The $Yb^{3+}$:YAG is an important high-power laser material used in the diode pumped solid state laser (DPSSL), which has high thermal conductivity, good mechanical property, high chemical and physical stability and excellent large crystal growth properties, and has received great attention. Rutherford *et al.* predicted that it can be used to obtain 100-kW edge-pumped DPSSL.[1] A continuous-wave average output power of 2.65 kW from a single composite Yb:YAG laser rod with (0.6 at.%) Yb:YAG of $\phi$4 mm×80 mm pumped with 9000 W from 940-nm InGaAs laser diodes has been reported.[2] Lucia laser in French could deliver a 100 Joules/10 ns/10 Hz pulse train from ceramic/crystal Yb:YAG,[3] and a 64-J at 10-ns output was demonstrated by diode-pumped cryogenically cooled Yb:YAG ceramic laser amplifier recently.[4] It has been also used to develop few-cycle, waveform-controlled light pulses with a repetition rate of 10 Hz and a peak power in the petawatt regime in petawatt field synthesizer in Germany.[5] A 7.07-kW Yb:YAG composite crystal zigzag slab amplifier at room temperature was developed and demonstrated with an optical-to-optical efficiency of about 29.5% and a slope efficiency of 39.5% by Li *et al.* in Institute of Applied Electronics, China Academy of Engineering Physics.[6]

Patel *et al.* reported the crystal growth, spectral, thermal and laser properties.[7] Due to the fact that $f$–$f$ transitions of $Yb^{3+}$ are very few in number, which is usually far less than the number of the crystal field parameters $B_{q}^{k}$ and also the number of intensity transition parameters $A_{tp}^{k}$, the crystal field parameters and transition intensity parameters ae rarely reported for $Yb^{3+}$, although they are important for understanding the electron structure and optical transition mechanism of $Yb^{3+}$ in solid. Recently, the method to fit crystal filed parameters with numerical derivative of matrix eigenvalues,[8] and full profile fitting the emission or absorption spectrum to obtain transition intensity parameters[9,10] has been developed, and applied to Yb:$Sc_2O_3$, Yb:$GdTaO_4$, etc. The crystal parameters of Yb:YAG have been determined in Ref. [7]. In this work, we apply the full-profile fitting method to the emission spectrum of Yb:YAG to determine its transition intensity parameters and Huang–Rhys factors, and explain why $Yb^{3+}$:YAG has high laser-efficiency.

## 2. Experiment

(10 at.%) Yb:YAG was grown by the Czockralski method in a furnace JGD60, made in the 26th Institute, China Electronics Technology Corporation (CETC) in China, and its photoluminescence spectra at 8 K and 300 K were measured under the light excitation of 914.8 nm with Fluorolog-3-Tau made in

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†Corresponding author. E-mail: zql@aiofm.ac.cn
‡Corresponding author. E-mail: 1678648615@qq.com

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JOBIN YVON in France. The photoluminescence decay curve was measured at a monitor wavelength of 1028.2 nm and an excitation wavelength of 914.8 nm, which is a single exponential decay with decay constant 643 μs and shown as in Fig. 1.

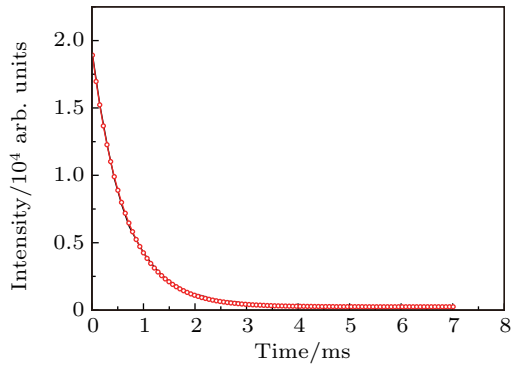



**Fig. 1.** (color online) Time-dependent intensity of (10 at.%) Yb:YAG at 8 K ($\lambda_{ex} = 914.8$ nm, $\lambda_{em} = 1028.2$ nm).

## 3. Transition intensity calculation and discussion

### 3.1. Transition intensity fitting of the emission spectrum of Yb:YAG at 8K

The $Yb^{3+}$ dopants in YAG replace $Y^{3+}$ ions and occupy the site symmetry $D_2$, which results in nine non-vanished pure imaginary intensity parameters $A^k_{tp}$ with $(k,t,p) = (2,3,2)$, (4,3,2), (4,5,4), (6,5,4), (4,5,2), (6,5,2), (6,7,6), (6,7,4), and (6,7,2), which are independent variables to be fitted, and the estimated initial values are shown as in Table 1.

The crystal field energy levels of $Yb^{3+}$ in YAG have been evaluated in Ref. [11], which is denoted as $|J,\Gamma_i\rangle(r)$, where $\Gamma_i$ is point group irreducible representation, $r$ represents the multiplicity of $|J,\Gamma_i\rangle$. $|J,\Gamma_i\rangle(r)$ refers to the dominant component of the corresponding wavefunctions. For convenience, the energy levels of Yb:YAG are labeled as $E_i$ ($i = 1$, 2, 3, 4, 5, 6 and 7) form low to high energy, which is 0, 564, 606, 787, 10329, 10629, and 10829 in unit $cm^{-1}$ (wavenumber). The calculated photoluminescence emission intensity of $Yb^{3+}$, including electric and magnetic dipole transitions, can be expressed as[9]

$$I(\lambda) \equiv I\left(\lambda, A^k_{tp}, \lambda_{IF}, w_{IF}, \sigma_I\right) = \sum_{I,F} I_{IF}(\lambda) = \sum\nolimits_{I,F} \sigma_I A_{IF}(A^k_{tp}) \phi_{IF}(\lambda, \lambda_{IF}, w_{IF}) \nu_{IF}, \quad (1)$$

where $I$ represents the initial state $|5/2,\Gamma_1\rangle(1)$ with $E_5$, $F$ denotes the final state $|7/2,\Gamma_6\rangle(1)$, $|7/2,\Gamma_6\rangle(2)$, $|7/2,\Gamma_7\rangle(1)$, $|7/2,\Gamma_7\rangle(2)$ with energies $E_1$, $E_2$, $E_3$, and $E_4$, and their vibronic transitions. Namely, the emission spectrum of Yb:YAG is composed of the transitions of $E_5 \to E_{1,2,3,4}$, $E_5 +$ phonon $\to E_2$, $E_5 \to E_2 +$ phonon. The $\sigma_I$ is a constant relating to the measurement instrument, initial state population and Planck constant, $\lambda$ is the wavelength variable, $A_{IF}$ is the transition probability of $I \to F$, $\lambda_{IF}$, and $\nu_{IF}$ are the transition center wavelength and frequency of $I \to F$, $\phi_{IF}$ is the profile function with full width at half maximum (FWHM) $w_{IF}$.

In order to eliminate the arbitrariness resulting from measurement, the transition intensity and energy life time of $|5/2,\Gamma_1\rangle(1)$ with $E_1$ are fitted at the same time to the following equation:[9]

$$\varphi = \frac{1}{2}\sum_{\lambda}\left(I(\lambda)_{\text{obs}} - I\left(\lambda, A^k_{tp}, \lambda_{IF}, w_{IF}, \sigma_I\right)\right)^2 + \frac{1}{2}\left(1 - \tau_I^{\text{obs}} \sum\nolimits_F A_{IF}(A^k_{tp})\right)^2, \quad (2)$$

where $I(\lambda)_{\text{obs}}$ is the observed spectral intensity, $\tau_I^{\text{obs}}$ is the observed fluorescence decay time, which is 643 μs or 950 μs here, $A^k_{tp}$, $\lambda_{IF}$, $w_{IF}$, $\sigma_I$ are variable parameters to be fitted. On the other hand, Huang–Rhys factors $s$ are also to be fitted for vibronic transitions. In order to obtain good convergence for the different spectral intensities, $\sigma_I$ is expressed as the product of tow parameters[9]

$$\sigma_I = C\sigma'_I, \quad (3)$$

where $C$ is an adjustable parameter manually, usually on the order of $10^{-19}$ s$\sim 10^{-14}$ s, and $\sigma'_I$ is a fitted parameter by computational routine.

Three pure electron state transitions, which correspond to the emission peaks at 1023.5 nm, 1028 nm, and 1046.1 nm, and three vibronic coupling transitions peaking at 1021.4 nm and 1028.28 nm, which mean that $Yb^{3+}$ absorbs a phonon of 22 $cm^{-1}$ and releases a phonon of 3.0 $cm^{-1}$, were used to fit the emission spectrum of Yb:YAG at 8-K temperature.

Fifteen variables, including nine transition intensity parameters $A^k_{tp}$, two Huang–Rhys factors $s$, three FWHMs $w$, and one $\sigma'_I$, are fitted to the emission spectrum and decay time of Yb:YAG. The initial values of $A^k_{tp}$ are evaluated with point charge model (PCM). The line profile function $\phi$ is Lorentz function, and the iteration method is the Levenberg–Marquardt method. After 20 iterations, $\varphi = 0.0086$, relative residual $R$ is 12.1%, and the calculated decay time is 638 μs, which is very close to experimental value 643 μs. The obtained parameters are listed in Table 1.

Due to the high dopant concentration, the fluorescence quenching is obvious. According to Ref. [12], the decay time of $Yb^{3+}$:YAG is 950 μs, and its temperature dependence is not obvious. So the decay time of 950 μs is also used to fit the transition intensity parameters of Yb:YAG again. After 20 iterations, $R$ reaches 12.1%, and the calculated decay time of $E_5$ is 953 μs. Next, the obtained $A^k_{tp}$ are fixed, and the Huang–Rhys factor, FHWM and $\sigma'_I$ are fitted again. The finally obtained values are listed in Table 2, the calculated and experimental emission spectra are shown in Fig. 2. It can be seen that the longer the decay time, generally the smaller the transition intensity parameters are, which results from the case where the decay time is inversely proportional to transition probability $A_{IF}$, and $A_{IF} = \left|\sum_{\text{even } k,\ \text{odd } t} D^k_{tp,q} \cdot A^k_{tp}\right|^2$ if

magnetic contribution is omitted. Omitting their signs, the ratios of $A^k_{tp}$ $((k,t,p) = (2,3,2)$, (4,3,2), (4,5,4), (6,5,4), (4,5,2), (6,5,2), (6,7,6), (6,7,4), (6,7,2)) obtained by fitting with the decay times of 950 μs and 643 μs, are 0.60, 0.93, 0.37, 1.60, 0.83, 0.70, 0.81, 8.97, 0.88, respectively, most of which are close to $\sqrt{643/950} = 0.82$, which indicates that $A^k_{tp}$ are approximately inversely proportional to the square root of the decay time, and the precise decay time is important for determining $A^k_{tp}$.

The transition probabilities and line strengths of Yb:YAG are listed in Table 2. It can be seen that magnetic dipole transitions are very weak in all of transitions, its total transition probability is 12.3/s, the total electric dipole ratio reaches up to 1050/s, and the ratio of magnetic transition is about 1%. It is surprised that the branch ratio of $E_5 \rightarrow E_1$ is very large, which is 43.6%, although it is very weak in the observed emission spectrum. The transitions of $E_5 \rightarrow E_3$ and phonon assisted transition $E_5 \rightarrow E_3 +$ phonon of 3 $cm^{-1}$with Huang–Rhys factor 0.25, which is usually used as laser channel, account for 29.0% and 16.8%, and their sum is about 45.8%, respectively. So 45.8% should be the possible highest efficiency of a diode-pumped solid state laser by using $Yb^{3+}$:YAG as a gain medium. On the other hand, during the procedure of $E_5 +$ phonon of 22 $cm^{-1}$ with Huang–Rhys factor 0.33$\rightarrow E_2$, $Yb^{3+}$:YAG can absorb a phonon and emit a higher-energy photon, which will reduce the thermal load during laser operation and result in its good thermal properties and high efficiency.

**Table 1.** Initial and fitted parameters of $A^k_{tp}$ in $10^{-13}$ m, Huang–Rhys factor $s$, FWHM in unit nm, and manual adjusted $C$ in $10^{-18}$ s.

| Parameters | Initial values | Fitted values with $\tau = 643$ μs | Fitted values with $\tau = 950$ μs |
|---|---|---|---|
| $A^2_{3,2}$ | –1.769 | –8.748 | 5.254 |
| $A^4_{3,2}$ | –1.988 | –4.923 | 4.570 |
| $A^4_{5,4}$ | –0. 368 | –0.816 | 0.302 |
| $A^6_{5,4}$ | –0. 998 | 0.367 | 0.586 |
| $A^4_{5,2}$ | –0. 637 | –19.735 | 16.339 |
| $A^6_{5,2}$ | – 1.728 | –1.348 | 0.940 |
| $A^6_{7,6}$ | 0. 17 | -16.54 | 13.315 |
| $A^6_{7,4}$ | 0. 533 | 0.151 | 1.355 |
| $A^6_{7,2}$ | 0. 156 | –6.601 | 5.824 |
| $s_1$ | 0.32 | 0.35 | 0.33 |
| $s_2$ | 0.12 | 0.31 | 0.25 |
| $w_1$ | 0.75 | 0.65 | 0.75 |
| $w_2$ | 0.42 | 0.52 | 0.53 |
| $w_3$ | 2.2 | 2.20 | 2.23 |
| $\sigma'_I$ | 1.1 | 0.687 | 0.341 |
| $C$ | / | 6.17 | 17.66 |

**Table 2.** Values of transition probability $A$, line strength $S$, fluorescence branch ratio $\beta$ of $Yb^{3+}$:YAG at 8 K.

| Transition | $\lambda$/nm | $A_{ed}$/(1/s) | $S_{ed}/10^{-0}$ $cm^2$ | $A_{md}$/(1/s) | $S_{md}/10^{-20}$ $cm^2$ | $A$/(1/s) | $\beta$/% |
|---|---|---|---|---|---|---|---|
| $E_5 \rightarrow E_1$ | 968 | 457.2 | 0.20 | 0 | 0 | 457.4 | 43.6 |
| $E_5 \rightarrow E_2$ | 1023.5 | 57.4 | 0.03 | 5.1 | $2.5 \times 10^{-3}$ | 62.5 | 6.0 |
| $E_5 +$ phonon of 22 $cm^{-1} \rightarrow E_2$ | 1021.2 | 26.7 | 0.014 | 2.4 | $1.0 \times 10^{-3}$ | 29.0 | 2.8 |
| $E_5 \rightarrow E_3$ | 1028 | 302.0 | 0.16 | 2.9 | $1.4 \times 10^{-3}$ | 305.0 | 29.0 |
| $E_5 \rightarrow E_3 +$ phonon of 3.0 $cm^{-1}$ | 1028.3 | 175.0 | 0.09 | 1.7 | $8.2 \times 10^{-4}$ | 176.6 | 16.8 |
| $E_5 \rightarrow E_4$ | 1046 | 19.4 | 0.01 | 0.2 | $8.2 \times 10^{-5}$ | 19.5 | 1.9 |

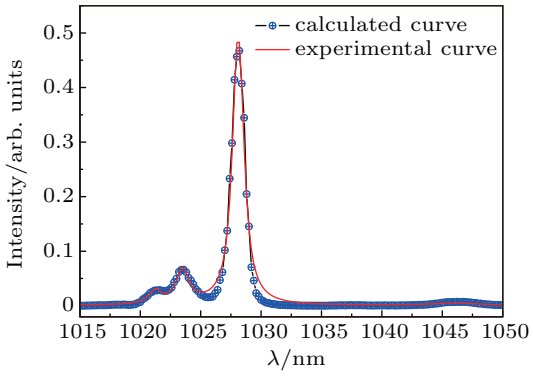



**Fig. 2.** (color online) Calculated and experimental emission spectra of (10 at.%) Yb:YAG at 8 K.

### 3.2. Transition intensity fitting of the emission spectrum of Yb:YAG at 300 K

The emission spectrum of (10 at.%) Yb:YAG excited by the light of 914.8 nm is shown in Fig. 3, which can be fitted by seven Lorentz functions, and their FWHMs, centers and areas are shown in Table 3.

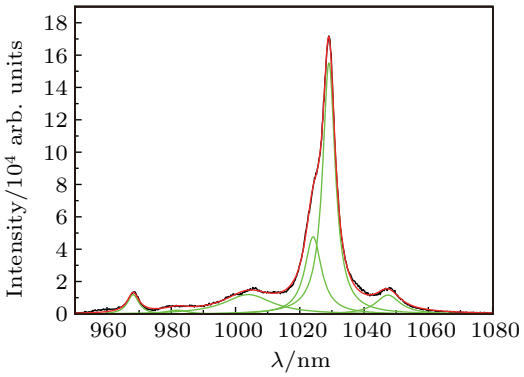



**Fig. 3.** (color online) Emission spectra of (10 at.%) Yb:YAG at 300 K ($\lambda_{ex} = 914.8$ nm).

The emission band with peak 981 nm should be from the vibronic band of $E_5 \rightarrow E_1$ of Yb:YAG, and has the same initial state as the emission band with peak 968.1 nm. When the temperature increases, the lattice vibration intensifies, and the

corresponding phonon energy should increase. So it is suggested that the emission bands of 1000.8 nm and 1006.5 nm are from the vibronic bands of $E_5 \rightarrow E_2$, which means that the transition with one-phonon assisted transition at low temperature has turned into two-phonon assisted transitions with energies of 172 $cm^{-1}$ and 228 $cm^{-1}$, respectively.

**Table 3.** Fitted results of the room temperature emission spectra of (10 at.%) Yb:YAG with Lorentz profile.

| Center/nm | FWHM/nm | Area/a.u.·nm |
|---|---|---|
| 968.1 | 4.46 | 85401 |
| 981.3 | 7.85 | 32021 |
| 1000.8 | 19.4 | 223355 |
| 1006.5 | 11.1 | 112526 |
| 1024.2 | 6.5 | 494926 |
| 1029.1 | 4.7 | $1.1 \times 10^6$ |
| 1047.4 | 9.4 | 176155 |

Due to the strong re-absorption near 968 nm, the emission spectrum as shown in Fig. 4, in which the emission band of 968 nm is stripped and not included in fitting. No obvious dependence of decay time on temperature is observed, and it is reasonable to assume that the transition probability of Yb:YAG nearly keeps constant during temperature changing from the low temperature to room temperature. Then $A_{tp}^k$ values obtained from the emission spectrum of 8 K are used and fixed, and Huang–Rhys factor, FWHM and $\sigma_l'$ are fitted to its emission spectrum of 300 K again. The 14 iterations are performed and final $R$ is 11.7%, the decay time of $E_2$ is calculated to be 952 μs and close to the experimental value 950 μs, and the values of $s_1$ and $s_2$, which correspond to the phonon assisted transitions of $E_5 \rightarrow E_2$ and $E_3$, are 0.129 and 0.006, respectively. The other fitted parameters are listed in Table 4. The fitted curve is shown is Fig. 4, the consistency between the calculated and experimental curves near 100 nm is not so good, which may be due to the fact that only two-phonon assisted transition procedures are considered, more-phonon assisted transitions at room temperature are needed in order to reach good consistency. But the emission bands near 1000 nm are weak, and the present difference between calculated and experimental values is not dominant, so no more-phonon assisted transitions are considered and incorporated into the fitting.

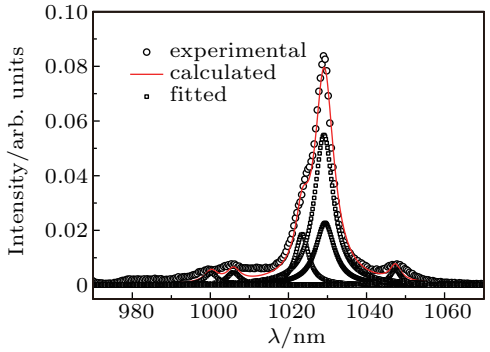



**Fig. 4.** (color online) Fitted and experimental emission spectra of (10 at.%) Yb:YAG at 300 K ($\lambda_{ex} = 914.8$ nm).

**Table 4.** Parameters fitting the emission spectrum of (10 at.%) Yb:YAG at room temperature.

| | Initial value | Final value |
|---|---|---|
| $s_1$ | 0.1 | 0.1291 |
| $s_2$ | 0.02 | 0.0063 |
| $w_1$/nm | 1.88 | 2.05 |
| $w_2$/nm | 3.24 | 2.94 |
| $w_3$/nm | 1.63 | 1.64 |
| $\sigma_l'$ | 1.1 | 0.295654 |
| $C/10^{-18}$ s | $1.766 \times 10^{-17}$ | |

**Table 5.** Values of transition probability $A$, line strength $S$, fluorescence branch ratio $\beta$ of $Yb^{3+}$:YAG at 300 K.

| Transition | $\lambda$/nm | $A_{ed}$/(1/s) | $S_{ed}/10^{-20}$ $cm^2$ | $A_{md}$/(1/s) | $S_{md}/10^{-20}$ $cm^2$ | $A$/(1/s) | $\beta$/% |
|---|---|---|---|---|---|---|---|
| $E_5 \rightarrow E_1$ | 968 | 457.2 | 0.02 | 0 | 0 | 457.2 | 43.5 |
| $E_5$ + phonon of 172 $cm^{-1} \rightarrow E_2$ | 1000.2 | 15.0 | $7.2 \times 10^{-3}$ | 1.3 | $6.0 \times 10^{-4}$ | 16.4 | 1.6 |
| $E_5$ + phonon of 228 $cm^{-1} \rightarrow E_2$ | 1005.8 | 16.4 | $8 \times 10^{-3}$ | 1.5 | $6.7 \times 10^{-4}$ | 17.8 | 1.7 |
| $E_5 \rightarrow E_2$ | 1023.5 | 66.4 | 0.03 | 5.9 | $2.8 \times 10^{-3}$ | 72.3 | 6.9 |
| $E_5 \rightarrow E_3$ | 1029.1 | 327.1 | 0.22 | 3.1 | $1.5 \times 10^{-3}$ | 330.2 | 31.4 |
| $E_5 \rightarrow E_3$ + phonon of 3.0 $cm^{-1}$ | 1029.4 | 135.6 | 0.07 | 1.3 | $6.4 \times 10^{-4}$ | 136.9 | 13.0 |
| $E_5 \rightarrow E_4$ | 1047.4 | 19.4 | 0.01 | 0.2 | $8.2 \times 10^{-5}$ | 19.5 | 1.9 |

### 3.3. Emission cross section of Yb:YAG

Emission cross section is a gain ability parameter of a laser medium and can be used to evaluate its laser performance directly. Because there exists an obvious concentration quenching in (10 at.%) $Yb^{3+}$:YAG, so its emission cross section is calculated by $A_{tp}^k$ fitted with the decay time 950 μs. The emission cross section of $Yb^{3+}$ is calculated by[10]

$$\sigma_{em} = \frac{\lambda^4}{8\pi c n^2} A \phi(\lambda), \tag{4}$$

where $c$ is the light speed in vacuum, $n$ is the refractive index of $Yb^{3+}$:YAG, $A$ is the transition probability, $\phi(\lambda)$ is a normalized profile function, which satisfies the following normalization:

$$\int \phi(\lambda)\, \mathrm{d}\lambda = 1. \tag{5}$$

In Eq. (4), $\phi(\lambda)$ is a Lorentz function and calculated with FWHM directly.

As a comparison, the emission cross section of $Yb^{3+}$:YAG is calculated by Fuchtbauer–Ladenburg (F–L)

equation[7]

$$\sigma_{\rm em} = \frac{\lambda^4 \phi'(\lambda)}{8\pi n^2 \tau_{\rm rad}}, \tag{6}$$

where $\tau_{\rm rad}$ is the decay time, and profile function $\phi'(\lambda)$ is calculated from emission spectrum $I(\lambda)$ as follows:

$$\phi'(\lambda) = \frac{I(\lambda)}{\int I(\lambda)\,{\rm d}\lambda}. \tag{7}$$

The emission cross sections at 8 K calculated by Eqs. (4) and (6) are shown in Fig. 5 and Table 6. It can be seen that emission cross section from $E_5 \rightarrow E_1$, with a peak value of $0.07 \times 10^{-20}$ cm$^2$ at 968 nm, calculated by the F–L formula is very close to zero due to the strong re-absorption of $Yb^{3+}$, but the corresponding value calculated by Eq. (4) with intensity parameters $A^k_{tp}$ is $5.07 \times 10^{-20}$ cm$^2$. The emission cross sections obtained with the F–L formula at 1021.2 nm, 1023.5 nm, 1028 nm, 1028.3 nm, 1046 nm, which correspond to the transitions $E_5 + {\rm phonon} \rightarrow E_2$, $E_5 \rightarrow E_2$, $E_5 \rightarrow E_3$, $E_5 \rightarrow E_3 + {\rm phonon}$, and $E_5 \rightarrow E_4$, are 1.29, 3.06, 22.09, 21.57, 0.36 in $10^{-20}$ cm$^2$, and the corresponding values calculated by intensity parameters and profile functions are 0.81,1.61, 12.04, 11.95, 0.18 in $10^{-20}$ cm$^2$. It can be seen that the later values are much less than the former values. According to the emission cross sections of $Yb^{3+}$:YAG at cryogenic temperatures,[7] the emission cross sections are about $11 \times 10^{-20}$ cm$^2$ at 80 K, which is close to $12.04 \times 10^{-20}$ cm$^2$ of 1028 nm, and $12.04 \times 10^{-20}$ cm$^2$ of 1028.3 $10^{-20}$ cm$^2$ at 8 K, respectively, which indicates that our calculated cross sections are reasonable. The overrated cross sections through F–L formula result from the case where the strong reabsorption of $E_1 \rightarrow E_5$ causes the $\int I(\lambda)\,{\rm d}\lambda$ to be underrated, and profile value is overrated in the wavelength at which there exists no reabsorption.

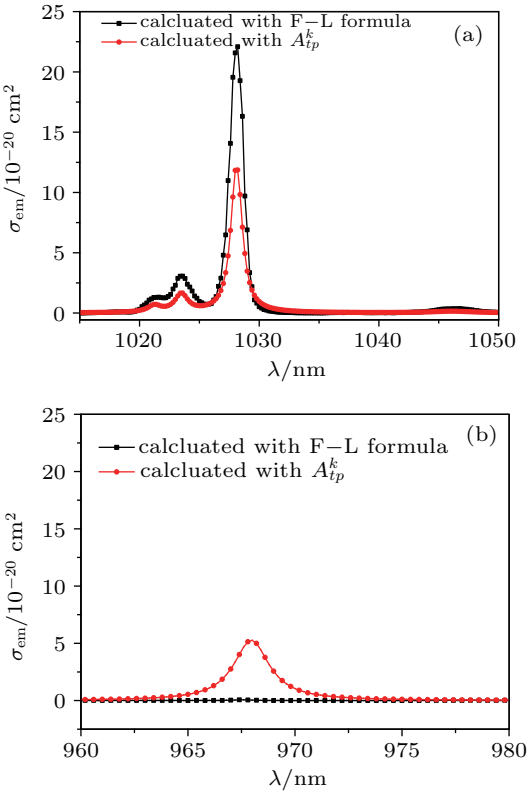



**Fig. 5.** (color online) Emission cross sections of Yb:YAG, calculated by intensity parameters $A^k_{tp}$ (a) and F–L formula (b) at 8 K.

**Table 6.** Values of emission cross section $\sigma_{\rm em}$ (in $10^{-20}$ cm$^2$) of $Yb^{3+}$:YAG, calculated with $A^k_{tp}$ and F–L formula at 8 K and 300 K.

| Transition | at 8 K | | | at 300 K | | |
|---|---|---|---|---|---|---|
| | $\lambda$/nm | with $A^k_{tp}$ | with F–L | $\lambda$/nm | with $A^k_{tp}$ | with F–L |
| $E_5 \rightarrow E_1$ | 968 | 5.07 | 0.07 | 968.1 | 1.16 | 0.24 |
| $E_5 + {\rm phonon}^{\rm a} \rightarrow E_2$ | 1021.4 | 0.81 | 1.29 | 1000.8 | 0.16 | 0.24 |
| $E_5 + {\rm phonon}^{\rm b} \rightarrow E_2$ | / | / | / | 1006.5 | 0.18 | 0.32 |
| $E_5 \rightarrow E_2$ | 1023.5 | 1.61 | 3.05 | 1024.1 | 1.03 | 1.72 |
| $E_5 \rightarrow E_3$ | 1028 | 12.04 | 22.09 | 1029.1 | 2.34 | 3.78 |
| $E_5 \rightarrow E_3 + {\rm phonon}^{\rm c}$ | 1028.3 | 11.95 | 21.57 | 1029.4 | 2.33 | 3.74 |
| $E_5 \rightarrow E_4$, | 1046.1 | 0.18 | 0.36 | 1047.4 | 0.25 | 0.38 |

[a]Phonon energies are 22 cm$^{-1}$ at 8 K and 228 cm$^{-1}$ at 300 K; [b]172 cm$^{-1}$ at 300 K, and [c]3 cm$^{-1}$ at 8 K and 300 K.

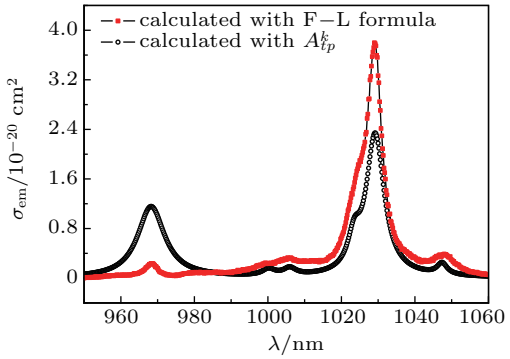



**Fig. 6.** (color online) Emission cross sections of Yb:YAG at 300 K.

The emission cross sections at 300 K are also calculated by $A^k_{tp}$ and F–L formula is shown as in Fig. 6. The emission spectra are broadened as sample temperature increases. Then the profile function peak values decrease, and the cross section peak values also decrease. For example, the fitted FWHMs of $E_5 \rightarrow E_2$, $E_3$ are 0.61 nm and 0.52 nm at 8 K and 2.05 nm and 2.94 nm at 300 K, and the cross sections calculated with $A^k_{tp}$ are 1.61 and 12.04 in $10^{-20}$ cm$^2$ at 8 K, and 1.03 and 2.33 in $10^{-20}$ cm$^2$ at 300 K, respectively.

### 3.4. Transition intensity parameters $\Omega_t$ of Yb:YAG

According to Judd–Ofelt triple parameter intensity formula, if the crystal field energy level splitting is omitted, the dipole transition line strength of $^2F_{5/2} \rightarrow ^2F_{7/2}$ can be calculated by

$$S_{\mathrm{ed}} = \sum_{t=2,4,6} \Omega_t (\langle\, ^2F_{7/2} \parallel U^{(t)} \parallel {}^2F_{5/2} \rangle)^2, \tag{8}$$

where $\Omega_t$ can be calculated with $A_{tp}^k$ directly, $U^{(t)}$ is the single electron operator, $\Omega_{2,4,6} = 0.20$, 2.45, 1.86 in $10^{-20}$ cm$^2$. With the values of $\langle ^2F_{7/2} \parallel U^{(t)} \parallel {}^2F_{5/2} \rangle$,[9] $S_{\mathrm{ed}}$ is calculated to be $3.4 \times 10^{-20}$ cm$^2$, which is much higher than $0.514 \times 10^{-20}$ cm$^2$ obtained from the fitting to emission spectrum of $Yb^{3+}$:YAG, and is similar to the case of $Yb^{3+}$:$GdTaO_4$. This indicates that triple parameter intensity formula is unsuitable for rare-earth ion transition at the cryogenic temperature again.

In summary, the emission spectrum of $Yb^{3+}$:YAG at 8 K consists of four pure electron state transitions and two-phonon assisted transitions, one vibronic transition releases a phonon, and the other vibronic transition absorbs a phonon in the frame of the single frequency approximation. In the emission spectrum at 300 K, there exist one vibronic transition releasing a phonon, and two vibronic transitions absorbing a phonon. The optical transition procedure absorbing phonon can reduce the thermal load of $Yb^{3+}$:YAG and improve its thermal management. Emission cross sections determined by F–L formula are remarkably different from those calculated by $A_{tp}^k$, F–L formula underrates the emission cross sections in a reabsorption wavelength region, and overrates emission cross sections in other wavelength regions.

## 4. Conclusions

Transition intensity parameters $A_{tp}^k$ and Huang–Rhys factor are fitted to the emission spectrum of $Yb^{3+}$:YAG by the full-profile fitting method. The emission spectrum of $Yb^{3+}$:YAG at cryogenic temperature consists of three pure electron state transitions and two- or three-phonon assisted transitions, one vibronic transition releases phonon, and the other one or three vibronic transitions absorb phonon in the frame of the single frequency approximation, the later procedure can reduce the thermal load of $Yb^{3+}$:YAG and improve its thermal management, which may be one of the reasons why the $Yb^{3+}$:YAG is an excellent high-average power laser medium. Emission cross sections determined by F–L formula are remarkably different from those calculated with $A_{tp}^k$, F–L formula underrates emission cross section which is nearly equal to zero in the reabsorption wavelength region, and overrates remarkably emission cross section in the other wavelength region, which suggests that it is necessary for Yb-doped laser medium to determine intensity parameters in order to evaluate a proper laser performance. The emission bands of Yb:YAG are broadened thermally, which results in the emission cross section peak values at 300 K being much less than those at 8 K.

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