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Classical-driving-assisted coherence dynamics and its conservation*

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We investigate the quantum coherence and quantum entanglement dynamics of a classical driven single atom coupled to a single-mode cavity. It is shown that the transformation between the atomic coherence and the atom-field entanglement exists, and can be improved by adjusting the classical driving field. The joint evolution of two identical single-body systems is also studied. The results show the quantum coherence transfers among composite subsystems, and the coherence conservation of composite subsystems is obtained. Moreover, the classical driving field can be used to suppress the decay of the coherence and entanglement, owing to considering the leaky cavity. The non-Markovian dynamics of the system is also discussed finally.

Keywords: quantum coherence, classical driving field, decoherence

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1. Introduction

Quantum entanglement which indicates a very subtle relationship among subsystems, is a basic concept in quantum physics and plays an important role in quantum information processing.[1–5] Quantum coherence is also one of the most important concepts in quantum physics and is usually described as the interposition of the wave functions. It is generally believed that quantum coherence is a necessary precondition for various non-classical effects, such as quantum entanglement, quantum phase transition,[6,7] and quantum interference.[8,9] Recently, a physical resource theory of quantum coherence has been proposed and a rigorous framework to describe quantum coherence has also been provided.[10–14] On the basis of satisfying the constraints above, the l_1-norm, the relative entropy of coherence,[10] coherence of formation,[13] and trace distance of coherence[15] have been suggested to measure coherence. The research shows that the coherence of an open quantum system may be totally unaffected by noise and can be frozen under different conditions.[16,17] Another study shows that quantum coherence can be protected by weak measurement and the atom–cavity coupling.[18]

Quantum coherence and quantum entanglement are the basic resources in quantum information processing, and the intrinsic link between them arouses a great deal of research interest. Studies have shown that any degree of coherence with respect to some reference basis can be converted into entanglement by incoherent operations.[19] Another study has presented the connection of quantum coherence and quantum correlations such as quantum discord, quantum entanglement by determining the relative quantum coherence of two states.[20]

The atom–cavity system based on a cavity quantum electrodynamics(QED) framework is a powerful tool for entanglement engineering[21–24] and quantum information processing.[25,26] Schemes have been proposed for implementing the single and multi-atom entanglement and quantum information processing via resonant[27] as well as non-resonant interaction of the atoms with a cavity mode.[28] Besides, the atomic system is driven by non-resonant external periodic (laser) fields,[29,30] and applying the external classical driving field to the atom can preserve entanglement and improve the robustness of entanglement.[31–34] So in the atom–cavity evolution process, the relationship between the quantum coherence and the quantum entanglement by considering a classical field to drive the atom, should be studied both physically and methodologically.

In this paper, we will concentrate on the following questions. How does the classical driving field affect the transformation between the coherence and the entanglement, and the conservation relationship of the composite atom–cavity coherence? How can the decay of the coherence and entanglement dynamics be inhibited by the classical driving field? How does the non-Markovian dynamics of a system evolve? For the case of an ideal cavity, the atomic coherence can be protected by adjusting the driving field, and increasing the frequency detuning can increase the transformation. For two identical single-body systems, the conservation of coherence of different partitions can be obtained. For the case of a leaky cavity, the classical driving field can suppress the negative influences of the

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leaky cavity on coherence and entanglement. The rest of this paper is organized as follows. In Section 2, we describe the physical model of a two-level atom coupled to a single mode cavity, and the expressions of quantum coherence and quantum entanglement are obtained for the ideal and leaky cavity. In Section 3, the effects of perfect and leaky cavities on coherence and entanglement and the non-Markovian dynamics of a system are discussed. Finally, the conclusion is drawn from the present study in Section 4.

2. Theoretical model

The system considered here consists of a two-level atom coupled to a single mode cavity. The atom is driven by a classical driving field additionally. The Hamiltonian of the total system is given by (setting $\hbar = 1$)[34,35]

$$\begin{aligned} H &= \omega a^{\dagger}a + \frac{\omega_0}{2}\sigma_z + g(\sigma_+ a + \sigma_- a^{\dagger}) \\ &\quad + \Omega(\mathrm{e}^{-\mathrm{i}\omega_c t}\sigma_+ + \mathrm{e}^{\mathrm{i}\omega_c t}\sigma_-), \end{aligned} \tag{1}$$

where $a^{\dagger}$, a are the creation and annihilation operators of the cavity, and ω is the frequency of the cavity. The operators $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ denote the raising, lowering, and inversion operators of the atom, $|e\rangle$, $|g\rangle$ are the excited and ground states of the atom, while ω_0, ω_c are the frequencies of the atom and the classical driving field, respectively. In addition, g is the coupling constant of the atom with the cavity, Ω is the Rabi frequency of the classical driving field and denotes the coupling strength of the atom with the classical driving field.

The Hamiltonian of the system is transformed into H_1 under a unitary transformation $U_1 = \mathrm{e}^{-\mathrm{i}\omega_c\sigma_z t/2}$

$$H_1 = \mathrm{i}\hbar\frac{\mathrm{d}U_1^{+}}{\mathrm{d}t}U_1 + U_1^{+}HU_1 = H_1^{(1)} + H_1^{(2)}, \tag{2}$$

with $H_1^{(1)} = \omega a^{+}a + g(\mathrm{e}^{\mathrm{i}\omega_c t}\sigma_+ a + \mathrm{e}^{-\mathrm{i}\omega_c t}\sigma_- a^{+})$, $H_1^{(2)} = (\Delta/2)\sigma_z + \Omega(\sigma_+ + \sigma_-)$, $\Delta = \omega_0 - \omega_c$ referring to the frequency detuning of atom and classical driving field.

Using a method similar to that used in Ref. [36], introducing the dressed-state $|+\rangle = \cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle$, $|-\rangle = \cos(\theta/2)|g\rangle - \sin(\theta/2)|e\rangle$ with $\theta = \arctan((2\Omega/\Delta))$, diagonalizing the Hamiltonian $H_1^{(2)}$, and neglecting the terms which do not conserve energies (rotating wave approximation), we can obtain the Hamiltonian of the system as follows:

$$\begin{aligned} H_1 &= \frac{\Lambda}{2}S_z + \omega a^{\dagger}a \\ &\quad + g\cos^2\frac{\theta}{2}(\mathrm{e}^{\mathrm{i}\omega_c t}S_+ a + \mathrm{e}^{-\mathrm{i}\omega_c t}S_- a^{\dagger}), \end{aligned} \tag{3}$$

with $\Lambda = \sqrt{\Delta^2 + 4\Omega^2}$, $S_z = |+\rangle\langle+| - |\rangle\langle-|$, $S_+ = |+\rangle\langle-|$, $S_- = |-\rangle\langle+|$ being the inversion, raising, and lowering operators of the atom in the presentation of the dressed-states.

The Hamiltonian of the system is transformed into H_2 under a unitary transformation $U_2 = \mathrm{e}^{\mathrm{i}\omega_c t S_z/2}$:

$$\begin{aligned} H_2 &= \mathrm{i}\hbar\frac{\mathrm{d}U_2^{+}}{\mathrm{d}t}U_2 + U_2^{+}H_1U_2 \\ &= \omega a^{\dagger}a + \frac{\Lambda + \omega_c}{2}S_z + g\cos^2\frac{\theta}{2}(S_+ a + S_- a^{\dagger}) \\ &= H_2^{(0)} + H_2^{(1)}, \end{aligned} \tag{4}$$

with $H_2^{(0)} = \omega a^{\dagger}a + [(\Lambda + \omega_c)/2]S_z$, $H_2^{(1)} = g\cos^2(\theta/2)(S_+ a + S_- a^{\dagger})$.

In the interaction representation, the Hamiltonian is

$$\begin{aligned} H_3 &= \mathrm{e}^{\mathrm{i}H_2^{(0)}t}H_2^{(1)}\mathrm{e}^{-\mathrm{i}H_2^{(0)}t} \\ &= g\cos^2\frac{\theta}{2}(\mathrm{e}^{\mathrm{i}\Lambda' t}S_+ a + \mathrm{e}^{-\mathrm{i}\Lambda' t}S_- a^{\dagger}), \end{aligned} \tag{5}$$

with $\Lambda' = \Lambda + \omega_c - \omega$.

The purpose of introducing the dressed-state and unitary transformation U_1, U_2 is to solve the Hamiltonian of the quantum system.

It is worth noting that unitary transformations U_1, U_2 do not change the eigenvalues of the system, and the purpose of introducing the dressed-state and unitary transformation U_1 is to remove the driving term from Hamiltonian (1), then its effect is concealed in the augmented atomic frequency Λ of Hamiltonian (3). The unitary transformation U_2 can diagonalize Hamiltonian (3) and simplify the system Hamiltonian.

2.1. Case of ideal cavity

Assuming that the atom is initially prepared in the state $\cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle$ and the ideal cavity field is prepared in the Fock state $|n\rangle$, the whole system is initially prepared in the state $|\psi(0)\rangle = (\cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle)\otimes|n\rangle$, via the atom–photon interaction inside the cavity, and the state vector at time t is

$$|\psi(t)\rangle = \alpha_n(t)|+,n\rangle + \beta_{n+1}(t)|-,n+1\rangle. \tag{6}$$

Using the Schrödinger equation $\mathrm{i}\partial\psi(t)/\partial t = H_3\psi(t)$ and the initial conditions of the system, the quantum state coefficient of the system is obtained as

$$\begin{aligned} \alpha_n(t) &= \left(\cos\frac{\Gamma_n t}{2} - \frac{\mathrm{i}\Lambda'}{\Gamma_n}\sin\frac{\Gamma_n t}{2}\right)\exp\left(\frac{\mathrm{i}}{2}\Lambda' t\right), \\ \beta_{n+1}(t) &= \frac{-2\mathrm{i}g\cos^2\frac{\theta}{2}\sqrt{n+1}}{\Gamma_n}\sin\frac{\Gamma_n t}{2}\exp\left(-\frac{\mathrm{i}}{2}\Lambda' t\right), \end{aligned}$$

with

$$\begin{aligned} \Lambda' &= \sqrt{\Delta^2 + 4\Omega^2} + \omega_c - \omega, \\ \Gamma_n &= \sqrt{\Lambda'^2 + 4(n+1)\left(g\cos^2\frac{\theta}{2}\right)^2}. \end{aligned} \tag{7}$$

Considering the atom and cavity field as a bipartite system, in the basis $(|+,n+1\rangle, |+,n\rangle, |-,n+1\rangle, |-,n\rangle)$, the density matrix of the system can be obtained.

Tracing over the degrees of the freedom of the cavity, the reduced density matrix of the atom can be obtained as

$$\rho_a = \mathrm{Tr}_f(\rho) = \begin{pmatrix} \alpha_n(t)\alpha_n^*(t) & 0 \\ 0 & \beta_{n+1}(t)\beta_{n+1}^*(t) \end{pmatrix}. \quad (8)$$

Then by the basic vector transformation

$$|+\rangle = \cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle, \quad |-\rangle = \cos\frac{\theta}{2}|g\rangle - \sin\frac{\theta}{2}|e\rangle.$$

In the basis $(|e\rangle, |g\rangle)$, the reduced density matrix element of the atom can be obtained as follows:

$$\begin{aligned} \rho_{ee} &= \alpha_n(t)\alpha_n^*(t)\cos^2\frac{\theta}{2} + \beta_{n+1}(t)\beta_{n+1}^*(t)\sin^2\frac{\theta}{2}, \\ \rho_{eg} &= \rho_{ge}^* = (\alpha_n(t)\alpha_n^*(t) - \beta_{n+1}(t)\beta_{n+1}^*(t))\sin\frac{\theta}{2}\cos\frac{\theta}{2}, \\ \rho_{gg} &= \alpha_n(t)\alpha_n^*(t)\sin^2\frac{\theta}{2} + \beta_{n+1}(t)\beta_{n+1}^*(t)\cos^2\frac{\theta}{2}. \end{aligned} \quad (9)$$

In order to quantify the atomic coherence described by the reduced density matrix (9), we adopt the l_1 norm of coherence

$$C_{l_1}(\rho) = \sum_{i,j,i\neq j} |\rho_{ij}|. \quad (10)$$

It can easily be seen that the atomic coherence described by the reduced density matrix (9) is the absolute value of the non-diagonal element of the density matrix.

According to Eqs. (9) and (10), it is easy to calculate the atomic coherence

$$C_{l_1}(\rho) = 2\left|\left(\alpha_n(t)\alpha_n^*(t) - \beta_{n+1}(t)\beta_{n+1}^*(t)\right)\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right|. \quad (11)$$

The linear entropy of the two-level atom can measure the entanglement of the atom and the cavity field[37]

$$S = 1 - \mathrm{Tr}(\rho_a(t)^2) = (1 - |\alpha_n(t)|^4 - |\beta_{n+1}(t)|^4). \quad (12)$$

2.2. Case of leaky cavity

Now we consider the more realistic case of a leaky cavity in the system. The reduced density operator $\rho(t)$ of the atom and the leaky cavity is described by the standard master equation:

$$\begin{aligned} \frac{\mathrm{d}\rho(t)}{\mathrm{d}t} &= -\mathrm{i}[H, \rho(t)] \\ &\quad + \Gamma\left[a\rho(t)a^+ - \frac{1}{2}a^+a\rho(t) - \frac{1}{2}\rho(t)a^+a\right], \end{aligned}$$

where $H = g\cos^2(\theta/2)(\mathrm{e}^{\mathrm{i}\Lambda' t}S_+a + \mathrm{e}^{-i\Lambda' t}S_-a^+)$ and Γ is the photon escape rate of the cavity.

Suppose that the initial state of the system is

$$|\varphi(0)\rangle = \left(\cos\frac{\eta}{2}|e\rangle + \sin\frac{\eta}{2}|g\rangle\right)_a \otimes |0\rangle_f, \quad (13)$$

by the basic vector transformation

$$|e\rangle = \cos\frac{\theta}{2}|+\rangle - \sin\frac{\theta}{2}|-\rangle, \quad |g\rangle = \sin\frac{\theta}{2}|+\rangle + \cos\frac{\theta}{2}|-\rangle,$$

equation (13) is turned into

$$\begin{aligned} |\phi(0)\rangle &= \left(\cos\frac{\eta}{2}\cos\frac{\theta}{2} + \sin\frac{\eta}{2}\sin\frac{\theta}{2}\right)|+0\rangle \\ &\quad + \left(\sin\frac{\eta}{2}\cos\frac{\theta}{2} - \cos\frac{\eta}{2}\sin\frac{\theta}{2}\right)|-0\rangle, \end{aligned} \quad (14)$$

$|-0\rangle$ does not evolve with time, and by the quantum trajectory approach,[38,39] $|+0\rangle$ has two possible quantum states $|\phi_{\mathrm{no}}(t)\rangle = (c_1(t)|+0\rangle + c_2(t)|-1\rangle)$ and $|\phi_{\mathrm{yes}}(t)\rangle = c_3(t)|-0\rangle$: $|\phi_{\mathrm{no}}(t)\rangle$ indicates that no photon jump from the cavity to the environment occurs between time t_0 and t, and $|\phi_{\mathrm{yes}}(t)\rangle$ indicates the fact that one photon jump appears.

The density operator is obtained as

$$\begin{aligned} \rho(t) &= P_{\mathrm{no}}(t)\rho_{\mathrm{no}}(t) + P_{\mathrm{yes}}(t)\rho_{\mathrm{yes}}(t) == |c_1(t)|^2|+0\rangle\langle+0| \\ &\quad + |c_2(t)|^2|-1\rangle\langle-1| + c_1(t)c_2^*(t)|+0\rangle\langle-1| \\ &\quad + c_1^*(t)c_2(t)|-1\rangle\langle+0| + |c_3(t)|^2|-0\rangle\langle-0|, \end{aligned} \quad (15)$$

and $|c_3(t)|^2 = 1 - |c_1(t)|^2 - |c_2(t)|^2$ can easily be obtained from Eq. (15).

To cope with this quantum feature it is assumed that there are no jumps up to time t. Then the "jumpless" state $|\phi_{\mathrm{no}}(t)\rangle$ obeys the nonunitary Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\phi_{\mathrm{no}}(t)\rangle = H_{\mathrm{eff}}|\phi_{\mathrm{no}}(t)\rangle, \quad (16)$$

with $H_{\mathrm{eff}} = H_3 - \frac{\mathrm{i}\Gamma}{2}a^\dagger a$.

By Eq. (16), with the initial conditions $c_1(0) = 1$, $c_2(0) = 0$, the solutions of $c_1(t)$ and $c_2(t)$ are found, respectively, to

$$\begin{aligned} c_1(t) &= \exp\left[\frac{-t}{4}(\Gamma - 2\mathrm{i}\Lambda')\right]\left(\cos\frac{\lambda t}{4} + \frac{\Gamma - 2\mathrm{i}\Lambda'}{\lambda}\sin\frac{\lambda t}{4}\right), \\ c_2(t) &= \frac{-4\mathrm{i}g\cos^2(\theta/2)}{\lambda}\exp\left[\frac{-t}{4}(\Gamma + 2\mathrm{i}\Lambda')\right]\sin\frac{\lambda t}{4}, \end{aligned} \quad (17)$$

with $\lambda = \sqrt{16(g\cos^2(\theta/2))^2 - (\Gamma - 2\mathrm{i}\Lambda')^2}$.

It is worth noting that the evolutionary process will display the Markovian behavior with the weak coupling regime $g < 0.5\Gamma$, and the non-Markovian dynamics occurs, accompanied by an oscillatory reversible decay with the strong coupling regime $g > 0.5\Gamma$.

The density operator of the system at time t, in the basis $(|+1\rangle, |+0\rangle, |-1\rangle, |-0\rangle)$, can be obtained as

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d_1(t)d_1^*(t) & d_1(t)d_2^*(t) & d_1(t)b_1 \\ 0 & d_2(t)d_1^*(t & d_2(t)d_2^*(t) & d_2(t)b_1 \\ 0 & b_1d_1^*(t) & b_1d_2^*(t) & d_3(t)d_3^*(t) + b_1b_1 \end{pmatrix}, \quad (18)$$

with

$$d_1(t) = \left(\cos\frac{\eta}{2}\cos(\theta/2) + \sin\frac{\eta}{2}\sin(\theta/2)\right)c_1(t),$$

$$d_2(t) = \left(\cos\frac{\eta}{2}\cos(\theta/2) + \sin\frac{\eta}{2}\sin(\theta/2)\right)c_2(t),$$
$$d_3(t) = \left(\cos\frac{\eta}{2}\cos\frac{\theta}{2} + \sin\frac{\eta}{2}\sin\frac{\theta}{2}\right)c_3(t),$$
$$b_1 = \sin\frac{\eta}{2}\cos\frac{\theta}{2} - \cos\frac{\eta}{2}\sin\frac{\theta}{2}.$$

Using the same method as the above, the reduced density matrix elements of the atom are

$$\begin{aligned}
\rho_{ee} &= \sin^2\frac{\theta}{2} - b_1 d_1^*(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\
&\quad - b_1 d_1(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} + |d_1(t)|^2\cos\theta, \\
\rho_{eg} &= \rho_{ge}^* = -\sin\frac{\theta}{2}\cos\frac{\theta}{2} - b_1 d_1^*(t)\sin^2\frac{\theta}{2} \\
&\quad + b_1 d_1(t)\cos^2\frac{\theta}{2} + |d_1(t)|^2\sin\theta, \\
\rho_{gg} &= \cos^2\frac{\theta}{2} + b_1 d_1^*(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\
&\quad + b_1 d_1(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - |d_1(t)|^2\cos\theta. \qquad (19)
\end{aligned}$$

The reduced density matrix element of the cavity in the basis $(|0\rangle, |1\rangle)$ is

$$\begin{aligned}
&\rho_{00} = d_1(t)d_1^*(t) + d_3(t)d_3^*(t) + b_1 b_1, \\
&\rho_{01} = b_1 d_2^*(t) = \rho_{10}^*, \quad \rho_{11} = d_2(t)d_2^*(t). \qquad (20)
\end{aligned}$$

The l_1 norm of coherence for the atom,

$$\begin{aligned}
C_{l_1}(\rho) &= 2\Big| -\sin\frac{\theta}{2}\cos\frac{\theta}{2} - b_1 d_1^*(t)\sin^2\frac{\theta}{2} \\
&\quad + b_1 d_1(t)\cos^2\frac{\theta}{2} + |d_1(t)|^2\sin\theta\Big|, \qquad (21)
\end{aligned}$$

and for the cavity,

$$C_{l_1}(\rho) = 2|b_1 d_2^*(t)|. \qquad (22)$$

By using the same method as the above, the linear entropy of the two-level atom can be obtained as follows:

$$S = 2|d_1(t)|^2(1 - b_1^2 - |d_1(t)|^2). \qquad (23)$$

We should point out that the quantum system used to measure the coherence is a single qubit system and an X-type quantum system, for this kind of system, l_1 norm coherence and other methods, such as trace distance measurement[15] and the robustness of coherence,[40] are the same. The conclusions are independent of the choice of measurement.

3. Discussion and results

3.1. Effects of perfect cavity on coherence and entanglement

In Fig. 1, with the increase of the driving field strength, the value of the atomic coherence becomes larger, and the atom–field entanglement is smaller. That is to say, the atomic coherence can be protected by adjusting the driving field strength.

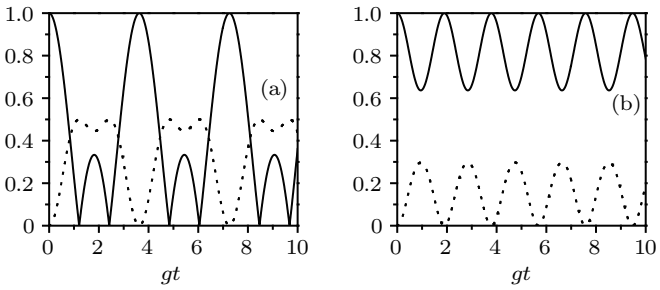


Fig. 1. Plots of atomic coherence (solid line) and atom–field entanglement (dot line) versus rescaled time gt with $\omega = 2$, $\omega_0 = 1$, $\omega_c = 1$, $n = 1$, (a) $\Omega = 1$, (b) $\Omega = 2$.

In Fig. 2, the atomic coherence and the atom–field entanglement evolve periodically with rescaled time gt. With the increase of the frequency detuning Δ, the oscillation amplitudes of the atomic coherence and the atom–field entanglement become larger. This phenomenon is interpreted as follows. The larger the detuning Δ, the weaker the coupling between the driving field and the atom is, and the stronger the effective coupling between the atom and the cavity field becomes, and the more the resources of these two kinds (the atomic coherence and the atom–field entanglement) are transformed, so the larger their oscillation amplitudes become.

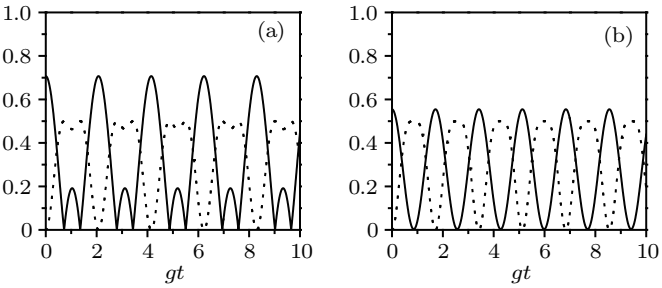


Fig. 2. Curves of atomic coherence (solid) and atom–field entanglement (doted) versus rescaled time gt with $\omega = 2$, $\Omega = 1$, $n = 1$, (a) $\Delta = 2$, (b) $\Delta = 3$.

From Figs. 1 and 2, it is interesting to find that at the initial time, the atomic coherence has a maximum value while the atom–field entanglement is zero, and the atomic coherence decreases to zero. Meanwhile the atom–field entanglement increases and reaches a maximum value. Moreover, the more the atomic coherence increases, the more the atom–field entanglement decreases. This shows that the atomic coherence itself can be transformed into the atom–field entanglement through the atom–field interaction for this system. Furthermore, the increasing frequency detuning can strengthen the transformation.

Now, we come to study the case of a pair of separate cavities each with a single atom inside and with the initial density matrix $\rho(0) = \rho_\Phi(0) \otimes |0\rangle_1 |0\rangle_2 \langle 0|_2 \langle 0|$, where $\rho_\Phi(0) = r|\Phi\rangle\langle\Phi| + [(1-r)/4]I$, $|\Phi\rangle = \cos\phi|-+\rangle +$

$\sin\phi\,|+-\rangle$, $|0\rangle_1\,|0\rangle_2$ denotes the vacuum state for two cavities, with r indicating the purity of initial states and being a real number varying from 0 to 1. According to Eqs. (6) and (7) with $n=0$, the density matrix of the system at time t can be obtained as

$$\begin{aligned}\rho(t) = & \frac{1-r}{4}|\varphi\rangle_1|\varphi\rangle_2{}_1\langle\varphi|_2\langle\varphi| \\ & + \left(r\sin^2\phi+\frac{1-r}{4}\right)|\varphi\rangle_1|-0\rangle_2{}_1\langle\varphi|_2\langle-0| \\ & + \left(r\cos^2\phi+\frac{1-r}{4}\right)|-0\rangle_1|\varphi\rangle_2{}_1\langle-0|_2\langle\varphi| \\ & + \frac{1-r}{4}(|--00\rangle\langle--00|) \\ & + r\sin\phi\cos\phi\,|\varphi\rangle_1|-0\rangle_2{}_1\langle-0|_2\langle\varphi| \\ & + r\sin\phi\cos\phi\,|-0\rangle_1|\varphi\rangle_2{}_1\langle\varphi|_2\langle-0|,\end{aligned}$$

where $|\varphi\rangle_i=\alpha_0(t)\,|+\rangle_i|0\rangle_i+\beta_1(t)\,|-\rangle_i|1\rangle_i$ $(i=1,2)$ denotes the i-th atom–cavity evolution, in a way of the l_1 norm coherence, we can obtain the following quantum coherence for composite subsystems, i.e., $q_1\otimes q_2$, $c_1\otimes c_2$, $q_1\otimes c_2$, $q_2\otimes c_1$ and $q_1\otimes c_1$, respectively:

$$\begin{aligned}C_{q_1q_2} &= 2\,|r\sin\phi\cos\phi\,\alpha_0(t)\alpha_0^*(t)|, \\ C_{c_1c_2} &= 2\,|r\sin\phi\cos\phi\,\beta_1(t)\beta_1^*(t)|, \\ C_{q_1c_2} &= 2|\sin\phi\cos\phi\,\alpha_0(t)\beta_1^*(t)|, \quad C_{q_2c_1}=C_{q_1c_2}, \\ C_{q_1c_1} &= 2\left|\left(r\sin^2\phi+\frac{1-r}{2}\right)\alpha_0(t)\beta_1^*(t)\right|, \end{aligned} \tag{24}$$

where $q_i(c_i)$ represents the i-th atom (cavity).

Furthermore, for the initial density matrix $\rho(0)=\rho_\Psi(0)\otimes|0\rangle_1|0\rangle_2{}_1\langle0|_2\langle0|$, with $\rho_\Psi(0)=r|\Psi\rangle\langle\Psi|+[(1-r)/4]I$, $|\Psi\rangle=\cos\phi\,|--\rangle+\sin\phi\,|++\rangle$, the quantum coherence for composite subsystems can also be obtained, and its results are the same as those from Eq. (24). This indicates that quantum coherence of composite subsystems has the same evolution for the two initial states.

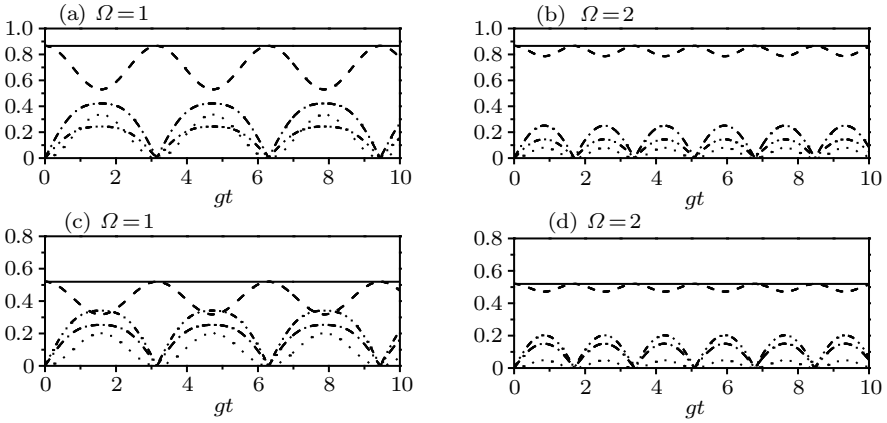


Fig. 3. Plots of quantum coherence for different partitions $C_{q_1q_2}$ (dash line), $C_{c_1c_2}$ (dot line), $C_{q_1c_2}=C_{q_2c_1}$ (dash–dot line), $C_{q_1c_1}$ (dash–dot–dot line) versus gt with $\phi=\pi/6$. Solid line denotes the coherence invariant of $C_{q_1q_2}+C_{c_1c_2}$, panels (a) and (b) show the purity of initial states $r=1$, and panels (c) and (d) display the purity of initial states $r=0.6$.

The bipartite coherence of five partitions: $q_1\otimes q_2$, $c_1\otimes c_2$, $q_1\otimes c_2$, $q_2\otimes c_1$, $q_1\otimes c_1$ is shown in Fig. 3. In the initial period of time, $C_{q_1q_2}$ decreases from the original value to zero, and then increases due to atom–photon interaction. The values of $C_{c_1c_2}$, $C_{q_1c_2}$, $C_{q_2c_1}$, $C_{q_1c_1}$ firstly increase from zero to a maximum, and then decrease from the maximum to zero. The phenomena mentioned above indicate that the $C_{q_1q_2}$ does not disappear, but transfers to other subsystems, no matter whether they are local $q_1\otimes c_1$ or non-local $c_1\otimes c_2$, $q_1\otimes c_2$, $q_2\otimes c_1$. Furthermore, based on the analysis of the results, $Q_1(t)=C_{q_1q_2}+C_{c_1c_2}$ is conserved for any value of r as shown in Fig. 3, and the $Q_1(t)$ value and the initial value of $C_{q_1q_2}$ are equal. In other words, there is mutual conversion between $C_{q_1q_2}$ and $C_{c_1c_2}$. Besides, we find that the $Q_2(t)=C_{q_1q_2}+C_{c_1c_2}+2C_{q_1c_1}\,|\cot\phi|-2C_{q_1c_2}$ is also conserved in the case of $r=1$. Interestingly, its value is also equivalent to the initial value of $C_{q_1q_2}$. Furthermore, we find that $Q_3(t)=C_{q_1q_2}^2+C_{c_1c_2}^2+C_{q_1c_2}^2+C_{q_2c_1}^2$ is also held no matter what the value of r is. It indicates that for two identical single-body systems there is a rich phenomenon of quantum coherence, and it is very meaningful to study the theory associated with coherence swapping and coherence transfer in quantum information processing.

To better understand the effects of the driving field strength Ω on the conservation rules, it follows from Figs. 3(a) and 3(b), or Figs. 3(c) and 3(d) that Ω only influences the relative number of the bipartite coherence of partitions, and does not change the conservation rules.

It is worth pointing out that there is also a similar conservative relation of entanglement to $Q_i(t)$ $(i=1,2,3)$ in this system,[41–44] such as, for the initial density matrix $\rho(0)=\rho_\Phi(0)\otimes|0\rangle_1|0\rangle_2{}_1\langle0|_2\langle0|$ with $r=1$, $Q_i(t)$ $(i=1,2,3)$ of entanglement is always true, however, for the initial density matrix $\rho(0)=\rho_\Psi(0)\otimes|0\rangle_1|0\rangle_2{}_1\langle0|_2\langle0|$ with $r=1$, only $Q_2(t)$ of entanglement would be conserved in the case of $|\cos\phi|>|\sin\phi|$.

In the process of atom–cavity interaction, the entanglement sudden death and entanglement sudden birth between atom–atom/atom–cavity will appear. However, the coherence will always exist and will not disappear. So the entanglement cannot keep the conservation law like quantum coherence. On the other hand, for different initial states, the times of entanglement sudden death and birth between atom-atom/atom–cavity are different, so the conservation of entanglement depends on the initial state.

3.2. Effects of leaky cavity on coherence and entanglement

Without the driving field, as displayed in Fig. 4(a), the atomic coherence decreases from the maximum to zero and then increase to a certain value, and its evolution is of damped decay. Meanwhile, the atom–cavity entanglement increases from zero to a certain value and then reduces to zero with atomic coherence. The above phenomenon indicates that the atomic coherence transforms into the atom–cavity entanglement and the cavity coherence, and the three resources eventually disappear as the cavity is dissipative. This is more visible in the weak coupling regime as displayed in Fig. 4(c). However, it can be seen clearly through Figs. 4(b) and 4(d) that the atomic coherence decreases slowly by increasing the driving field strength, no matter whether it is in the strong coupling regime or in the weak coupling regime. So for the realistic case of cavity loss in the system, although the atomic coherence and the atom–field entanglement are very sensitive to the cavity decay, they can be protected by adjusting the driving field strength.

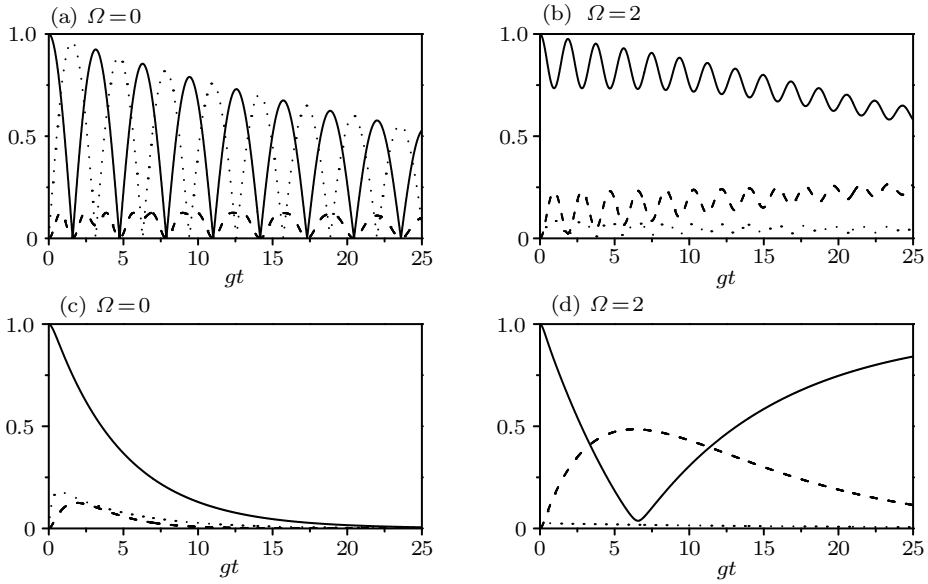


Fig. 4. Plots of atomic coherence (solid line), atom–reservoir entanglement (dash line), reservoir coherence (dot line) versus gt with $\omega_0 = 2$, $\omega_c = 1$, $\omega = 2$, $\eta = \pi/2$ in panels (a) and (b) strong coupling regime $g = 10\Gamma$, and in panels (c) and (d) weak coupling regime $g = 0.1\Gamma$.

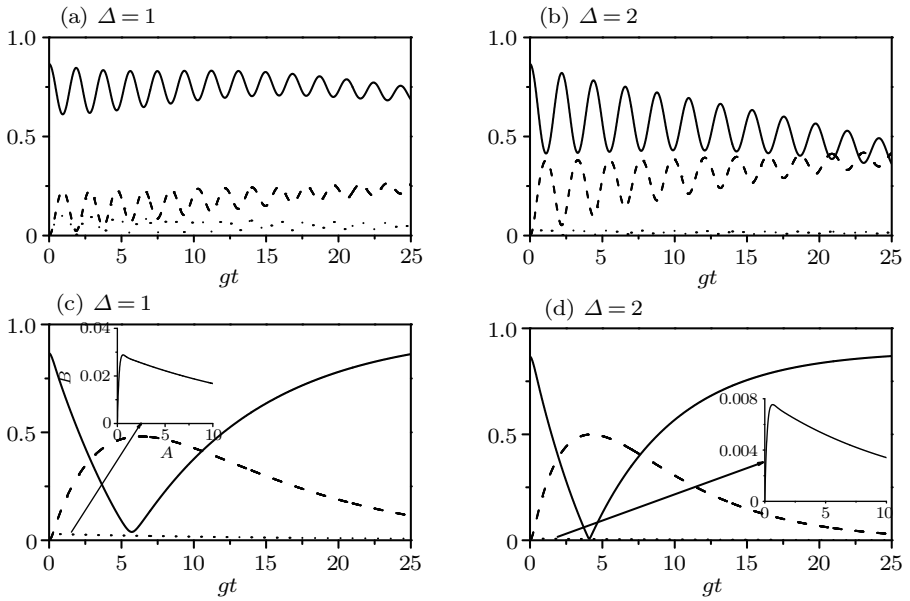


Fig. 5. Plots of atomic coherence (solid line), atom–reservoir entanglement (dash line), reservoir coherence (dot line) versus gt with $\Omega = 2$, $\eta = \pi/3$ in panels (a) and (b) strong coupling regime $g = 10\Gamma$, and in panels (c) and (d) weak coupling regime $g = 0.1\Gamma$.

With the increase of detuning Δ, no matter whether in the strong coupling regime or in the weak coupling regime as displayed in Fig. 5, we can see the atomic coherence decreases more, and the atom–cavity entanglement increases more. As the detuning increases, the effective coupling between the atom and the cavity increases, so we can see that the atomic coherence and the atom–cavity entanglement transform more in the case of larger detuning. In order to protect the atomic coherence, it is necessary to reduce the atom-driving field detuning Δ.

3.3. Non-Markovian dynamics

Considering the case of a leaky cavity in the system, there is a backflow of information from the environment to the system under suitable conditions, which means that the dynamics of the system is non-Markovian. The degree of non-Markovian dynamics can be characterized by the non-Markovianity, which can be defined as[45]

$$N = \max_{\rho_1(0),\rho_2(0)} \int_{\sigma>0} \sigma[t,\rho_1(0),\rho_2(0)]\,\mathrm{d}t, \tag{25}$$

where $\sigma[t,\rho_1(0),\rho_2(0)] = \mathrm{d}D[\rho_1(t),\rho_2(t)]/\mathrm{d}t$ denotes the rate of change of the trace distance $D[\rho_1(t),\rho_2(t)]$ expressed as

$$D[\rho_1(t),\rho_2(t)] = \frac{1}{2}\mathrm{Tr}\,|\rho_1(t)-\rho_2(t)|, \tag{26}$$

where $|A| = \sqrt{A^{+}A}$ (A denotes $\rho_1(t)-\rho_2(t)$), and the trace distance D describes the ability to distinguish between $\rho_1(t)$ and $\rho_2(t)$.

A Markovian evolution can never increase the trace distance, which means that the trace distance is a monotonically decreasing function for a Markovian progress and the $\sigma[t,\rho_1(0),\rho_2(0)] \le 0$. In other words, the trace distance is non-monotonic, which corresponds to a non-Markovian progress and the non-Markovianity $N > 0$. In order to calculate the degree of non-Markovianity, a specific pair of optimal initial states to maximize the $\sigma[t,\rho_1(0),\rho_2(0)]$ are $\rho_1(0) = |0\rangle\langle 0|$ and $\rho_2(0) = |1\rangle\langle 1|$.[45]

Figure 6 shows the trace distance $D[\rho_1(t),\rho_2(t)]$ as a function of Γ and gt with the optimal initial states $\rho_1(0) = |0\rangle\langle 0|$, $\rho_2(0) = |1\rangle\langle 1|$. As shown in Fig. 6, with the increase of Γ, the trace distance $D[\rho_1(t),\rho_2(t)]$ becomes monotonic and a Markovian process occurs. In contrast, with the decrease of Γ, the trace distance $D[\rho_1(t),\rho_2(t)]$ turns non-monotonic and a non-Markovian progress occurs. The smaller the value of Γ, the bigger the change of the trace distance $D[\rho_1(t),\rho_2(t)]$, so the larger the degree of non-Markovianty is. Furthermore, from Fig. 4, we can find the coherence and the entanglement decay slowly with the value of Γ decreasing. So there is a positive association between the non-Markovianty and coherence or entanglement.

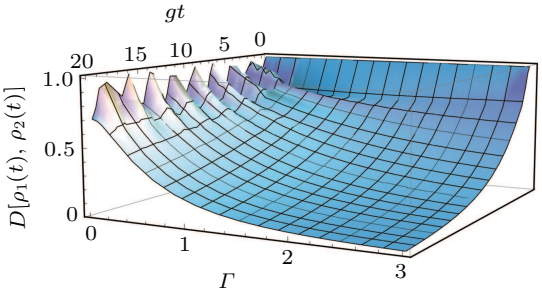


Fig. 6. (color online) Trace distance $D[\rho_1(t),\rho_2(t)]$ as a function of Γ and gt with the optimal initial states $\rho_1(0) = |0\rangle\langle 0|$, $\rho_2(0) = |1\rangle\langle 1|$. The other parameters $\Omega = 1$, $\omega_0 = 2$, $\omega_c = 1$.

4. Conclusions

In this work, we investigate the interaction between a single-mode cavity and a single atom with a classical driving field. It is shown that the atomic coherence can be protected by adjusting the driving field, and the transformation between the atomic coherence and the atom–field entanglement exists, which can be increased by increasing the atom-driving field detuning. Then, the systems are extended to a pair of separate cavities each with a single atom inside, and the quantum coherence transforms among composite subsystems are found. Moreover, the conservation of coherence for different partitions can be obtained. For the case of a leaky cavity, the classical driving field can suppress the negative influences of the leaky cavity on coherence and entanglement. By the non-Markovian dynamics of the system, the non-Markovianty and coherence or entanglement have a positive correlation. We believe that our results contribute to shedding light on the behaviors of quantum coherence and quantum entanglement in realistic conditions, that is, when the classical driving field and the perfect or leaky cavity on the quantum system are taken into account.

The scheme with a classical driving field is feasible for implementation in the microwave and optical regimes in cavity QED experiments.[46] The external classical driving can control the effective Hamiltonian of a system which contains the driving strength of the classical field and the atom-photon detuning, and the above can be realized in an experimental setup.[47,48] Besides, the experimental realization of the above results is also suitable in the context of trapped ions.[49]

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