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Quantum estimation of detection efficiency with no-knowledge quantum feedback*

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We investigate that no-knowledge measurement-based feedback control is utilized to obtain the estimation precision of the detection efficiency. We show that no-knowledge measurement is the optimal way to estimate the detection efficiency. The higher precision can be achieved for the lower or larger detection efficiency. It is found that no-knowledge feedback can be used to cancel decoherence. No-knowledge feedback with a high detection efficiency can perform well in estimating frequency and detection efficiency parameters simultaneously; simultaneous estimation is better than independent estimation given by the same probes.

Keywords: no-knowldege feedback, parameter estimation, simultaneous measurement

PACS: 03.65.Yz, 03.65.Ud, 42.50.Pq **DOI:** 10.1088/1674-1056/27/6/060303

1. Introduction

Quantum metrology is a fundamental and important subject, which concerns the estimation of parameters under the constraints of quantum mechanics.[1–4] There are widespread applications such as in timing, healthcare, defence, navigation, and astronomy.[5–11] For the mean-square error criterion, the Cramér–Rao bound[12–14] is the most well-known analytic bound on the error. The estimation precision of the parameter is inversely proportional with quantum Fisher information (QFI).

The detection efficiency is a crucial physical quantity to judge the detector quality; it is becoming more and more important to improve the estimation accuracy of the detection efficiency.[15] Quantum feedback control[16–19] can be used to improve the detection efficiency. In Ref. [15], the use of the different measurement-based quantum feedback types to enhance the QFI about the detection efficiency of the detector has been investigated. In general, more signal, less noise from the detection can be used to better perform feedback control.[20,21] However, Szigeti *et al.*[22] showed significant and surprising results: performing a no-knowledge measurement (no signal, only noise) can be advantageous in canceling decoherence. It is due to that a system undergoing no-knowledge monitoring has reversible noise, which can be canceled by directly feeding back the measurement signal. For a perfect detection efficiency $\eta = 1$, no-knowledge feedback control can be used to completely cancel Markovian decoherence .

In this paper, we consider that the information of detection efficiency is encoded by a no-knowledge measurement-based quantum feedback. When one uses the optimal feedback operator to cancel decoherence, no-knowledge quantum feedback control is the optimal way to measure the detection efficiency. For the low or high detection efficiency, the precision can be dramatically high. Finally, we show that simultaneous estimation of the frequency and detection efficiency parameters has an advantage over estimating the noise and phase parameters individually, bringing us into the field of multi-parameter quantum metrology.[23,24] Also, no-knowledge quantum feedback can perform well in estimating the frequency with a high detection efficiency. Due to that the decoherence can be significantly canceled by the no-knowledge quantum feedback control with the high detection efficiency.

The rest of this article is arranged as follows. In Section 2, we briefly introduce the quantum metrology of a single parameter and multi-parameter, and the formula of Fisher information. In Section 3, we detail the physical model: a qubit interacting with a dephasing or dissipation reservoir and the feedback to the qubit by the no-knowledge homodyne measurement. Then, we obtain the precision of detection efficiency by the no-knowledge feedback in Section 4. Simultaneous estimation of the frequency and detection efficiency parameters is studied in Section 5. A conclusion and outlook are presented in Section 6.

2. Review of quantum metrology

The famous Cramér–Rao bound[12,13] offers a very good parameter estimation under the constraints of quantum physics

$$(\delta x)^2 \geq \frac{1}{N\mathcal{F}_Q[\hat{\rho}(x)]}, \tag{1}$$

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where N represents the total number of experiments. $\mathcal{F}_Q[\hat{\rho}(x)]$ denotes QFI, which can be generalized from classical Fisher information. The classical Fisher information of probability distribution $\{p_k(x)\}_k$ is defined by

$$f(x) = \sum_k p_k(x) \left[\frac{\mathrm{d} \ln[p_k(x)]}{\mathrm{d}x} \right]^2, \tag{2}$$

where $p_k(x)$ is the probability of obtaining the set of experimental results k for the parameter value x. Furthermore, the QFI is given by the maximum of the Fisher information over all measurement strategies allowed by quantum physics:

$$\mathcal{F}_Q\left[\hat{\rho}(x)\right] = \max_{\{\hat{E}_k\}} f\left[\hat{\rho}(x); \{\hat{E}_k\}\right], \tag{3}$$

where $f[\hat{\rho}(x); \{\hat{E}_k\}]$ denotes the classical Fisher information of the probability distribution $\{p_k(x)\}_k$ with $p_k(x) = \mathrm{tr}(\hat{\rho}(x)\hat{E}_k)$ and positive operator-valued measure $\{\hat{E}_k\}$ represents a specific measurement device.

If the probe state is pure, $\hat{\rho}_S(x) = |\psi(x)\rangle\langle\psi(x)|$, the corresponding expression of QFI is

$$\mathcal{F}_Q[\hat{\rho}(x)] = 4 \left[\frac{\mathrm{d}\langle\psi(x)|}{\mathrm{d}x} \frac{\mathrm{d}|\psi(x)\rangle}{\mathrm{d}x} - \left| \frac{\mathrm{d}\langle\psi(x)|}{\mathrm{d}x} \middle| \psi(x)\rangle \right|^2 \right]. \tag{4}$$

If the probe state is a mixed state, $\hat{\rho}(x) = \sum_k \lambda_k |k\rangle\langle k|$, the concrete form of QFI is given by

$$\mathcal{F}_Q[\hat{\rho}(x)] = \sum_{k,\lambda_k>0} \frac{(\partial_x \lambda_k)^2}{\lambda_k} + \sum_{k,k',\lambda_k+\lambda_k'>0} \frac{2(\lambda_k - \lambda_k')^2}{\lambda_k + \lambda_k'} \langle k|\partial_x|k'\rangle. \tag{5}$$

In general, it is complicated to calculate QFI. In this paper, we only consider a two-dimensional system. The QFI can be calculated explicitly in the following way[25,26]

$$\mathcal{F}_Q[\hat{\rho}(x)] = \mathrm{Tr}[(\partial_x \hat{\rho}(x))^2] + \frac{1}{\mathrm{Det}(\hat{\rho}(x))} \mathrm{Tr}[(\hat{\rho}(x)\partial_x \hat{\rho}(x))^2]. \tag{6}$$

For the classical multi-parameter Cramér–Rao bound:

$$\mathrm{Cov}(\widetilde{\boldsymbol{x}}) \geq F^{-1}, \tag{7}$$

where $\widetilde{\boldsymbol{x}} = \{x_1, x_2, \ldots, x_m\}$, $\mathrm{Cov}(\widetilde{\boldsymbol{x}})$ refers to the covariance matrix for a locally unbiased estimator $\widetilde{\boldsymbol{x}}(k)$, $\mathrm{Cov}(\widetilde{\boldsymbol{x}})_{jk} = \langle(\widetilde{x}_j - x_j)(\widetilde{x}_k - x_k)\rangle$ and $\langle\cdots\rangle$ represents the average with respect to the probability distribution $p_k(\boldsymbol{x})$. The classical Fisher matrix for m parameters as the $m \times m$ matrix with entries given by

$$F_{jk} = \sum_k p_k(x) \left(\frac{\partial \ln[p_k(x)]}{\partial x_j} \right) \left(\frac{\partial \ln[p_k(x)]}{\partial x_k} \right). \tag{8}$$

The multi-parameter QFI Cramér–Rao bound is given by

$$\mathrm{Cov}(\widetilde{\boldsymbol{x}}) \geq F_Q^{-1}, \quad F_{Q_{ij}} = \frac{1}{2} \mathrm{Tr}\left[\hat{\rho}(x)\{L_i, L_j\}\right], \tag{9}$$

where the symmetric logarithmic derivative is obtained by

$$L_i = 2 \sum_{m,n} \frac{\langle\psi_m|\partial_{x_i}\hat{\rho}(x)|\psi_n\rangle}{p_m + p_n} |\psi_m\rangle\langle\psi_m|. \tag{10}$$

Here, $p_{m,n}$ and $|\psi_{m,n}\rangle$ denote the eigenvalues and eigenvectors of density operator $\hat{\rho}(x)$. For a two-dimensional system, the multi-parameter QFI matrix is expressed by[27]

$$F_{Q_{ij}} = (\partial_{x_i}\boldsymbol{r}) \cdot (\partial_{x_j}\boldsymbol{r}) + \frac{(\boldsymbol{r} \cdot \partial_{x_i}\boldsymbol{r})(\boldsymbol{r} \cdot \partial_{x_j}\boldsymbol{r})}{1 - |\boldsymbol{r}|^2}, \tag{11}$$

where $\boldsymbol{r}$ denotes the Bloch vector of $\hat{\rho}(x)$.

3. A physical model of feedback control

Consider a two-dimensional system with Hamiltonian H interacting with a Markovian reservoir via the coupling operator L. The system density operator, $\rho(t)$, evolves according to the master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}[H, \rho(t)] + \mathcal{D}[L]\rho(t), \tag{12}$$

where

$$\mathcal{D}[L]\rho(t) = L\rho(t)L^\dagger - (L^\dagger L\rho(t) + \rho(t)L^\dagger L)/2,$$

and we have set $\hbar = 1$ throughout the article.

For a homodyne measurement of the environment at angle θ, the conditional system dynamics are described by a stochastic master equation (SME)[28,29]

$$\begin{aligned} \mathrm{d}\rho_{\mathrm{c}}(t) = &-\mathrm{i}\left[H, \rho_{\mathrm{c}}(t)\right]\mathrm{d}t + \mathcal{D}[L]\rho_{\mathrm{c}}(t)\mathrm{d}t \\ &+ \sqrt{\eta}\,\mathrm{d}W(t)\mathcal{H}\left[L\mathrm{e}^{\mathrm{i}\theta}\right]\rho_{\mathrm{c}}(t), \end{aligned} \tag{13}$$

where $\mathrm{d}W(t)$ is the standard Wiener increment with mean zero and variance $\mathrm{d}t$. In the following, we consider the continuous feedback control, and take the Markovian feedback of the white-noise measurement record via a Hamiltonian. The continuous measurement record can be described by the homodyne detection photocurrent[30]

$$I(t) = \sqrt{\eta}\left\langle L\mathrm{e}^{\mathrm{i}\theta} + L^\dagger \mathrm{e}^{-\mathrm{i}\theta}\right\rangle_t + \xi(t), \tag{14}$$

where $\xi(t) = \mathrm{d}W(t)/\mathrm{d}t$ is Stratonovich noise. When the coupling operator L is Hermitian, such as dephasing in qubits ($L = \sigma_z$), no-knowledge measurement occurs for quadrature angle $\theta = \pm\pi/2$. The measurement signal $I_{\pi/2}(t) = \xi(t)$ returns only noise. For a non-Hermitian operator, such as dissipation reservoir ($L = \sigma_-$), no-knowledge measurement requires an extra reservoir with coupling operator $L^\dagger$,[22] giving the unconditional dynamics

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}[H, \rho(t)] + \mathcal{D}[L]\rho(t) + \mathcal{D}[L^\dagger]\rho(t), \tag{15}$$

where

$$\mathcal{D}[L]\rho(t) + \mathcal{D}[L^\dagger]\rho(t) = \mathcal{D}[L_+]\rho(t) + \mathcal{D}[L_-]\rho(t)$$

with

$$L_{\pm} = \mathrm{i}^{(1\mp1)/2}(L \pm L^{\dagger})/\sqrt{2}.$$

Thus $L_{\pm}$ are effective Hermitian coupling operators that admit no-knowledge measurements. The corresponding conditional system dynamics are described by stochastic master equation (SME)

$$\begin{aligned}\mathrm{d}\rho_{\mathrm{c}}(t) = &-\mathrm{i}\left[H,\rho_{\mathrm{c}}(t)\right]\mathrm{d}t + \mathcal{D}\left[L_{+}\right]\rho_{\mathrm{c}}(t)\mathrm{d}t + \mathcal{D}\left[L_{-}\right]\rho_{\mathrm{c}}(t)\mathrm{d}t \\ &+ \mathrm{d}W_{+}(t)\mathcal{H}\left[L_{+}\,\mathrm{e}^{\mathrm{i}\theta}\right]\rho_{\mathrm{c}}(t) \\ &+ \mathrm{d}W_{-}(t)\mathcal{H}\left[L_{-}\,\mathrm{e}^{\mathrm{i}\theta}\right]\rho_{\mathrm{c}}(t), \end{aligned} \tag{16}$$

where $\mathrm{d}W_{\pm}(t)$ are independent Wiener increments. Performing homodyne detection at two outputs, this yields the two corresponding measurement signals

$$I_{\pm}(t) = 2\cos\theta\sqrt{\eta}\langle L_{\pm}\rangle_t + \xi_{\pm}(t). \tag{17}$$

Then the control Hermitian can be written as $H_{\mathrm{fb}} = I(t)F$ where F is the feedback Hermitian operator, as shown in Fig. 1. For Hermitian coupling operator L, the stochastic equation for the conditioned system state including feedback is:

$$\begin{aligned}\mathrm{d}\rho_{\mathrm{c}}(t) = &-\mathrm{i}\left[H,\rho_c(t)\right]\mathrm{d}t + \mathcal{D}[L]\rho_{\mathrm{c}}(t)\mathrm{d}t - \mathrm{i}\sqrt{\eta}\left[F, L\,\mathrm{e}^{\mathrm{i}\theta}\rho_{\mathrm{c}}(t)\right. \\ &\left. + \rho_{\mathrm{c}}(t)L\,\mathrm{e}^{-\mathrm{i}\theta}\right]\mathrm{d}t + \mathrm{d}W(t)\mathcal{H}\left[\sqrt{\eta}L\,\mathrm{e}^{\mathrm{i}\theta}\right. \\ &\left. - \mathrm{i}F\right]\rho_{\mathrm{c}}(t) + \mathcal{D}[F]\rho_{\mathrm{c}}(t)\mathrm{d}t. \end{aligned} \tag{18}$$

The unconditional master equation becomes[29]

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = &-\mathrm{i}[H,\rho(t)] + \mathcal{D}[L]\rho(t) \\ &- \mathrm{i}\sqrt{\eta}\left[F, L\,\mathrm{e}^{\mathrm{i}\theta}\rho(t) + \rho(t)L\,\mathrm{e}^{-\mathrm{i}\theta}\right] \\ &+ \mathcal{D}[F]\rho(t). \end{aligned} \tag{19}$$

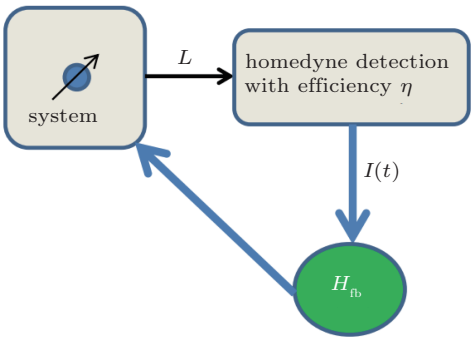


Fig. 1. (color online) Schematic diagram for a measurement-based feedback control protocol. Information about the system is extracted by monitoring the decoherence channel L. The feedback Hamiltonian is applied to the system according to the homodyne current $I(t)$ from the homodyne detection. Here, η denotes the detection efficiency.

For non-Hermitian operator L, the corresponding unconditional master equation is given by

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = &-\mathrm{i}[H,\rho(t)] + \mathcal{D}[L_{+}]\rho(t) - \mathrm{i}\sqrt{\eta}\left[F_{+}, L_{+}\,\mathrm{e}^{\mathrm{i}\theta}\rho(t) + \rho(t)L_{+}\,\mathrm{e}^{-\mathrm{i}\theta}\right] \\ &+ \mathcal{D}[F_{+}]\rho(t) + \mathcal{D}[L_{-}]\rho(t) - \mathrm{i}\sqrt{\eta}\left[F_{-}, L_{-}\,\mathrm{e}^{\mathrm{i}\theta}\rho(t) + \rho(t)L_{-}\,\mathrm{e}^{-\mathrm{i}\theta}\right] + \mathcal{D}[F_{-}]\rho(t), \end{aligned} \tag{20}$$

where $F_{\pm}$ denote independent feedback operators.

4. The precision of detection efficiency

For canceling decoherence, the optimal feedback operator is $F = L$ in the case of Hermitian operator L.[22] In the case of non-Hermitian operator L, the optimal feedback operators are $F_{\pm} = L_{\pm}$.[22] We consider that the feedback operator is proportional to the coupling operator: $F = \lambda L$ ($F_{\pm} = \lambda L_{\pm}$), where λ denotes a real number.

Firstly, we consider the Markoivian dephasing operator $L = (1/\sqrt{2})\sigma_z$. The Hamiltonian of the system is given by $H = (\omega/2)\sigma_z$. The master equation includes the coupling operator: $F = \lambda L$, which is given by

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = &-\mathrm{i}\left[\frac{\omega}{2}\sigma_z, \rho(t)\right] \\ &+ (1 + \lambda^2 - 2\sin\theta\lambda\sqrt{\eta})\mathcal{D}\left[\frac{\sigma_z}{\sqrt{2}}\right]\rho(t). \end{aligned} \tag{21}$$

Choosing $|\varphi_0\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ as the initial state, we can obtain the evolved density matrix

$$\rho_{\eta}(t) = \frac{1}{2}\begin{pmatrix} 1 & \mathrm{e}^{-\mathrm{i}\omega t - (1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} \\ \mathrm{e}^{\mathrm{i}\omega t - (1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} & 1 \end{pmatrix}. \tag{22}$$

The QFI of η is derived by Eq. (6),

$$\mathcal{F}_Q\left[\rho_{\eta}(t)\right] = \frac{\sin^2\theta\lambda^2 t^2\,\mathrm{e}^{-2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t}}{\eta\left(1 - \mathrm{e}^{-2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t}\right)}. \tag{23}$$

From the above equation, we can see that for $\lambda > 0$, $\sin\theta = 1$ (or for $\lambda < 0$, $\sin\theta = -1$) is chosen to obtain the maximal QFI. Namely, no-knowledge measurement ($\cos\theta = 0$) is the optimal way. The optimal precision of detection efficiency η is obtained by Eq. (1)

$$(\delta\eta)^2 \geq \frac{\eta\left[1 - \mathrm{e}^{-2(1+\lambda^2-2\lambda\sqrt{\eta})t}\right]}{N\lambda^2 t^2\,\mathrm{e}^{-2(1+\lambda^2-2\lambda\sqrt{\eta})t}}. \tag{24}$$

We can see that the uncertainty of η can be 0 for $\eta = 0$. When $\lambda = 1$, the uncertainty of η can also be 0 for $\eta = 1$. Then, we can achieve the maximal QFI by taking the derivative of t and λ. However, it is difficult to calculate the exact analytical solution. We make an approximation for $\eta < 1$:

$$\frac{\eta\left[1-\mathrm{e}^{-2(1+\lambda^2-2\lambda\sqrt{\eta})t}\right]}{N\lambda^2 t^2\,\mathrm{e}^{-2(1+\lambda^2-2\lambda\sqrt{\eta})t}} \approx \frac{\eta}{N\lambda^2 t^2\,\mathrm{e}^{-2(1+\lambda^2-2\lambda\sqrt{\eta})t}}. \quad (25)$$

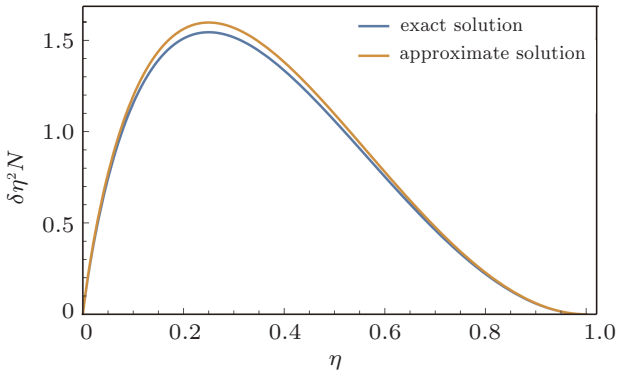


Fig. 2. (color online) Diagram of the exact minimum precision and approximate analytical precision $N\delta\eta^2$ changing with the detection efficiency η. The exact minimum precision comes from numerical calculation. The approximate analytical precision comes from Eq. (27); it shows that the approximate analytical precision is close to the exact precision.

Then we can obtain the approximate optimal feedback constant $\lambda = 1$ and the interrogation time

$$t_{\mathrm{opt}} = \frac{1}{2-2\sqrt{\eta}}. \quad (26)$$

Substituting them into Eq. (24), we can obtain the uncertainty of detection efficiency

$$(\delta\eta)^2 \geq 4\eta(1-\sqrt{\eta})^2(\mathrm{e}^2-1)/N. \quad (27)$$

An exact numerical solution is shown in Fig. 2.

We can find that the approximate solution in Eq. (27) is close to the exact solution. Namely, equation (27) represents a good approximate analytical form.

In general, the optimal feedback operator can be chosen to be the coupling operator ($\lambda = 1$).

Next, we consider the Markoivian dissipative operator $L = (1/\sqrt{2})\sigma_-$.[31]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\mathrm{i}\left[\frac{\omega}{2}\sigma_z, \rho(t)\right] + (1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})\mathcal{D}\left[\frac{\sigma_x}{2}\right]\rho(t) + \left(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta}\right)\mathcal{D}\left[\frac{\sigma_y}{2}\right]\rho(t). \quad (28)$$

Without loss of generality, it is simply to choose $\omega = 0$. Choosing $(1/\sqrt{2})(|0\rangle + |1\rangle)$ as the initial state, we can obtain the density matrix

$$\rho_\eta(t) = \frac{1}{2}\begin{pmatrix} 1 & \mathrm{e}^{-\mathrm{i}\omega t-(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} \\ \mathrm{e}^{\mathrm{i}\omega t-(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} & 1 \end{pmatrix}.$$

So it is similar with the above case in estimating the detection efficiency.

5. Simultaneous estimation of the frequency and detection efficiency parameters

In general, simultaneous estimation of multi-parameter can perform better than estimating each parameter independently.[32] We consider simultaneously estimating frequency ω and detection efficiency η. The information of ω and η is encoded onto the evolved density matrix as shown in Eq. (22).

Utilizing Eqs. (7) and (11), we can achieve the precision of ω and η, which is given by

$$\mathrm{Cov}(\widetilde{\omega},\widetilde{\eta}) \geq \frac{1}{t^2}\begin{pmatrix} \mathrm{e}^{2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} & 0 \\ 0 & \frac{\eta[\mathrm{e}^{2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t}-1]}{\lambda^2} \end{pmatrix}. \quad (29)$$

Considering the balanced importance of ω and η, the simultaneous estimation precision is given by

$$\delta\omega^2+\delta\eta^2 \geq \frac{1}{t^2}\left[\mathrm{e}^{2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t} + \frac{\eta[\mathrm{e}^{2(1+\lambda^2-2\sin\theta\lambda\sqrt{\eta})t}-1]}{\lambda^2}\right]. \quad (30)$$

Obviously, no-knowledge measurement ($\sin\theta = 1$) is the optimal way.

We numerically calculate the optimal precision of simultaneous and independent estimation with the same probes by choosing the optimal t and λ, as shown in Fig. 3. The optimal t and λ can be different in independently estimating ω and η, which is more flexible than the simultaneous estimation. However, the independent estimation consumes twice the number of probes that the simultaneous estimation uses. Hence, we can see that the simultaneous estimation can provide a better precision than the independent estimation. Also, the precision of estimating ω and η is becoming smaller with the increase

of detection efficiency. It is because the no-knowledge feedback can better resist the decoherence with the larger detection efficiency.

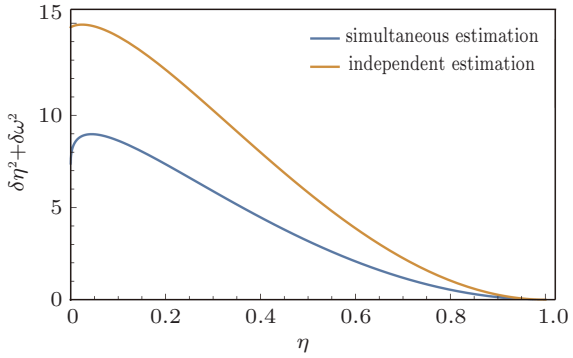


Fig. 3. (color online) Diagram of simultaneous and independent estimation $\delta\eta^2+\delta\omega^2$ changing with detection efficiency η, given by the same probes. Here, we calculate the precision with an average of one probe. It shows that the simultaneous estimation performs better than the independent estimation.

6. Conclusion and outlook

We have utilized the no-knowledge feedback control to estimate detection efficiency. The results show that when the feedback operator F is proportional to the coupling operator L, the no-knowledge measurement is the optimal way to encode the information of detection efficiency onto the probe state. By the exact numerical and approximate analytical calculation, we find that the high precision of detection efficiency can be obtained for low or large detection efficiency. Finally, we show that a simultaneous estimation frequency and detection efficiency with no-knowledge feedback control can perform better than independent estimation.

Whether no-knowledge measurement is the optimal way for any feedback operator will be our further investigation. In this article, we only consider the Markovian feedback. Non-Markovian phenomena are very common.[33,34] Hence, it is worth utilizing non-Markovian feedback to estimate the precision of detection efficiency.

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