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Quantum speed-up capacity in different types of quantum channels for two-qubit open systems*

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A potential acceleration of a quantum open system is of fundamental interest in quantum computation, quantum communication, and quantum metrology. In this paper, we investigate the "quantum speed-up capacity" which reveals the potential ability of a quantum system to be accelerated. We explore the evolutions of the speed-up capacity in different quantum channels for two-qubit states. We find that although the dynamics of the capacity is varying in different kinds of channels, it is positive in most situations which are considered in the context except one case in the amplitude-damping channel. We give the reasons for the different features of the dynamics. Anyway, the speed-up capacity can be improved by the memory effect. We find two ways which may be used to control the capacity in an experiment: selecting an appropriate coefficient of an initial state or changing the memory degree of environments.

Keywords: quantum speed-up capacity, quantum speed limit, two-qubit open systems, quantum channel, memory effect

PACS: 03.65.Yz, 03.65.Ta, 03.67.Mn **DOI:** 10.1088/1674-1056/27/6/060302

1. Introduction

Whether a quantum system has a potential capacity to be accelerated is an important question in a quantum process. A concept which may weigh this question is

$$\mathrm{Cap} = \frac{(\tau - \tau_{\mathrm{QSL}})}{\tau} \times 100\%, \tag{1}$$

where τ_{QSL} is a quantum speed limit (QSL) time[1–6] and τ is an actual evolution time. Equation (1) gives a percentage of the potential acceleration of a quantum system, and its physical meaning will be revealed in the next section: it may play a decisive role in quantum computation,[7,8] quantum communication,[9,10] quantum optimal control,[11–14] and quantum metrology.[15–17]

Using Eq. (1), it is obvious that the QSL time which has been studied in recent years[18–25] needs to be calculated to obtain the speed-up capacity. A unified lower bound of a QSL time involving Mandelstam–Tamm (MT)[26] and Margolus–Levitin (ML)[27] types in open systems has been derived by Deffner and Lutz.[22] It has also been confirmed that the ML type bound based on the operator norm provides the sharpest bound of the QSL time in an open system and non-Markovianity leads to a smaller QSL time. The QSL time of a multi-qubit open system has now caught increasing attention from researchers. Since it was found that the QSL time can be reduced into a special class of multi-qubit states in an amplitude-damping channel even in a memoryless environment,[28] three interesting questions have arisen. 1) What is a common case for the speed-up capacity of multi-qubit open systems in different types of quantum channels? 2) Does the memory effect play the same role as the speed-up capacity in different channels? 3) What will happen when the memory degree is changed? In this paper we consider these queries in a two-qubit system and show that even different quantum channels lead to different speed-up capacities that exist in most cases. We demonstrate that the capacity will benefit from a memory environment. Moreover, we find two key factors which may be helpful in controlling the speed-up capacity of a quantum process in experiments.

The rest of this paper is organized as follows. In Section 2 we explain the physical meaning of the speed-up capacity and deal with the dynamics of multi-qubit open systems by Kraus operators. Evolutions of the speed-up capacity of typical two-qubit states in different quantum channels without the memory effect are explored and explained in Section 3. The memory and memory degree effect on the speed-up capacity are discussed in Section 4. Finally, the results obtained in this work are summarized in Section 5.

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2. Quantum speed-up capacity and dynamics of multi-qubit open system

With the form of Eq. (1), it can be seen that the speed-up capacity is just a ratio of difference between τ and $\tau_{\rm QSL}$ to τ. Since $\tau_{\rm QSL}$ and τ are the minimal evolution time and the actual evolution time respectively, the difference between them represents a time length which may be potentially reduced in evolution. Therefore, greater difference causes longer potential time, so that the evolution process may be speeded up. In this sense, equation (1) gives a maximal percentage of the evolution of a system, by which the process can be accelerated in theory within a given actual evolution time. This is the exact physical meaning of Eq. (1), serving as the definition of the quantum speed-up capacity. To obtain the capacity, the QSL time which can be derived by combining the results of MT and ML bounds[3,4,14,29] need to be calculated. A definition of the QSL time in an open system is[22,24,28,30,31]

$$\tau_{\rm QSL} = \frac{\sin^2[B(\rho_0,\rho_\tau)]}{\min\{\Lambda_\tau^{\rm op},\Lambda_\tau^{\rm tr},\Lambda_\tau^{\rm hs}\}}, \tag{2}$$

where $B(\rho_0,\rho_\tau) = \arccos(\sqrt{\langle\Psi_0|\rho_\tau|\Psi_0\rangle}$ denotes the Bures angle between an initial state $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$ and its target state ρ_τ, $\Lambda_\tau^{\rm op,tr,hs} = (1/\tau)\int_0^\tau \|L_t(\rho_t)\|_{\rm op,tr,hs}\,{\rm d}t$ with L_t being a superoperator which satisfies ${\rm d}\rho_t/{\rm d}t = L_t(\rho_t)$. Here $\|A\|_{\rm op} = \alpha_1$, $\|A\|_{\rm tr} = \sum_i \alpha_i$ and $\|A\|_{\rm hs} = \sqrt{\sum_i \alpha_i^2}$ are the operator norm, trace norm and the Hilbert–Schmidt norm, respectively, α_i being the singular value of A.[32]

A popular and convenient description which indicates the dynamics of a state in a quantum channel is the Kraus representation.[33] With this description, the evolution of a state ρ can be written in the form of

$$\rho(t) = \sum_\mu K_\mu(t)\rho(0)K_\mu^\dagger(t), \tag{3}$$

where the operators K_μ are the so-called Kraus operators and satisfy $\sum_\mu K_\mu^\dagger K_\mu = 1$ for all t's. When the system is composed of N subsystems with independent environments respectively, equation (3) is replaced by[34]

$$\rho(t) = \sum_{\mu\ldots\nu} K_\mu^1 \otimes\cdots\otimes K_\nu^N \rho(0) K_\mu^{1\dagger}\otimes\cdots\otimes K_\nu^{N\dagger}. \tag{4}$$

By using Eq. (4), the evolution of a multi-qubit system can be evaluated.

3. Evolutions of the speed-up capacity in different quantum channels

Now we focus on the evolutions of the speed-up capacity of two-qubit states in different quantum channels where the N in Eq. (4) equals 2. The operator norm which has been proved to provide the sharpest bound[22] is used here. Two classes of typical Bell-type initial states, $|\Psi_1\rangle$ and $|\Psi_2\rangle$ with coefficient $a \in [0,1]$, are considered as the initial states respectively. The evolving state $\rho(t)$ is used as a target state to show the dynamics of the capacity.

3.1. Amplitude-damping channel

This channel represents the dissipative interaction between a qubit and its environment. The Hamiltonian model for the process can be written as[35]

$$H_{\rm AD} = \omega_0\sigma_+\sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (\sigma_+ g_k a_k + \sigma_- g_k^* a_k^\dagger), \tag{5}$$

where $\sigma_\pm$ are the raising and lowering operator with ω_0 being the transition frequency of the qubit, ω_k denotes different field modes of the reservoir where $a_k(a_k^\dagger)$ is the annihilation (creation) operator and g_k is the coupling constant. A damped Jaynes–Cummings model is considered with

$$J(\omega) = \frac{\gamma_0\lambda^2}{2\pi[(\omega_0-\omega)^2+\lambda^2]}, \tag{6}$$

where λ defines the spectral width and γ_0 quantifies the coupling strength. The decoherence function of the model is

$$G(t) = {\rm e}^{-\lambda t/2}\left[\cosh\left(\frac{{\rm d}t}{2}\right) + \frac{\lambda}{\rm d}\sinh\left(\frac{{\rm d}t}{2}\right)\right], \tag{7}$$

where $d = \sqrt{\lambda^2 - 2\gamma_0\lambda}$. The environment is Markovian (memoryless) when $\gamma_0 < \lambda/2$, otherwise a non-Markovian (memory effect) environment is caused.[36–40] The Kraus operators of this model are given as

$$K_1(t) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{p(t)} \end{pmatrix}, \quad K_2(t) = \begin{pmatrix} 0 & \sqrt{1-p(t)} \\ 0 & 0 \end{pmatrix}, \tag{8}$$

where the damping parameter $p(t)$ equals $G^2(t)$. The evolution can be easily expanded to two-qubit systems by using Eq. (4).

The speed-up capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ each as a function of the scaled time λt and coefficient a without memory effect are shown in Fig. 1. It can be found that $|\Psi_1\rangle$ always has no speed-up capacity no matter how long it evolves. Yet $|\Psi_2\rangle$ has a nonzero speed-up capacity at the initial time (except $a = 1$) and then it increases to an invariant value within a short time of evolving. The capacity is in inverse proportion to coefficient a. These phenomena mean that $|\Psi_1\rangle$ reaches the best accelerated performance in this channel, while $|\Psi_2\rangle$ can obtain a further acceleration even when the environment is memoryless. It is an important character for state selecting in an experiment.

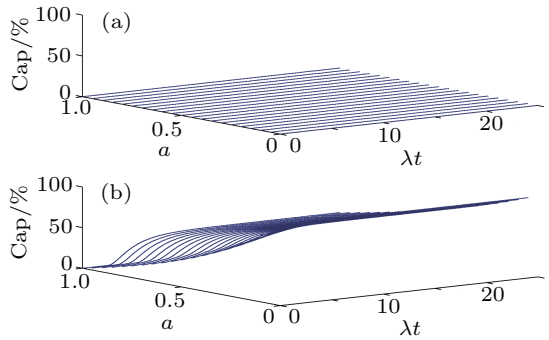


Fig. 1. Evolutions of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and coefficient a with $\lambda = 50$.

3.2. Phase-damping channel

The phase damping process describes a pure dephasing type of interaction between a qubit and a bosonic reservoir. The Hamiltonian is written as follows:[35]

$$H_{\mathrm{PD}} = \omega_0\sigma_z + \sum_k \omega_k a_k^\dagger a_k + \sum_k \sigma_z(g_k a_k + g_k^* a_k^\dagger). \tag{9}$$

A spectral density of an Ohmic-like form is considered here:

$$J(\omega) = \frac{\omega^s}{\omega_{\mathrm{c}}^{s-1}} \mathrm{e}^{-\omega/\omega_{\mathrm{c}}}, \tag{10}$$

where s is the Ohmic parameter and ω_{c} is the cutoff frequency of the environment. By changing the relationships for s and constant 1, we obtain different Ohmic spectra which correspond to sub-Ohmic environment ($s < 1$), Ohmic environment ($s = 1$), and super-Ohmic environment ($s > 1$), respectively. Besides, $s > 2$ may cause a memory effect with zero T.[41] Kraus operators of this channel are given as

$$K_1(t) = \begin{pmatrix} 1 & 0 \\ 0 & p(t) \end{pmatrix}, \quad K_2(t) = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1-p^2(t)} \end{pmatrix}, \tag{11}$$

where $p(t)$ is a dephasing parameter and can be calculated from

$$p(t) = \exp\left[-\int_0^t \gamma(t')\mathrm{d}t'\right], \tag{12}$$

with $\gamma(t)$ being the dephasing rate, and it can be expressed as follows:

$$\gamma(t) = \omega_{\mathrm{c}}[1+(w_{\mathrm{c}}t)^2]^{-s/2}\Gamma(s)\sin[s\arctan(\omega_{\mathrm{c}}t)], \tag{13}$$

with $\Gamma(s)$ representing the Euler function. It can also be easily expanded to two-qubit systems by using Eq. (4).

Evolutions of the speed-up capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in this channel are very different from those in an amplitude-damping channel. A biggest distinction is that neither $|\Psi_1\rangle$ or $|\Psi_2\rangle$ has a different speed-up capacity any more. As shown in Fig. 2, the two classes of states have an identical invariant speed-up capacity when coefficient a is the same in a memoryless environment. There is a non-monotonic relationship between the capacity and the coefficient a. This relationship can also be used to select an appropriate state for experiment aims.

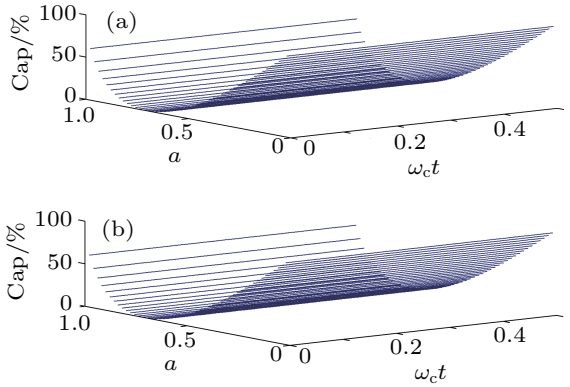


Fig. 2. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase-damping channel as a function of scaled time $\omega_{\mathrm{c}}t$ and coefficient a with $s = 1$, $\omega_{\mathrm{c}} = 1$.

3.3. Bit flip, phase flip, and bit-phase flip channels

These three channels are all under the Markov approximation in which the memory effect does not exist. Their unified Lindblad operator in a single-qubit system is expressed as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \gamma[\sigma_i\rho(t)\sigma_i - \rho(t)], \tag{14}$$

where γ is the time-independent dephasing rate and σ_i is the Pauli matrix with $i = x, y, z$ denoting bit flip, phase-bit flip, and phase flip channels, respectively. The set of Kraus operators for each one of these channels is given as[34]

$$K_1(t) = \begin{pmatrix} \sqrt{1-\dfrac{p(t)}{2}} & 0 \\ 0 & \sqrt{1-\dfrac{p(t)}{2}} \end{pmatrix},$$
$$K_2^i(t) = \sqrt{\frac{p(t)}{2}}\sigma_i, \tag{15}$$

where $p(t) = 1 - \exp[-\gamma t]$.

In these three channels the speed-up capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are the same. The evolution in a bit flip channel is shown in Fig. 3. It can be seen that the capacity in this channel rises from zero to a very small value within a short time. The relationship between the final value and coefficient a is still non-monotonic but it is different from that in a phase-damping channel. The dynamics in a phase flip channel is similar to that in a phase-damping channel as shown in Fig. 4. This phenomenon is easy to be understood since it has been known that a phase-damping channel and a phase flip channel have exactly the same quantum operation.[42] In Fig. 5 it is found that the speed-up capacity in a bit-phase flip channel is the same as that in a bit flip channel. This phenomenon can be easily confirmed mathematically by using Eqs. (2), (4), and (15).

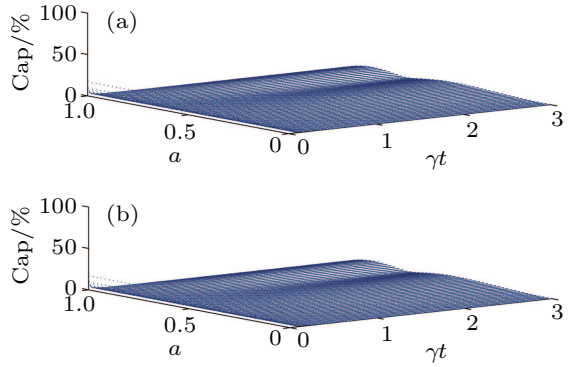


Fig. 3. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a bit flip channel as a function of scaled time γt and coefficient a with $\gamma = 10$.

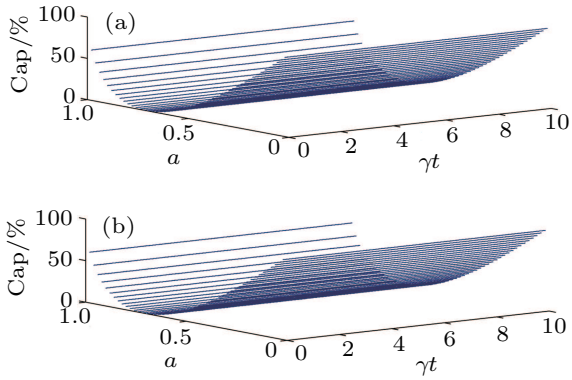


Fig. 4. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase flip channel as a function of scaled time γt and coefficient a with $\gamma = 10$.

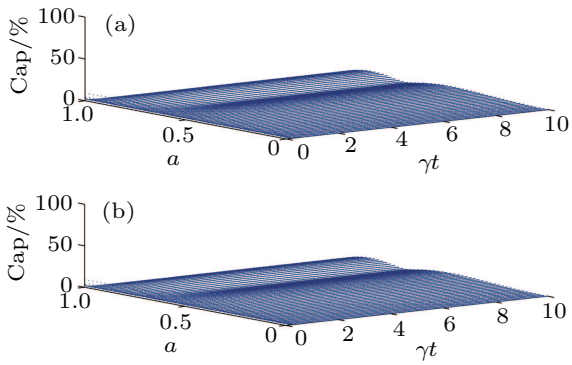


Fig. 5. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a bit-phase flip channel as a function of scaled time γt and coefficient a with $\gamma = 10$.

3.4. Explanations for the different features of the speed-up capacities in different channels

Now we investigate the reason why the speed-up capacity has such different features in different channels. Two main different characters are explained here. i) Why are the capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ vastly different in an amplitude-damping channel but exactly the same in other channels we have considered? ii) Why does the capacity always exist in most situations we have considered and what conditions should be satisfied for the capacity to disappear, such as $|\Psi_1\rangle$ in an amplitude-damping channel? By answering question ii), the results can be generalized to general scenarios.

Substituting Eq. (2) into Eq. (1), we have

$$\begin{aligned} C_{\rm ap} &= \left(1-\frac{\tau_{\rm QSL}}{\tau}\right)\times 100\% \\ &= \left(1-\frac{\sin^2[B(\rho_0,\rho_\tau)]}{\int_0^\tau \|L_t(\rho_t)\|_{\rm op}\,{\rm d}t}\right)\times 100\% \\ &= \left(1-\frac{1-\langle\Psi_0|\rho_\tau|\Psi_0\rangle}{\int_0^\tau \|L_t(\rho_t)\|_{\rm op}\,{\rm d}t}\right)\times 100\%. \end{aligned} \tag{16}$$

The reduced density matrices of the states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ at time t can be written as

$$\rho_1(t)=\begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix},$$

$$\rho_2(t)=\begin{pmatrix} \rho'_{11} & 0 & 0 & \rho'_{14} \\ 0 & \rho'_{22} & \rho'_{23} & 0 \\ 0 & \rho'^*_{23} & \rho'_{33} & 0 \\ \rho'^*_{14} & 0 & 0 & \rho'_{44} \end{pmatrix}. \tag{17}$$

Then we can obtain

$$\langle\Psi_1|\,\rho_1(t)\,|\Psi_1\rangle=\alpha^2\rho_{22}+\alpha\sqrt{1-\alpha^2}(\rho_{23}+\rho_{23}^*)+(1-\alpha^2)\rho_{33}$$

and

$$\langle\Psi_2|\,\rho_2(t)\,|\Psi_2\rangle=\alpha^2\rho'_{44}+\alpha\sqrt{1-\alpha^2}(\rho'_{14}+\rho'^*_{14})+(1-\alpha^2)\rho'_{11}.$$

In phase-damping, bit flip, phase flip, and bit-phase flip channels we find that $\rho_{11}=\rho'_{33}$, $\rho_{22}=\rho'_{44}$, $\rho_{33}=\rho'_{11}$, $\rho_{44}=\rho'_{22}$, $\rho_{14}=\rho'_{23}$, and $\rho_{23}=\rho'_{14}$ always hold during the evolution. Therefore $\langle\Psi_1|\,\rho_1(t)\,|\Psi_1\rangle$ always equals $\langle\Psi_2|\,\rho_2(t)\,|\Psi_2\rangle$ in these channels. On the other hand, $L_t(\rho_1(t))$ and $L_t(\rho_2(t))$ have the same singular values[32,43] due to the symmetry between $\rho_1(t)$ and $\rho_2(t)$. This is the exact reason why the capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are the same in these channels by using Eq. (16). When we come to the case of an amplitude-damping channel, it is found that the symmetry between $\rho_1(t)$ and $\rho_2(t)$ is destroyed in diagonal elements. Accordingly, the capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are different in this channel.

Next, we come to explain the second character. The definition of $\tau_{\rm QSL}$ is originally derived from the inequality which has been obtained as the nonunitary generalization[22]

$$\begin{aligned} &2\cos(B(\rho_0,\rho_\tau))\sin(B(\rho_0,\rho_\tau))\frac{{\rm d}B(\rho_0,\rho_\tau)}{{\rm d}t} \\ &\le |\langle\Psi_0|L_t(\rho_t)|\Psi_0\rangle|. \end{aligned} \tag{18}$$

By using the relationship $\langle\Psi_0|\,L_t(\rho_t)\,|\Psi_0\rangle$ and the von Neumann trace inequality for operators,[44,45] we obtain

$$2\cos(B(\rho_0,\rho_\tau))\sin(B(\rho_0,\rho_\tau))\frac{{\rm d}B(\rho_0,\rho_\tau)}{{\rm d}t}\le\|L_t(\rho_t)\|_{\rm op}. \tag{19}$$

Integrating Eq. (19) over time, it is found that

$$\frac{\sin^2[B(\rho_0,\rho_\tau)]}{\int_0^\tau \|L_t(\rho_t)\|_{\rm op}\,{\rm d}t}\le 1. \tag{20}$$

By substituting Eq. (20) into Eq. (16), $C_{\rm ap} \geq 0$ is obtained. This is the reason why in most situations the capacity is positive. Obviously, the condition which causes the capacity to equal zero is the same as the condition which leads to an equal sign in the von Neumann trace inequality. From this we can see that the evolution of state $|\Psi_1\rangle$ in an amplitude-damping channel reaches the condition and the capacity disappears in this case. Moreover, since equation (20) is a general equation which is independent of the values of the parameters of quantum channels and always holds in different dynamical processes, the results may be generalized to general occasions and more specific researches will be conducted in future.

4. Memory and memory degree effect

In this section we explore the scenario in which the memory effect appears. How to distinguish whether the environment is memory or memoryless has been described in the previous section. In an amplitude-damping channel we see that things become a little different from previous ones when the system is affected by memory. As shown in Fig. 6, $|\Psi_1\rangle$ still has no speed-up capacity at the beginning, but suddenly rises to an invariant value which is unrelated to a. For $|\Psi_2\rangle$ the circumstance is similar to what happens in a memoryless environment, however it takes a shorter time to reach the invariant value which is higher than that in a memoryless environment.

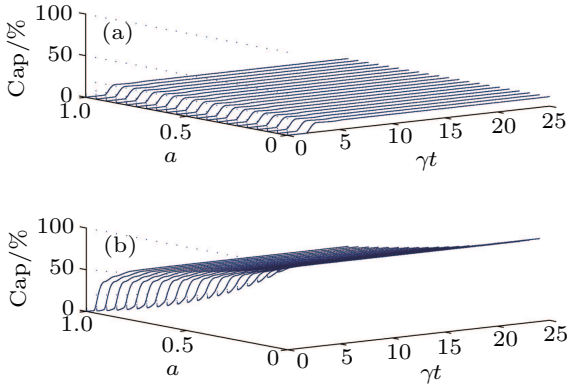


Fig. 6. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and coefficient a with $\lambda = 50$.

Dynamics of the speed-up capacity in a phase-damping channel with memory effect is shown in Fig. 7. Although it seems to be the same as that in a memoryless environment, it is found that the capacity slightly rises to a higher value within a short time of evolving when the environment is a memory one as shown in Fig. 8.

Overall, though the dynamics of the capacity is different in different channels, it can be concluded that the memory effect causes a more potentially accelerated ability in two-qubit open systems, which in fact is a powerful supplement to the result that non-Markovianity may reduce the QSL time in a Jaynes–Cummings model of a single qubit open system.[22] It also testifies to the generalization of the relationship between the non-Markovianity and the QSL time. Since the essential reason why quantum speedup connects directly with the non-Markovianity in a single qubit open system has been revealed in Ref. [46], our result regarding the relationship between the quantum speed-up capacity and the non-Markovianity in a two-qubit open system may also be explained by using the conclusion of Ref. [46].

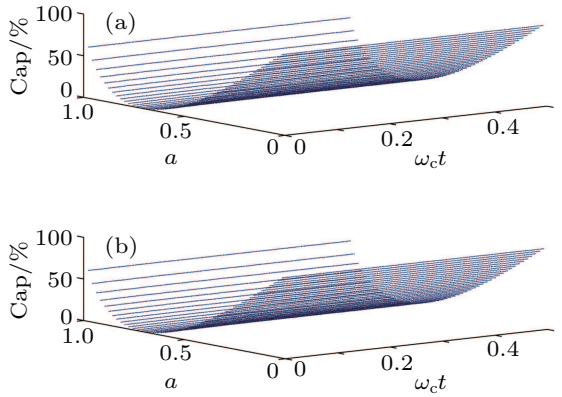


Fig. 7. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase-damping channel as a function of scaled time $\omega_c t$ and coefficient a with $s = 5$ and $\omega_c = 1$.

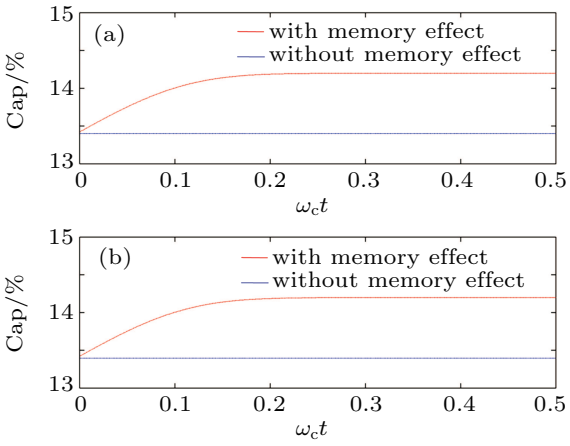


Fig. 8. (color online) Comparison of speed-up capacity between (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a memory and a memoryless environment in a phase-damping channel with $a = 0.5$.

Finally, we deal with the question over what will happen if the memory degree is changed. We know that the memory effect comes from the non-Markovianity of the environment. So if the degree of non-Markovianity is higher, the memory degree is higher. It has been proved that the degree of non-Markovianity is in proportion to the coupling strength γ_0 in a damped Jaynes–Cummings model.[37] Therefore the memory degree is also in proportion to the γ_0 in a damped Jaynes–Cummings model. The speed-up capacities of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as a function of scaled time λt and γ_0 are shown in Fig. 9. It is found that the higher the memory degree, the stronger the speed-up capacity is. Namely, the stronger the memory effect, the greater the potential acceleration is. It is remarkable that when γ_0 is high enough, there is a short fluctuation for $|\Psi_1\rangle$

before it reaches an invariant speed-up capacity. Some methods which may control the evolution speed of a single qubit system in theory and experiment have been presented,[46–57] so our results may make some contribution to experiments in further work.

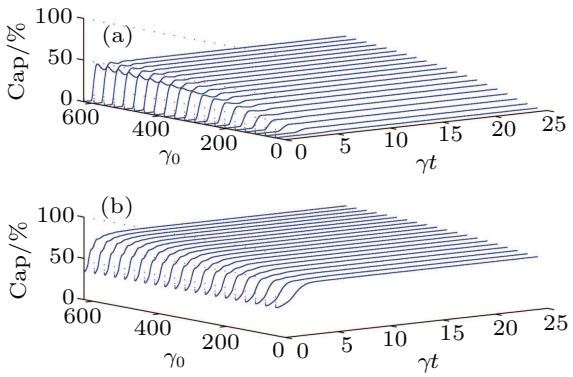


Fig. 9. (color online) Evolutions of the speed-up capacities of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and the coupling strength γ_0 (in proportion to memory degree) with $\lambda = 50$ and $a = 0.5$.

5. Conclusions

In this work, we have presented a formula to describe the speed-up capacity of a quantum system. We find that the capacity has different dynamic behaviors in different channels and exists in most situations of two-qubit open systems. We explain the characters of the capacity and demonstrate that the memory effect can always improve the capacity. We also find that the coefficient a of the initial state and the memory degree are two key factors which may be useful in controlling the capacity experimentally.

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