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Monogamy quantum correlation near the quantum phase transitions in the two-dimensional XY spin systems*

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We investigate the role of quantum correlation around the quantum phase transitions by using quantum renormalization group theory. Numerical analysis indicates that quantum correlation as well as quantum nonlocality can efficiently detect the quantum critical point in the two-dimensional XY systems. The nonanalytic behavior of the first derivative of quantum correlation is observed at the critical point as the size of the model increases. Furthermore, we discuss the quantum correlation distribution in this system based on the square of concurrence (SC) and square of quantum discord (SQD). The monogamous properties of SC and SQD are obtained. Particularly, we prove that the quantum critical point can also be achieved by monogamy score.

Keywords: XY model, quantum phase transitions, quantum correlation, monogamy relation

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1. Introduction

In order to exhibit internal contradictions in quantum mechanics, Einstein, Podolsky, and Rosen (EPR) proposed a thought experiment in 1935.[1] The mysterious phenomenon demonstrated in EPR experiments is now known as entanglement. The investigation on the entanglement in quantum states has formed a new discipline namely quantum information science. It is generally recognized that entanglement is the key resource in quantum information processing.[2] Recently, researchers have found that the entangled state have shown monogamy properties, i.e., quantum entanglement cannot be freely shared among the constituents of a multipartite system.[3,4] Monogamy is one of the basic rules in making quantum cryptography secure and plays an indispensable role in superdense coding.[5] Many achievements have been obtained after it was introduced by Coffman *et al.*[4] However, according to recent progress, there are also some separable states containing quantum discord that are also very effective in quantum information processing.[6] So, entanglement cannot signify all the quantum correlation in a quantum system. Quantum discord or discord-like measures may be the most fundamental resource in quantum information protocols. Therefore, the monogamy relation of quantum correlation also deserves more attention.[7–10] Motivated by the development of the monogamy relation of quantum correlation, we also want to ask whether the monogamy relation can be used to investigate some fundamental physics problems, such as quantum phase transitions (QPT).

QPT indicates that the ground state of a many-body system changes abruptly when varying a physical parameter such as magnetic field or pressure at absolute zero temperature.[11] Contrary to thermal phase transitions, QPT is completely induced by quantum fluctuations.[11] Generally, researchers adopt order parameter, correlation functions, and other concepts in thermal phase transitions to investigate QPT. Though many gratifying results have been achieved, there are still some shortcomings in it. The rapid development of quantum information science provides us with a good means to understand the nature of QPT. A lot of studies indicate that entanglement and quantum correlation can be used to detect QPT or describe the property near the critical point.[12–21] In addition, the renormalization group theory has also been a powerful tool to study QPT for many years. Recently, researchers have begun to study QPT in low-dimensional spin systems by combining quantum information concepts and quantum renormalization group (QRG) theory. It has been shown that the entanglement behavior in the vicinity of the critical point is directly connected with the quantum critical phenomena.[22–26] Quantum correlation also can be used to detect the quantum critical point.[27–30] However, these studies mainly concentrate on one-dimensional systems. Two-dimensional spin systems, such as lanar quadrilateral spin crystal, triangular spin grid, and kagome spin lattice are also important systems. The research on these models will promote the understanding of ground-state properties, correlation length, and critical point in low-dimensional systems.

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Recently, Xu[31] investigated the quantum entanglement around the quantum critical point in the Ising model on a square lattice. Usman[32] gave an analysis of two-dimensional XY systems by using entanglement theory. Nevertheless, as mentioned before, quantum entanglement is not adequate to represent all the quantum correlation contained in a quantum system; this inspired us to apply the quantum correlation to study the two-dimensional system.[27] Furthermore, we also want to know whether the monogamy relation exists in the two-dimensional spin system, and whether the monogamy relation can detect the quantum critical point. To answer these questions, the critical behavior and the monogamy property of the two-dimensional XY systems will be studied by the quantum correlation measures.

This article is organized in the following way. In the next section, we will give an introduction on the two-dimensional XY model. In Section 3, the quantum correlation measures and the analytical results of this model are given. In Section 4, the dynamical behavior, nonanalytic results, and scaling behavior are presented. Section 5 gives the monogamy relation of this system. The last section is a summary of our work.

2. Model description

The Hamiltonian of a two-dimensional XY model can be written as[32]

$$H(J,\gamma) = \frac{J}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}[(1+\gamma)(\sigma_{i,j}^{x}\sigma_{i+1,j}^{x}+\sigma_{i,j}^{x}\sigma_{i,j+1}^{x}) + (1-\gamma)(\sigma_{i,j}^{y}\sigma_{i+1,j}^{y}+\sigma_{i,j}^{y}\sigma_{i,j+1}^{y})], \quad (1)$$

where J is the exchange interaction, γ is the anisotropy parameter, and $\sigma^{\tau}(\tau = x,y)$ are standard Pauli operators at site i,j. In order to ensure the symmetry of the system and get a self-similar Hamiltonian, we select a five-site as one block . Such five-site blocks can be viewed as one-site in renormalized subspace. A schematic diagram can be seen in Fig. 1.

So, the Hamiltonian can be separated as block Hamiltonian H^{B} and interacting Hamiltonian H^{BB} respectively.

$$H^{\mathrm{B}} = \frac{J}{4}\sum_{L=1}^{N/5}[(1+\gamma)(\sigma_{L,1}^{x}\sigma_{L,2}^{x}+\sigma_{L,1}^{x}\sigma_{L,3}^{x}+\sigma_{L,1}^{x}\sigma_{L,4}^{x} + \sigma_{L,1}^{x}\sigma_{L,5}^{x}) + (1-\gamma)(\sigma_{L,1}^{y}\sigma_{L,2}^{y}+\sigma_{L,1}^{y}\sigma_{L,3}^{y} + \sigma_{L,1}^{y}\sigma_{L,4}^{y}+\sigma_{L,1}^{y}\sigma_{L,5}^{y})], \quad (2)$$

$$H^{\mathrm{BB}} = \frac{J}{4}\sum_{L=1}^{N/5}[(1+\gamma)(\sigma_{L,2}^{x}\sigma_{L+1,3}^{x}+\sigma_{L,2}^{x}\sigma_{L+1,4}^{x}+\sigma_{L,2}^{x}\sigma_{L+2,5}^{x} + \sigma_{L,3}^{x}\sigma_{L+2,4}^{x}+\sigma_{L,3}^{x}\sigma_{L+2,5}^{x}+\sigma_{L,4}^{x}\sigma_{L+3,5}^{x}) + 1-\gamma)(\sigma_{L,2}^{y}\sigma_{L+1,3}^{y}+\sigma_{L,2}^{y}\sigma_{L+1,4}^{y}+\sigma_{L,2}^{y}\sigma_{L+2,5}^{y} + \sigma_{L,3}^{y}\sigma_{L+2,4}^{y}+\sigma_{L,3}^{y}\sigma_{L+2,5}^{y}+\sigma_{L,4}^{y}\sigma_{L+3,5}^{y})]. \quad (3)$$

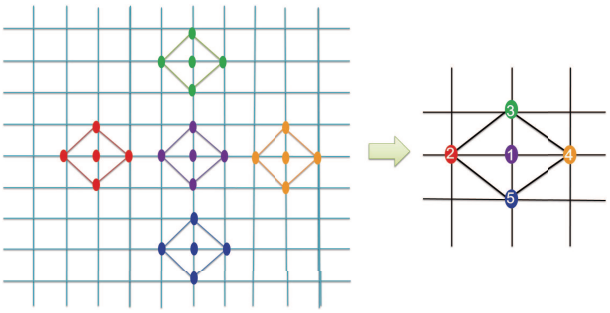


Fig. 1. (color online) A schematic description of QRG for five sites in a block.

The two lowest eigenvectors of the corresponding L-th block

$$|\varphi_0\rangle = \gamma_1(|\uparrow\uparrow\uparrow\uparrow\downarrow\rangle+|\uparrow\uparrow\uparrow\downarrow\uparrow\rangle+|\uparrow\uparrow\downarrow\uparrow\uparrow\rangle+|\uparrow\downarrow\uparrow\uparrow\uparrow\rangle) + \gamma_2(|\uparrow\uparrow\downarrow\downarrow\downarrow\rangle+|\uparrow\downarrow\uparrow\downarrow\downarrow\rangle+|\uparrow\downarrow\downarrow\uparrow\downarrow\rangle+|\uparrow\downarrow\downarrow\downarrow\uparrow\rangle) + \gamma_3|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle+\gamma_4(|\downarrow\uparrow\uparrow\uparrow\uparrow\rangle+|\downarrow\uparrow\uparrow\downarrow\downarrow\rangle+|\downarrow\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\downarrow\uparrow\uparrow\downarrow\rangle+|\downarrow\downarrow\uparrow\downarrow\uparrow\rangle+|\downarrow\downarrow\downarrow\uparrow\uparrow\rangle)+\gamma_5|\downarrow\downarrow\downarrow\downarrow\downarrow\rangle), \quad (4)$$

$$|\varphi_0'\rangle = \gamma_6|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle+\gamma_7(|\uparrow\uparrow\uparrow\downarrow\downarrow\rangle+|\uparrow\uparrow\downarrow\uparrow\downarrow\rangle+|\uparrow\uparrow\downarrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\uparrow\downarrow\rangle+|\uparrow\downarrow\uparrow\downarrow\uparrow\rangle+|\uparrow\downarrow\downarrow\uparrow\uparrow\rangle)+\gamma_8|\uparrow\downarrow\downarrow\downarrow\downarrow\rangle + \gamma_9(|\downarrow\uparrow\uparrow\uparrow\downarrow\rangle+|\downarrow\uparrow\uparrow\downarrow\uparrow\rangle+|\downarrow\uparrow\downarrow\uparrow\uparrow\rangle+|\downarrow\downarrow\uparrow\uparrow\uparrow\rangle) + \gamma_{10}(|\downarrow\uparrow\downarrow\downarrow\downarrow\rangle+|\downarrow\downarrow\uparrow\downarrow\downarrow\rangle+|\downarrow\downarrow\downarrow\uparrow\downarrow\rangle+|\downarrow\downarrow\downarrow\downarrow\uparrow\rangle) \quad (5)$$

can be used to establish the projection operator

$$P_0^L = |\varphi_0\rangle_L\langle\Uparrow| + |\varphi_0'\rangle_L\langle\Downarrow|.$$

Analytical expressions of the γ_i's can be found in Appendix A,[32] $\langle\Uparrow|, \langle\Downarrow|$ are renamed states of each block to represent the effective site degrees of freedom. Now we can derive the effective Hamiltonian

$$H^{\mathrm{eff}}(J',\gamma') = P_0^{\dagger}(H^{\mathrm{B}}+H^{\mathrm{BB}})P_0 = \frac{J'}{4}\sum_{p=1}^{N/5}\sum_{q=1}^{N/5}[(1+\gamma')(\sigma_{p,q}^{x}\sigma_{p+1,q}^{x}+\sigma_{p,q}^{x}\sigma_{p,q+1}^{x}) + (1-\gamma')(\sigma_{p,q}^{y}\sigma_{p+1,q}^{y}+\sigma_{p,q}^{y}\sigma_{p,q+1}^{y})], \quad (6)$$

where the renormalized couplings are

$$\begin{aligned} J' &= J(\gamma_{10}^2(9\gamma_4^2+6\gamma\gamma_4\gamma_5+\gamma_5^2)+9\gamma_2^2\gamma_7^2+\gamma_1^2(\gamma_6^2+6\gamma\gamma_6\gamma_7+9\gamma_7^2)+6\gamma\gamma_2^2\gamma_7\gamma_8+\gamma_2^2\gamma_8^2+6\gamma\gamma_2\gamma_3\gamma_7\gamma_9 \\ &+18\gamma\gamma_2\gamma_4\gamma_7\gamma_9+2\gamma_2\gamma_3\gamma_8\gamma_9+6\gamma\gamma_2\gamma_4\gamma_8\gamma_9+\gamma_3^2\gamma_9^2+6\gamma\gamma_3\gamma_4\gamma_9^2+9\gamma_4^2\gamma_9^2+2\gamma_1\{\gamma_2[3\gamma_7(3\gamma\gamma_7+\gamma_8) \\ &+\gamma_6(3\gamma_7+\gamma\gamma_8)]+(\gamma\gamma_3\gamma_6+3\gamma_4\gamma_6+3\gamma_3\gamma_7+9\gamma\gamma_4\gamma_7)\gamma_9\}+2\gamma_{10}\{\gamma_1(\gamma_5\gamma_6+9\gamma_4\gamma_7)+\gamma[9\gamma_2\gamma_4\gamma_7 \\ &+3\gamma_1(\gamma_4\gamma_6+\gamma_5\gamma_7)+\gamma_2\gamma_5\gamma_8+9\gamma_4^2\gamma_9+\gamma_3\gamma_5\gamma_9]+3[\gamma_2(\gamma_5\gamma_7+\gamma_4\gamma_8)+\gamma_4(\gamma_3+\gamma_5)\gamma_9]\}), \quad (7) \\ \gamma' &= [2(3\gamma_{10}\gamma_4+3\gamma_1\gamma_7+\gamma_2\gamma_8+\gamma_3\gamma_9)(\gamma_{10}\gamma_5+\gamma_1\gamma_6+3\gamma_2\gamma_7+3\gamma_4\gamma_9)+\gamma(\gamma_{10}^2(9\gamma_4^2+\gamma_5^2)+9\gamma_2^2\gamma_7^2 \\ &+\gamma_1^2(\gamma_6^2+9\gamma_7^2)+\gamma_2^2\gamma_8^2+18\gamma_2\gamma_4\gamma_7\gamma_9+2\gamma_2\gamma_3\gamma_8\gamma_9+\gamma_3^2\gamma_9^2+6\gamma_1[\gamma_2\gamma_7(\gamma_6+\gamma_8)+(\gamma_4\gamma_6+\gamma_3\gamma_7)\gamma_9] \end{aligned}$$

$$+2\gamma_{10}\{\gamma_1(\gamma_5\gamma_6+9\gamma_4\gamma_7)+3[\gamma_2(\gamma_5\gamma_7+\gamma_4\gamma_8)+\gamma_4(\gamma_3+\gamma_5)\gamma_9]\})]/[\gamma_{10}^2(9\gamma_4^2+6\gamma\gamma_4\gamma_5+\gamma_5^2)+9\gamma_2^2\gamma_7^2$$
$$+\gamma_1^2(\gamma_6^2+6\gamma\gamma_6\gamma_7+9\gamma_7^2)+6\gamma\gamma_2^2\gamma_7\gamma_8+\gamma_2^2\gamma_8^2+6\gamma\gamma_2\gamma_3\gamma_7\gamma_9+18\gamma_2\gamma_4\gamma_7\gamma_9+2\gamma_2\gamma_3\gamma_8\gamma_9+6\gamma\gamma_2\gamma_4\gamma_8\gamma_9$$
$$+\gamma_3^2\gamma_9^2+6\gamma\gamma_3\gamma_4\gamma_9^2+9\gamma_4^2\gamma_9^2+2\gamma_1\{\gamma_2[3\gamma_7(3\gamma\gamma_7+\gamma_8)+\gamma_6(3\gamma_7+\gamma\gamma_8)]+(\gamma\gamma_3\gamma_6+3\gamma_4\gamma_6$$
$$+3\gamma_3\gamma_7+9\gamma\gamma_4\gamma_7)\gamma_9\}+2\gamma_{10}(\gamma_1(\gamma_5\gamma_6+9\gamma_4\gamma_7)+\gamma[9\gamma_2\gamma_4\gamma_7+3\gamma_1(\gamma_4\gamma_6+\gamma_5\gamma_7)+\gamma_2\gamma_5\gamma_8$$
$$+9\gamma_4^2\gamma_9+\gamma_3\gamma_5\gamma_9]+3[\gamma_2(\gamma_5\gamma_7+\gamma_4\gamma_8)+\gamma_4(\gamma_3+\gamma_5)\gamma_9])]. \tag{8}$$

The ground state density matrix is given by

$$\rho=|\varphi_0\rangle\langle\varphi_0|. \tag{9}$$

Owing to the symmetry, the bipartite state between the center block and every corner block is identical, i.e., $\rho_{12}=\rho_{13}=\rho_{14}=\rho_{15}$. Similarly, $\rho_{23}=\rho_{34}=\rho_{45}=\rho_{25}$. After some algebra, we can derive the bipartite states ρ_{12} and ρ_{23} by tracing out particles 3, 4, 5 or 1, 4, 5.

$$\rho_{12}=\begin{pmatrix} 3\gamma_1^2+\gamma_2^2 & 0 & 0 & 3\gamma_1\gamma_4+\gamma_2\gamma_5 \\ 0 & \gamma_1^2+3\gamma_2^2 & \gamma_1\gamma_3+3\gamma_2\gamma_4 & 0 \\ 0 & \gamma_1\gamma_3+3\gamma_2\gamma_4 & \gamma_3^2+3\gamma_4^2 & 0 \\ 3\gamma_1\gamma_4+\gamma_2\gamma_5 & 0 & 0 & 3\gamma_4^2+\gamma_5^2 \end{pmatrix}, \tag{10}$$

$$\rho_{23}=\begin{pmatrix} 2\gamma_1^2+\gamma_3^2+\gamma_4^2 & 0 & 0 & 2\gamma_1\gamma_2+\gamma_3\gamma_4+\gamma_4\gamma_5 \\ 0 & \gamma_1^2+\gamma_2^2+2\gamma_4^2 & \gamma_1^2+\gamma_2^2+2\gamma_4^2 & 0 \\ 0 & \gamma_1^2+\gamma_2^2+2\gamma_4^2 & \gamma_1^2+\gamma_2^2+2\gamma_4^2 & 0 \\ 2\gamma_1\gamma_2+\gamma_3\gamma_4+\gamma_4\gamma_5 & 0 & 0 & 2\gamma_2^2+\gamma_4^2+\gamma_5^2 \end{pmatrix}. \tag{11}$$

3. Different quantum correlation measures

3.1. Negativity

We first select an easily computable entanglement method named negativity (*Ne*) to study this model.[33] *Ne* was introduced for testing the violation degree of positive partial transpose criterion in entangled states. *Ne* has been proved to be monotone and convex under local operations and classical communication. For a bipartite system ρ_{AB}, the partial transpose of ρ_{AB} on A can be described as $(\rho_{AB}^{T_A})_{ij,kl}=(\rho)_{kj,il}$. So, for a given state ρ_{AB}, the *Ne* is defined as

$$Ne(\rho_{AB})=\frac{\|\rho_{AB}^{T_A}\|-1}{2}, \tag{12}$$

where $\|\rho_{AB}^{T_A}\|=\mathrm{Tr}\sqrt{\rho_{AB}^{T_A}\rho_{AB}^{T_A\dagger}}$ denotes the trace norm. For the bipartite $2\otimes2$ and $2\otimes3$ quantum systems, *Ne* is the necessary and sufficient inseparable condition.

The analytical results of *Ne* for states ρ_{12} and ρ_{13} are

$$\begin{aligned} Ne_{12} &= (9\gamma_1^4+6\gamma_1^2\gamma_2^2+4\gamma_1^2\gamma_3^2-18\gamma_1^2\gamma_4^2-6\gamma_1^2\gamma_5^2 \\ &\quad +24\gamma_1\gamma_2\gamma_3\gamma_4+\gamma_2^4+30\gamma_2^2\gamma_4^2-2\gamma_2^2\gamma_5^2+9\gamma_4^4 \\ &\quad +6\gamma_4^2\gamma_5^2+\gamma_5^4)^{1/2}/2-(\gamma_2^2+3\gamma_4^2+\gamma_5^2+3\gamma_1^2)/2, \end{aligned} \tag{13}$$

$$\begin{aligned} Ne_{23} &= (8\gamma_1^4+4\gamma_1^2\gamma_3^2+16\gamma_1^2\gamma_4^2-4\gamma_1^2\gamma_5^2+8\gamma_2^4-4\gamma_2^2\gamma_3^2 \\ &\quad +16\gamma_2^2\gamma_4^2+4\gamma_2^2\gamma_5^2+\gamma_3^4-2\gamma_3^2\gamma_5^2+16\gamma_4^4+\gamma_5^4)^{1/2}/2 \\ &\quad -(\gamma_1^2+\gamma_2^2+\gamma_3^2/2+\gamma_4^2+\gamma_5^2/2). \end{aligned} \tag{14}$$

3.2. Quantum discord

Quantum discord (QD) is proposed from the perspective of the information-theoretic paradigm. It is defined by the following expression[34]

$$\mathrm{QD}(\rho_{AB})=I(\rho_{AB})-CC(\rho_{AB}), \tag{15}$$

where $I(\rho_{AB})$ is the total correlation and measured by the quantum mutual information

$$I(\rho_{AB})=\sum_{i=A,B}S(\rho_i)-S(\rho_{AB}),$$

while $CC(\rho_{AB})=S(\rho_A)-\min_{\Pi_k^B}S(S_{A|B}\{\Pi_k^B\})$, in which Π_k^B is a positive operator-valued measure performed on the subsystem B. $S(\rho)=-\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy, while ρ_A and ρ_B denote the reduced density matrices of state ρ_{AB} by tracing out A or B.

Since ρ_{12} and ρ_{13} are X-type states, it is easy to get the quantum discord result.[2,35–37] But the analytical result is too complicated to express it here. We mainly show the numerical result in Section 4.

3.3. Measurement-induced disturbance

Measurement-induced disturbance (MID)[38] was defined by the difference between the quantum mutual information of a quantum state ρ_{AB} and the corresponding post-measurement classical state $\Pi(\rho_{AB})$

$$\mathrm{MID}(\rho_{AB})=I(\rho_{AB})-I(\Pi(\rho_{AB})), \tag{16}$$

here $I(\rho_{AB})$ is the same as in Eq. (15). $I(\Pi(\rho_{AB}))=\sum_{i,j}(\Pi_i^A\otimes\Pi_i^{\mathrm{B}})\rho_{AB}(\Pi_i^A\otimes\Pi_i^B)$ measures the classical correlation of a given state ρ_{AB}. The analytical result of the MID for ρ_{12} and ρ_{13} are also very complicated and no detailed results are given here.

3.4. Measurement-induced nonlocality

The measurement-induced (MIN) nonlocality is based on the trace norm for a bipartite state ρ_{AB}[39]

$$\mathrm{MIN}(\rho_{AB}) = \max_{\Pi^{\mathrm{A}}} \|\rho_{\mathrm{AB}} - \Pi^{\mathrm{A}}(\rho_{\mathrm{AB}})\|_1, \tag{17}$$

here $\|R\|_1 = \mathrm{Tr}\sqrt{R^{\dagger}R}$, and the maximum is taken over the full set of local projective measurements Π^A that is $\Pi^A(\rho_A) = \rho_A$.

The analytical results of the MIN of ρ_{12} and ρ_{13} are

$$\mathrm{MIN}_{12} = \max(|\tau_1|, |\tau_2|, |\tau_3|), \tag{18}$$

$$\mathrm{MIN}_{23} = (|8\gamma_1\gamma_2 + 4\gamma_4(\gamma_3+\gamma_5)| + |4\gamma_1^2 + 4\gamma_2^2 + 8\gamma_4^2|)/2, \tag{19}$$

where $\tau_1 = 2\gamma_1\gamma_3 + 6\gamma_4(\gamma_1+\gamma_2) + 2\gamma_2\gamma_5$, $\tau_2 = 2\gamma_1\gamma_3 + 6\gamma_4(\gamma_2 - \gamma_1) - 2\gamma_2\gamma_5$, $\tau_3 = 2\gamma_1^2 - 2\gamma_2^2 - \gamma_3^2 + \gamma_5^2$, $\iota_1 = 2(\gamma_1+\gamma_2)^2 + 2\gamma_4(\gamma_3+\gamma_5) + 4\gamma_4^2$, $\iota_2 = 2(\gamma_1-\gamma_2)^2 - 2\gamma_4(\gamma_3+\gamma_5) + 4\gamma_4^2$, $\iota_3 = \gamma_3^2 - 2\gamma_4^2 + \gamma_5^2$.

3.5. Geometric quantum discord

The geometric measure of quantum discord is defined as[40]

$$G\mathrm{QD}(\rho_{AB}) := \min_{\mathrm{x}\in\Omega} \|\rho - \chi\|, \tag{20}$$

where Ω means the set of zero-discord states, whose general form is defined by $\chi = \sum_k p_k \Pi_k^A \otimes \rho_k^B$ with $0 \le p_k \le 1 (\sum_k p_k = 1)$, and $\|\rho - \chi\|^2 = tr(\rho - \chi)^2$ means the square of the Hilbert–Schmidt norm. The analytical results of the present model are expressed as

$$\mathrm{GQD}_{12} = (\tau_1^2 + \tau_2^2 + \tau_3^2 + \chi_1^2 - \max(\tau_1^2, \tau_2^2, \tau_3^2 + \chi_1^2))/4, \tag{21}$$

$$\mathrm{GQD}_{23} = (\iota_1^2 + \iota_2^2 + \iota_3^2 + \chi_2^2 - \max(\iota_1^2, \iota_2^2, \iota_3^2 + \chi_2^2))/4, \tag{22}$$

where $\tau_1, \tau_2, \tau_3, \iota_1, \iota_2, \iota_3$ are the same as Eq. (18) and Eq. (19), and $\chi_1 = 4\gamma_1^2 + 4\gamma_2^2 - \gamma_3^2 - 6\gamma_4^2 - \gamma_5^2$, $\chi_2 = 2\gamma_1^2 - 2\gamma_2^2 + \gamma_3^2 - \gamma_5^2$.

3.6. Bell violation

The violation of Bell inequality is accepted as the existence of quantum nonlocality. The following equation is the Bell inequality corresponding to the Clauser–Horne–Shimony–Holt (CHSH) form[41,42]

$$B = a\cdot\sigma\otimes(b+b')\cdot\sigma + a'\cdot\sigma\otimes(b-b')\cdot\sigma, \tag{23}$$

where a, a', b, b' are the unit vectors in $\mathbb{R}^3$. The CHSH inequality can be written as

$$B = |\langle B_{\mathrm{CHSH}}\rangle| = |\mathrm{Tr}(\rho_{\mathrm{CHSH}})| \le 2,$$

in which the maximum violation of CHSH inequality is defined by

$$B_{\mathrm{CHSH}}^{\max} = \max_{a,a',b,b'} \mathrm{Tr}(\rho B_{\mathrm{CHSH}}). \tag{24}$$

The analytical result B of ρ_{12} and ρ_{13} is given by

$$B_{12} = \sqrt{\tau_1^2 + \tau_2^2 + \tau_3^2 - \lambda_{\min}}, \tag{25}$$

$$B_{23} = \sqrt{\iota_1^2 + \iota_2^2 + \iota_3^2 - \kappa_{\min}}, \tag{26}$$

here $\tau_1, \tau_2, \tau_3, \iota_1, \iota_2, \iota_3$ are also the same as before, and $\lambda_{\min} = \min(\tau_1^2, \tau_2^2, \tau_3^2)$, $\kappa_{\min} = \min(\iota_1^2, \iota_2^2, \iota_3^2)$.

4. Renormalized quantum correlation

According to the above-mentioned quantum correlation measures, the critical behavior of every quantity can be found by implementing the QRG method.

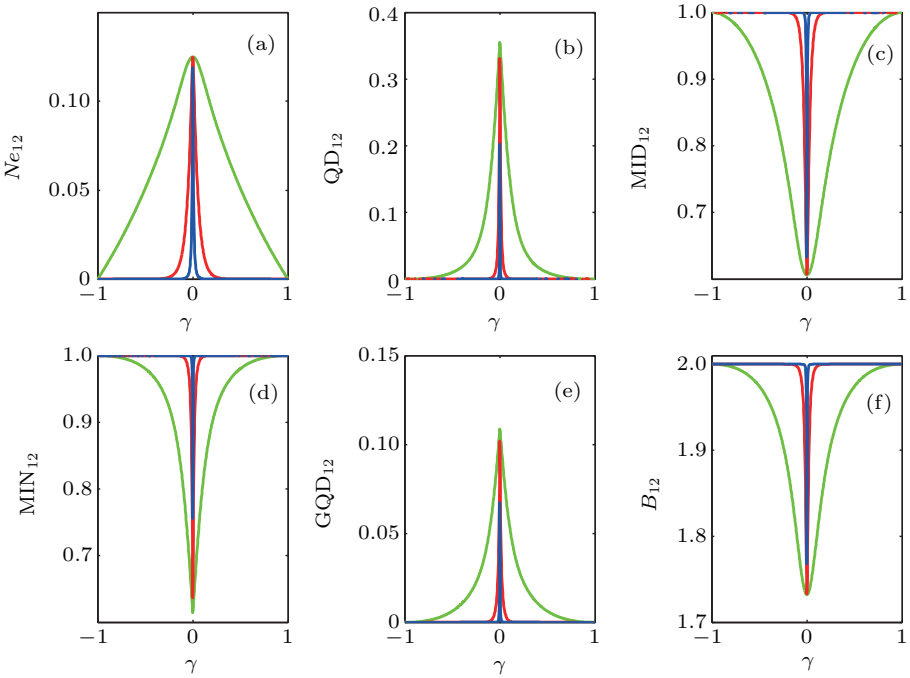


Fig. 2. (color online) Different quantum correlation measures as a function of γ for ρ_{12}. The green lines indicate 0th step QRG, the red lines indicate 1st step QRG, and the blue ones 2nd step QRG.

4.1. Behavior of different quantum correlation

The properties of different quantum correlation measures versus γ in terms of QRG iterations are plotted in Fig. 2. The plots cross each other at the critical point $\gamma_c = 0$. After two steps of renormalization, negativity will develop two saturated values, one that is nonzero for $\gamma_c = 0$ and one that is zero for $\gamma_c \neq 0$. QD and GQD have a similar property. The saturated value of MID, MIN and Bell inequality are different with them, namely one that is nonzero for $\gamma_c = 0$ and one that is 1 for $\gamma_c \neq 0$. We also have found that the block–block correlations of ρ_{12} will demonstrate QPT at the critical point $\gamma_c = 0$ and it cannot violate the CHSH inequality.

In Fig. 3, we illustrate the quantum correlation evolution of ρ_{23} versus γ for different QRG steps. The behavior of different quantum correlation measures are roughly the same with Fig. 2; however, there are also small differences, such as the saturated value.

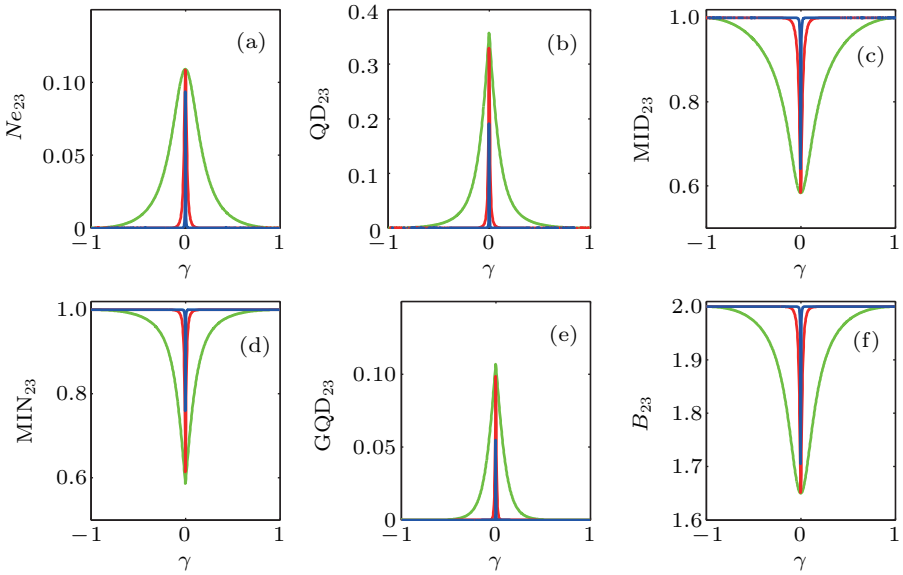


Fig. 3. (color online) Different quantum correlation measures as a function of γ for ρ_{23}. The plot colors mean the same as before.

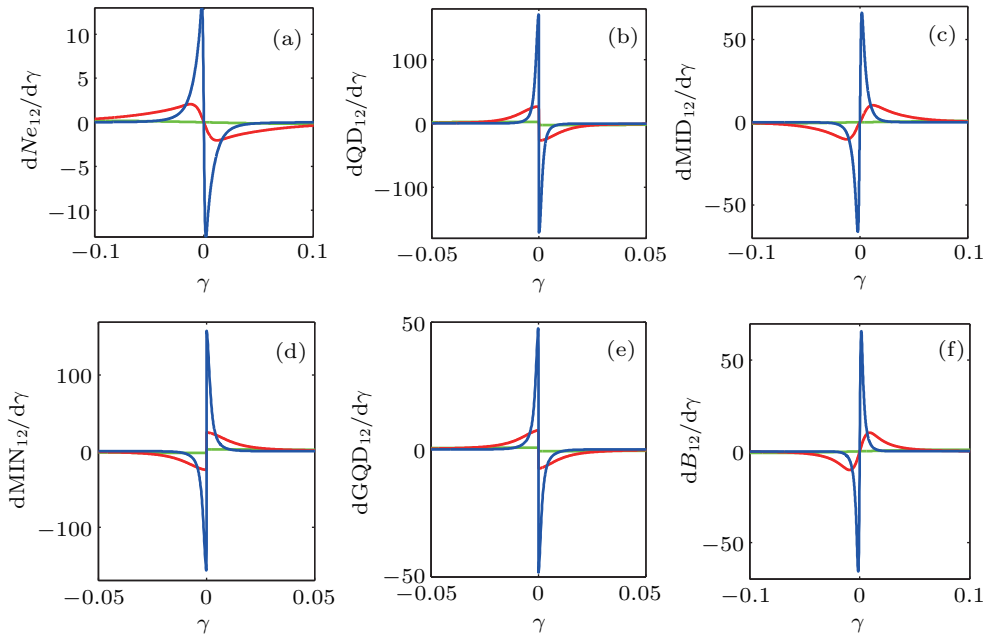


Fig. 4. (color online) Evolution of the first derivative of quantum correlation measures under QRG for state ρ_{12}. The plot colors mean the same as before.

4.2. Nonanalytic and scaling behavior

We have shown the first derivative of different quantum correlation measures (DQCM) versus γ in Fig. 4. From this figure, we notice that the derivative of quantum correlation diverges at the critical point $\gamma = 0$.[24] All the plots in the figure are the antisymmetrical function about $\gamma = 0$. There is a maximum and a minimum value for each plot. The peak value

becomes more pronounced near the critical point $\gamma = 0$. This property indicates that the two-dimensional XY system displays a second-order QPT. Comparing the six subgraphs, we find that the absolute peak values of QD and MIN are larger than the others.

Figure 5 shows the first derivatives of the quantum correlation measures as a function of γ after tracing out blocks 1, 4, and 5. We can find that the change rate and peak value of Fig. 5 are larger than those of Fig. 4.

We have found that the first derivative of DQCM will show the nonanalytic behavior at the critical point. We also display the property of $\ln \mathrm{dDQCM}/\mathrm{d}\gamma$ versus $\ln N$ in Fig. 6. The results shows that these quantities demonstrate scaling behavior. The scaling law is approximately $\mathrm{dDQCM}/\mathrm{d}\gamma_{\max} \sim N^{1.13}$ and $\mathrm{dDQCM}/\mathrm{d}\gamma_{\min} \sim N^{1.13}$ with exponent $\theta = 1.13$. At the critical point, the correlation length exponent ν reflects the behavior of correlation length ξ by $\xi \sim (\gamma - \gamma_c)^{-\nu}$. For the n-th QRG iteration the correlation length changes to $\xi_n \sim (\gamma_n - \gamma_c)^{-\nu}$, which leads to an expression

$$\left|\frac{\mathrm{d}\gamma_n}{\mathrm{d}\gamma}|_{\gamma_c} \sim N^{1/\mathrm{d}\nu}\right.$$

in terms of ν and the number of sites in each block.[43] Xu *et al.* pointed out that the exponent directly associates with the correlation length exponent, i.e., $\theta = 1/\nu \mathrm{d}$.[43] We have also noticed that the two exponents are generally identical, which means that the exponent will not change with the variation of quantum correlation measures. These results establish the relation between quantum information theory and condensed matter physics.[24]

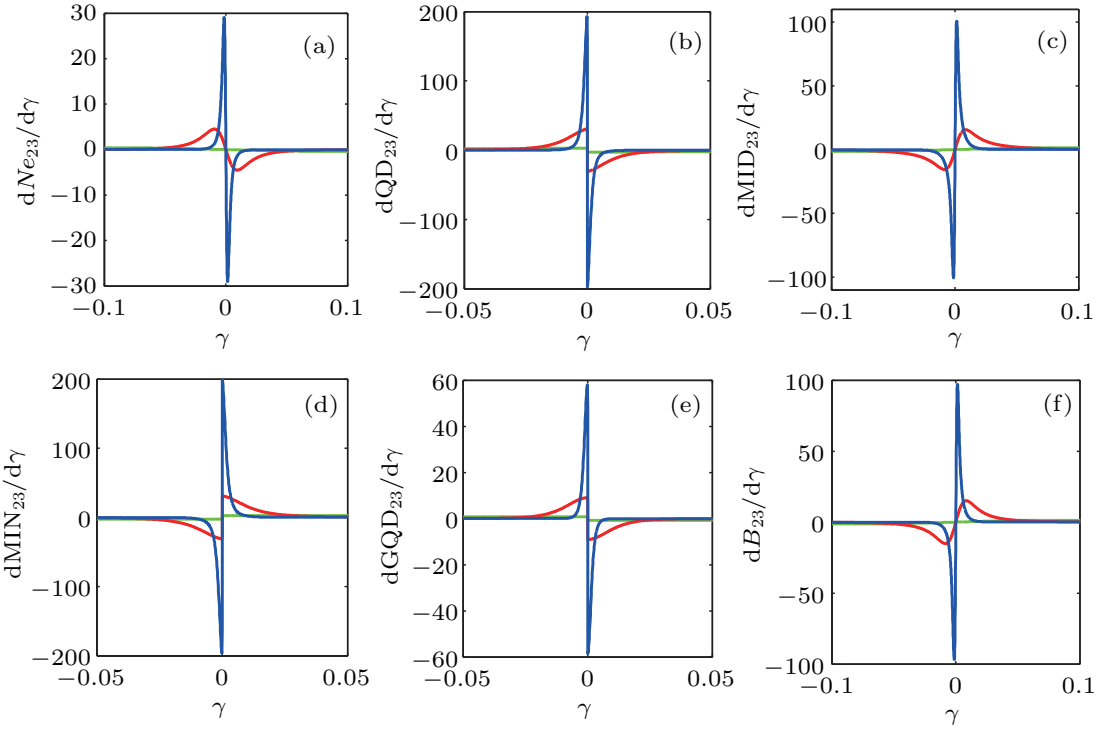


Fig. 5. (color online) Evolution of the first derivative quantum correlation measures under QRG for state ρ_{23}. The plot colors mean the same as before.

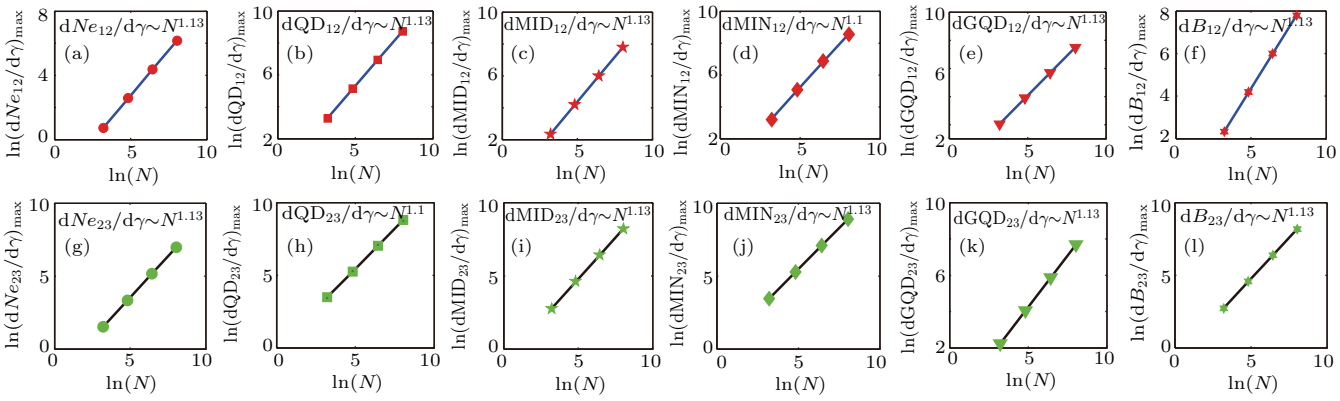


Fig. 6. (color online) The scaling behavior of different quantum correlations in terms of system size $\ln(N)$.

5. Monogamy relation in two-dimensional *XY* systems

Monogamy relation of entanglement[4] has been a subject in the quantum information processing over the years. It is worthwhile investigating whether the two-dimensional systems obey it. We also want to know the performance of the monogamy relation in detecting QPT. Here we select concurrence and QD as the quantity to investigate it. The concurrence[44] for a bipartite state is

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\},$$

where λ_k $(k = 1,2,3,4)$ are the square roots of the eigenvalues in descending order of the operator $R_{AB} = \rho_{AB}\widetilde{\rho}_{AB}$, $\widetilde{\rho}_{AB} = (\sigma_1^y \otimes \sigma_2^y)\rho_{AB}^*(\sigma_1^y \otimes \sigma_2^y)$. For the five-site block state, two different kinds of inequalities in terms of concurrence are[45–49]

$$C_{12}^2 + C_{13}^2 + C_{14}^2 + C_{15}^2 \leq C_{1|2345}^2, \tag{27}$$

$$C_{21}^2 + C_{23}^2 + C_{24}^2 + C_{25}^2 \leq C_{2|1345}^2, \tag{28}$$

where C_{ij} stands for the concurrence of the density matrix ρ with blocks other than i, j traced out, and $C_{i|jklm}$ stands for the concurrence between the subsystems ρ_i and ρ_{jklm}. The analytical expressions for C_{ij} can be computed through the above formula and $C_{i|jklm} = 2\sqrt{\rho_i}$. The difference between the two sides of inequality relation can be set as the residual entanglement that is

$$\delta_{1(2345)} = C_{1|2345}^2 - C_{12}^2 - C_{13}^2 - C_{14}^2 - C_{15}^2,$$

$$\delta_{2(1345)} = C_{2|1345}^2 - C_{21}^2 - C_{23}^2 - C_{24}^2 - C_{25}^2.$$

Similarly, we can derive the monogamy inequality of QD[7]

$$\mathrm{QD}_{12}^2 + \mathrm{QD}_{13}^2 + \mathrm{QD}_{14}^2 + \mathrm{QD}_{15}^2 \leq \mathrm{QD}_{1|2345}^2, \tag{29}$$

$$\mathrm{QD}_{21}^2 + \mathrm{QD}_{23}^2 + \mathrm{QD}_{24}^2 + \mathrm{QD}_{25}^2 \leq \mathrm{QD}_{2|1345}^2, \tag{30}$$

here QD_{ij} have a similar meaning to concurrence but stand for the quantum correlation, $\mathrm{QD}_{i|jklm} = S(\rho_i)$. So it is easy to obtain

$$\Delta_{1(2345)} = \mathrm{QD}_{1|2345}^2 - \mathrm{QD}_{12}^2 - \mathrm{QD}_{13}^2 - \mathrm{QD}_{14}^2 - \mathrm{QD}_{15}^2,$$

$$\Delta_{2(1345)} = \mathrm{QD}_{2|1345}^2 - \mathrm{QD}_{21}^2 - \mathrm{QD}_{23}^2 - \mathrm{QD}_{24}^2 - \mathrm{QD}_{25}^2.$$

Numerical simulations are performed for δ and Δ in Fig. 7. The difference of the monogamy relation is defined as the monogamy score. The curves of $\delta_{1(2345)}$, $\delta_{2(1345)}$, $\Delta_{1(2345)}$, and $\Delta_{2(1345)}$ also cross each other at $\gamma_c = 0$. This means that the residual entanglement and residual quantum correlation can be used as a good tool to investigate the QPT problem.[7] Moreover, we find that the concurrence and QD obey monogamously for this system.

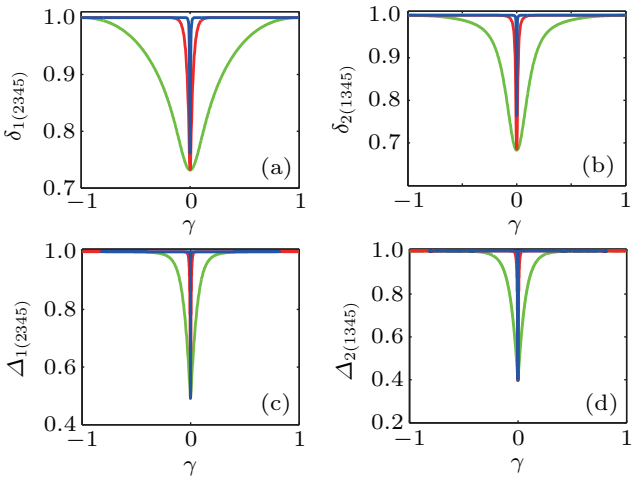


Fig. 7. (color online) The change of δ (a) and Δ (b) of the model versus γ at different QRG steps. The plot colors mean the same as before.

6. Conclusion

To summarize, we have studied the renormalization of quantum correlation and monogamy relation in the two-dimensional *XY* systems. As opposed to the one-dimensional case, the size of two-dimensional systems increase rapidly because we select a five-site as one block. Therefore, the critical point and the saturated values can be reached in the lesser number of the QRG iterations. The scaling behavior is investigated as the size of the model becomes large. Remarkably, we have obtained the identical critical exponent for entanglement, discord-like quantity, and Bell violation. Moreover, we have studied the multipartite quantum correlations with the monogamy of concurrence and monogamy of the quantum discord. It is shown that the two quantities are monogamous in these two-dimensional *XY* systems. Our results will help us deeply understand the quantum critical phenomena in the low-dimensional system by combining the quantum information theory.

Appendix A: Analytical expressions of γ

Analytical expressions of γ in Eqs. (4) and (5) are given below.

$$\gamma_1 = -\frac{(\alpha_1 + \gamma^2 - 1)\sqrt{\alpha_1 + 5\gamma^2 + 5}}{4\sqrt{2\alpha_2}},$$

$$\gamma_2 = -\frac{3\gamma\sqrt{\alpha_1 + 5\gamma^2 + 5}}{2\sqrt{2\alpha_2}}, \qquad \gamma_3 = \frac{\alpha_1 + \gamma^2 - 1}{\sqrt{2\alpha_2}},$$

$$\gamma_4 = \frac{\gamma(\alpha_1 + \gamma^2 + 5)}{2\sqrt{2\alpha_2}}, \quad \gamma_5 = 3\gamma^2\sqrt{\frac{2}{\alpha_2}},$$

$$\gamma_6=\frac{\sqrt{\dfrac{\gamma^2(\alpha_1+5\gamma^2+5)}{1+\alpha_1+34\gamma^2-\alpha_1\gamma^2+\gamma^4}}(-2-2\alpha_1+17\gamma^2-3\alpha_1\gamma^2+3\gamma^4)}{4(3+2\gamma^2+3\gamma^4)},$$

$$\gamma_7=-\frac{\sqrt{\dfrac{\gamma^2(\alpha_1+5\gamma^2+5)}{1+\alpha_1+34\gamma^2-\alpha_1\gamma^2+\gamma^4}}(1+2\alpha_1-\gamma^2+6\gamma^4)}{4\gamma(3+2\gamma^2+3\gamma^4)},\quad \gamma_8=-\frac{3\sqrt{\dfrac{\gamma^2(\alpha_1+5\gamma^2+5)}{1+\alpha_1+34\gamma^2-\alpha_1\gamma^2+\gamma^4}}(5-\alpha_1+5\gamma^2)}{4(3+2\gamma^2+3\gamma^4)},$$

$$\gamma_9=\frac{1+\alpha_1-\gamma^2}{4\gamma\sqrt{34-\alpha_1+\frac{1+\alpha_1}{\gamma^2}+\gamma^2}},\qquad \gamma_{10}=\frac{3}{2\sqrt{34-\alpha_1+\frac{1+\alpha_1}{\gamma^2}+\gamma^2}},$$

where

$$\alpha_1=\sqrt{1+34\gamma^2+\gamma^4},\qquad \alpha_2=2-2\alpha_1+71\gamma^2+17\alpha_1\gamma^2+104\gamma^4+3\alpha_1\gamma^4+3\gamma^6.$$

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