

Distance-based formation tracking control of multi-agent systems with double-integrator dynamics

Zixing Wu(吴梓杏), Jinsheng Sun(孙金生), Ximing Wang(王希铭)
Citation: Chin. Phys. B . 2018, 27(6): 060202. **doi:** 10.1088/1674-1056/27/6/060202

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Consensus for second-order multi-agent systems with position sampled data

Rusheng Wang(王如生), Lixin Gao(高利新), Wenhai Chen(陈文海), Dameng Dai(戴大蒙)
Chin. Phys. B . 2016, 25(10): 100202. **doi:** 10.1088/1674-1056/25/10/100202

Rigidity based formation tracking for multi-agent networks

Bai Lu, Chen Fei, Lan Wei-Yao
Chin. Phys. B . 2015, 24(9): 090206. **doi:** 10.1088/1674-1056/24/9/090206

Distributed H_∞ control of multi-agent systems with directed networks

Liu Wei, Liu Ai-Li, Zhou Shao-Lei
Chin. Phys. B . 2015, 24(9): 090208. **doi:** 10.1088/1674-1056/24/9/090208

Containment consensus with measurement noises and time-varying communication delays

Zhou Feng, Wang Zheng-Jie, Fan Ning-Jun
Chin. Phys. B . 2015, 24(2): 020203. **doi:** 10.1088/1674-1056/24/2/020203

Leaderless and leader-following consensus of linear multi-agent systems

Zhang Xiao-Jiao, Cui Bao-Tong, Lou Ke
Chin. Phys. B . 2014, 23(11): 110205. **doi:** 10.1088/1674-1056/23/11/110205

Distance-based formation tracking control of multi-agent systems with double-integrator dynamics*

Zixing Wu(吴梓杏), Jinsheng Sun(孙金生)†, and Ximing Wang(王希铭)

School of Automation, Nanjing University of Science and Technology, Nanjing 210094, *China*

(Received 26 December 2017; revised manuscript received 25 March 2018; published online 10 May 2018)

This paper addresses the distance-based formation tracking problem for a double-integrator modeled multi-agent system (MAS) in the presence of a moving leader in d-dimensional space. Under the assumption that the state of leader can be obtained over fixed graphs, a distributed distance-based control protocol is designed for each double-integrator follower agent. The protocol consists of three terms: a gradient function term, a velocity consensus term, and a leader tracking term. Different shape stabilizing functions proposed in the literature can be applied to the gradient function term. The proposed controller allows all agents to both achieve the desired shape and reach the same velocity with moving leader by controlling the distances and velocity. Finally, we analyze the local asymptotic stability of the equilibrium set with center manifold theory. We validate the effectiveness of our approach through two examples.

Keywords: multi-agent system, formation control, distributed control

PACS: 02.30.Yy, 02.30.Em, 05.65.+b, 02.10.Ox **DOI:** 10.1088/1674-1056/27/6/060202

1. Introduction

Over the past decade, formation control of MAS is an active research area in cooperative control and robotics due to its widespread applications in a variety of areas, such as satellite formation flying, search missions using flying agents, cooperative transportation, sensor networks, and so forth.[1,2] The aim of multi-agent formation control is to drive a group of agents to achieve and to maintain a desired graphic pattern with (or without) references.

As reviewed in Ref. [3], the existing control strategies for multi-agent formation control can be classified into position-based, displacement-based, and distance-based strategies. Among these formation strategies, distance-based control strategy assumes that agents sense and control the relative distance of their neighboring agents, which allows the reduced requirements for the sensing capabilities for each agent. In other words, compared with the other two strategies, the main advantages of the distance-based control lie in that agents need less global information and do not require expensive sensors.

We focus only on works based on distance-based formations in this paper. A gradient control law based on the potential function was proposed in Refs. [4] and [5], and the target undirected formation was defined by a set of inter-agent distances. After that, the authors in Ref. [6] extended the result in Ref. [5] to n-dimensional space and proposed gradient-based formation controllers for single-integrator and double-integrator modeled agents. A unified approach formation controller with general forms of gradient functions was developed in Ref. [7] to guarantee the exponential stability of the distance-based rigid formation control MASs. In Ref. [8], the authors considered the formation tracking problem under a rigidity framework for single-integrator modeled MAS. Among the aforementioned literature, one common feature is that the controllers employ objective functions to share shape control tasks between agents. Designing a potential function is a good way to show the constraint relation between each agent in the formation and easy to achieve real-time control.

Note that the results above primarily concentrate on the ideas of distance-based formations for single-integrator modeled agents. The research on distance-based control of double-integrator modeled agents has attracted attention, but most of them require the velocities of agents to decay to zero (e.g., Refs. [6] and [9]). Motivated by the pioneering work,[10] recent papers[11,12] combined the flocking strategy and the distance-based shape control strategy so as to show that the desired formation of double-integrator modeled agents can be achieved while all agents share a common velocity. As listed in Table 1, the distance-based formation for single-integrator modeled MAS are optimally solved via a gradient function that depends on formation distance error systems, and then the results can be extended to double-integrator modeled MAS by adding velocity attenuation or consensus term. It should be pointed out that the extension to distance-based formation tracking control has not been addressed in the literature. Motivated by the fact that formations in movement have more practical applications in various areas such as security patrols,[23,24] the formation tracking control problem will be considered in this paper.

Therefore, the motivation is to develop a new formation

*Project supported by the National Natural Science Foundation of China (Grant No. 61603188).

†Corresponding author. E-mail: jssun67@163.com

© 2018 Chinese Physical Society and IOP Publishing Ltd

tracking control strategy with a gradient function for followers. The gradient function will be considered as a general form which can encompass the most distance-based shape stabilizing controllers with different forms proposed in the literature. The main contributions of this paper can be summarized as follows. Firstly, a procedure to design distance-based control laws for the formation tracking problem will be constructed. We will give a characterization of shape stabilizing gradient functions for undirected distance-based formation and list the required conditions for distributed gradient-based controllers over a given undirected graph $\mathcal{G}$. Hence if there exists a potential function which satisfies the qualifications, the corresponding controller can achieve formation tracking. The recent findings in Table 1 will be extended to agents modeled by double integrator dynamics with a moving leader. Also, a novel control function will be illustrated and validated. Secondly, to address the problem of motion and formation control as a whole, we will propose a strategy based on the gradient method to design a motion not only in a plane but also in three-dimensional (3D) space. The asymptotic stability of the double-integrator MAS with a leader under the proposed distributed controller will be investigated. The LaSalle invariance principle will be employed to analyze the equilibrium set. Since the equilibrium set is not uniquely defined, the center manifold theory will be used to determine the stability when linearizing an equilibrium point, such that the distance-based formation control with a leader is solvable.

Table 1. Review of distance-based undirected formation with gradient-flow method.

System model	Objective function V	References
Single-integrator	$\frac{1}{2}\sum_{(i,j)\in\varepsilon}(\|q_i-q_j\|^2-d_{ij}^{*})^2$	[4], [5], [18], [19]
	$\frac{1}{4}\sum_{(i,j)\in\varepsilon}(\|q_i-q_j\|^2-d_{ij}^{*})^2$	[13], [16], [17], [21]
	$\frac{1}{2}\sum_{(i,j)\in\varepsilon}\frac{\|q_i-q_j\|^2-d_{ij}^2}{\|q_i-q_j\|^2}$	[14]
	$\sum_{(i,j)\in\varepsilon}\frac{(\|q_i-q_j\|^2-d_{ij}^2)^2}{\|q_i-q_j\|^2}$	[15]
	$\frac{1}{2(\alpha+1)}\sum_{(i,j)\in\varepsilon}\|\|q_i-q_j\|^2-d_{ij}^2\|^{1+\alpha}$	[20]
Double-integrator	$\frac{1}{2}\sum_{(i,j)\in\varepsilon}(\|q_i-q_j\|^2-d_{ij}^{*})^2+\frac{1}{2}\sum_{i\in\mathcal{V}}\|p_i\|^2$	[4], [5], [6]
	$\frac{1}{4}\sum_{(i,j)\in\varepsilon}(\|q_i-q_j\|^2-d_{ij}^{*})^2+\frac{1}{2}\sum_{i\in\mathcal{V}}(p_i-p_r)^{\mathrm{T}}(p_i-p_r)$	[21]
	$\frac{1}{4}\sum_{(i,j)\in\varepsilon}(\|q_i-q_j\|^2-d_{ij}^{*})^2+\frac{1}{2}\sum_{i\in\mathcal{V}}\|p_i\|^2$	[22]

This paper is organized as follows. Section 2 recalls some basic concepts on graphs and shape control. Section 3 formulates the main problem to be addressed. In Sections 4 and 5, detailed analysis on the proposed controller and stability of the formation tracking algorithm are provided, respectively. Section 6 presents two sets of simulation examples to demonstrate the controller performance. A conclusion is then given in Section 7.

2. Preliminaries

2.1. Basic concepts in graph theory and rigidity theory

An undirected graph with n agents and m edges is denoted by $\mathcal{G}=(\mathcal{V},\varepsilon)$ with nonempty node set $\mathcal{V}=\{1,\ldots,n\}$ and edges set $\varepsilon\subset\mathcal{V}\times\mathcal{V}$. j is called a neighbour of i if $(i,j)\in\varepsilon$, and the neighbor set of node i is denoted by $\mathcal{N}_i=\{j|(i,j)\in\varepsilon\}$. The weighted adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined such that a_{ij} is a positive weight if $(i,j)\in\varepsilon$, and $a_{ij}=0$ otherwise. For an undirected graph, $a_{ij}=a_{ji},\forall i\neq j$. The Laplacian matrix of $\mathcal{G}$ is defined as $\boldsymbol{L}_n=[l_{ij}]\in\mathbb{R}_{n\times n}$, where $l_{ii}=\sum_{i=1}^{n}a_{ij}$, $l_{ij}=-a_{ij}$, $i\neq j$. Note that if $(j,i)\notin\varepsilon$ then $l_{ij}=-a_{ij}=0$.

Consider n single-integrator modeled agents in d-dimensional space

$$\dot{q}_i=u_i,\quad i\in\mathcal{V},\tag{1}$$

where $q_i\in\mathbb{R}^d$ and $u_i\in\mathbb{R}^d$ denote the position and control input, respectively, of agent i with respect to a global coordinate system.

A graph that has a unique realization cannot be susceptible to continuous flexing and must be rigid.[25] A detailed discussion of graph rigidity can be found in Ref. [25]. One useful tool to characterize some other concepts related to graph rigidity is the rigidity matrix $J_{g_q}(q)$. We define the rigidity function associated with the framework $(\mathcal{G},q)$ as the function $g_{\mathcal{G}}(q)$: $\mathbb{R}^{dn}\to\mathbb{R}^m$ given by

$$g_{\mathcal{G}}(q)=\frac{1}{2}\left[\ldots,z_{kij}^2,\ldots\right]^{\mathrm{T}}\quad(i,j)\in\varepsilon.\tag{2}$$

The k-th component of $g_{\mathcal{G}}(q)$, $\|q_i-q_j\|^2$, coincides with the

relative distance for the k-th edge $z_{kij} = \|q_i - q_j\|$, which connects agents i and j. The Jacobian of function $g_{\mathcal{G}}(q)$ is the gradient of $g_{\mathcal{G}}(q)$ and we denoted it by $J_{g_{\mathcal{G}}}(q) = \partial(g_{\mathcal{G}}(q))/\partial q$. The matrix $J_{g_{\mathcal{G}}}(q)$ is referred to as the rigidity matrix of $(\mathcal{G}, q)$.

Lemma 1 [26] A framework $(\mathcal{G}, q)$ is infinitesimally rigid in n-dimensional space if and only if

$$\operatorname{rank}\big(J_{\mathcal{G}_q}(q)\big) = dn - d(d+1)/2. \tag{3}$$

Let $q^* \in \mathbb{R}^{dn}$ be given. Let $d_{ij}^* := \|q_j^* - q_i^*\|$ and $e_{ij} := \|q_i - q_j\|^2 - d_{ij}^{*2}$ denote, respectively, the desired distance and distance error of edge k which links the agents i and j in graph $\mathcal{G}$. The desired formation frame can be defined as a set of inter-agent distance realizations that are related to q^*:

$$E_{q^*} := \{q \in \mathbb{R}^{dn} : \|q_j - q_i\| = d_{ij}^*, (i,j) \in \varepsilon\}.$$

Then, the distance-based formation shape control problem of the single-integrator (1) MAS is equivalent to designing a decentralized control law such that E_{q^*} is asymptotically stable under the control law.

The gradient-based method is widely used to achieve the desired formation E_{q^*}. One can construct a local potential function V_i for agent i:

$$V_i = \frac{1}{2}\sum_{j\in\mathcal{N}_i} \psi_{ij}(\|q_j - q_i\|), \tag{4}$$

where the function ψ_{ij} is defined as follows.

Definition 1 The function ψ_{ij} is a differentiable, nonnegative function of $\|q_i - q_j\|$ satisfying the following conditions:

(i) $\psi_{ij}(e_{ij}) = 0$ when $\|q_i - q_j\|$ is equal to the desired distance d_{ij}^*, and $\psi_{ij} > 0$ for all $e_{ij} \neq 0$;

(ii) $\psi_{ii} = c$ for $i = 1, \ldots, n$, where c is a positive constant.

Define

$$\rho_{ij} := \frac{\partial \psi_{ij}(e_{ij})}{\partial e_{ij}}. \tag{5}$$

Note that $\rho_{ij} = \rho_{ji}$, for all $(i,j) \in \mathcal{V}, i \neq j$. Thus, the potential function V can be regarded as a common objective function which is employed to share a task between agents. The generalized gradient control law for the single-integrator (1) takes the following form:

$$\begin{aligned} u_i = -\nabla_{q_i} V_i = & -\frac{1}{2}\sum_{j\in\mathcal{N}_i} \frac{\partial \psi_{ij}(e_{ij})}{\partial e_{ij}} \frac{\partial e_{ij}}{\partial q_i} \\ = & -\sum_{j\in\mathcal{N}_i} (q_i - q_j)\rho_{ij}, i \in \mathcal{V}. \end{aligned} \tag{6}$$

Each agent may take different potential function ψ_{ij} by controlling inter-agent distances to achieve an anticipated target formation, which means that the function ρ_{ij} in controller (6) can take different forms. Some typical gradient-based control laws proposed in the literature are summarized in Table 1. The most commonly-used controller takes the following form

$$u_i = -\sum_{j\in\mathcal{N}_i} (q_i - q_j)(\|q_j - q_i\|^2 - d_{ij}^{*2}), \tag{7}$$

where the potential function in the form of $\psi_{ij} = (1/2)e_{ij}^2$ and ρ_{ij} is in the form of $\rho_{ij}(e_{ij}) = e_{ij}$.

3. Problem statement

3.1. Agent dynamics

Consider that the MAS consists of n double-integrator modeled agents and an additional agent labeled $n+1$, which acts as the collective objective or the unique leader of the team. For ease of development, let the first n agents indexed by $1, 2, \ldots, n$ be the followers and the agent indexed by $n+1$ be the team leader. The motion of follower i is given by

$$\dot{q}_i = p_i, \quad \dot{p}_i = u_i, \quad i \in \mathcal{V}, \tag{8}$$

where $q_i \in \mathbb{R}^d$, $p_i \in \mathbb{R}^d$, and $u_i \in \mathbb{R}^d$ denote the position, velocity, and control input, respectively, of follower i with respect to a global coordinate system. The motion of the leader is given a priori as a reference, and it is assumed that the velocity of the leader is piecewise continuously differentiable,

$$\dot{q}_r = p_r, \quad \dot{p}_r = u_r(q_r, p_r). \tag{9}$$

The pair $(q_r, p_r) \in \mathbb{R}^d \times \mathbb{R}^d$ is the state of the leader. The selection of $u_r(q_r, p_r)$ is part of the tracking control design for the collective objective of a group of followers. The choice of u_r leads to a corresponding motivation. For example, $u_r \equiv 0$ means the leader moves along a straight line with the desired velocity $p_r(0)$.

3.2. The distance-based formation tracking control problem

Denote by $\overline{\mathcal{G}}_{n+1} \triangleq (\overline{\mathcal{V}}, \overline{\varepsilon})$ the interaction topology among $n+1$ agents. Similarly, $\overline{\mathcal{A}}_{n+1}$ and $\overline{L}_{n+1}$ are, respectively, the adjacency matrix and the Laplacian matrix associated with $\overline{\mathcal{G}}_{n+1}$. Furthermore, the Laplacian matrix of graph $\overline{\mathcal{G}}_{n+1}$ can be written as follows:

$$\overline{\boldsymbol{L}}_{n+1} := \begin{bmatrix} \boldsymbol{L}_n & \boldsymbol{L}_{12} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix}, \tag{10}$$

where $\boldsymbol{L}_{12} \in \boldsymbol{R}^{n\times 1}$, $\boldsymbol{L}_{21} \in \mathbb{R}^{1\times n}$, and $\boldsymbol{L}_{22} \in \mathbb{R}^{1\times 1}$.

Let $p^* \in \mathbb{R}^{dn}$ be given and define the desired formation E_{q^*,p^*} for followers (8) as

$$E_{q^*,p^*} := \{[q^{\mathrm{T}}, p^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{2dn} : \|q_j - q_i\| = d_{ij}^*, p = 0, (i,j) \in \varepsilon\}.$$

Denote the desired distance and the distance error between the leader and follower i as: $d_{ir}^* := \|q_i^* - q_r\|$ and $e_{ir} := \|q_i - q_r\|^2 - d_{ir}^{*2}$, respectively. In order to achieve the goal of tracking the leader while maintaining the relative inter-follower position, it is assumed that each follower can detect the relative position of the leader, thus agents in target formation should satisfy the following constraints.

1) Inter-follower positions: $\|q_j - q_i\| = d_{ij}^*, \forall\, (i,j) \in \varepsilon$;

2) Follower-to-leader positions: $\|q_i - q_r\| = d_{ir}^*$, $\forall\, i \in \mathcal{V}$.

Let

$$\overline{p} = \left[p^{\mathrm{T}}, p_r^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{d(n+1)}, \quad \overline{q} = \left[q^{\mathrm{T}}, q_r^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{d(n+1)},$$

if the real formation $\overline{q}(t)$ converges to the target formation $\overline{q}^*(t)$, the desired formation pattern can be simultaneously achieved. The control objective is to design the suitable control laws for followers to drive $\overline{e}_{ij}(t) = \|\overline{q}_i - \overline{q}_j\|^2 - d_{ij}^{*2} \to 0$ and velocity error $p_i(t) - p_r(t) \to 0$ as $t \to \infty$. For a given realization

$$\overline{q}^{\,*} = \left[q_1^{*\mathrm{T}} \cdots q_n^{*\mathrm{T}} q_r^{*\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{d(n+1)},$$

we define the desired formation

$$\begin{aligned} \Omega \;&= \left\{ \left[\overline{q}^{\mathrm{T}} \overline{p}^{\mathrm{T}}\right] \in \mathbb{R}^{2d(n+1)} : \|\overline{q}_j - \overline{q}_i\| \right. \\ &\quad \left. = d_{ij}^*,\ \overline{p}_i = p_r, i, j \in \overline{\mathcal{V}} \right\}. \end{aligned} \tag{11}$$

In this paper, the control objective for a group of double-integrator modeled agents is to achieve a formation that satisfies a given set of desired distance constraints and moving with the leader. Before starting the main problem, we list the assumption as follows.

Assumption 1 In the framework of the formation tracking problem, the tracking objective defined by the leader is given in advance, which is global information. The sensing graph of the n followers and the leader is given by an undirected graph $\overline{\mathcal{G}}_{n+1}$.

Then, the distance-based formation tracking control problem is stated as follows.

Problem 1 For n double-integrator followers (8) and a leader modeled with Eq. (9) in the d-dimensional space. Given a realization $\overline{p}^* \in \mathbb{R}^{d(n+1)}$, satisfying Assumptions 1, design a suitable distributed tracking control law on the basis of the target formation $\overline{q}^*$ and reference motion in Eq. (9) such that Ω is asymptotically stable under the control law.

4. Distance-based distributed formation tracking control law

4.1. Description of the desired formation tracking control law

We employ the gradient control laws and construct distance potential functions for stabilizing rigid formation shapes. To achieve the tracking control target, the control input for follower $i \in \mathcal{V}$ can be divided into three terms

$$u_i = f_i^{\mathrm{g}} + f_i^{\mathrm{c}} + f_i^{\mathrm{r}}, \tag{12}$$

where f_i^{g} is a gradient-based term, f_i^{c} is a velocity consensus term, and f_i^{r} is a tracking term that acts as a navigational feedback due to the leader, respectively. More specifically, the details are as follows.

i) The gradient-based control term can be constructed as

$$f_i^{\mathrm{g}} = -k_q \nabla_{q_i} V_i = -k_q \sum_{j \in \mathcal{N}_i} \rho_{ij}(q_i - q_j), \tag{13}$$

where k_q is a positive constant, and ρ_{ij} is defined in Eq. (5).

ii) For each agent, the velocity consensus equation, defining how the neighbor of each agent influences and changes its velocity, is

$$f_i^{\mathrm{c}} = -k_p \sum_{j \in \mathcal{N}_i} a_{ij}(p_i - p_j), \tag{14}$$

where k_p is a positive constant.

iii) The tracking term f_i^{r} is considered as a navigational feedback, which is given by

$$f_i^{\mathrm{r}} = u_r - c_1(q_i - q_r)\rho_{ir} - c_2(p_i - p_r), \tag{15}$$

where c_1, c_2 are positive constant control gains and $\rho_{ir} = \partial \psi_{ir} / \partial e_{ir}$ is a gradient function regarding the distance error between leader and followers with $e_{ir} = \|q_r - q_i\|^2 - d_{ir}^{*2}$. The accelerated velocity of leader u_r is globally known.

Remark 1 The velocity consensus term f_i^{c} can be written in a matrix form, which is

$$f^{\,\mathrm{c}} = -k_p \mathcal{L}_n p. \tag{16}$$

where $\mathcal{L}_n = L_n \otimes I_d$ and $\otimes$ denotes the Kronecker product.

Remark 2 The potential function V, defined in Eq. (4), can be regarded as a common objective function which is employed to share a task between agents. The gradient-based term in Eq. (13) is to achieve the shape control between the followers. Besides the potential functions in the existing work shown in Table 1, other types of potential function for formation shape control can be constructed. For example, one can consider the following gradient-based controller

$$f_i^{\mathrm{g}} = -\sum_{j \in \mathcal{N}_i} (q_i - q_j) tanh(\|q_j - q_i\|^2 - d_{ij}^{*2}). \tag{17}$$

In this case, the function ρ is constructed as

$$\rho_{ij} = \tanh(e_{ij}), \tag{18}$$

which is continuously differentiable and strictly monotone.

4.2. Collective dynamics

To facilitate the subsequent analysis, we use the following artificial potential function to show the sum of energy of all agents and the leader:

$$\begin{aligned} Q_i(q) &= k_q V_i + c_1 V_{ri} \\ &= k_q \sum_{j \in \mathcal{N}_i} \psi(e_{ij}) + c_1 \sum_{(i,n+1) \in \varepsilon} \psi(e_{ir}). \end{aligned} \tag{19}$$

Then, the controller (12) can be expressed as

$$u_i = u_r - \nabla_{q_i} Q_i(q) - k_p \sum_{j \in \mathcal{N}_i} a_{ij}(p_i - p_j) - c_2(p_i - p_r). \tag{20}$$

Let the following vector Φ be the gradients of potential functions of edges

$$\Phi(\bar{e}) = [k_q\rho_1(e_1), k_q\rho_2(e_2), \ldots, k_q\rho_m(e_m), c_1\rho_{m+1}(e_{1r}), \ldots, c_1\rho_l(e_{nr})], \tag{21}$$

where m and l are the numbers of edges in graph $\mathcal{G}$ and $\overline{\mathcal{G}}$. Noting that

$$\frac{\partial \psi_k(e_k)}{\partial q_i} = \rho_k(e_k)\frac{\partial e_k}{\partial q_i}$$

and combining the expression of $Q(x)$ with the rigidity matrix $J_{g_{\mathcal{G}}}(\overline{q})$, one can derive the following form of the overall shape controller

$$\nabla Q(q) = k_q \nabla_q V(q) + c_1 \nabla_q V_r(q) = J_{g_q}(\overline{q})^{\mathrm{T}} \Phi(\bar{e}). \tag{22}$$

Then, the MAS model combining the controller (12) can be written as

$$\begin{cases} \dot{\overline{q}} = \overline{p}, \\ \dot{\overline{p}} = u_r - \left(\begin{bmatrix} \boldsymbol{L}_n & \boldsymbol{L}_{12} \\ 0 & 0 \end{bmatrix} \otimes I_d \right) \overline{p} - \begin{bmatrix} I_{dn} & 0 \\ 0 & 0 \end{bmatrix} \nabla Q(\overline{q}). \end{cases} \tag{23}$$

The system (23) is aligned to a global system. However, followers do not necessarily know the origin of the global coordinate system. One can further transform the origin into a moving frame according to the leader. The position and velocity of follower i in the moving frame are given by

$$\begin{cases} x_i = q_i - q_r, \\ v_i = p_i - p_r. \end{cases} \tag{24}$$

Let $q_{ij} = q_i - q_j$ and $x_{ij} = x_i - x_j$. The relative positions and velocities remain the same in the moving frame, i.e., $x_{ij} = q_{ij}$, $v_j - v_i = p_j - p_i$. Thus, $e_{ij}(q_{ij}) = e_{ij}(x_{ij})$ and $\dot{v}_i = u_i - u_r$.

Using the control protocol (12) in the moving frame, the closed-loop MAS can be expressed as

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = -k_q \sum\limits_{j\in\mathcal{N}_i} \nabla_{x_i} \psi(e_{ij}(x_{ij})) - c_1 \nabla_{x_i} \psi(e_{ir}(x_i)) \\ \qquad - k_p \sum\limits_{j\in\mathcal{N}_i} a_{ij}(v_i - v_j) - c_2 v_i. \end{cases} \tag{25}$$

The system is converted to a matrix form:

$$\begin{cases} \dot{x} = v, \\ \dot{v} = -k_q \nabla \tilde{V}(x) - k_p \mathcal{L}_n v - c_1 \nabla \tilde{V}_r(x) - c_2 v, \end{cases} \tag{26}$$

where $x \in \mathbb{R}^{dn}$ and $v \in \mathbb{R}^{dn}$ are respectively the stacked vectors of all x_i and v_i. Similarly, the collective potential functions satisfy $\tilde{V}_i(x_{ij}) = V_i(q_{ij})$ and $\tilde{V}_{ri}(x_i) = V_{ri}(q_{ir})$, respectively.

Let

$$\tilde{Q}(x) = k_q \tilde{V}(x) + c_1 \tilde{V}_r(x), \tag{27}$$

then the collective dynamics of agents can be decomposed as the following structural dynamics:

$$\begin{cases} \dot{x} = v, \\ \dot{v} = -\nabla \tilde{Q}(x) - Dv, \end{cases} \tag{28}$$

where $D = c_2 I_{dn} + k_p \mathcal{L}_n$.

5. Stability analysis

5.1. Equilibrium set

Before presenting the result on the stability analysis of distance-based formation behavior under control algorithm (12), we define the Hamiltonian function of system (25) as

$$H(x,v) = 1/2(\tilde{Q}(x) + K(v)),$$

where $K(v) = \sum_i \|v_i\|^2$ is a velocity mismatch function. Note that $H(x,v)$ is a nonnegative valued function. Then, we present the following theorem which summarizes the stability analysis result.

Theorem 1 Consider a group of double-integrator agents in the form of Eqs. (8) and (9), and apply the tracking control law (12) to the structural dynamics (25). Suppose that $\tilde{Q}(x(0))$ and $K(v(0))$ are finite. $\Omega_c = \{(x,v) : H(x,v) \le c\}$ is defined as a level-set of Hamilton function $H(x,v)$ of system (28) such that for any solution starting in Ω_c the agents achieve the target formation in $t \geqslant 0$. Then, the following statements hold.

(I) All agents asymptotically move with the same velocity.

(II) Almost all solutions of system (25) asymptotically converge to an equilibrium point x_λ^*, where x_λ^* is minima of $\tilde{Q}(x)$.

Proof Part (I) Note that by Eq. (27), we have

$$\nabla_x \left(\frac{1}{2} \sum_{i=1}^{n} \tilde{Q}_i \right) = \sum_{i=1}^{n} (k_q v_i^{\mathrm{T}} \nabla_x \tilde{V}_i(x_{ij}) + c_1 v_i^{\mathrm{T}} \nabla_x \tilde{V}_{ri}(x_i)).$$

Hence, the Hamilton $H(x,v) = 1/2(\tilde{Q}(x) + K(v))$ of structure dynamics (28) is a strictly dissipative particle system due to the fact that

$$\begin{aligned} \dot{H}(x,v) &= \frac{1}{2} \sum_{i=1}^{n} \dot{\tilde{Q}}_i + \sum_{i=1}^{n} v_i^{\mathrm{T}} \dot{v}_i \\ &= -v^{\mathrm{T}} [(c_2 I_n + \boldsymbol{L}) \otimes I_d] v \le 0 \end{aligned} \tag{29}$$

for all $v \neq 0$. Therefore, the structure energy function is monotone decreasing for all (x,v) and its maximum value equals to its initial value

$$H(x,v) \le H_0 := H(x(0), v(0)) < \infty.$$

This means that $H(x(t), v(t)) \le c$ for all $t \ge 0$. Thus, one has that $\left\{ \left[x^{\mathrm{T}}, v^{\mathrm{T}} \right] \in \mathbb{R}^{2dn} : H \le H_0 \right\}$ is a compact invariant set.

From LaSalle's Invariance principle, all the solutions of structure dynamics (28) starting from Ω_c converge to the largest invariant set in $\{(x,v)\in\Omega_c : \dot{H}=0\}$. From Eq. (29), we have

$$\dot{H}(x,v) = -v^{\mathrm{T}}\left[(c_2 I_n + \boldsymbol{L})\otimes I_d\right]v, \tag{30}$$

where $\boldsymbol{L}$ is a positive semi-definite matrix. Hence $\dot{H}=0$ is equivalent to $v=0$, which occurs only when

$$p_1 \equiv p_2 \equiv \cdots \equiv p_n \equiv p_r. \tag{31}$$

Thus, all agents asymptotically match in the reference frame.

Part (II) From Eqs. (9) and (31), we see that, there holds

$$\dot{p}_1 \equiv \dot{p}_2 \equiv \cdots \equiv \dot{p}_n \equiv \dot{p}_r = u_r(q_r, p_r), \tag{32}$$

which implies that $u_i = u_r(q_r,p_r)$ in steady state. It thus follows from Eqs. (20) and (19) that

$$\nabla_q Q(q) = \nabla_x \tilde{Q}(x) = 0. \tag{33}$$

This implies followers show that all trajectories coverage to the set Ω_c:

$$\Omega_c = \{(x,v) : \nabla_x \tilde{Q}(x) = 0, K(v) = 0\}. \tag{34}$$

If the solution of Eq. (33) starts at the equilibrium, which is local maxima, the solution will be invariant at all times. Thus, the configuration converges asymptotically to a fixed configuration that is an extremum of $Q(q)$. This proves that almost all solutions of the particle system asymptotically converge to an equilibrium point $(x_\lambda^*, 0)$, where x_λ^* is a local minima of aggregate potential function $\tilde{Q}(x)$. ■

Unfortunately, when $\nabla\tilde{Q}(x)=0$, the interagent distances do not necessarily meet expectations, which means that the invariant set Ω_c cannot amount to the target equilibrium set Ω (shown in Eq. (11)). The desired formation Ω in Eq. (11) can be presented as

$$\begin{aligned}\Omega_1 :&= \{(x,v) : \bar{e}(x) = 0 \quad v = 0\}\\ &\equiv \{(q,p) : \Phi(\bar{e}(q)) = 0 \quad p_i = p_r\}.\end{aligned} \tag{35}$$

These are not the only equilibria of Eq. (28). Since $\nabla Q(q) = J_{g_q}(q)^{\mathrm{T}}\Phi(\bar{e}(q))$, there is a larger set of equilibria

$$\Omega_2 :=\{(q,p) : J_{g_{\bar{q}}}(\bar{q})^{\mathrm{T}}\Phi(\bar{e}(x)) = 0 \quad v=0\}. \tag{36}$$

The matrix $J_{g_{\bar{q}}}(\bar{q}) = J_{g_x}(x)$ has a kernel if and only if rank $(\bar{q}) < d$, that is, some edges $z_{kij} = \|\bar{q}_i - \bar{q}_j\| = 0$. Hence, if a point q is an equilibrium in Ω_2, for all $i\in\mathcal{V}$, then one has that $\|z_{kij}\| = d_{ij}^*$ or $\|z_{kij}\| = 0$.

Remark 3 Actually, for different initial conditions, the final formation can be flipped over, as shown in Fig. 1. Since the target formation cannot be uniquely defined only by interagent distances, the formation control system exists in multiple equilibrium orbits. Thus, due to the realization of flip formations, in general, $\Omega_2 \neq \Omega_c$. Obviously, we have $\Omega_1 \subset \Omega_2 \subset \Omega_c$.

It should also be mentioned that if the agents are initialized on a collinear configuration in two-dimensional (2D) space, or on a coplanar configuration in 3D space, then they remain collinear or coplanar. These realizations and the flip vision are described as degenerate equilibrium in Ref. [11]. These extra equilibria are not expected. In this paper, since we focused on the local convergence of a gradient-based system, it is feasible to avoid the degenerate equilibrium by assuming that the initial formation is closer to the target formation than the degenerate equilibrium.

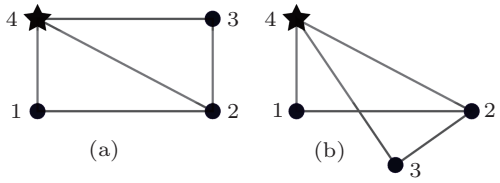


Fig. 1. Flipped situation for the fixed topology with four agents and of five edges.

Remark 4 In the formation tracking control problem, the leader-follower technique consists of specifying one or several leader agents and several followers. Specifically, there are the following three situations. Firstly, there may be one single leader which specifies the trajectory for the formation and the rest are set to follow the leader. The single leader is given in advance and is defined as global information. This situation is studied in this paper. Secondly, there is more than one agent acting as leaders. If the motion of leaders is given a priori as a global reference, the controller for followers in this paper can also handle the formation tracking task. Moreover, if only a subset of agents can have access to the objective, the desired motion can be regarded as a virtual leader. When only a fraction of agents are informed, the distance-based controller in this paper still enables all the informed agents and the uninformed agents to move with the desired motion by changing the tracking-term in Eq. (15) as

$$f_i^{\mathrm{r}} = -b_i[c_1(q_i - q_r)\rho_{ir} - c_2(p_i - p_r)], \tag{37}$$

where $b_i = 1$ if agent i is informed and $b_i = 0$ otherwise. Analogously, modify the sum of total artificial potential energy in Eq. (19) as

$$Q_i(q) = k_q \sum_{j\in\mathcal{N}_i}\psi(e_{ij}) + b_i c_1 \sum_{(i,n+1)\in\varepsilon}\psi(e_{ir}). \tag{38}$$

Let

$$\boldsymbol{B}(t) = \begin{bmatrix} b_1 & 0 & \cdots & 0\\ 0 & b_2 & \cdots & 0\\ \vdots & 0 & \ddots & \vdots\\ 0 & \cdots & 0 & b_n\end{bmatrix}. \tag{39}$$

The Hamilton function holds the following equation

$$\begin{aligned}\dot{H}(x,v) &= \frac{1}{2}\sum_{i=1}^{n}\dot{\tilde{Q}}_i + \sum_{i=1}^{n} v_i^{\mathrm{T}}\dot{v}_i \\ &= -v^{\mathrm{T}}[(c_2\boldsymbol{B}(t)+\boldsymbol{L})\otimes I_d]v \leq 0. \end{aligned} \tag{40}$$

Thus all agents asymptotically move with the same velocity of the virtual leader. The proof is omitted here, as it follows a similar process to Theorem 1.

5.2. Stability analysis of formation

In the following, we concentrate on the analysis of asymptotic stability of the set Ω_1. In this section, we first briefly introduce some basic concepts in center manifold theory that could be used to determine the stability of a nonlinear system when linearizing an equilibrium point. More details are available in Ref. [28]. We then consider the corresponding stability analysis using the center manifold theory.

Consider a system in normal form

$$\begin{cases} \dot{\theta} = \boldsymbol{A}\theta + f_1(\theta,\rho), \\ \dot{\rho} = \boldsymbol{B}\rho + f_2(\theta,\rho), \end{cases} \tag{41}$$

where $\theta \in \mathbb{R}^k$, and $\rho \in \mathbb{R}^{v-k}$. $\boldsymbol{A}$ is a matrix having eigenvalues only on the imaginary axis, $\boldsymbol{B}$ is a matrix having eigenvalues with negative real parts, and the functions $f_1(\theta,\rho)$ and $f_2(\theta,\rho)$ satisfy $f_1(0,0)=0$, $f_2(0,0)=0$, $J_{f_1}(0,0)=0$, $J_{f_2}(0,0)=0$, where $J_f(x)$ denotes the Jacobian of $f(x)$ evaluated at a point x. An invariant manifold $\mathcal{M}$ is a center manifold for Eq. (41) if it can be locally represented as

$$\mathcal{M} := \{(\theta,\rho)\in\mathcal{U} : \rho = h(\theta)\}, \tag{42}$$

where $\mathcal{U}$ is a sufficiently small neighborhood of the origin, $h(0)=0$ and $J_h(0)=0$.

It can be shown that a center manifold always exists and the dynamic (41) restricted to the center manifold are

$$\dot{\varepsilon} = \boldsymbol{A}\varepsilon + f_1(\varepsilon,h(\varepsilon)), \varepsilon \in \mathbb{R}^{(v-k)}. \tag{43}$$

The stability of the system (41) can be analyzed from the dynamics on the center manifold using the following lemma.

Lemma 2[27] If the origin is stable under Eq. (43), then the origin of system (41) is also stable. Then if $(\theta(t),\rho(t))$ is a solution of Eq. (41) starting with a sufficiently small $(\theta(0),\rho(0))$, there is a solution of Eq. (43) such that as $t\to\infty$

$$\begin{cases} \theta(t) = \varepsilon(t) + O(\mathrm{e}^{-\gamma t}), \\ \rho(t) = h(\varepsilon(t)) + O(\mathrm{e}^{-\gamma t}). \end{cases} \tag{44}$$

Next, it can be shown that the set of correct equilibria is locally exponentially stable by using center manifold theorem. Define the matrix $\boldsymbol{T}\in\mathbb{R}^{dn\times d(n+1)}$ as $x=\boldsymbol{T}\overline{q}$ and $v=\boldsymbol{T}\overline{p}$. From the definition of matrix $\boldsymbol{T}$, it can be found that $\mathrm{rank}\,(\boldsymbol{T}) = dn$.

The set of correct equilibrium points can be written as

$$\hat{\Omega}_1 = \{(z,v): \ \|z_{kij}\| = d_{ij}^{*} \ \text{ and } \ v_i = 0\}. \tag{45}$$

The advantage of using $\hat{\Omega}_1$ rather than Ω_1 in the ensuing stability analysis is that $\hat{\Omega}_1$ is a compact set.

Consider Eq. (28) and define $F(x,v)$ as

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix} v \\ -Dv - \nabla\tilde{Q}(x)\end{bmatrix} := F(x,v). \tag{46}$$

Then by linearizing Eq. (46) at a point in $\hat{\Omega}_1$, one has

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix} 0 & I_{dn} \\ -J_{g_x}(x)^{\mathrm{T}}J_{g_x}(x) & D\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} := J_F(x,v)\begin{bmatrix}x\\ v\end{bmatrix}, \tag{47}$$

where $J_{g_{\overline{q}}}(\overline{q}) = J_{g_x}(x)\boldsymbol{T}$. Thus, $\mathrm{rank}\,(J_{g_{\overline{q}}}(\overline{q})) = \mathrm{rank}\,(J_{g_x}(x)\boldsymbol{T})$, which implies that

$$\mathrm{rank}\,(J_{g_{\overline{q}}}(\overline{q})) \leqslant \min\{\mathrm{rank}\,(J_{g_x}(x)), \mathrm{rank}\,(T)\}.$$

According to Lemma 1, the framework $(\overline{G},\overline{q})$ is infinitesimally rigid, we have that

$$\dim(\mathrm{Ker}\,(J_{g_x}(x))) = \begin{cases} 2n-1, & d=2, \\ 3n-3, & d=3. \end{cases} \tag{48}$$

Thus, $\mathrm{Ker}\,(J_{g_x}(x)) = \mathrm{Ker}\,(J_{g_x}(x)^{\mathrm{T}}J_{g_x}(x)) = d(d-1)/2$, which means that $J_{g_x}(x)^{\mathrm{T}}J_{g_x}(x)$ has $d(d-1)/2$ eigenvalues.

Next, it can be shown that $J_F(x,v)$ in Eq. (47) has $d(d-1)/2$ zero eigenvalues and the rest have negative real parts. The proof is omitted here, as it follows a similar idea to the proof of Lemma 5 in Ref. [10].

We are now in a position to give the main result.

Theorem 2 Suppose framework $(\overline{\mathcal{G}},\overline{q})$ is infinitesimally rigid. Consider the double-integrator model in the form of Eqs. (8) and (9), and the transformation to the moving frame in Eq. (24). Let the potential function $\tilde{Q}(x)$ be defined as Eq. (27) in system (46). Then $\hat{\Omega}_1$ defined in Eq. (45) is locally asymptotically stable. Moreover, there exists a neighbourhood $\mathcal{N}_0$ of the set $\hat{\Omega}_1$ such that any trajectory of $(x(t),v(t))$ starting in $\mathcal{N}_0$ converges to a point in $\hat{\Omega}_1$.

Proof To prove that equation (45) is locally asymptotically stable we define $\hat{z} := [x^{\mathrm{T}}, v^{\mathrm{T}}]^{\mathrm{T}}$ and consider the dynamics of $(x(t),v(t))$. Note that there exists an orthonormal transformation K such that $KJ_F(x,v)K^{-1}$ is in block diagonal form with zero for the first $d(d-1)/2$ terms. Rewrite the $\hat{z}$ dynamic near a point $\hat{z}_0$ in $\hat{\Omega}_1$ as

$$\dot{\hat{z}} = J_F(\hat{z}_0)\hat{z} + (F(\hat{z}) - J_F(\hat{z}_0)\hat{z}). \tag{49}$$

Define $(\theta,\rho) = K\hat{z}$. Then it is easily verified that the (θ,ρ) dynamics have the following form

$$\begin{cases} \dot{\theta} = h_1(\theta,\rho), \\ \dot{\rho} = A_s\rho + h_2(\theta,\rho). \end{cases} \tag{50}$$

Without loss of generality, we assume that $\hat{z}_0 = 0$. Then $h_1(\theta,\rho)$ and $h_2(\theta,\rho)$ in Eq. (50) satisfy $h_1(0,0) = 0$, $h_2(0,0) = 0$, $J_{h_1}(0,0) = 0$, and $J_{h_2}(0,0) = 0$. Moreover, one has that $\hat{\mathcal{M}} = \{(\theta,\rho) : (\theta,\rho) = K\hat{z} \text{ and } \hat{z} \in \hat{\Omega}_1\}$ is a center manifold for Eq. (50) because $\hat{\mathcal{M}}$ is invariant as well as it consists of equilibria of Eq. (50) and is tangent to the center subspace of Eq. (50) at the origin. The proof is similar to Ref. [4].

Since $\hat{\mathcal{M}}$ is an equilibrium manifold, there exists a function $h(\theta)$ such that

$$\hat{\mathcal{M}} \cap \mathcal{N}_0 = \{(\theta,\rho) : \rho = h(\theta)\}$$

in a neighborhood $\mathcal{N}_0$ of $\hat{z}_0$. On the other hand, one has that $h_1(\theta,h(\theta)) = 0$ on $\mathcal{N}_0$. Thus, the dynamics restricted to $\hat{\mathcal{M}}$ are $\dot{\xi} = 0$. Applying Theorem 1, the solutions for (θ,ρ) starting in $\mathcal{N}_0$ are

$$\begin{cases} \theta(t) = \varepsilon(t) + O(\mathrm{e}^{-\gamma t}), \\ \rho(t) = h(\varepsilon(t)) + O(\mathrm{e}^{-\gamma t}). \end{cases} \tag{51}$$

Therefore, $\lim_{t\to\infty}(\theta,h(\theta)) = (\varepsilon(0),h(\varepsilon(0))) \in \hat{\mathcal{M}}$, which implies that $\hat{z}$ converges to $K^{\mathrm{T}}(\varepsilon(0),h(\varepsilon(0))) \in \hat{\Omega}_1$ exponentially fast. Then $q(t)$ and $p(t)$ converge to $\hat{\Omega}_1$ exponentially fast. ■

In summary, the local stability of the formation implies that if the agents experience small perturbations away from an equilibrium formation, they can converge back to another nearby equilibrium point in the target set.

6. Illustrative examples

In this section, two illustrative examples are provided to verify the effectiveness of the proposed distance-based formation tracking control method for four double-integrator agents in two-dimensional (2D) space and eight double-integrator agents in three-dimensional (3D) space, respectively. The local convergence of the formation system with the tracking controllers is demonstrated.

Example 1 The sensing graph for all agents is illustrated in Fig. 1. We assume that the desired distances are given by $d_{12}^* = d_{34}^* = 10$, $d_{14}^* = d_{23}^* = 5$ and $d_{24}^* = 5\sqrt{5}$, which correspond to a rectangular shape as Fig. 1(a). The initial positions $q_i(0)$ for all agents are randomly perturbed from those of desired formation q^* by a random variable uniformly distributed on $[2,2]$. The accelerated velocity is taken as $u_r = [1;1]$. The positive coefficients are chosen as $k_q = k_p = c_1 = c_2 = 2$. We assume that for different edges the agents are taking different forms of potential functions, as shown in Table 2 specifically.

Figures 2 and 3 show the interagent distance errors and velocity errors of agents. The trajectories of all agents under heterogeneous controller are depicted in Fig. 4.

Table 2. Selection for each edge of the corresponding potential function ρ_{ij}.

Edge	Function ρ_{ij}
(1,2)	$\|q_i - q_j\|^2 - d_{ij}^{*2}$
(1,4)	$\dfrac{\|q_i - q_j\| - d_{ij}^*}{\|q_i - q_j\|}$
(2,4)	$\dfrac{\|q_i - q_j\|^4 - d_{ij}^{*4}}{\|q_i - q_j\|^4}$
(2,3)	$\dfrac{\|q_i - q_j\|^2 - d_{ij}^{*2}}{\|q_i - q_j\|^2}$
(3,4)	$\tanh(\|q_i - q_j\|^2 - d_{ij}^{*2})$

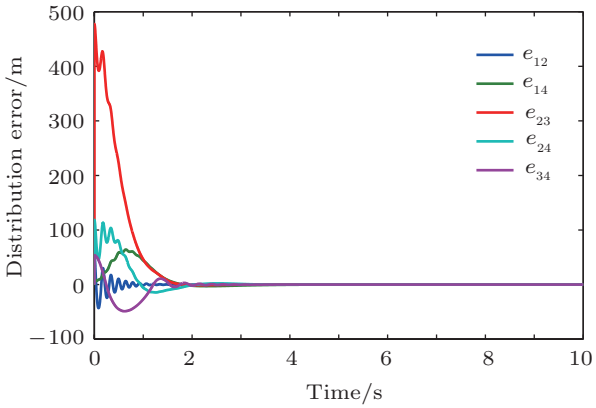


Fig. 2. (color online) Asymptotic convergence of the distance errors for 2D rectangular formation tracking control.

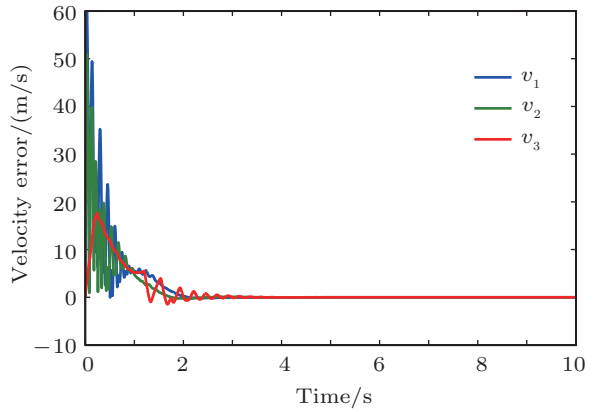


Fig. 3. (color online) Asymptotic convergence of the velocity errors for 2D rectangular formation tracking control.

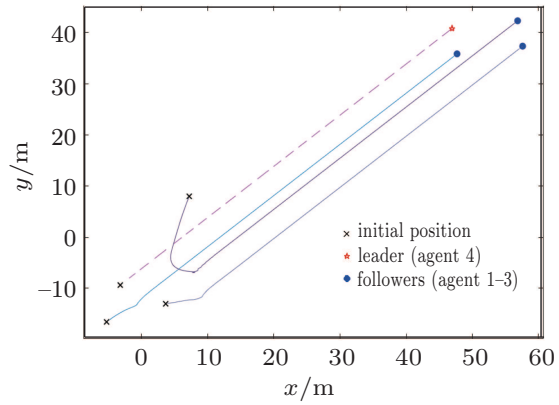


Fig. 4. (color online) The simulation on tracking control of a 4-agent rectangular formation shape.

Example 2 In this example, the target formation shape is considered as a cube formation with 8 agents in 3D space. The length of the side and the diagonal line of the target cube are designed as 5 and $5\sqrt{2}$. We adopt the potential function

$$\rho_{ij} = \tanh(\|q_i - q_j\|^2 - d_{ij}^{*2})$$

proposed in Eq. (18). The initial positions are also given as the same as those for the case in the 2D space. Here $k_q = c_1 = 4$, $k_p = c_2 = 2$, and the accelerated velocity is given by $u_r = [1,1,0.5]^{\mathrm{T}}$. As depicted in Figs. 5–7, the formation $\overline{q}$ converges to the target realization $\overline{q}^*$, and the interagent distance errors and velocity errors converge to zero.

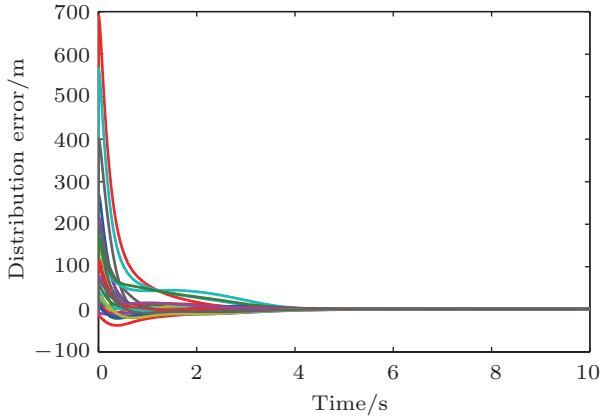


Fig. 5. (color online) Asymptotic convergence of the distance errors for 3D cube formation tracking control.

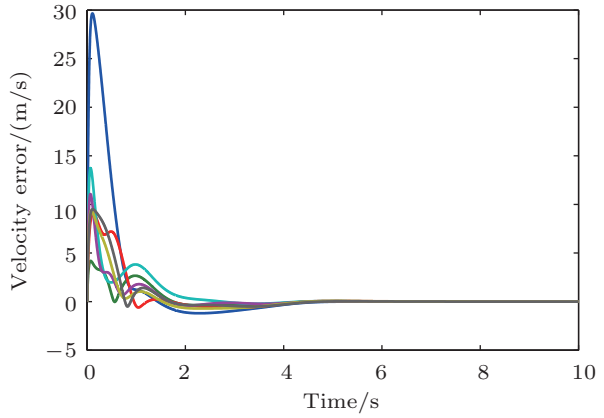


Fig. 6. (color online) Asymptotic convergence of the velocity errors for 3D cube formation tracking control.

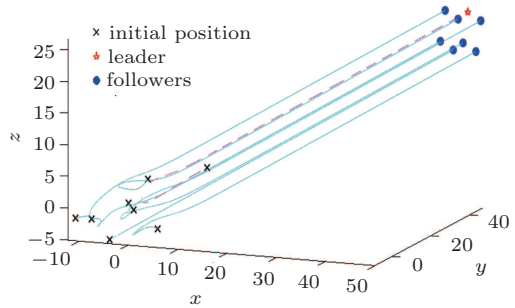


Fig. 7. (color online) The simulation on tracking control of an 8-agent cube formation shape in 3D space.

Remark 5 In distance-based formation control, formation shape control has been primarily concerned while research on the formation tracking problem is scanty. This paper is similar to the existing papers in Table 1 based on a gradient-flow approach. Compared with recent work on a distance-based formation tracking problem in Ref. [29], the proposed controller is more general as the potential function proposed in previous literature can be included in this controller (as shown in Example 1). This enables a group of heterogeneous agents to take different formation control strategies to achieve formation tracking. Note that the objective function in Ref. [29] is

$$V = \frac{1}{2}\sum_{(i,j)\in\varepsilon}(\|q_i - q_j\|^2 - d_{ij}^*)^2, \tag{52}$$

which is also experimented with edge $(1,2)$ in Example 1 of this section. Under the same original conditions, positive coefficients and objective function on the 2D plane, the controller (20) performs better as shown in Ref. [29] because the velocity consensus term in Eq. (14) improves the convergence speed. Moreover, our formation tracking controller can be applied not only to a plane like Ref. [29], but also to the 3D space.

7. Conclusion

We have investigated the distance-based formation tracking control for a group of double-integrator modeled agents. It is assumed that the state of the leader is time-varying and all agents with the leader communicate with each other according to an undirected fixed topology. With the target formation defined by inter-agent distances, the formation tracking controller is developed. The proposed controller can be decomposed to three terms, where the gradient-based term with potential function exhibited the formation target. Next, the asymptotic stability of the system with the proposed control law has been demonstrated. Two examples of the 2D plane and 3D space are provided to illustrate the validity of the theoretical results. The research on time-varying graphs and global stability analysis is ongoing.

References

[1] Ren W 2007 *IET Control Theory Appl.* **1** 505
[2] Ma G F, Sun Y C and Chen L M 2017 *Chin. Phys. B* **26** 068703
[3] Oh K K, Park M and, Ahn H S 2015 *Automatica* **53** 424
[4] Krick L, Broucke M E and Francis B A 2009 *Int. J. Control* **82** 423
[5] Dorfler F and Francis B 2009 *Proceedings of IEEE European Control Conference (ECC)*, "2009 European", pp. 2432–2437
[6] Oh K K and Ahn H S 2014 *Int. J. Robust Nonlinear Control* **24** 1809
[7] Sun Z, Mou S, Anderson B D O and Cao M 2016 *Systems and Control Letters* **93** 50
[8] Lu B, Chen F and Lan W Y 2015 *Chin. Phys. B* **24** 090206
[9] Olfati-Saber R and Murray R M 2002 *IFAC World Congress* **1** 242
[10] Deghat M, Anderson B and Lin Z 2016 *IEEE Trans. Automatic Control* **61** 1824

[11] Sun Z, Anderson B D, Deghat M and Ahn H S 2017 *Int. J. Control* **90** 1403
[12] Olfati-Saber R 2006 *IEEE Trans. Automatic Control* **51** 401
[13] Oh K K and Ahn H S 2011 *Automatica* **47** 2306
[14] Dimarogonas D V and Johansson K H 2008 *Proceedings of 47th IEEE Conference on Decision and Control CDC* pp. 1200–1205
[15] Dimarogonas D V, Johansson K H 2010 *Automatica* **46** 695
[16] Anderson B D O, Helmke U 2014 *SIAM Journal on Control and Optimization* **52** 219
[17] Dorfler F and Francis B 2010 *IEEE Trans. Automatic Control* **55** 2379
[18] de Marina H G, Jayawardhana B and Cao M 2017 arXiv preprint arXiv: 1703.07960
[19] Rozenheck O, Zhao S, Zelazo D 2015 *Proceedings of Control Conference* pp. 1693–1698
[20] Sun Z, Mou S, Deghat M and Anderson B D O 2016 *Int. J. Robust & Nonlinear Control* **26** 2824
[21] Cai X and de Queiroz M 2015 *Proceedings of ASME* 2014 *Dynamic Systems and Control Conference: American Society of Mechanical Engineers* V001T14A003–V001T14A003
[22] Marina H G D, Jayawardhana B and Cao M 2016 arXiv preprint arXiv: 1604.02943
[23] Dong L J, Chai S C, Zhang B H and Nguang S K 2014 *Chin. Phys. B* **23** 060502
[24] Zhang X, Wang J H, Yang D D and Xu Y 2017 *Chin. Phys. B* **26** 070501
[25] Hendrickson B 1992 *SIAM Journal on Computing* **21** 65
[26] Asimow L and Roth B 1979 *J. Math. Anal. Appl.* **68** 171
[27] Sastry S S 1999 *Springer Science and Business Media* **10**
[28] Carr J 1991 *Springer Science and Business Media* **35**
[29] Oh K K and Ahn H S 2017 *Int. J. Control, Automation and Systems* **15** 1738