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Controlling flexural waves in thin plates by using transformation acoustic metamaterials

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In this study, we design periodic grille structures on a single homogenous thin plate to achieve anisotropic acoustic metamaterials that can control flexural waves. The metamaterials can achieve the bending control of flexural waves in a thin plate at will by designing only one dimension in the thickness direction, which makes it easier to use this metamaterial to design transformation acoustic devices. The numerical simulation results show that the metamaterials can accurately control the bending waves over a wide frequency range. The experimental results verify the validity of the theoretical analysis. This research provides a more practical theoretical method of controlling flexural waves in thin-plate structures.

Keywords: acoustic metamaterials, coordinate transformation, flexural wave, grille structure

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1. Introduction

Acoustic metamaterials are artificial acoustic structural materials that consist of specially designed acoustic structural units that are arranged in a certain period in an elastic medium. This type of structural material has a low-frequency band gap and extraordinary material parameters in the subwavelength band of the structure size, and it can guide the propagation of sound waves in gas, liquid and solid. In recent years, the study of acoustic metamaterials has attracted the attention of scholars worldwide.[1–8]

Coordinate transformation theory is one of the most important theoretical methods applied to the design of acoustic metamaterials, which is an innovative theoretical method proposed in 2006 by Pendry *et al.*[9] in the field of electromagnetics. The propagation of an electromagnetic wave can be redirected at will with this method. The basic idea of coordinate transformation theory is to consider the arbitrary wave propagation path to be the result of the linear propagation path following the specific coordinate transformation. By using the same coordinate transformation in the material space, the homogeneous medium is compressed into a specific non-homogeneous medium and the medium can achieve the bending control of an arbitrary set of waves. Schurig *et al.*[10] and Zhang *et al.*[11] designed electromagnetic stealth cloaking to achieve the diffraction of electromagnetic waves by using this method. Cummer *et al.*[12,13] compared the acoustic wave equation in fluids with the equation of TE mode electromagnetic wave and found consistency between these two equations. Based on coordinate transformation theory, acoustic cloaking was designed to realize the regulation of acoustic waves. Rahm *et al.*[14] extended coordinate transformation theory into finite embedded coordinate transformations, which provides more flexibility for coordinate transformation design and can help to design various devices, such as curved waveguides, beam splitters and imaging lenses.[15,16]

Milton *et al.*[17] explored the feasibility of applying coordinate transformation theory to elastic wave regulation and Farhat *et al.*[18,19] designed elastic cloaking in a thin plate on the basis of Milton's research work. The effectiveness of regulating elastic waves by using coordinate transformation theory has been theoretically proven. The homogeneous material in the original coordinate space is transformed into a non-homogeneous anisotropic material. Such an anisotropic non-homogeneous material does not exist in nature. However, it is possible to convert the material into a multi-layer homogeneous material by continuous changing of material parameters by using the equivalent material theory of the material parameters.[20] Nevertheless, it is still difficult to obtain a material with continuously changing material parameters or according to a certain gradient. Therefore, many new electromagnetic devices and acoustic devices can be theoretically designed. However, there are great difficulties in preparing experimental samples, which lead to difficulties of its experimental verification. Few cases have thus been applied to engineering practice. In 2012, Nicolas *et al.*[21] designed and prepared a type of elastic cloak on the basis of the theoretical research of Farhat. However, the elastic cloaking proposed by

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Nicolas *et al.* is composed of 20 layers of a circular composite material and the elastic modulus of each layer varies from 0.002 GPa to 2.22 GPa. Moreover, each layer is made of PVC and PDMS compounds and neither of the compositions of the two materials occurs in the same proportion. Thus, the sample preparation process is complex, which is not conducive to experimental verification and engineering applications.

An anisotropic acoustic metamaterial based on a homogeneous thin plate was designed to achieve a transformation acoustic design that can control flexural wave propagation along an arbitrary curved path. Compared with the metamaterials proposed by Farhat and Nicolas, the metamaterial in this paper is very easy to prepare. The theoretical and experimental results are consistent with each other, which confirms the validity of the theoretical design.

2. Theoretical design of metamaterial

Consider the coordinate transformation shown in Fig. 1.

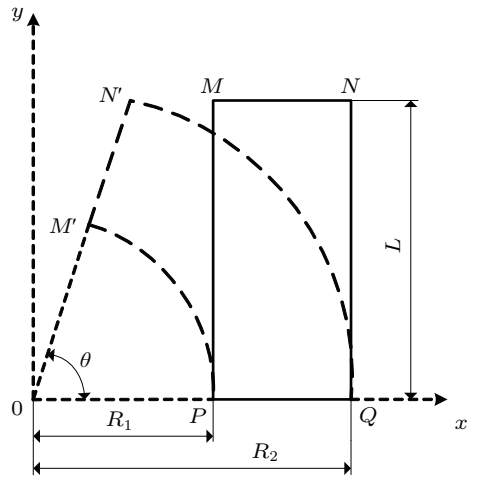


Fig. 1. Coordinate transformation.

The transformation equation can be written as[14]

$$\begin{cases} r'(x,y) = x, \\ \theta'(x,y) = (y/L)\cdot\theta = y/C, \quad C = L/\theta. \end{cases} \tag{1}$$

As shown in Fig. 1, the area $MNQP$ is translated into the area $M'N'QP$ through the coordinate transformation. Let $\boldsymbol{\Lambda}$ be the transformation matrix of the coordinate transformation and it will be expressed as

$$\boldsymbol{\Lambda} = \boldsymbol{R}(\theta')\,\mathrm{diag}\left(\frac{C}{r'}, \frac{r'}{C}\right)\boldsymbol{R}^{-1}(\theta'),$$
$$\boldsymbol{R}(\theta') = \begin{bmatrix} \cos(\theta') & -\sin(\theta') \\ \sin(\theta') & \cos(\theta') \end{bmatrix}. \tag{2}$$

Assuming that the area shown in Fig. 1 is a homogeneous elastic thin plate, its elastic modulus and mass density are E_0 and ρ_0, respectively. As a result of space compression due to coordinate transformation, the distribution of the material parameters in area $M'N'QP$ is anisotropic and unhomogeneous. Combining Eq. (2) with the dynamic equations of flexural waves,[18,19] we can obtain the spatial distribution of the materials in the space after the coordinate transformation:

$$E'_r = \frac{C^2}{r^2}\cdot E_0, \quad E'_\theta = \frac{r^2}{C^2}\cdot E_0, \quad \rho' = \frac{C^2}{r^2}\cdot\rho_0. \tag{3}$$

It can be seen from Eq. (3) that the spatial distribution of the materials appears anisotropic in the new coordinate system. According to the equivalent parameter theory of layered periodic composites,[20,22,23] anisotropic nonhomogeneous materials can be equilibrated by using isotropic alternating A and B layered media and the equivalent relationship can be written as[18]

$$\frac{1}{E'_r} = \frac{1}{1+\eta}\left(\frac{1}{E_A} + \frac{\eta}{E_B}\right), \quad E'_\theta = \frac{E_A + \eta E_B}{1+\eta},$$
$$\rho' = \frac{\rho_A + \eta\rho_B}{1+\eta}, \quad \eta = \frac{d_A}{d_B}. \tag{4}$$

If $\eta = 1$, we can obtain

$$E_A = \left(\frac{r^4}{C^4} + \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right)\cdot E_0,$$
$$E_B = \left(\frac{r^4}{C^4} - \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right)\cdot E_0, \quad \rho_A = \rho_B = \rho_0. \tag{5}$$

The flexural wave control based on coordinate transformation theory can be realized by using the above-mentioned multi-layer isotropic medium structure. However, as the elastic modulus of each layer is different and continuously varies according to a certain gradient, such materials are difficult to obtain in practice, which causes great difficulties of experimental verification. Therefore, to prepare experimental samples more easily and to reduce the difficulty of experimental verification, we must seek a new theoretical design method.

The flexural wave in a thin plate is a dispersion wave and the wave velocity changes with its frequency. The thickness of the thin plate and the material parameters influence the wave velocity of the flexural wave and its phase velocity expression is $C_f = \left[12\left(1-\nu^2\right)\right]^{-1/4}(\omega\cdot h)^{1/2}(E/\rho)^{1/4}$,[24,25] where ω is the angular frequency, h is the thickness of the plate, E is the elastic modulus, ρ is the mass density and ν is Poisson's ratio. The phase velocity of the flexural wave that propagates in A and B layered media are

$$C_{fA} = \left[12\left(1-\nu^2\right)\right]^{-1/4}\omega^{1/2}h_0^{1/2}\left(\alpha_A\cdot E_0/\rho_0\right)^{1/4}$$

and

$$C_{fB} = \left[12\left(1-\nu^2\right)\right]^{-1/4} \omega^{1/2} h_0^{1/2} \left(\alpha_B \cdot E_0/\rho_0\right)^{1/4},$$

$$\alpha_A = \left(\frac{r^4}{C^4} + \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right),$$

and

$$\alpha_B = \left(\frac{r^4}{C^4} - \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right).$$

For flexural waves propagating in the coordinate transformation region in the new coordinate system, under the premise that the propagation speed in each layer of A and B layered media in this area is not changed, if we change the respective variables in A and B arbitrarily for all beams having the same initial phase without changing the propagation speed in each layer of A and B in this area, the time for waves to reach a certain position in the region remains constant. Under this condition, the wave front in the region does not change and the mode of vibration of the coordinate transformation region remains unchanged with the change in the variables A and B. That is to say, when C_{fA} and C_{fB} keep constant, even if the size or material parameters of the space region change, the propagation mode of the flexural wave in the new spatial region is still the same as that in the space described in Eq. (5). As shown from the expressions of C_{fA} and C_{fB}, the variables affecting the propagation velocity of flexural waves in A and B layered media are α_A and α_B, respectively. Therefore, we can make the deformation of the wave velocity as

$$C_{fA/B} = \left[12\left(1-\nu^2\right)\right]^{-1/4} \omega^{1/2} \left(\alpha_{A/B}^{1/2} \cdot h_0\right)^{1/2} \left(E_0/\rho_0\right)^{1/4}. \quad (6)$$

After the deformation according to Eq. (6), the variables affecting the elastic moduli of A and B layered media are transferred to their thickness. Therefore, while maintaining the elastic modulus of A and B layered media, the propagation velocity of the flexural wave can hold constant by changing the thickness of each layer. Then the propagation mode of the flexural wave remains constant. The flexural wave can also propagate in the coordinate transformation area according to the theoretical design angle. In this case, the elastic moduli of A and B media are both E_0 and the thickness expression is written as

$$h_A = \alpha_A^{1/2} \cdot h_0 = \left(\frac{r^4}{C^4} + \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right)^{1/2} \cdot h_0,$$

$$h_B = \alpha_B^{1/2} \cdot h_0 = \left(\frac{r^4}{C^4} - \sqrt{\frac{r^8}{C^8} - \frac{r^4}{C^4}}\right)^{1/2} \cdot h_0. \quad (7)$$

3. Finite element model and numerical simulation results

The finite element model of the metamaterial grille structure is built according to Eq. (6), as shown in Fig. 2.

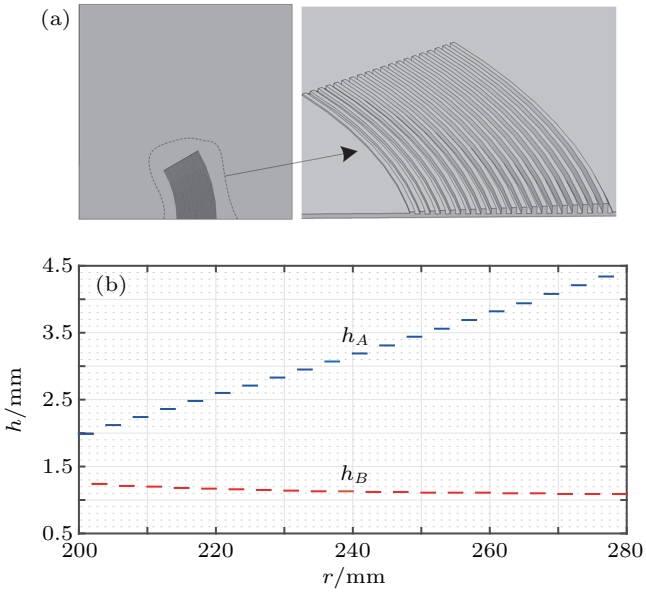


Fig. 2. Grille model and structural design parameters. (a) Simulation model of the grille and (b) thickness variation of grille model.

The dimensions of the thin plate are as follows: length of sides $D = 400$ mm, $L = 100$ mm, $h_0 = 1.5$ mm, $R_1 = 200$ mm, $R_2 = 280$ mm, $\theta = \pi/6$, elastic modulus $E_0 = 72$ GPa, and mass density $\rho_0 = 2700$ kg/m^3. The anisotropic non-homogeneous medium of the coordinate transformation region is dispersed over 20 layers, each of which contains isotropic homogeneous media A and B.

According to the flexural wave velocity expression, the relationship between the flexural wave wavelength λ and the excitation frequency f can be expressed as $\lambda = \left[12\left(1-\nu^2\right)\right]^{-1/4} (2\pi h)^{1/2} (E/\rho)^{1/4} f^{-1/2}$, because the propagation of flexural waves in thin plates must satisfy $h_0 \ll \lambda \ll D$[18,19] and the long wave approximation condition of $\lambda \gg d_A\,(d_B)$. Thus, we choose a wavelength range of $10h_0 \leq \lambda \leq 0.1D$, $\lambda \geq 10d_A\,(d_B)$ to obtain 20 mm$\leq \lambda \leq$ 40 mm, and the corresponding excitation frequency range should meet 9500 Hz$\leq f \leq$ 37000 Hz.

The numerical simulations of the above model are conducted at 9500 Hz and 37000 Hz, respectively. The excitation is between R_1 and R_2, which is perpendicular to the plane of the thin plate.

As shown in Figs. 3(a) and 3(c), the flexural waves in the thin-plate model propagate along the curved path according to the theoretical design. As shown in Figs. 3(b) and 3(d), the change in the direction of propagation of the flexural wave occurs in the coordinate transformation area. In this area, the wave surface of the flexural wave deflects neatly and there is

no reflection at the left or right borders of the coordinate transformation region. Very few waves scatter from the bottom of the right border, which might be caused by the discretization of continuous material parameters and the mismatch of mechanical impedance. The vibration energy is mainly concentrated in the coordinate transformation region, which indicates that the deflection propagation of the flexural wave is not caused by the reflection of the boundary of the coordinate transformation region. Instead, it is caused by the coordinate transformation design method. When the flexural waves pass through the coordinate transformation area, they continue to propagate along the straight line. These phenomena coincide with the theoretical analysis and design, which shows that the grille structure can regulate flexural waves in a highly accurate manner.

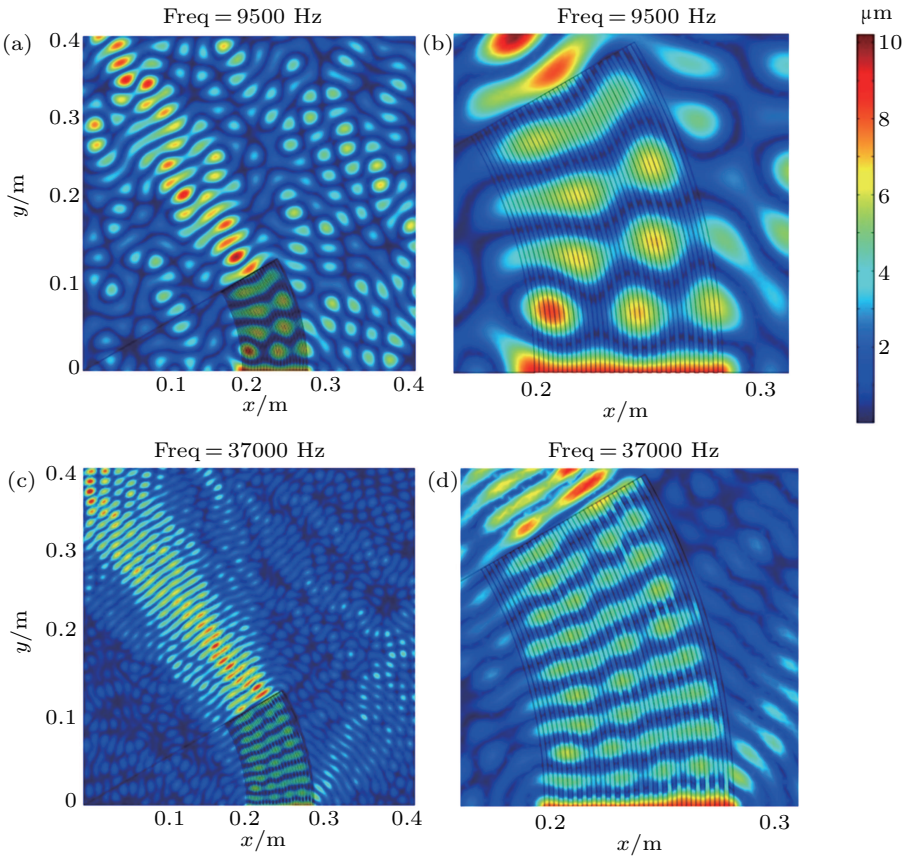


Fig. 3. Numerical simulation results of the grille model for the vibration displacement of (a) the thin plate at 9500 Hz in the normal direction of the plate plane, (b) the coordinate transformation area at 9500 Hz, (c) the thin plate at 37000 Hz in the normal direction of the plate plane and (d) the coordinate transformation area at 37000 Hz.

4. Experimental results and discussion

According to the grille structure thickness parameters shown in Fig. 2(b), we fabricate the corresponding experimental sample by using an aluminum plate that has a size of 400 mm×400 mm×5 mm. We test the experimental sample by using a laser vibration measurement system at a frequency of 9500 Hz and the measured datum is the vibration velocity of the sample at steady state.

In the experimental test system shown in Fig. 4(a), the excitation signal is first emitted by the laser vibration meter. The signal is amplified by the power amplifier and input into the exciter. The exciter generates the excitation of the sample according to the input signal and the sample is scanned by the laser scanning head to measure the displacement of the thin plate in the normal direction of the plate plane. The test results are returned to the laser vibration meter. Finally, the test results are analyzed and processed by the laser vibration meter and the vibration velocity of the thin plate in the normal direction of the plate plane under steady state is obtained. As the excitation frequency of the exciter can only reach 10000 Hz, we are more concerned about the low-frequency vibration characteristics of structures in practical engineering applications. So, we conduct an experiment at 9500 Hz to verify the results of the simulation calculations.

As shown in Fig. 4(b), the propagation directions of the flexural wave in the thin plate are deflected. As shown in Fig. 4(c), the vibration of the central region of the grille region is more intense than that of the boundary region and the flexural waves are mainly concentrated in the central region, which indicates that the change in the flexural wave propagation direction is not caused by the reflection of the bound-

ary of the grille structure. When flexural waves pass through the grille region, they continue to propagate along the straight line, which shows that the grille structure maintains the flexural wave propagation directivity. The numerical simulation results in Fig. 3(a) are slightly different from the experimental results in Fig. 4(b), which is caused by the difference in boundary condition between them. The experimental phenomena are consistent with the theoretical design and with the numerical simulations presented in Figs. 3(a) and 3(b). Moreover, the experimental results verify that the grille structure can be used to guide the flexural waves in propagating in any direction, which proves the accuracy and feasibility of the theoretical design method.

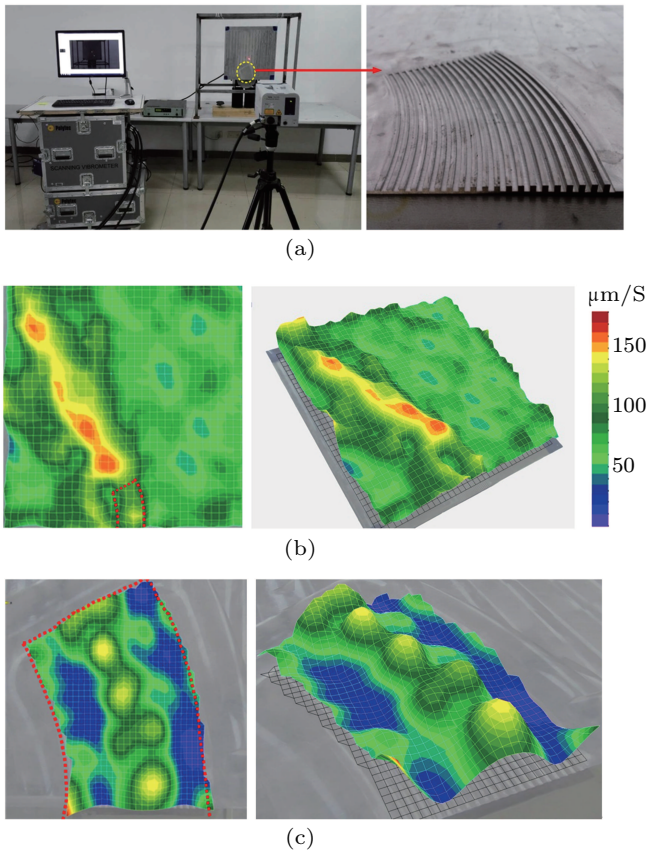


Fig. 4. Experimental set up and test results. (a) Experimental test system and sample. (b) Vibration velocity of the sample in the normal direction of the plate plane at 9500 Hz and its side view. (c) Vibration velocity of the grille structure area in the normal direction of the plate plane at 9500 Hz and its side view.

5. Conclusion

Using the coordinate transformation design method and combining thin-plate flexural dynamic analysis with the equivalent medium parameter theory, a new type of metamaterial grille structure is demonstrated. We use a homogenous medium and manipulate the structural dimensions instead of the parameters of the material to achieve anisotropic metamaterials. Then we achieve control of the flexural waves. A finite element model is established and the corresponding experimental sample is prepared. The numerical simulation and experimental results show that the one-dimensional height-tunable grille can guide flexural waves to propagate at will. The theoretical design method of the metamaterial proposed in this paper can be used to regulate the flexural waves in thin-plate structures made of different types of materials and it has a wide range of applications. With this kind of metamaterial, we can achieve the design of vibration and noise reduction in a specific area of a thin plate. The theoretical method is simple and feasible and the corresponding sample is easy to manufacture. Therefore, this method has strong practicality. This paper contributes to the development of microstructural design of metamaterial plates with a tunable anisotropic modulus and density distribution, thus paving the way for the physical realization of transformation acoustic designs for flexural waves.

References

[1] Liu Z Y, Zhang X X, Mao Y W, Zhu Y Y, Yang Z Y, Chan C T and Sheng P 2000 *Science* **289** 1734
[2] Li J and Chan C T 2004 *Phys. Rev. E* **70** 055602
[3] Fang N, Xi D, Xu J, Ambati M, Srituravanich W, Sun C and Zhang X 2006 *Nat. Mater.* **5** 452
[4] Qian J, Xia J P, Sun H X, Yuan S Q, Ge Y and Yu X Z 2017 *J. Appl. Phys.* **122** 244501
[5] Lu W J, Jia H, Bi Y F, Yang Y Z and Yang J 2017 *J. Acoust. Soc. Am.* **142** 84
[6] Xia J P, Sun H X and Yuan S Q 2017 *Sci. Rep.* **7** 8151
[7] Wang Y Y, Ding E L, Liu X Z and Gong X F 2016 *Chin. Phys. B* **12** 124305
[8] Yang Y Z, Jia H, Lu W J, Sun Z Y and Yang J 2017 *J. Appl. Phys.* **122** 054502
[9] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780
[10] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 *Science* **314** 977
[11] Zhang B L, Luo Y, Liu X G and Barbastathis G 2011 *Phys. Rev. Lett.* **106** 033901
[12] Cummer S A and Schurig D 2007 *New J. Phys.* **9** 45
[13] Cummer S A, Popa B I, Schurig D, Smith D R, Pendry J, Rahm M and Starr A 2008 *Phys. Rev. Lett.* **100** 024301
[14] Rahm M, Roberts D A, Pendry J B and Smith D R 2008 *Opt. Express* **16** 11555
[15] Christensen J and De Abajo F J G 2010 *Appl. Phys. Lett.* **97** 164103
[16] Jia H, Ke M Z, Hao R, Ye Y T, Liu F and Liu Z Y 2010 *Appl. Phys. Lett.* **97** 173507
[17] Milton G W, Briane M and Willis J R 2006 *New J. Phys.* **8** 248
[18] Farhat M, Guenneau S and Enoch S 2009 *Phys. Rev. Lett.* **103** 024301
[19] Farhat M, Guenneau S, Enoch S and Movchan A B 2009 *Phys. Rev. B* **79** 033102
[20] Schoenberg M and Sen P N 1983 *J. Acoust. Soc. Am.* **73** 61
[21] Nicolas S, Manfred W and Martin W 2012 *Phys. Rev. Lett.* **108** 014301
[22] Torrent D and Sánchez-Dehesa J 2008 *New J. Phys.* **10** 063015
[23] Chen H Y and Chan C T 2007 *Appl. Phys. Lett.* **91** 183518
[24] Graff K F 1992 *Wave Motion in Elastic Solids* (Shanghai: Tongji University Press) pp. 247–252
[25] Pao Y H and Mow C C 1993 *Diffraction of Elastic Waves and Dynamic Stress Concentrations* (Beijing: Science Press) pp. 52–55