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Generalized Drude model and electromagnetic screening in metals and superconductors*

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Electromagnetic screening is studied from the perspective of fluid mechanics by generalizing the Drude theory, which unifies three known results: Thomas–Fermi screening of the longitudinal field in both metals and superconductors, the skin effect of the transverse field in metals, and the Meissner effect in superconductors. In the special case of superfluid electrons, we slightly generalize the London equations to incorporate the longitudinal electric fields. Moreover, regarding the experiments, our study points out that the dynamical measurement may overestimate the superfluid density.

Keywords: electromagnetic screening, Thomas–Fermi screening, London penetration depth, Meissner effect

PACS: 74.25.N–, 74.20.De, 74.25.Gz, 71.10.Ca **DOI:** 10.1088/1674-1056/27/5/057401

1. Introduction

Electromagnetic fields can be screened by materials as a result of field–matter interactions. For example, a metal can screen the longitudinal electrostatic field called electrostatic shielding, while a superconductor can screen the transverse magnetostatic field called the Meissner effect. The former is governed by the Thomas–Fermi screening length $\lambda_{\rm TF}$,[1] while the latter is by the London penetration depth $\lambda_{\rm L}$.[2] Then, how about the longitudinal electric field screening in superconductors? It is commonly believed to be still governed by $\lambda_{\rm TF}$. However, in recent years, some theories[3–7] and experiments[8] suggested that the longitudinal electric field is screened by $\lambda_{\rm L}$ rather than $\lambda_{\rm TF}$, challenging the community. Unfortunately, when we resort to the London equations which are widely believed to be correct for ideal superconductors, we find that the longitudinal electrostatic field has to be zero immediately inside a superconductor, which means that the penetration depth must be zero. To the best of our knowledge, how to justify or reconcile these contradictions is still an open question.

On the other hand, the experimental measurement of the superfluid density $\rho_{\rm s}$ (defined as the inverse square of the magnetostatic field penetration depth) in the overdoped cuprates by mutual inductance also posed big challenges.[9,10] Firstly, the measured value is much smaller than the total carrier density. Secondly, the observed relation between $\rho_{\rm s}$ and $T_{\rm c}$ is different from the theoretical predictions of neither clean nor dirty superconductors within the Bardeen–Cooper–Schrieffer (BCS) theory. Thirdly, even without resorting to any microscopic theories, it is in conflict with another universal law[11,12] observed by optical experiments. In fact, noticing that the mutual inductance technique works at the alternating current (AC) frequency (of order 10 kHz) while the optical conductivity method works at the frequency of order THz or higher, it is reasonable to assume that the discrepancy comes from their different working frequencies and thus different electromagnetic (EM) screening behaviors.

Motivated by these questions, we revisit the old problem of EM screening in metals and superconductors. In this work, we choose to work with the quasiclassical perspective of fluid mechanics to describe the electron fluids. Under the relaxation time approximation, we arrive at a generalization of the Drude theory including both diffusive and magnetic field effects, from which we study the screenings of different EM waves/fields as functions of frequencies and dissipation times. Taking the infinite dissipation time limit, we obtain the results of superconductors. In addition, the London equations are slightly generalized to include the charge diffusion effect and thus are able to describe the longitudinal field screening, which is found to be still governed by the Thomas–Fermi length. On the other hand, applying our results to the overdoped cuprates, we propose a "hidden fluid" with small but nonzero dissipations to explain the discrepancies between the AC and optical experiments. Our proposal can be experimentally checked by performing a full-frequency measurement combining different techniques.

2. Generalized Drude theory

We start from the quasiclassical perspective of fluid mechanics, in which a fundamental equation is the Euler equation[13]

$$\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}+\frac{1}{nm}\nabla p=\frac{q}{m}\boldsymbol{E}+\frac{q}{mc}\boldsymbol{v}\times\boldsymbol{B}-\frac{\boldsymbol{v}}{\tau}, \quad (1)$$

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where $\boldsymbol{v}$ is the velocity field, p is the hydrodynamic pressure, n is the electron density, q and m represent electron charge and mass, and $\boldsymbol{E}$ and $\boldsymbol{B}$ are electric and magnetic fields respectively. The last term phenomenologically describes the dissipation with the collision time τ. Here, we have neglected the viscosity effect, which is usually very small compared with the dissipation effect except in very clean metals.[14] In order to apply the Euler equation, we need to first obtain the hydrodynamic pressure p via the thermodynamic relation $p = -\partial F/\partial V$. At low temperature far below the electron degenerate temperature, p is given by the degenerate pressure

$$p = \frac{2}{3}\frac{E}{V}. \tag{2}$$

Then, using relations $E/V = 3nE_{\rm F}/5$ and $3\pi^2 n = k_{\rm F}^3 = (mv_{\rm F})^3$ for a free fermi gas model, we have

$$\nabla p = \frac{mv_{\rm F}^2}{3}\nabla n. \tag{3}$$

Next, supposing $\boldsymbol{v}$ and n are spatially slowly varying functions, which is also required by our quasiclassical approximation, we make linearization to the Euler equation. Up to the linear order of $\boldsymbol{E}$, n, and $\boldsymbol{J} = n_0 q\boldsymbol{v}$ (n_0 is the uniform charge density), the Euler equation becomes

$$\frac{\partial \boldsymbol{J}}{\partial t} + \frac{\boldsymbol{J}}{\tau} - \boldsymbol{J}\times\frac{q}{mc}\boldsymbol{B}_0 + \frac{qv_{\rm F}^2}{3}\nabla n = \frac{n_0 q^2}{m}\boldsymbol{E}, \tag{4}$$

where $\boldsymbol{B}_0$ is the static magnetic field that must remain as an external parameter since it is independent of $\boldsymbol{E}$.[15] In this work, we only focus on a uniform $\boldsymbol{B}_0$ for simplicity (otherwise we have to solve the boundary value problem to determine $\boldsymbol{B}_0$ for special cases). Equation (4) is the constitutive equation and can be viewed as a generalization of the conventional Drude model (with $\boldsymbol{B}_0 = 0$ and $n = n_0$).[1] We argue that it can be used to study both metals ($\tau^{-1} \neq 0$) and superconductors ($\tau^{-1} = 0$ for superfluid electrons although the underlying physics[16] cannot be explained in our phenomenological model) effectively.

3. EM screenings in metals and superconductors

Combining the constitutive equation with the Maxwell equations

$$\nabla\cdot\boldsymbol{E} = 4\pi q(n - n_0) \tag{5}$$

and

$$\nabla^2\boldsymbol{E} - \frac{1}{c^2}\partial_t^2\boldsymbol{E} - \nabla(\nabla\cdot\boldsymbol{E}) = \frac{4\pi}{c^2}\partial_t\boldsymbol{J}, \tag{6}$$

after Fourier transformation and some straightforward algebra, we obtain all normal modes of the linear EM waves, as listed in Table 1.

Table 1. Lists of all normal modes of linear EM waves in the generated Drude model. In these expressions, $\tilde{\omega} = \omega + \frac{\rm i}{\tau}$, $\omega_{\rm p}^2 = \frac{4\pi n_0 q^2}{m}$, $\omega_{\rm c} = \frac{qB_0}{mc}$. The names of these modes are following the standard conventions in optical and plasma physics.[15] The major difference here is the inclusion of dissipation which is usually inevitable in condensed matter systems.

Condition	EM wave	Dispersion
$\boldsymbol{k} \parallel \boldsymbol{B}_0$ and $\boldsymbol{E} \parallel \boldsymbol{B}_0$	plasma oscillation (longitudinal)	$\omega\tilde{\omega} = \frac{v_{\rm F}^2}{3}k^2 + \omega_{\rm p}^2$
$\boldsymbol{k} \perp \boldsymbol{B}_0$ and $\boldsymbol{E} \parallel \boldsymbol{B}_0$	O-wave (transverse)	$\omega^2 = c^2k^2 + \frac{\omega}{\tilde{\omega}}\omega_{\rm p}^2$
$\boldsymbol{k} \parallel \boldsymbol{B}_0$ and $\boldsymbol{E} \perp \boldsymbol{B}_0$	L-wave and R-wave (transverse)	$\omega^2 = c^2k^2 + \frac{\omega}{\tilde{\omega}} \pm \omega_{\rm c}\omega_{\rm p}^2$
$\boldsymbol{k} \perp \boldsymbol{B}_0$ and $\boldsymbol{E} \perp \boldsymbol{B}_0$	X-wave (mixing of longitudinal and transverse)	$\omega^2 = c^2k^2 + \frac{\tilde{\omega}\omega - \omega_{\rm p}^2 - \frac{v_{\rm F}^2}{3}k^2}{\tilde{\omega}^2 - \omega_{\rm c}^2 - \frac{\tilde{\omega}}{\omega}(\omega_{\rm p}^2 + \frac{v_{\rm F}^2}{3}k^2)}\omega_{\rm p}^2$

After obtaining the dispersions of these linear EM waves, we are ready to discuss the EM screening phenomena. In fact, the inverse of the penetration depth λ is directly given by the imaginary part of the wave vector, i.e., $\lambda^{-1} = {\rm Im}(k)$, which is plotted as functions of ω and τ^{-1} in Fig. 1 for the plasma oscillation and O-wave, and in Fig. 2 for the L/R/X-waves.

For the plasma oscillation shown in Figs. 1(a) and 1(c), at zero frequency limit we obtain the Thomas–Fermi screening[1]

$$\lambda_{\rm TF}^{-2} = 3\omega_{\rm p}^2/v_{\rm F}^2 = \frac{12\pi n_0 q^2}{mv_{\rm F}^2}, \tag{7}$$

which is unaffected by the dissipation time τ. This is known as the electrostatic shielding effect. At finite frequency, the screening is suppressed and totally vanishes at $\omega > \omega_{\rm p}$ and $\tau^{-1} = 0$, since the electrons cannot follow up the fast oscillating longitudinal electric field any more. However, finite scattering rate $\tau^{-1} \neq 0$ will maintain the screening effect as a result of the dissipation process of the electrons which inevitably causes the EM wave damping.

For the O-wave shown in Figs. 1(b) and 1(d), we obtain two important results: the skin effect in normal metals and the Meissner effect in superconductors. At a given finite τ^{-1} by choosing the zero frequency limit, we obtain the classical skin depth[1]

$$\lambda_{\rm skin}^{-2} = \frac{\omega_{\rm p}^2\tau\omega}{2c^2} = \frac{2\pi\sigma_{\rm dc}\omega}{c^2}, \tag{8}$$

where $\sigma_{\rm dc} = n_0 q^2\tau/m$ is the direct current (DC) conductivity. Notice that the skin effect only implies screening at finite frequency but no screening in the static limit. On the other hand,

in order to describe superfluid electrons in superconductors, we first take $\tau^{-1}=0$ and then $\omega\to 0$. We obtain the Meissner effect with the London penetration depth[2]

$$\lambda_{\rm L}^{-2}=\frac{\omega_{\rm p}^2}{c^2}=\frac{4\pi n_0 q^2}{mc^2}. \tag{9}$$

Correspondingly, the dynamical penetration depth at finite frequency is given by

$$\frac{\lambda_{\rm L}^2}{\lambda^2}=1-\frac{\omega^2}{\omega_{\rm p}^2}. \tag{10}$$

Again, the screening vanishes at $\omega>\omega_{\rm p}$ due to the superfluid electrons and cannot follow up the fast oscillation of the external transverse field. From the above discussions, we see that the discrepancy between skin and Meissner effects when $\tau^{-1}\to 0$ and $\omega\to 0$ depends on the sequence of these two limits. This implies the existence of a singularity at $\tau^{-1}=\omega=0$ as shown in Fig. 1(d) and also in the dispersion

$$\lambda_{\rm L}^2 k^2=-\frac{\omega}{\omega+{\rm i}\tau^{-1}} \tag{11}$$

as $\omega\to 0$, from which we see clearly the singularity. In a sense, it implies that the superconductivity cannot be smoothly connected to the normal metals without crossing a singularity (phase transition). As a result, it is impossible to explain the Meissner effect from a purely classical way without involving symmetry breaking.[17–20]

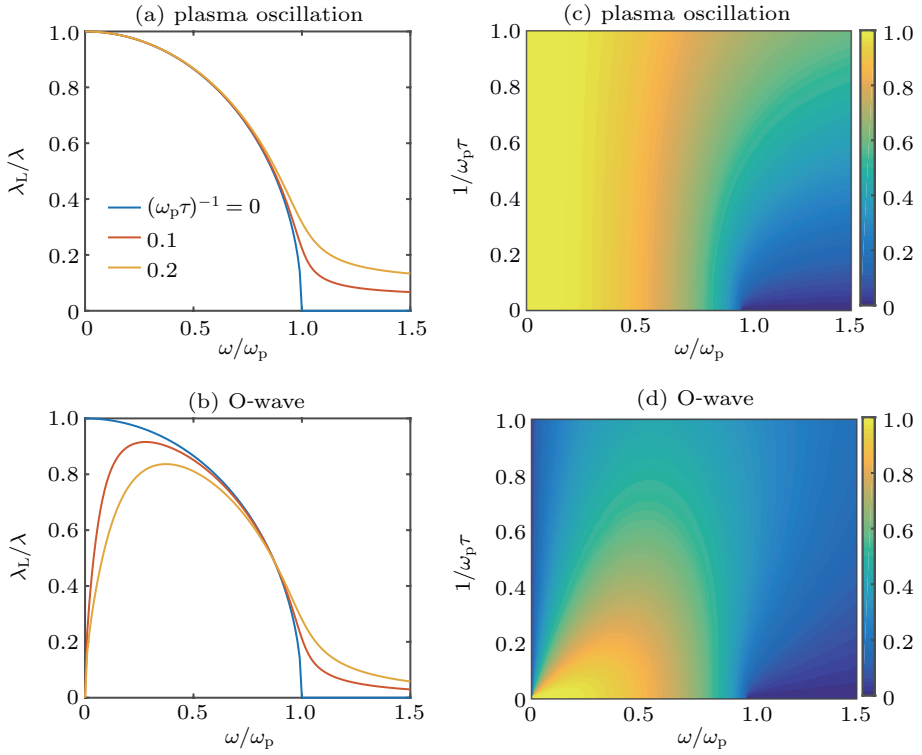


Fig. 1. (color online) Penetration depths as functions of frequency ω and dissipation rate τ^{-1} for ((a), (c)) the plasma oscillation and ((b), (d)) the O-wave. Notice that these plots are not affected by external static magnetic field.

For the L-, R-, and X-waves, when $\boldsymbol{B}_0=0$ they have no difference from the O-wave. But when $\boldsymbol{B}_0\neq 0$, they display different behaviors. The upper frequency limits of the screenings at $\tau^{-1}\to 0$ are shifted from $\omega_{\rm p}$ to $\omega_{\rm L/R}=\sqrt{\omega_{\rm p}^2+\omega_{\rm c}^2/4}\mp\omega_{\rm c}/2$ for L- and R-waves, respectively. In addition, the lower screening limit is shifted from 0 to $\omega_{\rm c}$ for the R-wave. More extraordinarily, two separated screening regimes emerge for the X-wave: one with $0<\omega<\omega_{\rm L}$ and the other with $\sqrt{\omega_{\rm p}^2+\omega_{\rm c}^2}<\omega<\omega_{\rm R}$. Here, when discussing the X-wave, we have chosen $v_{\rm F}/c=0$ for simplicity. It should be noted that the discussions of L-, R-, and X-waves are not relevant in superconductors since $\boldsymbol{B}_0$ must be zero as a result of the Meissner effect.

The above discussions of the EM screening are based on penetration depths. Next, let us revisit it from another perspective of the photon mass. In screening regions, the EM wave cannot propagate as freely as in a vacuum any more and feels a drag effect. In other words, we say that the quanta of the EM wave, i.e., photon, acquires a mass.[21] Sometimes, we only call the transverse component as a photon but the longitudinal one as plasmon. From the EM wave dispersions, at $k=0$, the real part of the frequency ${\rm Re}(\omega)$ gives the photon mass $m_{\rm ph}$, while the imaginary part ${\rm Im}(\omega)$ gives the decay rate $\Gamma_{\rm ph}$ (inverse of lifetime) of the photon. Accordingly, for both plasma oscillation and O-wave in small τ^{-1} limit, we obtain

$$m_{\rm ph}=\omega_{\rm p},\quad \Gamma_{\rm ph}=\frac{1}{2\tau}. \tag{12}$$

Therefore, we get a conclusion that only when $\tau^{-1}=0$ can we obtain a well-defined photon (otherwise the photon will decay) with a mass $\omega_{\rm p}$ regardless of whether the photon is longitudinal or transverse.[22] As a comparison, the longitudinal penetration depth is characterized by $\lambda_{\rm TF}$ while the transverse one is by $\lambda_{\rm L}$. Interestingly, such a dichotomy cannot be obtained by imposing the covariant gauge condition to London equations,[2,6] Ginzburg–Landau theory[3] or Proca action,[7] which all predict both longitudinal and transverse penetration depths are $\lambda_{\rm L}$.

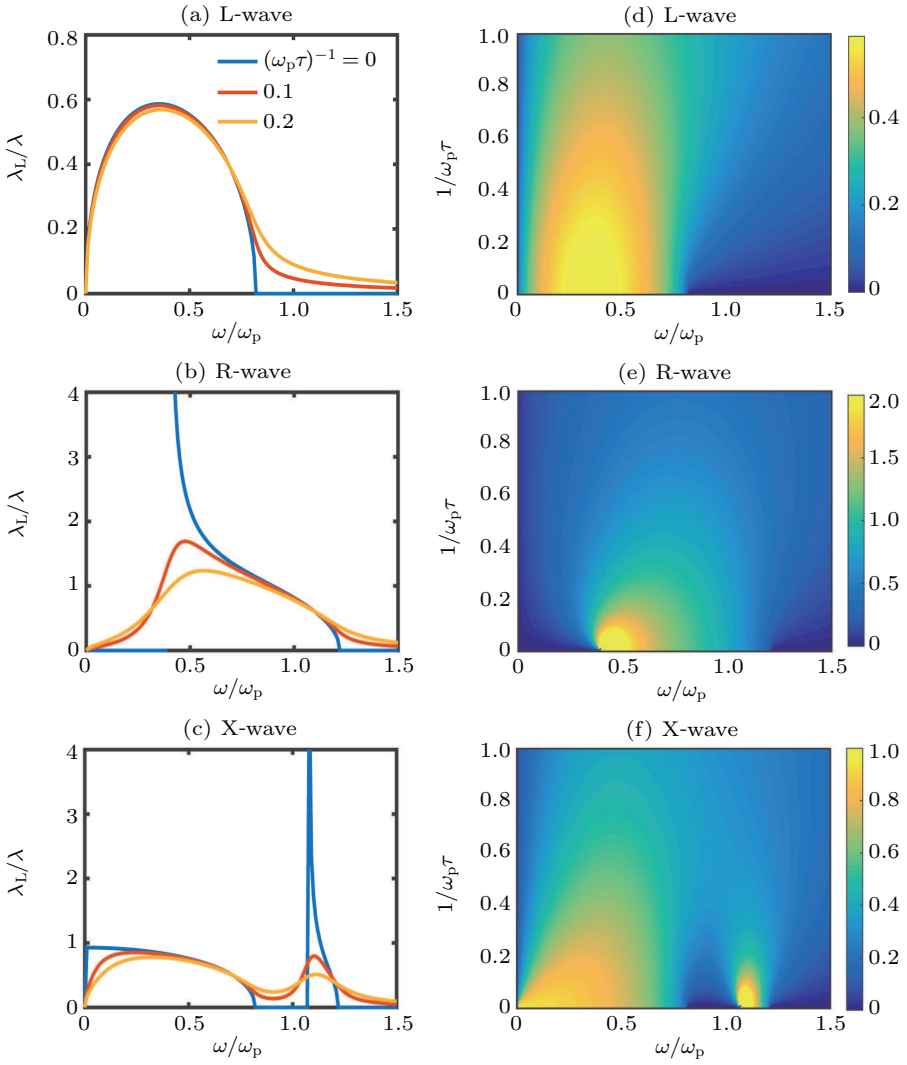


Fig. 2. (color online) Penetration depths as functions of frequency ω and dissipation rate τ^{-1} at a given external static magnetic field with the cyclotron frequency $\omega_{\rm c}=0.4\omega_{\rm p}$ for the L/R/X-waves in (a) and (d), (b) and (e), and (c) and (f), respectively. In the color plots, we have set the upper limit of $\lambda_{\rm L}/\lambda$ as (e) 2 and (f) 1, in order to obtain a better display effect. For the X-wave, we have chosen $v_{\rm F}/c=0$ as a good approximation in numerical calculations.

In fact, from Eq. (4) by setting $\tau^{-1}=0$ and $\boldsymbol{B}_0=0$, we immediately get

$$\frac{\partial \boldsymbol{J}}{\partial t}+\frac{qv_{\rm F}^2}{3}\nabla n=\frac{n_0q^2}{m}\boldsymbol{E}, \tag{13}$$

which can be viewed as a generalization of the first London equation. Here, the additional density gradient term is necessary to give the Thomas–Fermi screening. Taking curl on two sides, we obtain

$$\nabla\times\boldsymbol{J}=-\frac{n_0q^2}{mc}\boldsymbol{B} \tag{14}$$

as long as $\omega\neq 0$, which is the same as the second London equation except that the latter also applies to $\omega=0$. Following Londons' observation, we find that equation (13) can also be obtained from the gauge-fixing expression using the vector potential $\boldsymbol{A}$ as

$$\boldsymbol{J}=-\frac{n_0q^2}{mc}\boldsymbol{A}, \tag{15}$$

under a special gauge condition[23]

$$\nabla\cdot\boldsymbol{A}+\frac{3c^2}{v_{\rm F}^2}\frac{1}{c}\frac{\partial\phi}{\partial t}=0. \tag{16}$$

As comparisons, the conventional London equations are also described by Eq. (15) but under the Coulomb gauge $\nabla\cdot\boldsymbol{A}=0$, while the original London equations[2] and some other works[3,6,7] applied the covariant (or Lorentz) gauge $\nabla\cdot\boldsymbol{A}+(1/c)\partial_t\phi=0$.

4. Discussions

When applying to real materials, we may have several components of the electron fluids. We take the O-wave as an example. Suppose that we have several components, each of which with a dissipation time τ_i and plasma frequency $\omega_{\mathrm{p}i}^2 = 4\pi n_i q_i^2/m_i$. Then from Eq. (4) we have the total current

$$\boldsymbol{J} = \frac{1}{4\pi}\sum_i \frac{\omega_{\mathrm{p}i}^2}{-\mathrm{i}\omega + \tau_i^{-1}}\boldsymbol{E}. \tag{17}$$

Combining with Eq. (6) we get the dispersion of the EM wave

$$\omega^2 = c^2k^2 + \sum_i \frac{\omega}{\omega + \mathrm{i}\tau_i^{-1}}\omega_{\mathrm{p}i}^2, \tag{18}$$

from which we get a conclusion that at a finite frequency ω, all components with $\tau_i^{-1} \ll \omega$ contribute to λ^{-2}, while the others with $\tau_i^{-1} \gg \omega$ have no contribution. Accordingly, as $\omega \to 0$, only the component with $\tau_i^{-1} = 0$ (i.e., superfluid electrons) contributes to the screening and we obtain the superfluid density $\rho_{\mathrm{s}} = \frac{c}{4\pi}\lambda^{-2}$. In Fig. 3, we give a schematic example of a three-component model. Hence, we get a conclusion that if the measurement is at finite frequency, we may overestimate the superfluid density. For example, in the AC experiment of the overdoped cuprates,[9] the superfluid density is found to be much smaller than the value obtained by the optical experiment.[11,12] Notice that the working frequency of the AC experiment is of order 10 kHz while the optical experiments are of order THz or higher. Then according to our multi-component analysis, we propose that there is a "hidden fluid" with a dissipation time between 10^{-12} s and 10^{-4} s. It exists even at very low temperature and thus cannot be the normal fluid (quasiparticles) as in the two-fluid model.[24,25] However, in a recent work we have shown that the "hidden fluid" is in conflict with the BCS theory.[26] Therefore, if it can be confirmed experimentally by a full-frequency measurement combining different techniques, it will greatly challenge the BCS theory.

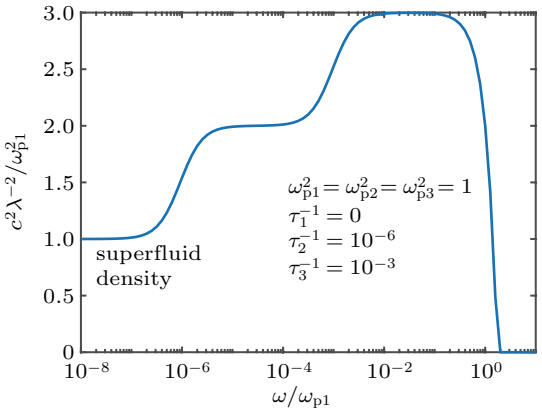


Fig. 3. (color online) Transverse field screening in a three-component model with $\omega_{\mathrm{p1}}^2 = \omega_{\mathrm{p2}}^2 = \omega_{\mathrm{p3}}^3 = 1$ as a schematic example. The dissipation times of the three components are chosen as $\tau_1^{-1} = 0$, $\tau_2^{-1} = 10^{-6}$, and $\tau_3^{-1} = 10^{-3}$. By definition, only the first component contributes to the superfluid density, but the dynamical λ^{-2} is given by all components with $\tau_i^{-1} \ll \omega$ and thus depends on the frequency.

5. Conclusion

We have studied the EM screening in metals and superconductors from the perspective of fluid mechanics by generalizing the Drude theory. The screenings of different EM waves are obtained under the linear approximation. Applying to superconductors, we slightly generalize the conventional London equations to incorporate the longitudinal electric fields. Our study confirms that the screenings of longitudinal and transverse fields are governed by Thomas–Fermi length and London penetration depth, respectively. Moreover, regarding experiments, we point out that the dynamical measurement may overestimate the superfluid density.

Acknowledgment

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Note added. After we submitted this manuscript, the "hidden fluid" we proposed in overdoped cuprates has been confirmed in a recent experiment,[27] although its microscopical explanation remains a challenge.

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