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Scattering of a single photon in a one-dimensional coupled resonator waveguide with a Λ-type emitter assisted by an additional cavity*

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We analyze the transport property of a single photon in a one-dimensional coupled resonator waveguide coupled with a Λ-type emitter assisted by an additional cavity. The reflection and transmission coefficients of the inserted photon are obtained by the stationary theory. It is shown that the polarization state of the inserted photon can be converted with high efficiency. This study may inspire single-photon devices for scalable quantum memory.

Keywords: one-dimensional waveguide, photon scattering, quantum memory

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1. Introduction

Realization of a quantum network requires accurate coherent control of single photons, since they play the role of a 'flying qubit' to transfer quantum information between different nodes in the quantum network.[1] A variety of methods are proposed for the realization of quantum control of single photons.[2,3] Among them, scattering of single photons in a one-dimensional waveguide or coupled resonator waveguide (CRW) is one of the most promising candidates.[4–20] In particular, to realize single photon quantum switching, Sun *et al.* used a two-level atom coupled to a one-dimensional (1D) coupled resonator waveguide to control the coherent transport of a single photon.[21] They also realized a single-photon quantum switch in cross-resonator arrays with a Λ-type atom which is localized in the intersectional resonator using the Fano–Feshbach effect based on the dark state of the Λ-type atom.[22] The single-photon routing scheme was proposed using a Δ-type Three-level atom embedded in quantum multichannels composed of a CRW or using a single atom with an inversion center coupled to quantum multichannels made of a CRW.[23,24] Controllable single-photon frequency converter was realized using a three-level V-type atom with a classical driving field.[25]

In this article, we theoretically analyze the transport property of a single photon in a 1D CRW with a Λ-type emitter assisted by an additional cavity. There are two kinds of polarization modes in the CRW. Each polarization mode is coupled to one transition of the Λ-type emitter, such that the emitter can absorb one kind of polarization photon and re-emit it with the same or different polarization. Assisted by an additional cavity, we show that the photon will be converted from one polarization to the other when it is scattered by a Λ-type emitter. This result has potential applications in scalable quantum memory.

2. Model

We consider a CRW with a Λ-type emitter embedded in one of the resonators. For convenience, we assume that the emitter is located in the n-th resonator. An assisted cavity is coupled with the 0-th resonator. The hopping between the neighbor cavity is evanescent field coupling,[4–6] as shown in Fig. 1.

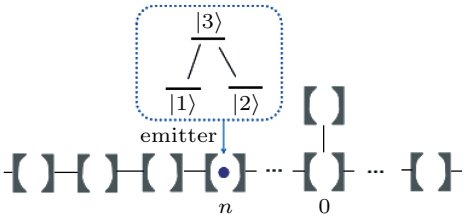


Fig. 1. (color online) Schematic diagram of our model.

Under the rotating wave approximation, the Hamiltonian for our model is

$$H = H_{\rm c} + H_{\rm emit} + H_{\rm ec}, \tag{1}$$

with $\hbar = 1$, where

$$\begin{aligned} H_{\rm c} &= \omega_{\rm c} \sum_{s={\rm h,v}} \sum_{j} c^{\dagger}_{j,{\rm s}} c_{j,{\rm s}} - \xi_{\rm h} \sum_{j} (c^{\dagger}_{j,{\rm h}} c_{j+1,{\rm h}} + {\rm H.c.}) \\ &\quad - \xi_{\rm v} \sum_{j} (c^{\dagger}_{j,{\rm v}} c_{j+1,{\rm v}} + {\rm H.c.}), \\ H_{\rm emit} &= \sum_{k=1,2,3} \omega_k |k\rangle\langle k| \\ &\quad + (J_1 |1\rangle\langle 3| c^{\dagger}_{n,{\rm h}} + J_2 |2\rangle\langle 3| c^{\dagger}_{n,{\rm v}} + {\rm H.c.}), \end{aligned} \tag{2}$$

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and

$$H_{\rm ec} = \omega_{\rm b} b_{\rm h}^{\dagger} b_{\rm h} + \omega_{\rm b} b_{\rm v}^{\dagger} b_{\rm v} + g(b_{\rm h}{}^{\dagger} c_{0,\rm h} + b_{\rm v}{}^{\dagger} c_{0,\rm v} + {\rm H.c.}),$$

where $H_{\rm c}$ is the Hamiltonian for the resonators in the CRW, $\omega_{\rm c}$ is the frequency of the resonator, $c_{j,\rm s}^{\dagger}$ and $c_{j,\rm s}$ are the creation and annihilation operators of the j-th cavity mode with s ($s = {\rm h}, {\rm v}$) polarizationand $\xi_{\rm s}$ is the intercavity coupling constant for s polarization, which describes photon hopping from one resonator to another (in our scheme we assume that different polarization modes cannot be coupled with each other). $H_{\rm emit}$ is the Hamiltonian for the emitter, which has two ground states, $|1\rangle$ and $|2\rangle$, and an excited state $|3\rangle$, and corresponding frequencies ω_1, ω_2 and ω_3, respectively. $|\mu\rangle\langle\nu|$ ($\mu, \nu = 1, 2, 3$) is the transition operator. The h(v) polarization mode can be only coupled with transition $|1\rangle \leftrightarrow |3\rangle$ ($|2\rangle \leftrightarrow |3\rangle$) with coupling constants $J_1(J_2)$. $H_{\rm ec}$ is the Hamiltonian of the assisted cavity. $b_{\rm s}^{\dagger}$ and $b_{\rm s}$ are the creation and annihilation operators of the empty cavity mode with s polarization ($s = {\rm h}, {\rm v}$), respectively.

Assuming that a photon comes from the left with h polarization and the emitter is in state $|1\rangle$, the stationary eigenstate for the Hamiltonian H is

$$\begin{aligned} |\psi\rangle = & \sum_j u_{\rm h}(j) c_{j,\rm h}^{\dagger} |\emptyset, 1\rangle \\ & + \sum_j u_{\rm v}(j) c_{j,\rm v}^{\dagger} |\emptyset, 2\rangle + u_{\rm e} |\emptyset, 3\rangle + u_{\rm b,h} b_{\rm h}{}^{\dagger} |\emptyset, 1\rangle \\ & + u_{\rm b,v} b_{\rm v}{}^{\dagger} |\emptyset, 2\rangle, \end{aligned}$$

where $|\emptyset\rangle$ represents no photon in the CRW.

We use the eigenequation $H|\psi\rangle = \Omega|\psi\rangle$ and get discrete scattering equations

$$\begin{aligned} & (\omega_1 + \omega - \Omega) u_{\rm h}(j) + \delta_{j,\rm n} u_{\rm e} J_1 + \delta_{j,0} u_{\rm b,h} g \\ = & \ \xi_{\rm h}(u_{\rm h}(j-1) + u_{\rm h}(j+1)), \end{aligned} \tag{3}$$

$$\begin{aligned} & (\omega_2 + \omega - \Omega) u_{\rm v}(j) + \delta_{j,\rm n} u_{\rm e} J_2 + \delta_{j,0} u_{\rm b,v} g \\ = & \ \xi_{\rm v}(u_{\rm v}(j-1) + u_{\rm v}(j+1)), \end{aligned} \tag{4}$$

$$u_{\rm h}(n) J_1 + u_{\rm v}(n) J_2 = (\Omega - \omega_3) u_3, \tag{5}$$

$$u_{\rm h}(0) g = (\Omega - \omega_1 - \omega_{\rm b}) u_{\rm b,h}, \tag{6}$$

$$u_{\rm v}(0) g = (\Omega - \omega_2 - \omega_{\rm b}) u_{\rm b,v}. \tag{7}$$

Using the form of the solution[21,23]

$$u_{\rm h}(j) = \begin{cases} {\rm e}^{{\rm i}kj} + r{\rm e}^{-{\rm i}kj} & (j \le n), \\ t_1 {\rm e}^{{\rm i}kj} + r_1 {\rm e}^{-{\rm i}kj} & (n \le j \le 0), \\ t{\rm e}^{{\rm i}kj} & (j \ge 0), \end{cases}$$

$$u_{\rm v}(j) = \begin{cases} r'{\rm e}^{-{\rm i}k'j} & (j \le n), \\ t_1' {\rm e}^{{\rm i}k'j} + r_1' {\rm e}^{-{\rm i}k'j} & (n \le j \le 0), \\ t'{\rm e}^{{\rm i}k'j} & (j \ge 0) \end{cases}$$

together with continuous conditions

$$u_{\rm h}(0^-) = u_{\rm h}(0^+), \tag{8}$$

$$u_{\rm v}(0^-) = u_{\rm v}(0^+), \tag{9}$$

$$u_{\rm h}(n^-) = u_{\rm h}(n^+), \tag{10}$$

$$u_{\rm v}(n^-) = u_{\rm v}(n^+), \tag{11}$$

we can obtain

$$t_1 = \frac{\xi_{\rm h}({\rm e}^{{\rm i}k(n-1)} + {\rm e}^{{\rm i}k(n+1)})}{(C_1 - \frac{A_1 D_1}{B_1}) - \frac{(\frac{J_1 J_2}{\Omega - \omega_3})^2 ({\rm e}^{{\rm i}kn} - \frac{A_2}{B_2}{\rm e}^{-{\rm i}kn})({\rm e}^{{\rm i}kn} - \frac{A_1}{B_1}{\rm e}^{-{\rm i}kn})}{C_2 - \frac{A_2 D_2}{B_2}}}, \tag{12}$$

$$t_1' = -\frac{(\frac{J_1 J_2}{\Omega - \omega_3})({\rm e}^{{\rm i}kn} - \frac{A_1}{B_1}{\rm e}^{-{\rm i}kn})}{C_2 - \frac{A_2 D_2}{B_2}} t_1, \tag{13}$$

$$r_1 = -\frac{A_1}{B_1} t_1, \tag{14}$$

$$r_1' = -\frac{A_2}{B_2} t_1', \tag{15}$$

$$t = t_1 + r_1, \tag{16}$$

$$t' = t_1' + r_1', \tag{17}$$

$$r = t_1 {\rm e}^{{\rm i}2kn} + r_1 - {\rm e}^{{\rm i}2kn}, \tag{18}$$

$$r' = t_1' {\rm e}^{{\rm i}2kn} + r_1'. \tag{19}$$

Here

$$A_1 = \omega_{\rm c} + \omega_1 - \Omega + \frac{g^2}{\Omega - \omega_{\rm b} - \omega_1} - \xi_{\rm h}({\rm e}^{-{\rm i}k} + {\rm e}^{{\rm i}k}), \tag{20}$$

$$A_2 = \omega_{\rm c} + \omega_2 - \Omega + \frac{g^2}{\Omega - \omega_{\rm b} - \omega_2} - \xi_{\rm v}({\rm e}^{-{\rm i}k} + {\rm e}^{{\rm i}k}), \tag{21}$$

$$B_1 = \omega_{\rm c} + \omega_1 - \Omega + \frac{g^2}{\Omega - \omega_{\rm b} - \omega_1} - 2\xi_{\rm h}{\rm e}^{{\rm i}k}, \tag{22}$$

$$B_2 = \omega_{\rm c} + \omega_2 - \Omega + \frac{g^2}{\Omega - \omega_{\rm b} - \omega_2} - 2\xi_{\rm v}{\rm e}^{{\rm i}k}, \tag{23}$$

$$C_1 = \left(\omega_{\rm c} + \omega_1 - \Omega + \frac{J_1^2}{\Omega - \omega_3}\right){\rm e}^{{\rm i}kn} - 2\xi_{\rm h}{\rm e}^{{\rm i}k(n+1)}, \tag{24}$$

$$C_2 = \left(\omega_{\rm c} + \omega_2 - \Omega + \frac{J_2^2}{\Omega - \omega_3}\right){\rm e}^{{\rm i}kn} - 2\xi_{\rm v}{\rm e}^{{\rm i}k(n+1)}, \tag{25}$$

$$\begin{aligned} D_1 = & \left(\omega_{\rm c} + \omega_1 - \Omega + \frac{J_1^2}{\Omega - \omega_3}\right){\rm e}^{-{\rm i}kn} \\ & - \xi_{\rm h}({\rm e}^{-{\rm i}k(n-1)} + {\rm e}^{-{\rm i}k(n+1)}), \end{aligned} \tag{26}$$

$$\begin{aligned} D_2 = & \left(\omega_{\rm c} + \omega_2 - \Omega + \frac{J_2^2}{\Omega - \omega_3}\right){\rm e}^{-{\rm i}kn} \\ & - \xi_{\rm v}({\rm e}^{-{\rm i}k(n-1)} + {\rm e}^{-{\rm i}k(n+1)}). \end{aligned} \tag{27}$$

3. Results

We can get transmissivity $T_1 = |t|^2$ ($T_2 = |t'|^2$) and reflectivity $R_1 = |r_1|^2$ ($R_2 = |r'|^2$) for the input photon (converted photon) from Eqs. (16)–(19). In Fig. 2, we plot T_1, T_2, R_1, and R_2 as a function of detuning $\Delta = \Omega - \omega_3$. From Fig. 2, we can see that in the near-resonance region, R_1, T_1, and T_2 decrease with the decrease of $|\Delta|$, but R_2 increases with the decrease of $|\Delta|$. In the resonated case $\Delta = 0$, R_2 approaches unity, i.e., the input photon is completely converted. It is noted that Obi and Shen have shown that the maximum conversion efficiency is

50% when a Λ-type emitter is coupled to a one-dimensional waveguide.[7] In our scheme assisted by an empty cavity coupled to the CRW, we could achieve high conversion efficiency approaching unity. Figure 3 plots R_1 and R_2 as a function of n in the resonated case. From Fig. 3 we can find that the resonated photon will be converted when n is odd number, but it will not be converted when n is an even number. The physical reason for the above results may be seen from the quantum interference. When $\Delta = 0$, the wave vector in our scheme is $k = m\pi + \pi/2$ (m is an integer), such that the phase difference between two reflection paths (one is reflected by the atom and the other is reflected by the additional empty cavity) is $2nk = n\pi + 2nm\pi$. When n is an odd number, the phase difference $n\pi + 2nm\pi$ induces destructive interference which causes R_1 to approach zero and R_2 to approach unity. When n is even number, the phase difference $n\pi + 2nm\pi$ induces constructive interference which causes R_1 to approach unity and R_2 approaches zero.

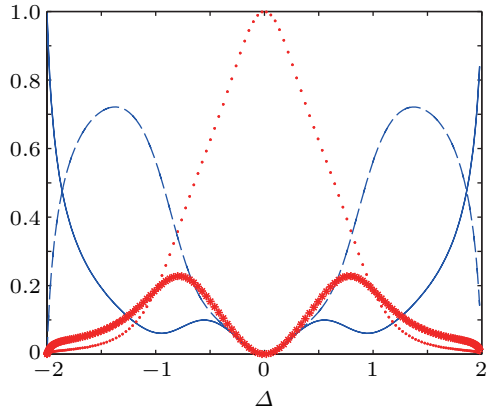


Fig. 2. (color online) The transmissivity and reflectivity as a function of detuning $\Delta = \Omega - \omega_3$. The blue solid line and blue dashed line are the reflectivity and the transmissivity of the input photon, respectively. The red dotted line and the red star line are the reflectivity and the transmissivity of the converted photon, respectively. We assume that the dispersion relation is $\Omega = \omega_c - 2\xi\cos(k)$[21] and $n = -1$. Other parameters in the units of g are $\omega_1 = \omega_2 = 0$, $\omega_3 = \omega_c = \omega_b = 5$, $\xi_h = \xi_v = 1$ and $J_1 = J_2 = 1$.

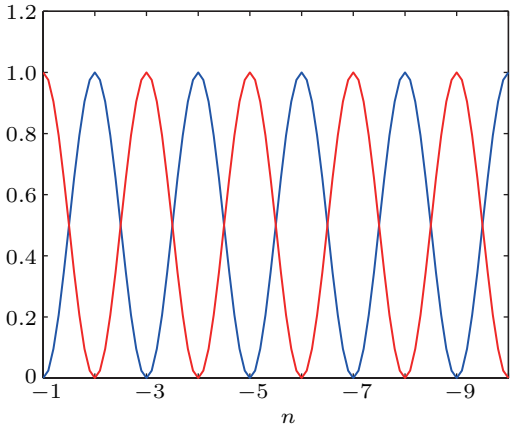


Fig. 3. (color online) Single-photon reflectivity as a function of n when $\Delta = 0$. The blue solid line is the reflectivity of the insert photon and the red dashed line is the reflectivity of the converted photon. Other parameters are the same as that in Fig. 2.

Next, we show a potential application of our model for scalable quantum memory. As shown in Fig. 4, there is a CRW has $2N+1$ resonators and resonator 0 is coupled with an empty cavity. We assume that each resonator with odd number trapping an emitter with five states, four ground states $|f1\rangle$, $|f2\rangle$, $|1\rangle$, $|2\rangle$ and an excited state $|3\rangle$. All the emitters are initially in state $|f1\rangle$. If we want to store a photon with any superposition state $\alpha|\mathrm{h}\rangle + \beta|\mathrm{v}\rangle$ into the j-th (j is odd number) resonator in the CRW, the steps are as follows: (i) use a classical field to map the quantum state of the emitter in j-th resonator from $|f1\rangle$ to $|1\rangle$. (ii) Insert the photon into the CRW and the evolution of the state of the system is $(\alpha|\mathrm{h}\rangle + \beta|\mathrm{v}\rangle)\otimes|1\rangle \to |\mathrm{v}\rangle\otimes(\alpha|2\rangle + \beta|1\rangle)$ according to the above calculations. When the inserted photon is in state $|\mathrm{v}\rangle$ and the emitter in state $|1\rangle$, the photon polarization state will not be converted since we assume the v polarization photon cannot be coupled to state $|1\rangle$. (iii) Use the classical fields to map the quantum state $\alpha|2\rangle + \beta|1\rangle$ to $\alpha|f2\rangle + \beta|f1\rangle$. Obviously, we can map the stored state back to photon polarization state by the opposite process. In this way, we can store many photons into the CRW and realize a scalable quantum memory.

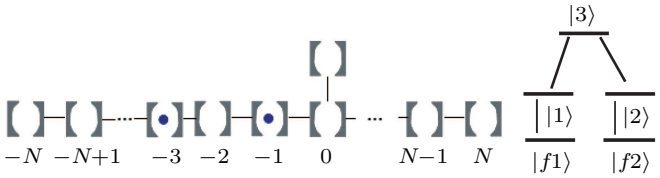


Fig. 4. (color online) (a) The schematic diagram for scalable quantum memory. (b) The energy level of the emitters.

4. Conclusion

We have studied the transport property of a single photon scattering off a Λ-type emitter in a 1D CRW. We find that, assisted by an empty cavity, the photon polarization can be converted with high efficiency approaching unity when n is odd and the photon is resonated to the emitter. Our work may inspire new optical devices for scalable quantum memory.

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