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A new control method based on the lattice hydrodynamic model considering the double flux difference*

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A new feedback control method is derived based on the lattice hydrodynamic model in a single lane. A signal based on the double flux difference is designed in the lattice hydrodynamic model to suppress the traffic jam. The stability of the model is analyzed by using the new control method. The advantage of the new model with and without the effect of double flux difference is explored by the numerical simulation. The numerical simulations demonstrate that the traffic jam can be alleviated by the control signal.

Keywords: traffic flow, lattice hydrodynamic model, control method, double flux difference

PACS: 05.70.Fh, 05.70.JK, 05.90.+m, 89.40.–a **DOI:** 10.1088/1674-1056/27/5/050503

1. Introduction

With the development of cities, people's daily lives are deeply affected by traffic problems, especially traffic congestionm which is one of the most serious problems. Many experts and scholars have paid more and more attention to alleviating traffic jams. A variety of traffic flow models[1–18] have been studied, such as the lattice hydrodynamic model[4–7] and the car-following model,[8–12] to work out the formation of the traffic jam and reduce traffic congestion. All models can be categorized as either a macroscopic model,[4,5] microscopic model,[9,10] or mesoscopic model.[19,20]

In 1961, a famous car-following model was proposed by Newell[8,9] with an optimal velocity (OV) function, which has a profound influence on later research. Scholars began to build models by considering the OV function to describe the traffic problems. In 1995, Bando *et al.*[21,22] put forward a classic car-following model called the optimal velocity model (OVM). The OVM shows that the driver adjusts the velocity based on the headway distance and exposes the essence of the traffic congestion. Later, many car-following models were developed based on the OVM. In 2006, Zhao *et al.* proposed a coupled-map (CM) car-following model,[23,24] and the velocity feedback control[25,26] was taken into account. According to the CM model, it is helpful to suppress the traffic congestion considering the feedback control signal in the models.

The lattice hydrodynamic model is a macroscopic model, which was put forward by Nagatani[27] based on the theory of Kerner and Konhauser.[28] Since then, more and more scholars have utilized the lattice hydrodynamic method to analyze the macroscopic traffic flow. The lattice hydrodynamic model draws lessons from the car-following model with a similar description of the density waves and stability analysis. In 2015, Ge[29] proposed a new lattice hydrodynamic model considering the feedback signal called flux difference to suppress the traffic jam in view of modern control theory.[30] In the same year, Redhu[31] presented a delayed-feedback control (DFC) model with the consideration of the driving behavior. Subsequently, the effect of the density change rate difference was considered as a feedback signal by Li.[32] In 2016, Zhu[33] proposed a new control signal called the variation rate of the optimal velocity in the lattice hydrodynamic model.

In the lattice hydrodynamic model, the traffic flow is divided into lots of lattices by the discretization method. Obviously, during the actual driving condition, the influence of multi-lattices is significant. The drivers adjust their behavior based on the flux difference of the preceding and local lattices. In 2012, Wang[34] considered the multiple flux difference effect in his new model. In Wang's research, the density wave analysis was adopted to explain the effect of the multiple flux difference. In addition, the flux of the two lattices in front of the local lattice has an important effect on driving, and the effect has diminished since the third lattice. As far as we know, the double flux difference which has an important effect on the real traffic, has not been studied. Up to date, most feedback signal methods were applied to the car-following model to alleviate traffic congestion. The feedback signal was seldom considered in the lattice hydrodynamic model. For these reasons, a new lattice hydrodynamic model with the consideration of the double flux difference is proposed. With the feedback signal of multi-lattices, a new optimal velocity func-

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tion is proposed to keep the traffic system stable. And we use the control method to explore the effect of the double flux difference on the traffic flow to explore its impact on the traffic flow.

The outline of the paper is as follows. In Section 2, the basic lattice hydrodynamic model is derived, and we introduce the control signal for the flux difference model. In Section 3, a control signal is added to the basic lattice hydrodynamic model and the feedback control theory is used to analyze the stability conditions. In Section 4, several numerical simulations are carried out to verify the theoretical results. Conclusions are given in Section 5.

2. Basic model

The basic lattice hydrodynamic model has the advantages of macroscopic models and car-following model. The equations are organized as follows:[28]

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v, \end{cases} \tag{1}$$

where ρ is the current density at time t and ρ_0 is the average density. δ is the average space headway and x is the position of the local lattice, so $\rho(x+\delta)$ is the current density at position $x+\delta$. Especially, ρ_0 and δ can be expressed as $\rho_0 = 1/\delta$. a represents the sensitivity coefficient of the driver. We set $\sigma = x/\delta$ and $x = \sigma$. Nagatani creatively put forward the discretization method to explain the equations as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0 \left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0, \\ \partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \end{cases} \tag{2}$$

where j represents the location of the lattice. In the above equations, ρ_j and v_j are the current density and the current velocity, respectively. By simplifying the equation, making $q = \rho v$, and considering the feedback signal, we have

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0 \left(q_{j+1} - q_j\right) = 0, \\ \partial_t(q_j) = a\rho_0 V(\rho_{j+1}) - aq_j + u_j, \end{cases} \tag{3}$$

where the control signal u_j is given as

$$u_j = k(q_{j+1} - q_j), \tag{4}$$

with k being the flux difference parameter.

The feedback control signal is similar to the velocity difference feedback signal indicating the flux difference of lattices j and $j+1$. Nagatani[27] also proposed the new optimal velocity function of the lattice hydrodynamic model

$$V(\rho) = (v_{\max}/2)\left[\tanh\left(1/\rho - 1/\rho_c\right) + \tanh\left(1/\rho_c\right)\right], \tag{5}$$

where $v_{\max}$ represents the maximum velocity of a vehicle on this road and ρ_c is the safety density.

3. Control scheme

In 2015, the control method was first introduced by Ge[29] to consider the flux difference. In reality, the traffic flow is a complex system which consists of many factors. In order to be closer to the actual traffic flow, we propose a lattice hydrodynamic model considering the double flux difference

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0 \left(q_{j+1} - q_j\right) = 0, \\ \partial_t(q_j) = a\rho_0 V(\rho_{j+1}, \rho_{j+2}) - aq_j + u_j, \\ u_j = \lambda[k\Delta q_j + (1-k)\Delta q_{j+1}], \end{cases} \tag{6}$$

where $\Delta q_j = q_{j+1} - q_j$, $\Delta q_{j+1} = q_{j+2} - q_{j+1}$, k is the ratio coefficient of the flux difference of lattices j, $j+1$, and $j+2$. Due to the influence of the multi-lattice feedback signal, the new optimal velocity function is given as

$$\begin{aligned} V(\rho) = {} & (v_{\max}/2)\left[\tanh\left(1/(2\rho_{j+1}) + 1/(2\rho_{j+2}) - 1/\rho_c\right)\right. \\ & \left. + \tanh\left(1/\rho_c\right)\right]. \end{aligned} \tag{7}$$

Based on the control theory, we compare the expected and real flux and density. ρ^* and q^* represent the theoretical data of flux and density. The following lattices have the steady state described as

$$[\rho_n(t), q_n(t)]^{\mathrm{T}} = [\rho_n^*, q_n^*]^{\mathrm{T}}. \tag{8}$$

We apply the linear stability criterion to analyze the system considering a small perturbation. The following equation can be derived by the control method:

$$\begin{cases} \partial_t \delta\rho_{j+1} + \rho_0 \left(\delta q_{j+1} - \delta q_j\right) = 0, \\ \partial_t(\delta q_j) = a\rho_0(\Lambda_{n+1}\delta\rho_{j+1} + \Lambda_{n+2}\delta\rho_{j+2}) - a\delta q_j - \delta u_j, \end{cases} \tag{9}$$

where $\Lambda_{n+1} = \partial V(\rho_{n+1})/\rho_{n+1}$, $\Lambda_{n+2} = \partial V(\rho_{n+2})/\rho_{n+2}$, $\delta\rho_j = \rho_j - \rho^*$, and $\delta q_j = q_j - q^*$.

Taking the Laplace transformation of Eq. (5), we can easily obtain

$$\begin{cases} sP_{j+1}(s) - \rho_{j+1}(0) + \rho_0[Q_{j+1}(s) - Q_j(s)] = 0, \\ sQ_j(s) - q_j(0) = a\rho_0[\Lambda_{n+1}P_{j+1}(s) + \Lambda_{n+2}P_{j+2}(s)] \\ -aQ_j(s) + \lambda[(1-k)Q_{j+2}(s) + (1-2k)Q_{j+1}(s) + kQ_j(s)], \end{cases} \tag{10}$$

where $L(\rho_{j+1}) = P_{j+1}(s)$, $L(q_{j+1}) = Q_{j+1}(s)$, $L(q_{j+2}) = Q_{j+2}(s)$, $L(q_j) = Q_j(s)$, and $L(p_{j+2}) = P_{j+2}$. L indicates the Laplace transform and s represents the transform function variable. Equation (10) can be written in the matrix form

$$\begin{aligned} & \begin{bmatrix} \rho_0 & -s \\ s+a+\lambda k & -a\rho_{\Lambda_{n+1}} \end{bmatrix} \begin{bmatrix} Q_j \\ P_j \end{bmatrix} \\ = {} & \begin{bmatrix} \rho_0 & 0 & 0 \\ -\lambda(1-2k) & -\lambda(1-k) & a\Lambda_{n+2} \end{bmatrix} \begin{bmatrix} Q_{j+1}(s) \\ Q_{j+2}(s) \\ P_{j+2}(s) \end{bmatrix}. \end{aligned} \tag{11}$$

By simplifying Eq. (11) with the control theory, the transfer function $G(s)$ is written as follows:

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -a\rho_0\Lambda_{n+1} & s \\ -(s+a+\lambda k) & \rho_0 \end{bmatrix}$$

$$\times \begin{bmatrix} \rho_0 & 0 & 0 \\ -\lambda(1-2k) & -\lambda(1-k) & a\Lambda_{n+2} \end{bmatrix} \frac{1}{p(s)}, \quad (12)$$

where the characteristic polynomial $p(s) = s^2 + as + s\lambda k - a\rho_0^2\Lambda_{n+1}$. From the deduction of Eq. (12), we can easily obtain

$$G(s) = \begin{bmatrix} -a\rho_0^2 - s\lambda + sk\lambda & s\lambda - sk\lambda & a\rho_0 s\Lambda_{n+2} \end{bmatrix} \frac{1}{p(s)}. \quad (13)$$

By the Hurwitz stability criterion, we can easily draw a conclusion that when the traffic flow is smooth, characteristic function $p(s)$ is greater than 0. To make the system stable, it can be confirmed that $p(s)$ is satisfied by the judgement of the control theory and inequalities $\|G(s)\|_\infty \leq 1$, $a + \lambda k > 0$, $-a\rho_0^2\Lambda_{n+1} > 0$. The following derivation provides an effective measure to work out the multi-variable system stable condition. $\|G(s)\|_\infty \leq 1$ must be smaller than 1 for all ω^2 to ensure the stability of the system

$$\begin{cases} \|G(s)\|_\infty = \sup\limits_{\omega\in[0,\infty)} |G(j\omega)| \leq 1, \\ |G(j\omega)| = \sqrt{G(jw)G(-jw)} \\ = \sqrt{\frac{a^2\rho_0^4\Lambda_{n+1}^2 + (2k\lambda-\lambda)^2\omega^2 + (k\lambda-\lambda)^2\omega^2 + a^2\rho_0^2\Lambda_{n+2}^2\omega^2}{\omega^4 + 2a\rho_0^2\Lambda_{n+1}\omega^2 + a^2\rho_0^4\Lambda_{n+1}^2 + (a+k\lambda)^2\omega^2}} \\ \leq 1. \end{cases} \quad (14)$$

The sufficient condition can be derived as

$$2a\rho_0^2\Lambda_{n+1} - a^2\rho_0^2\Lambda_{n+2}^2 - 4k^2\lambda^2 - 2\lambda^2 + 6k\lambda^2 + a^2 + 2k\lambda a \geq 0. \quad (15)$$

From the above analysis, the sensitivity coefficient of driver a in the traffic system should satisfy

$$a > \frac{-2k\lambda - 2\rho_0^2\Lambda_{n+1}}{1 - \rho_0^2\Lambda_{n+2}^2}. \quad (16)$$

4. Numerical simulations

In this section, several simulations are provided to verify the stability of the feedback control method for the traffic flow system. The original traffic flow is also investigated in our simulation for comparison. We assume that there are N lattice sites on a signal-way road and the specific parameters for the lattice hydrodynamic model are set as follows: $N = 140$, $\rho_c = 0.25\ \mathrm{m}^{-1}$, $v_{\max} = 2$ m/s, $\rho_0 = 0.5\ \mathrm{m}^{-1}$, $a = 2.5\ \mathrm{m}^{-1}$.

We suppose 280 vehicles in the traffic flow system at the steady state introduced by Eq. (6). Then we set the initial density of sites 50–55 as 0.5 and sites 56–60 as 0.2. The densities of the 5th, 25th, 55th and 80th lattices are depicted in detail.

Figure 1 presents the original lattice hydrodynamic model and figure 2 is an extended lattice hydrodynamic model with the consideration of the double flux difference. Comparing Fig. 1 with Fig. 2, it is clear that when the double flux difference is considered in the lattice hydrodynamic model, the traffic congestion is suppressed a great deal. From Figs. 2–5, with flux parameter λ increasing, the density–time curve becomes smoother. According to the patterns of the density in Figs. 5–8, we can conclude that when ratio coefficient k grows to 0.85, the traffic flow can become more stable. The traffic system will be most stable with $k = 0.85$ and $\lambda = 0.9$.

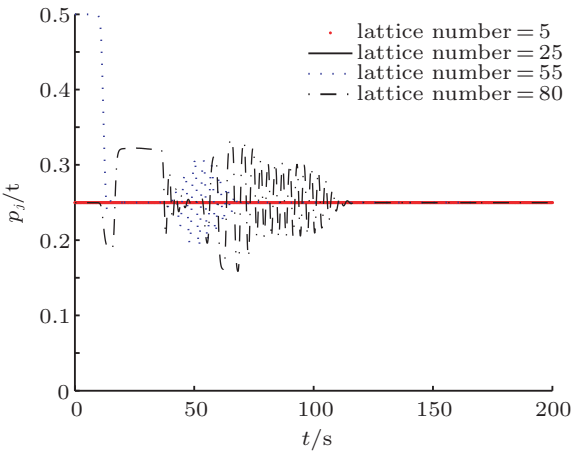


Fig. 1. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices without the control signal.

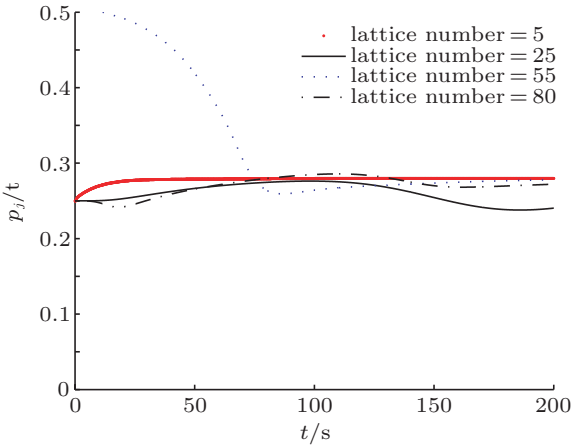


Fig. 2. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.6$ and $k = 0.85$.

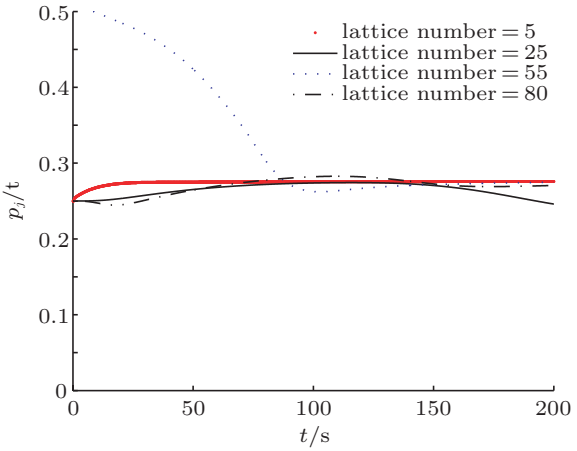


Fig. 3. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.7$ and $k = 0.85$.

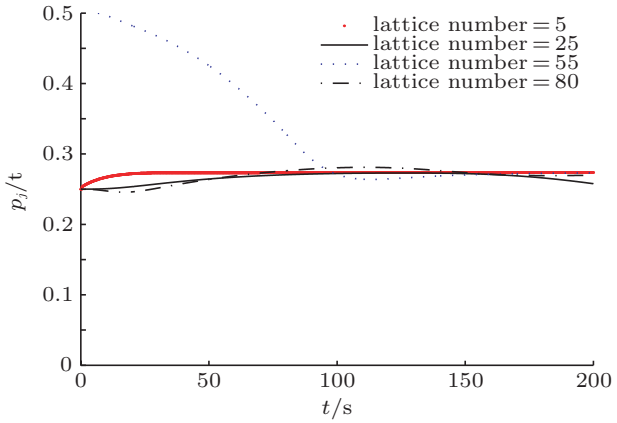


Fig. 4. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.8$ and $k = 0.85$.

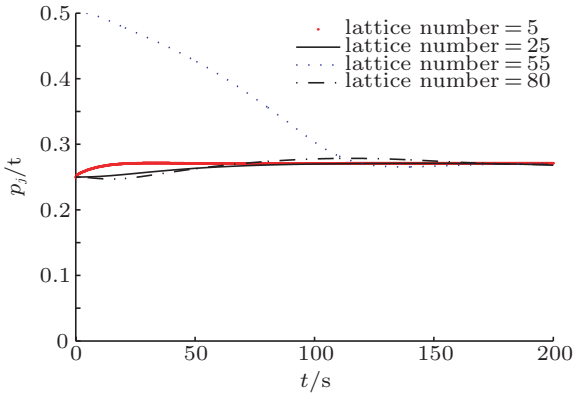


Fig. 5. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.9$ and $k = 0.85$.

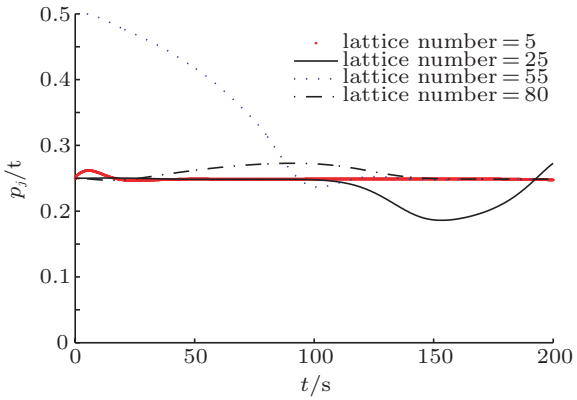


Fig. 6. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.9$ and $k = 0.75$.

Figures 9–12 show the relationship of the density difference and the densities at the 5th, 25th, 55th and 80th lattices. The Y axis indicates the density difference between time t and $t-1$ from $t = 1$ to $t = 500$, and the X axis represents the current density. Figures 9–12 are sets of dispersed points in the phase space plot which show the chaos of the traffic flow. The figures demonstrate that the densities of the 5th, 25th, 55th and 80th lattices will change with the feedback gain. Comparing patterns (a) with (b) of Figs. 9–12, we can clearly see that the amplitude of the density wave of every lattice approaches 0 under the influence of the positive feedback signal. To describe the stability of the system intuitively, local enlarged images are depicted in patterns (b) of the above figures.

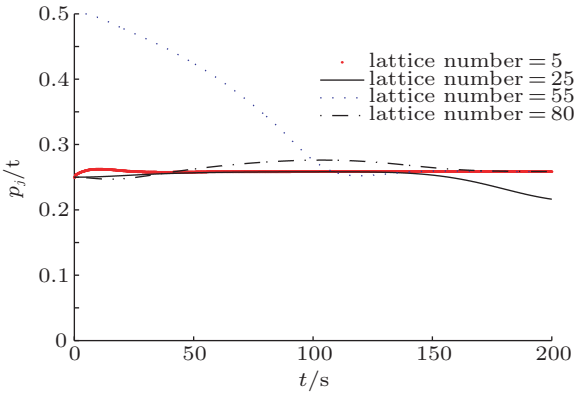


Fig. 7. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.9$ and $k = 0.8$.

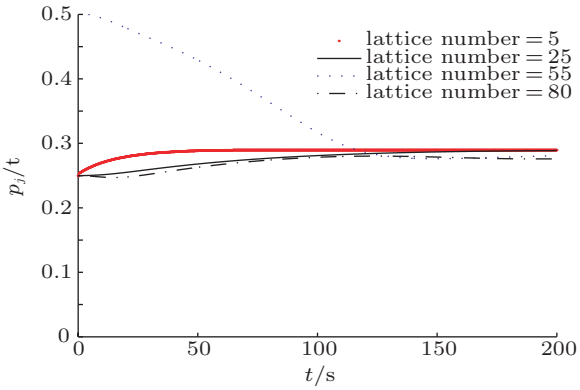


Fig. 8. (color online) The density–time curve of the 5th, 25th, 55th and 80th lattices with $\lambda = 0.9$ and $k = 0.9$.

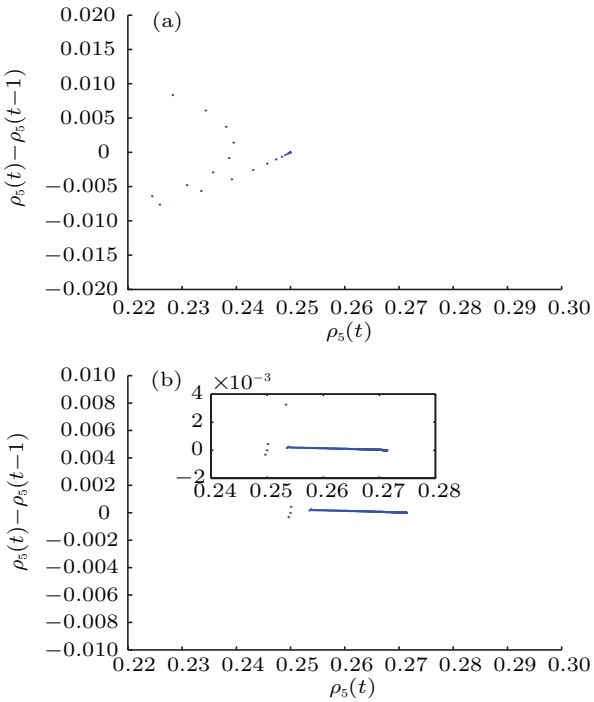


Fig. 9. (color online) Scatter plot of the 5th lattice at time $t = 1$–500 s (a) without control signal (b) and with $\lambda = 0.9$, $k = 0.85$.

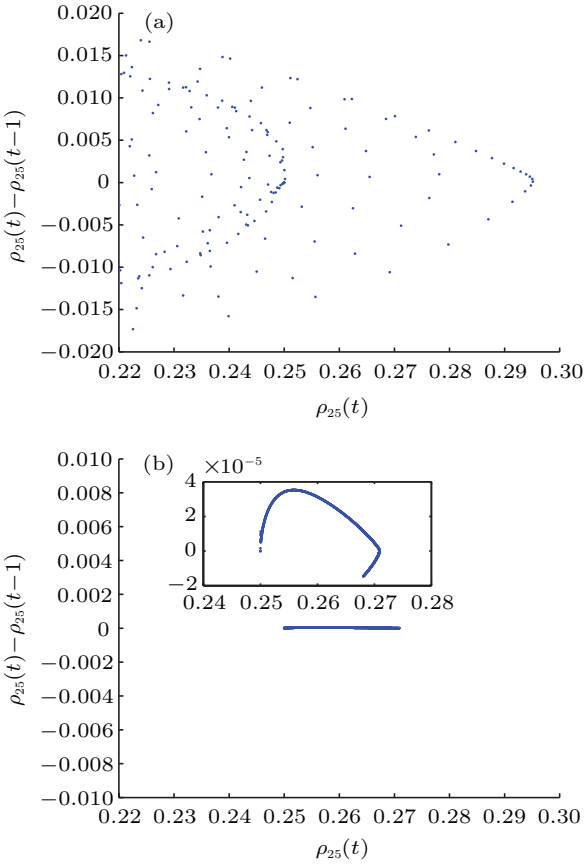


Fig. 10. (color online) Scatter plot of the 25th lattice at time $t = 1$–500 s (a) without control signal (b) and with $\lambda = 0.9$, $k = 0.85$.

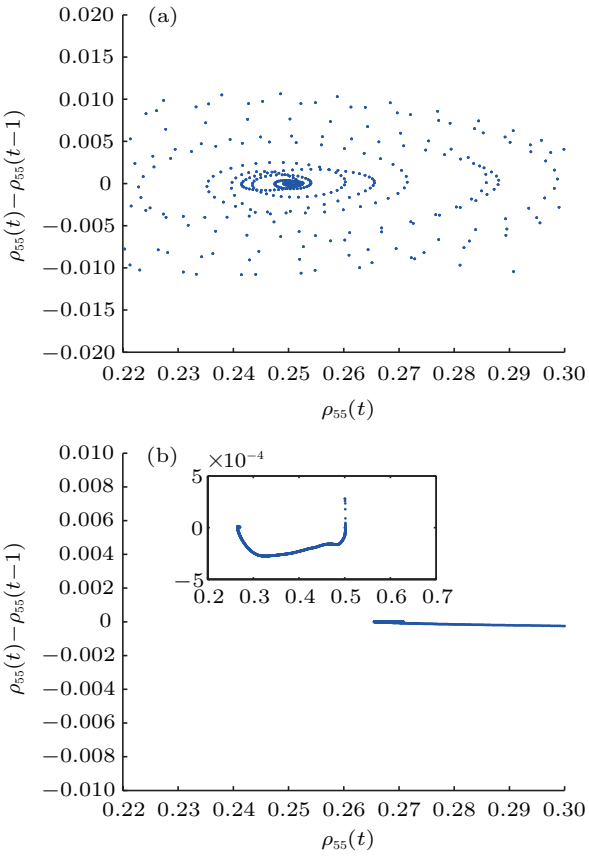


Fig. 11. (color online) Scatter plot of the 55th lattice at time $t = 1$–500 s (a) without the control signal (b) and with $\lambda = 0.9$, $k = 0.85$.

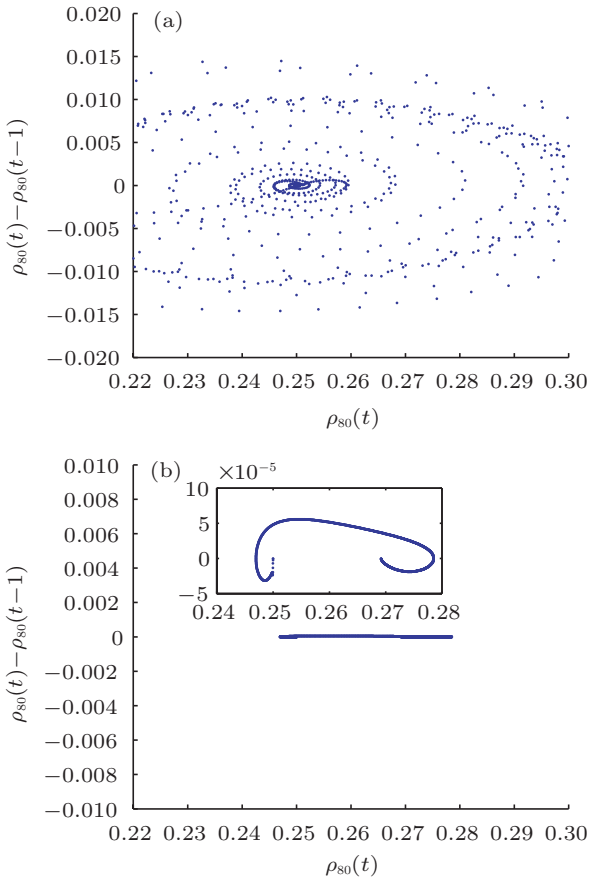


Fig. 12. (color online) Scatter plot of the 80th lattice at time $t = 1$–500 s (a) without the control signal (b) and with $\lambda = 0.9$, $k = 0.85$.

In brief, the simulation manifests the feasibility of taking into account the double flux difference to alleviate traffic congestion, and the control signal also plays a vital role in the lattice hydrodynamic model. Thus, it can be concluded that the proposed model is useful for suppressing the increasingly serious traffic jams.

5. Conclusion

A new feedback signal considering the double flux difference and the new optimal velocity function is proposed based on the control method for the lattice hydrodynamic model. The stability condition of the double flux difference model is derived by the control theory, and the transform function and the judgement of its norm expansion are deduced in the paper. The control coefficients k and λ can be computed by the numerical simulation of flux curve and density curve. In addition, the analysis of chaotic dispersed points validates the consistency between the theoretical analysis and the simulations. The result of simulations show that the double flux difference plays a vital role in alleviating traffic congestion. Therefore, the feedback control is helpful to suppress traffic jams and this factor should be considered in the traffic flow model. In reality, drivers adjust their driving behavior with a time delay. So research on the effect of the double flux difference model considering delay-time feedback is our next task.

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