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Symmetry and asymmetry rogue waves in two-component coupled nonlinear Schrödinger equations*

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By means of the modified Darboux transformation we obtain some types of rogue waves in two-coupled nonlinear Schrödinger equations. Our results show that the two components admits the symmetry and asymmetry rogue wave solutions, which arises from the joint action of self-phase, cross-phase modulation, and coherent coupling term. We also obtain the analytical transformation from the initial seed solution to unique rogue waves with the bountiful pair structure. In a special case, the asymmetry rogue wave can own the spatial and temporal symmetry gradually, which is controlled by one parameter. It is worth pointing out that the rogue wave of two components can share the temporal inversion symmetry.

Keywords: rogue wave, temporal inversion symmetry

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1. Introduction

The concept of rogue wave (RW) has been introduced in the investigation of oceanography,[1,2] (and references therein), starting with modeling a short-lived large amplitude wave in ocean. Furthermore, the rogue waves have also been studied in many fields, such as photonic crystal fibers,[3,4] space plasmas,[5–7] Bose–Einstein condensate,[8] water tanks,[9–11] and magnetic nanowires[12–14] driven by the external magnetic field or spin-polarized current. However, there is no exact definition for RW. In general, an RW means that the height of a rogue wave is two or more times those of the surrounding waves. The studies of RW in single-component system have indicated that the rational solution of the nonlinear Schrödinger equation can be used to describe this phenomenon well.[15–23] The lowest-order rational solutions[18,19] also can be obtained by Hirota technique for the discrete Ablowitz-Ladik and Hirota equations. Depending on the parameters chosen for the modulational unstable plane wave the lowest-order RW in the Sasa–Satsuma equation can feature a two-hump structure,[24,25] as well as an analog of the Peregrine soliton.[26–31]

However, many real complex systems usually involve more than one component, such as Bose–Einstein condensates, nonlinear optical fibers, and so on. Recently, the first-order rational solutions of a coupled nonlinear Schrödinger (CNLS) and Maxwell–Bloch equations have been reported in Refs. [32]– [36]. Some unique properties are obtained for the multicomponent system.[32,37–39] Dark RWs have been presented numerically.[39] and analytically.[40] Moreover, the emergence of two RWs[38,41,42] in the coupled-component system is quite distinct from the high-order RW in one-component system.[40,43,44] The interaction between RW and other nonlinear waves of the two-component coupled systems is also a hot topic of great interest.[37,38,40] Nonlinearly coupled dynamic systems with two or more degrees of freedom have attracted considerable interest for many years.[45,46] The localized waves and all kinds of soliton and soliton–soliton interaction solutions[42,47,48] have been observed in CNLS equation. It is well known that the CNLS equations has a wide range of physical backgrounds. It is a general equation describing wave propagation in weakly nonlinear media, widely used in plasma physics and nonlinear optics, condensed matter physics, biophysics, and other fields.[49–51] Therefore, the exploration of exact solutions in CNLS equations is an interesting subject both in mathematics and physics.

In this paper, we investigate some types of exact rogue wave solutions in the CNLS equations in terms of the developed Darboux transformation. As an example, we obtain the exact symmetry and asymmetry rogue waves. From these results we also find the two components share the temporal in-

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version symmetry owing to the joint action of self-phase and cross-phase modulation, as well as the coherent coupling term.

2. Exact symmetry and asymmetry rogue waves

The CNLS equations with negative coherent coupling can describe the propagation of the orthogonally polarized optical waves in an isotropic medium. In the paraxial approximation the CNLS equations takes the form[52,53]

$$\begin{aligned} &\mathrm{i}q_{1t}+q_{1xx}+2\left(|q_1|^2+2|q_2|^2\right)q_1-2q_1^*q_2^2=0,\\ &\mathrm{i}q_{2t}+q_{2xx}+2\left(|q_2|^2+2|q_1|^2\right)q_2-2q_2^*q_1^2=0, \end{aligned} \tag{1}$$

where q_1 and q_2 are the slowly varying envelopes of two interacting optical modes with the asterisk being the complex conjugate. The variables x and t correspond to the normalized distance and retarded time, respectively. In Eq. (1) the coefficients of the self-phase modulation and the cross-phase modulation are different, and the last terms in left hand are known as the coherent coupling term. In fact, equation (1) is integrable and its solutions can be obtained by the effective Darboux transformation and Hirota bilinear method. The Lax pair of Eq. (1) takes the form

$$\begin{aligned} &\Psi_x=(\lambda U_0+U_1)\Psi,\\ &\Psi_t=\left(\lambda^2V_0+\lambda V_1+V_2\right)\Psi, \end{aligned} \tag{2}$$

in which the compatibility condition $\Psi_{xt}=\Psi_{tx}$ yields the CNLS equations (1), here

$$\begin{aligned} &U_0=\mathrm{i}\begin{pmatrix}-I_{2\times2} & O\\ O & I_{2\times2}\end{pmatrix}, && U_1=\begin{pmatrix}O & Q\\ -Q^\dagger & O\end{pmatrix},\\ &Q=\begin{pmatrix}q_1 & q_2\\ -q_2 & q_1\end{pmatrix}, && V_0=\mathrm{diag}(-2\mathrm{i},-2\mathrm{i},2\mathrm{i},2\mathrm{i}),\\ &V_1=2\begin{pmatrix}O & Q\\ -Q^\dagger & O\end{pmatrix}, && V_2=\mathrm{i}\begin{pmatrix}QQ^\dagger & Q_x\\ Q_x^\dagger & -Q^\dagger Q\end{pmatrix},\\ &I_{2\times2}=\mathrm{diag}(1,1), && \Psi=\begin{pmatrix}\varphi_1 & \varphi_2 & \varphi_3 & \varphi_4\\ -\varphi_2 & \varphi_1 & -\varphi_4 & \varphi_3\end{pmatrix}^{\mathrm{T}}. \end{aligned}$$

Here the superscript T signifies the matrix transpose, Ψ is the matrix eigenfunction, φ_i $(i=1,2,3,4)$ are the function of x and t, $I_{2\times2}$ is the 2×2 unit matrix, O is the 2×2 zero matrix, $\dagger$ denotes the conjugate transpose, and λ is the eigenvalue parameter. If one assume that

$$\Psi=\begin{pmatrix}\varphi_1 & \varphi_2 & \varphi_3 & \varphi_4\\ -\varphi_2 & \varphi_1 & -\varphi_4 & \varphi_3\end{pmatrix}^{\mathrm{T}}$$

is a solution of Eqs. (2) corresponding to λ, and then

$$\begin{pmatrix}-\varphi_3^* & \varphi_4^* & \varphi_1^* & -\varphi_2^*\\ \varphi_4^* & \varphi_3^* & -\varphi_2^* & -\varphi_1^*\end{pmatrix}^{\mathrm{T}}$$

is also a solution of Eqs. (2) corresponding to the eigenvalue λ^*.

In this paper, we adopt the Darboux transformation method and mainly investigate the symmetry and asymmetric rogue waves. To this purpose we choose the initial seed solutions of Eq. (1) as

$$q_1^{(0)}=q_2^{(0)}=\mathrm{e}^{4\mathrm{i}t}. \tag{3}$$

Substituting Eq. (3) into Eq. (2) and employing the normal process of Darboux transformation, we obtain the eigenvalue $\lambda=\mathrm{i}\sqrt{2}$, and the corresponding eigenfunctions takes the form

$$\begin{aligned} \varphi_1&=\Gamma_1\mathrm{e}^{2\mathrm{i}t},\quad \varphi_2=\Gamma_2\mathrm{e}^{2\mathrm{i}t},\\ \varphi_3&=-\frac{1}{\sqrt{2}}(\Gamma_1-\Gamma_2)\mathrm{e}^{-2\mathrm{i}t}-\frac{1}{2}(C_4-C_2)\mathrm{e}^{-2\mathrm{i}t},\\ \varphi_4&=-\frac{1}{\sqrt{2}}(\Gamma_1+\Gamma_2)\mathrm{e}^{-2\mathrm{i}t}+\frac{1}{2}(C_2+C_4)\mathrm{e}^{-2\mathrm{i}t}, \end{aligned} \tag{4}$$

where $\Gamma_1=\mathrm{i}2\sqrt{2}C_2t+C_2x+C_1$, $\Gamma_2=\mathrm{i}2\sqrt{2}C_4t+C_4x+C_3$ with C_i $(i=1,2,3,4)$ being complex. With the help of eigenfunctions in Eq. (4) the rogue waves of Eq. (1) take the form

$$q_1=q_1^{(0)}-4\sqrt{2}\frac{f_1}{g_1},\quad q_2=q_2^{(0)}-4\sqrt{2}\frac{d_1}{g_1}, \tag{5}$$

where

$$\begin{aligned} f_1=&(\varphi_1\varphi_3+\varphi_2\varphi_4)\left((\varphi_3^*)^2+(\varphi_4^*)^2\right)\\ &+(\varphi_1^*\varphi_3^*+\varphi_2^*\varphi_4^*)\left(\varphi_1^2+\varphi_2^2\right),\\ d_1=&(\varphi_1\varphi_4-\varphi_2\varphi_3)\left((\varphi_3^*)^2+(\varphi_4^*)^2\right)\\ &+(\varphi_1^*\varphi_4^*-\varphi_2^*\varphi_3^*)\left(\varphi_1^2+\varphi_2^2\right),\\ g_1=&-(\varphi_1\varphi_1^*+\varphi_2\varphi_2^*+\varphi_3\varphi_3^*+\varphi_4\varphi_4^*)^2\\ &-(\varphi_1\varphi_2^*-\varphi_2\varphi_1^*+\varphi_3\varphi_4^*-\varphi_4\varphi_3^*)^2. \end{aligned}$$

Equation (4) indicates the rogue wave solution in Eq. (5) is combined by the rational polynomial dependence with temporal and spatial coordinates. Furthermore, we see that the rational polynomial form can be determined by the parameters C_i, $i=1,2,3,4$, which ultimately affect the types of rogue wave solutions, as shown in the following section.

2.1. Symmetry rogue waves

It is interesting to point out that the difference of the self-phase modulation and cross-phase modulation, and the coherent coupling term in Eq. (1) determines the types of rogue wave solution. Under their joint action two components q_1 and q_2 can take the symmetry rogue wave, i.e., q_1 and q_2 of Eq. (1) respectively shares the same expression which also has the spatial and temporal symmetry. These types of rogue wave solutions can be obtained by choosing the different values of the parameters C_i, $i=1,2,3,4$.

Firstly, we consider the case $C_i=0$, $i=1,2,3$, i.e., only $C_4\neq0$. Substituting Eq. (4) into Eq. (5) we find that rogue

wave q_1 and q_2 share the same form

$$q_1 = q_2 = \mathrm{e}^{4\mathrm{i}t}\left(-1+\frac{1+8\mathrm{i}t}{\Delta_1}\right), \tag{6}$$

where $\Delta_1 = 16t^2+1/4+2\left(x-\sqrt{2}/4\right)^2$.

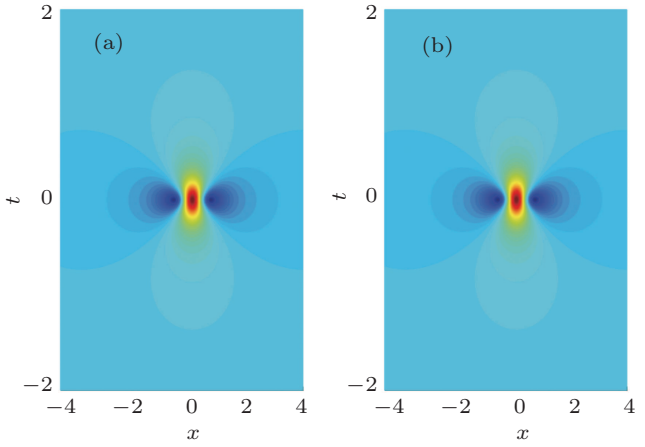


Fig. 1. (color online) The illustration of vector rogue wave envelope distribution for two component: (a) $|q_1|$, (b) $|q_2|$ in Eq. (6).

The solution in Eq. (6) has the normal form of rogue wave in the standard nonlinear Schrödinger equation, which has one hump and two valleys located at $x=\sqrt{2}/4$, $x=\sqrt{2}/4\pm\sqrt{6}/4$, respectively. Its illustration is shown in Fig. 1, which has the spatial and temporal symmetry.

Secondly, we consider the case $C_3=C_4=0$, and $C_2/C_1 \equiv \gamma_1\,\mathrm{e}^{\mathrm{i}\phi}$. By a tedious calculation we get the rogue wave

$$q_1 = q_2 = \mathrm{e}^{4\mathrm{i}t}\left(\frac{\sqrt{2}\gamma_1\left(1-\mathrm{e}^{2\mathrm{i}\phi}\right)+(1+8\mathrm{i}t)\,\gamma_1^2\,\mathrm{e}^{\mathrm{i}\phi}}{\left(\Delta_1\gamma_1^2+2\right)\mathrm{e}^{\mathrm{i}\phi}+\mathrm{i}\gamma_1\left(E^*\mathrm{e}^{2\mathrm{i}\phi}-E\right)}-1\right), \tag{7}$$

where $E=\left(4\sqrt{2}t+2\mathrm{i}x-\mathrm{i}\sqrt{2}/2\right)$. Equation (7) shows that this rogue wave also has the property of spatial and temporal symmetry, and the spatial and temporal location of rogue wave can be controlled by the parameters γ_1 and ϕ, as shown in Fig. 2. With the different parameters $\phi=0$, $\pi/2$, π, $3\pi/2$, we give the illustration in Fig. 2 for the rogue wave in Eq. (7). It is obvious that the rogue wave also has one hump and two valleys located at $x=\sqrt{2}/4-(\cos\phi)/\gamma_1$, $x=\sqrt{2}/4\pm\sqrt{6}/4$, and $t=\sqrt{2}\,(\sin\phi)/4\gamma_1$, respectively. When $\gamma_1\to\infty$, the rogue wave solution in Eq. (7) reduces into Eq. (6).

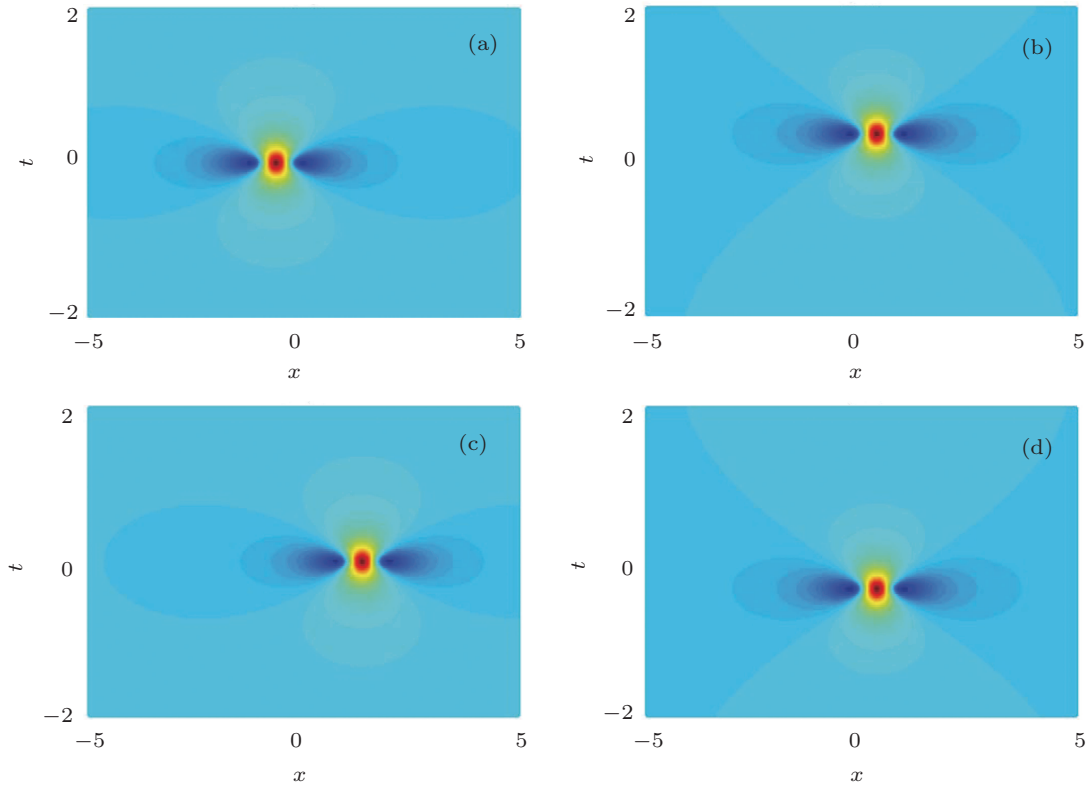


Fig. 2. (color online) The vector rogue wave envelope distribution for different ϕ with $q_1=q_2$ in Eq. (7) with the parameter $\gamma=1$, (a) $\phi=0$; (b) $\phi=\pi/2$; (c) $\phi=\pi$; (d) $\phi=3\pi/2$.

2.2. Asymmetry rogue waves

The very interesting phenomenon is that equation (1) admits the asymmetry rogue waves $q_1\neq q_2$ owing the joint action of the self-phase and cross-phase modulation, as well as the coherent coupling term in Eq. (1). In order to obtain the asymmetry rogue waves we can consider the case $C_1=C_4=0$ and $C_2/C_3\equiv\gamma_2\,\mathrm{e}^{\mathrm{i}\phi}$. With this condition the asymmetry rogue wave solution takes the following form

$$q_1 = \mathrm{e}^{4\mathrm{i}t}\left(1+\frac{F+N}{G}\right),$$

$$q_2 = \mathrm{e}^{4\mathrm{i}t}\left(1+\frac{F-N}{G}\right), \tag{8}$$

where

$$F = -\gamma_2^4 M B^* \Delta_1 - 8 + 2\gamma_2^2\left(E^* B^* \mathrm{e}^{2\mathrm{i}\phi} + EM\,\mathrm{e}^{-2\mathrm{i}\phi}\right),$$
$$N = \frac{\sqrt{2}}{2}\gamma_2\left[4\left(\mathrm{e}^{\mathrm{i}\phi}+\mathrm{e}^{-\mathrm{i}\phi}\right) - \gamma_2^2\left(B^{*2}\mathrm{e}^{\mathrm{i}\phi} + M^2\mathrm{e}^{-\mathrm{i}\phi}\right)\right],$$
$$G = \gamma_2^2\left(1+(\gamma_2\Delta_1)^2\right) - \gamma_2^2\left(E^2\mathrm{e}^{-2\mathrm{i}\phi} + E^{*2}\mathrm{e}^{2\mathrm{i}\phi}\right) + 4$$

with

$$B = E + \mathrm{i}\sqrt{2}/2,\ \ M = E - \mathrm{i}\sqrt{2}/2.$$

The rogue wave solution in Eq. (8) seems very complicated. However, there exists the natural intrinsic symmetry properties of rogue wave. Comparing the rogue wave solution for q_1 and q_2 in Eq. (8) carefully we find that $q_1(\phi = 0)$ takes the same expression of $q_2(\phi = \pi)$, as well as the same case for $q_1(\phi = \pi/2)$ and $q_2(\phi = 3\pi/2)$. It implies that the evolution of q_1 in fact has a π phase difference with q_2 in the ϕ-space, while the two components share the same illustration. This result comes from the action of the coherent coupling terms, that is, the last terms on the left-hand side of Eq. (1). It should be noted that when $\gamma_2 = 0$, the solution in Eq. (8) reduces to the initial seed solutions of Eq. (1), while with the phase difference π. This result implies that the variation of the parameter γ_2 can denote the transformation from the initial seed solution to the unique rogue wave solutions with the bountiful pair structure.

In order to discuss the detail illustration of rogue wave solutions in Eq. (8), we firstly consider the parameter $\phi = 0$. In this case, the solution in Eq. (8) represents a pair structure of rogue wave, which is determined by the parameter γ_2 shown in Fig. 3. When γ_2 is small each component has two peaks and four valleys, denoted in Figs. 3(a) and 3(e). From Figs. 3(a)–3(h) we find that the distance of peaks (valleys) for q_1 and q_2 will decreases with the increasing value γ_2. When $\gamma_2 \to \infty$, and equation (8) reduces to the rogue wave solution in Eq. (6), which implies that the pair structure of two peaks and four valleys vanishes and reduces to the structure with one peak and two valleys. Especially, the shape of q_1 in Fig. 3(a) likes the two anti-eye-shaped structure[54,55] of rogue wave solution. The solution of q_1 and q_2 in Eq. (8) also has the spatial and temporal symmetry, respectively.

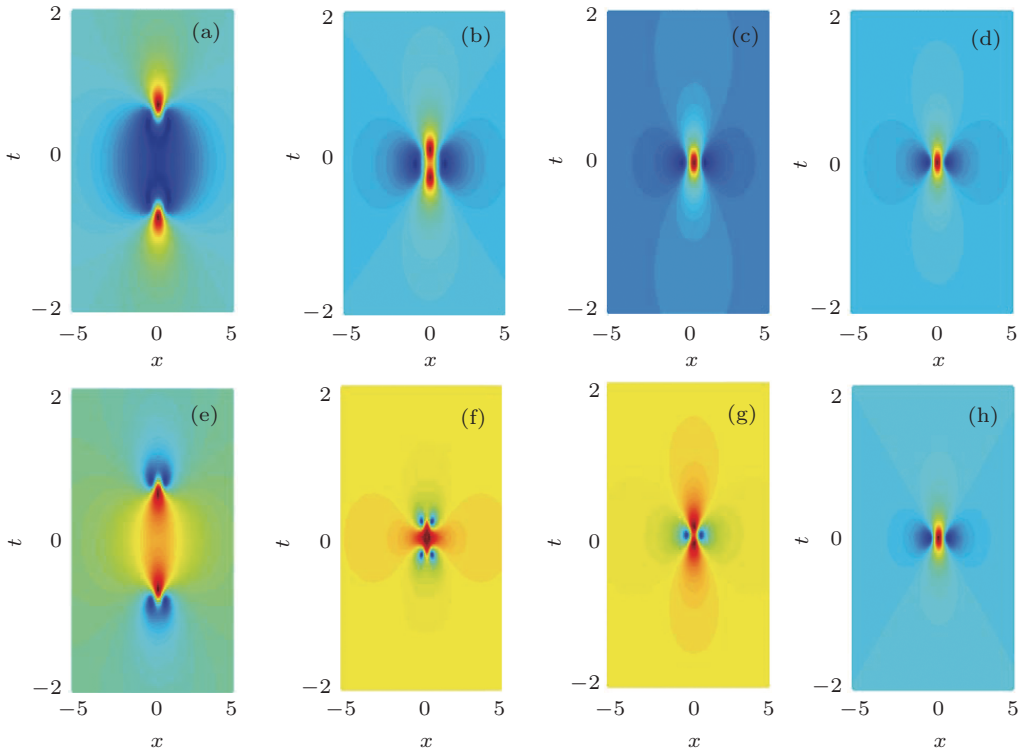


Fig. 3. (color online) Vector rogue wave envelope distributions for $|q_1|$ in panels (a)–(d) and $|q_2|$ in panels (e)–(h) in Eq. (8) with $\phi = 0$: (a) and (e) $\gamma = 0.5$; (b) and (f) $\gamma = 2$; (c) and (g) $\gamma = 8$; (d) and (h) $\gamma = 100$.

Secondly, we consider the case $\phi = \pi/2$. Similar the above discussion the rogue wave solution in Eq. (8) with $\phi = \pi/2$ also has the pair structure with the number changes of peak and valley, as shown in Fig. 4. From Figs. 4(a)–4(h) we find that a pair is formed by the combination of one peak and valley, and the distance of such pair decreases with the increasing value γ_2. When $\gamma_2 \to \infty$, equation (8) also reduces to the standard rogue wave solution in Eq. (6). It should be noted that the solution of q_1 and q_2 in Eq. (8) also loss the spatial and temporal symmetry when γ_2 is small. With γ_2 approach ∞ the solution for q_1 and q_2 own the spatial and temporal symmetry gradually. In terms of careful comparison we find that the rogue solution $q_1(t) = q_2^*(-t)$, i.e., $|q_1(t)| = |q_2(-t)|$. It is to say that the two components q_1 and q_2 share the temporal inversion symmetry.

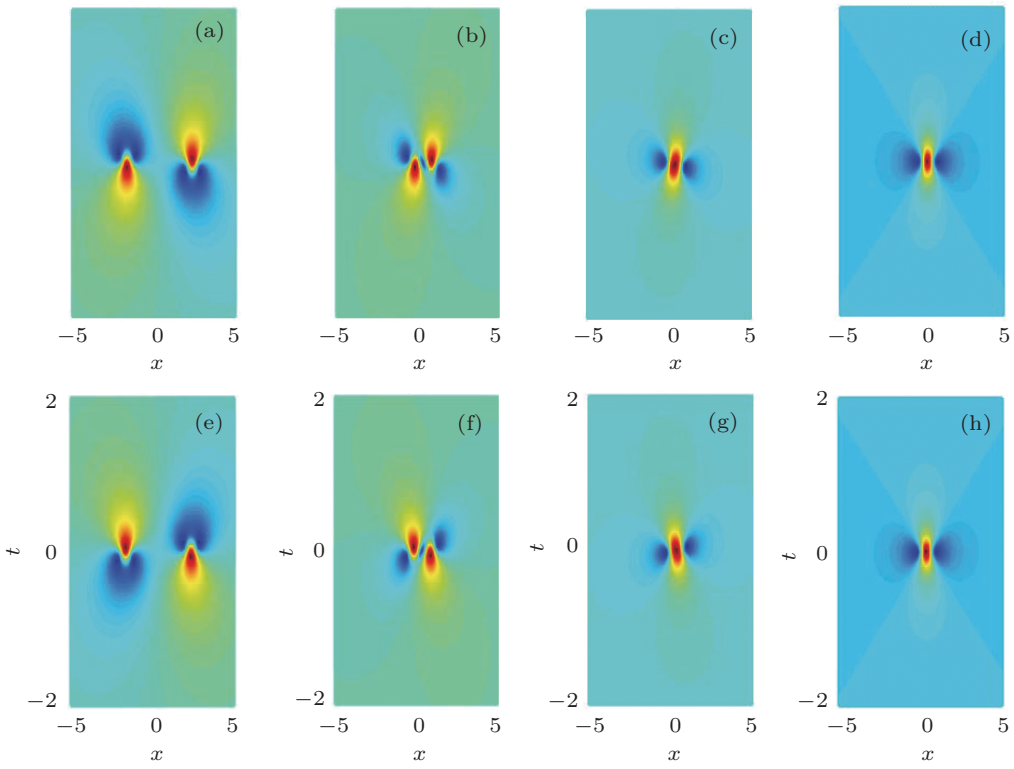


Fig. 4. (color online) Vector rogue wave envelope distributions for $|q_1|$ in panels (a)–(d) and $|q_2|$ in panels (e)–(h) in Eq. (8) with $\phi = \pi/2$: (a) and (e) $\gamma = 0.5$; (b) and (f) $\gamma = 2$; (c) and (g) $\gamma = 8$; (d) and (h) $\gamma = 100$.

3. Conclusion

In this paper, in terms of the developed the Darboux transformation we give some types of exact rogue wave solutions in the CNLS equations, which can be used to describe transition dynamics of a one-dimensional or two-component Bose–Einstein condensate system with particle transition in strong interaction regimes and other nonlinear systems. Under the affection of coherent coupling terms, the CNLS equations admits the symmetry and asymmetry rogue waves. We also give the different pair structure of rogue wave which can be adjusted by one parameter. In the special case, we find that the rogue wave solution q_1 and q_2 can share the temporal inversion symmetry.

References

[1] Kharif C and Pelinovsky E 2003 *Eur. J. Mech. B* **22** 603
[2] Kharif C, Pelinovsky E and Slunyaev A 2009 (Berlin, Heidelberg: Springer) 91
[3] Solli D R, Ropers C, Koonath P and Jalali B 2007 *Nature* **450** 1054
[4] Kibler B, Fatome J, Finot C, Millot G, Dias F, Genty G, Akhmediev N and Dudley J M 2010 *Nat. Phys.* **6** 790
[5] Ruderman M S 2010 *Eur. Phys. J. Spec. Top.* **185** 57
[6] Moslem W M, Shukla P K and Eliasson B 2011 *Euro. Phys. Lett.* **96** 25002
[7] Zeba I, Yahia M E, Shukla P K and Moslem W M 2012 *Phys. Lett. A* **376** 2309
[8] Bludov Y V, Konotop V V and Akhmediev N 2009 *Phys. Rev. A* **80** 2962
[9] Chabchoub A, Hoffmann N P and Akhmediev N 2011 *Phys. Rev. Lett.* **106** 204502
[10] Chabchoub A, Hoffmann N P and Akhmediev N 2012 *Geophys. J. Res.* **117** 100
[11] Chabchoub A, Hoffmann N, Onorato M and Akhmediev N 2012 *Phys. Rev. X* **2** 4089
[12] Zhao F, Li Z D, Li Q Y, Wen L, Fu G S and Liu W M 2012 *Ann. Phys.* **327** 2085
[13] Li Z D, Li Q Y, Xu T F and He P B 2016 *Phys. Rev. E* **94** 042202
[14] Li Z D, Li Q Y and Liu W M 2017 *J. Phys.: Conf. Ser.* **827** 012002
[15] Liu Y K and Li B 2017 *Chin. Phys. Lett.* **34** 10202
[16] Qian C, Liu Y B and He J S 2016 *Chin. Phys. Lett.* **33** 110201
[17] Voronovich V V, Shrira V I and Thomas G 2008 *J. Fluid Mech.* **604** 263
[18] Akhmediev N, Ankiewicz A and Taki M 2009 *Phys. Lett. A* **373** 675
[19] Ankiewicz A, Soto-Crespo J M and Akhmediev N 2010 *Phys. Rev. E* **81** 046602
[20] Akhmediev N, Ankiewicz A and Sotocrespo J M 2009 *Phys. Rev. E* **80** 026601
[21] He J S, Zhang H R, Wang L H, Porsezian K and Fokas A S 2012 *Phys. Rev. E* **87** 052914
[22] Wang L H, He J S, Xu H, Wang J and Porsezian K 2017 *Phys. Rev. E* **95** 042217
[23] Wang L H, Yang C H, Wang J and He J S 2017 *Phys. Lett. A* **381** 1714
[24] Bandelow U and Akhmediev N 2012 *Phys. Rev. E* **86** 026606
[25] Chen S 2013 *Phys. Rev. E* **88** 023202
[26] Peregrine D H 1983 *Anziam. Journal* **25** 16
[27] Chabchoub A, Hoffmann N P and Akhmediev N 2011 *Phys. Rev. Lett.* **106** 204502
[28] Wang Y, Song L J, Li L and Malomed Boris A 2015 *J. Opt. Soc. Am. B* **32** 2257
[29] Wang Y, Song L J and Li L 2016 *Appl. Opt.* **55** 7241
[30] Liu B, Li L and Malomed B A 2017 *Eur. Phys. D* **71** 140
[31] Ankiewicz A, Akhmediev N and Soto-Crespo J M 2010 *Phys. Rev. E* **82** 026602
[32] He J S, Xu S W and Porsezian K 2012 *J. Phys. Soc. Jpn.* **81** 4007
[33] He J S, Xu S W and Porsezian K 2012 *Phys. Rev. E* **86** 066603
[34] Xu S W, Porsezian K, He J S and Cheng Y 2013 *Phys. Rev. E* **88** 062925
[35] Xu S W, Porsezian K, He J S and Cheng Y 2016 *Romanian Reports in Physics* **68** 316
[36] He J S, Xu S, Porsezian K, Cheng Y and Dinda P T 2016 *Phys. Rev. E* **93** 062201
[37] Guo B L and Ling L M 2011 *Chin. Phys. Lett.* **28** 110202
[38] Baronio F, Degasperis A, Conforti M and Wabnitz S 2012 *Phys. Rev. Lett.* **109** 044102

[39] Bludov Y V, Konotop V V, Akhmediev N 2010 *Eur. Phys. J. Special Topics* **185** 169
[40] Zhao L C and Liu J 2012 *J. Opt. Soc. Am. B* **29** 3119
[41] Baronio F, Conforti M, Degasperis A, Lombardo S, Onorato M and Wabnitz S 2014 *Phys. Rev. Lett.* **113** 034101
[42] Zhang G Q, Yan Z Y and Wen X Y 2017 *Proc. Math. Phys. Eng. Sci.* **473** 0243
[43] Yang G Y, Li L and Jia S 2012 *Phys. Rev. E* **85** 046608
[44] Yang G Y, Wang Y, Qin Z Y, Malomed B A, Mihalache D and Li L 2014 *Phys. Rev. E* **90** 062909
[45] Mikhailovet A V, Shabat A B and Yamilov R I 1987 *Russ. Math. Surveys.* **42** 3
[46] Hietarinta J 1987 *Phys. Rep.* **147** 87
[47] Zhao L C, Ling L M, Yang Z Y and Liu J 2015 *Commun. Nonlinear Sci. Numer. Simulat.* **23** 21
[48] Ling L M, Feng B F and Zhu Z N 2018 *Nonlinear Analysis: Real World Applications* **40** 185
[49] Dysthe K B and Pecseli H L 1977 *Plasma Physics* **19** 931
[50] Chen Y Y, Wang Q and Shi J L 2004 *Acta Phys. Sin.* **53** 1070
[51] Sakuma T and Kawanami Y 1984 *Phys. Rev. B* **29** 880
[52] Malomed B A 1992 *Phys Rev A* **22** 403
[53] Nakkeeran K 2001 *J. Mod. Opt.* **48** 1863
[54] Ling L M and Zhao L C 2015 *Phys. Rev. E* **92** 022924
[55] Sun W R, Tian B, Jiang Y and Zhen H L 2015 *Phys. Rev. E* **91** 023205