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Time-varying formation for general linear multi-agent systems via distributed event-triggered control under switching topologies*

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This paper investigates the time-varying formation problem for general linear multi-agent systems using distributed event-triggered control strategy. Different from the previous works, to achieve the desired time-varying formation, a distributed control scheme is designed in an event-triggered way, in which for each agent the controller is triggered only at its own event times. The interaction topology among agents is assumed to be switching. The common Lyapunov function as well as Riccati inequality is applied to solve the time-varying formation problem. Moreover, the Zeno behavior of triggering time sequences can be excluded for each agent. Finally, a simulation example is presented to illustrate the effectiveness of the theoretical results.

Keywords: multi-agent systems, time-varying formation, switching topologies, event-triggered control

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1. Introduction

In recent years, the research on formation control of multi-agent systems has received significant attention from both scientific and engineering communities, which has great potential applications in broad areas such as load transportation,[1] static and dynamic obstacle avoidance,[2] and unmanned aerial vehicles formation.[3–5]

Formation control is a significant issue in cooperative control for multi-agent systems. A formation is defined as a predetermined special configuration (such as desired positions or orientations) formed by a cluster of interconnected agents, in which a global goal is achieved cooperatively.[6] Generally speaking, formation can be divided into fixed formation and time-varying formation. Most existing results considered fixed formation control, for example in Refs. [7] and [8] and the literature therein, to name a few. However, in many real applications, the environment may often change and the target is always moving. This motivates several researches on time-varying formation.[9–11] In addition, the derivative of time-varying formation vector may help to analyze and design the formation control strategy, which is the main difference between fixed formation control and time-varying formation control. Thus it is more utility to study time-varying formation control problem. It is worth mentioning that not all time-varying formation can be directly achieved. Hence a description of the feasible constraints (called the feasibility conditions) for time-varying formation control is usually given.[11–13]

Recently, the consensus problem for multi-agent systems has been greatly studied.[14–20] It is found that consensus method can be used to deal with formation control problem. The consensus-based method for formation problem has been developed in such as Refs. [12], [13], [21]–[24]. In Ref. [12], the distributed adaptive time-varying formation control problem for multi-agent systems with fixed topology was introduced. Other relevant researches on formation control under fixed topology can be found in Refs. [22]– [24]. Considering the fact that the interaction topology among agents may change dynamically, formation control for multi-agent systems under switching topologies was addressed in Refs. [11], [13], and [21]. Compared with Refs. [13] and [21], the feasibility condition in Ref. [11] is easier to test and the complexity of the proof is reduced.

In most existing formation control strategies, controller always updates continuously. However, such control strategies might be inefficient or impractical in multi-agent systems since they can lead to large communication load and waste excessive resources. Recently, researchers exhibit novel interests on event-triggered control since it is beneficial to coordination of multi-agent systems and can facilitate the efficient usage of the shared resources. In event-triggered control, data transmission or controller actuation times are determined by an event-triggering mechanism. The event-triggered control often depends on a well-defined event-triggering condition in which the measurement error plays an essential role. Many works on event-triggered control for multi-agent systems focused on the consensus problem.[25–28] In Refs. [25]–[27], the

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consensus can be achieved asymptotically via event-triggered control under fixed topology. The consensus problem under switching topologies with event-triggered strategy was studied in Ref. [28]. The event-triggering containment control of multi-agent networks with fixed and switching topologies was investigated in Ref. [29]. In practice, communication delays are inevitable. In Ref. [30], event-triggered tracking control of leader-follower multi-agent systems with communication delays was discussed. However, to our best knowledge, there are few results on time-varying formation problem for multi-agent systems via event-triggered control . This motivates our study.

Motivated by the above considerations, in this paper, we aim at dealing with the time-varying formation problem of multi-agent systems with general linear dynamics via distributed event-triggered control under switching topologies. The contributions of our work are four-fold: (i) The formation is time-varying, which has better flexibility. A feasibility condition for time-varying formation is given. In Ref. [31], the formation is assumed to be fixed. The results for time-invariant formation cannot be directly applied to time-varying formation. And in Ref. [21], the feasibility condition is more difficult to test than ours. (ii) A distributed event-triggered control scheme is designed. For each agent, the controller is triggered only at its own event times. In Refs. [12], [13], [21]– [24], their controllers are updated continuously, which may lead to large communication load and waste excessive resources. (iii) The interaction topology among agents is switching, which is more practical and challenging than fixed topology. In this case, the formation problem of the multi-agent system is changed into a stabilization problem of a switched system. A common Lyapunov function method is applied, which is more concise than the multiple Lyapunov functions method in Ref. [21]. (iv) It is further shown that the Zeno behavior can be avoided in the distributed event-triggered scheme, which means that two inter-execution instants have a positive lower bound.

The rest of this paper is organized as follows. In Section 2, some preliminaries and the problem formulation are provided. Section 3 presents the main results. The distributed event-triggered control law is designed and the time-varying formation problem under switching topologies is solved via common Lyapunov function method. The Zeno phenomena is further verified to be avoided. To illustrate the theoretical results, an example is shown in Section 4. Finally, Section 5 concludes this paper.

2. Preliminaries and problem formulation

2.1. Graph theory

An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ contains a vertex set $\mathcal{V} = \{1, 2, \ldots, N\}$, an edge set $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$, where $\alpha_{ii} = 0$ and $\alpha_{ij} = \alpha_{ji} \geq 0$. $\alpha_{ij} > 0$ if and only if there is an edge between vertex i and vertex j and the two vertices are called adjacent (or they are mutual neighbors). The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E},\ j \neq i\}$.

Define the Laplacian matrix L with respect to graph $\mathcal{G}$ as

$$L = D - \mathcal{A}, \tag{1}$$

where the diagonal matrix $D = \mathrm{diag}\{d_1, \ldots, d_N\} \in \mathbb{R}^{N\times N}$ is called degree matrix with the diagonal elements $d_i = \sum_{j=1}^{N} \alpha_{ij}$ for $i = 1, \ldots, N$.

By the definition, for undirected graph, L is symmetric and every row sum of it is zero. Zero 0 is an eigenvalue of L and $\mathbf{1}_N$ is the associated eigenvector, that is, $L\mathbf{1}_N = 0$, where $\mathbf{1}_N = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^N$. If $\mathcal{G}$ is undirected and connected, then 0 is the algebraically simple eigenvalue of L and all the other eigenvalues are positive.

2.2. Problem formulation

In this paper, the dynamics of each agent is described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ are the state and the control input of the i-th agent, $i = 1, \ldots, N$, respectively. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are constant matrices and (A, B) is assumed stabilizable.

We define that the vector $h(t) = [h_1^{\mathrm{T}}(t), h_2^{\mathrm{T}}(t), \ldots, h_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}$ is the desired time-varying formation and $h_i(t)$ $(i = 1, 2, \ldots, N)$ is piecewise continuous differentiable. It should be pointed out that $h(t)$ is only used to characterize the time-varying formation rather than to provide reference trajectory for each agent to follow (see, e.g., Ref. [9] for more details).

Definition 1 The multi-agent system (2) is said to achieve the desired time-varying formation $h(t)$, if for any given initial states, there exists a controller $u_i(t)$ such that

$$\lim_{t\to\infty} \|(x_i(t) - h_i(t)) - (x_j(t) - h_j(t))\| = 0, \quad i, j = 1, 2, \ldots, N.$$

The following lemmas will be used in the theoretical analysis.

Lemma 1[32] Assume that $x = 0$ is an equilibrium point for the system $\dot{x}(t) = f(x(t))$ and the existence and uniqueness conditions of solutions on $[t_0, +\infty)$ are satisfied. If $x = 0$ is locally asymptotically stable in $\mathcal{D} \subset \mathbb{R}^N$, then for any given signal $\bar{x}(t)$ satisfying $\lim_{t\to\infty} \bar{x}(t) = 0$, one can conclude that the trajectory of the differential system $\dot{x}(t) = f(x(t)) + \bar{x}(t)$ with initial state in $\mathcal{D}$ satisfies $\lim_{t\to\infty} x(t) = 0$.

Lemma 2[33] For Laplacian matrix $L \in \mathbb{R}^{N\times N}$ of graph $\mathcal{G}$ and a full row rank matrix $E \in \mathbb{R}^{(N-1)\times N}$ defined as

$$E = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \tag{3}$$

there exists a matrix $M \in \mathbb{R}^{N\times(N-1)}$ such that $L = ME$. Furthermore, if the graph is connected, M is full column rank and all $\lambda_i(EM) > 0$, $i = 1,2,\ldots,N-1$, where $\lambda_i(EM)$ is the eigenvalue of the matrix EM.

According to the definition of E in Eq. (3), the following property holds immediately.

Lemma 3 EE^{T} is positive definite, where E is defined in Eq. (3).

Proof Let $\hat{E} = \begin{bmatrix} E^{\mathrm{T}} & \mathbf{1}_N \end{bmatrix}$, then $\hat{E}$ is nonsingular and

$$\hat{E}^{\mathrm{T}}\hat{E} = \begin{bmatrix} EE^{\mathrm{T}} & E\mathbf{1}_N \\ \mathbf{1}_N^{\mathrm{T}}E^{\mathrm{T}} & \mathbf{1}_N^{\mathrm{T}}\mathbf{1}_N \end{bmatrix}.$$

Considering the property of E, we have $E\mathbf{1}_N = 0$ and $\mathbf{1}_N^{\mathrm{T}}E^{\mathrm{T}} = 0$. Then we have

$$\hat{E}^{\mathrm{T}}\hat{E} = \begin{bmatrix} EE^{\mathrm{T}} & 0 \\ 0 & N \end{bmatrix}.$$

According to the fact that $\hat{E}$ is nonsingular, $\hat{E}^{\mathrm{T}}\hat{E}$ and I_N have the same inertia by the Sylvester Law Inertia. Note that I_N is positive definite, one can obtain that $\hat{E}^{\mathrm{T}}\hat{E}$ is also positive definite, which implies that EE^{T} is positive definite. □

Lemma 4[34] For any given $x, y \in \mathbb{R}^n$, any matrices $P > 0$, D and S with appropriate dimensions, one has $2x^{\mathrm{T}}DSy \le x^{\mathrm{T}}DPD^{\mathrm{T}}x + y^{\mathrm{T}}S^{\mathrm{T}}P^{-1}Sy$.

3. Main results

3.1. Time-varying formation under switching topologies

In this section, we will consider the time-varying formation problem of multi-agent systems by distributed event-triggered control strategy.

We assume the adjacency graph is undirected and connected and the topology structure between agents is switching. Define the switching signal $\sigma(t) : [0,+\infty) \to \Lambda = \{1,2,\ldots,p\}$, which is a piecewise constant right continuous function. Assume the switching is arbitrary and $0 < \tau_1 < \cdots < \tau_l < \cdots$ are the switching moments of $\sigma(t)$. $\mathcal{G}_{\sigma(t)}$ denotes the topology graph at time t. During the time interval $[\tau_l, \tau_{l+1})$, the graph $\mathcal{G}_{\sigma(t)}$ is fixed. For simplicity, we assume the weights of all the interaction links are time-invariant. That is, when agent $j \in \mathcal{N}_i(t)$, the weight $\alpha_{ij}^{\sigma(t)} = \alpha_{ij} > 0$ is a constant, where $\mathcal{N}_i(t)$ is the neighbors' set of agent i at time t.

For saving energy, we construct the control law based on event-triggering strategy and assume the triggering mechanism is in the distributed style that the triggering instants are different for each agent. The event-triggering time sequence for agent i is denoted by $\{t_0^i, t_1^i, \ldots\}$ and t_k^i denotes the k-th triggering time of agent i.

In event-triggered strategy, agents only update their states at the time when some event conditions are triggered. Therefore, when $t \in [t_k^i, t_{k+1}^i)$, only at the triggering instant t_k^i, the topology graph $\mathcal{G}_{\sigma(t_k^i)}$ affects the update of the controller.

Let $z_i(t) = x_i(t) - h_i(t)$, $i = 1,2,\ldots,N$. The combinational measurement for agent i is defined as

$$q_i(t) = \sum_{j\in\mathcal{N}_i(t)} \alpha_{ij}^{\sigma(t_k^i)}(z_j(t) - z_i(t)), \quad t \in [t_k^i, t_{k+1}^i), \tag{4}$$

where t_k^i is the k-th event-triggering time of agent i.

At time t_k^i, agent i measures the information of $q_i(t)$ and will keep it unchanged until the next time t_{k+1}^i comes. Therefore, in the time interval $[t_k^i, t_{k+1}^i)$, the measurement of agent i is $q_i(t_k^i)$.

The measurement error can be defined as

$$e_i(t) = \sum_{j\in\mathcal{N}_i(t)} \alpha_{ij}^{\sigma(t_k^i)}(z_j(t_{k'(t)}^j) - z_i(t_k^i)) - q_i(t),\ t \in [t_k^i, t_{k+1}^i), \tag{5}$$

where t_k^i is the k-th event-triggering time of agent i, and $k'(t) = \arg\min_{l\in\mathbb{N}: t\ge t_l^j}\{t - t_l^j\}$. For each $t \in [t_k^i, t_{k+1}^i)$, $t_{k'(t)}^j$ is the latest event-triggering instant of agent j.

The event-triggering times $\{t_0^i, t_1^i, \ldots\}$ for agent i are specified when the following triggering condition

$$\|F_q e_i(t)\| \le \rho \,\|\, q_i(t)\| \tag{6}$$

is violated, that is,

$$t_0^i = 0,$$
$$t_{k+1}^i = \inf\{t > t_k^i : \|F_q e_i(t)\| > \rho\|q_i(t)\|\},\ \ k \in \mathbb{N},$$

where F_q and ρ will be determined later.

Then the distributed control law for agent i is designed as

$$u_i(t) = K\sum_{j\in\mathcal{N}_i(t)} \alpha_{ij}^{\sigma(t_k^i)}(z_j(t_{k'(t)}^j) - z_i(t_k^i)),$$
$$t \in [t_k^i, t_{k+1}^i),\ i = 1,2,\ldots,N, \tag{7}$$

where K is the feedback gain matrix to be designed.

In the time interval $[t_k^i, t_{k+1}^i)$, based on Eqs. (5) and (7), the closed-loop of system (2) is

$$\dot{x}_i(t) = Ax_i(t) + BK(e_i(t) + q_i(t)),\ \ i = 1,2,\ldots,N. \tag{8}$$

Denote

$$x(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$
$$e(t) = [e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$
$$h(t) = [h_1^{\mathrm{T}}(t), h_2^{\mathrm{T}}(t), \ldots, h_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$z(t) = [z_1^{\mathrm{T}}(t), z_2^{\mathrm{T}}(t), \ldots, z_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

where $z_i(t) = x_i(t) - h_i(t)$. Then equation (8) can be rewritten as

$$\begin{aligned}\dot{x}(t) = {} & (I_N \otimes A - L_{\sigma(t_k^i)} \otimes BK)x(t) + (I_N \otimes BK)e(t) \\ & + (L_{\sigma(t_k^i)} \otimes BK)h(t) \end{aligned} \tag{9}$$

or the error system

$$\begin{aligned}\dot{z}(t) = {} & (I_N \otimes A - L_{\sigma(t_k^i)} \otimes BK)z(t) + (I_N \otimes BK)e(t) \\ & + (I_N \otimes A)h(t) - (I_N \otimes I_n)\dot{h}(t). \end{aligned} \tag{10}$$

Define $\xi(t) = (E \otimes I_n)z(t)$, where E is defined in Eq. (3), then $\xi_i(t) = z_i(t) - z_{i+1}(t)$, $i = 1, 2, \ldots, N-1$. One has

$$\begin{aligned}\dot{\xi}(t) = {} & (I_{N-1} \otimes A - EM_{\sigma(t_k^i)} \otimes BK)\xi(t) \\ & + (I_{N-1} \otimes BK)\hat{e}(t) + (I_{N-1} \otimes A)H(t) - \dot{H}(t), \end{aligned} \tag{11}$$

where $\hat{e}(t) = (E \otimes I_n)e(t), H(t) = (E \otimes I_n)h(t)$, and $M_{\sigma(t_k^i)}$ is the corresponding matrix satisfying $L_{\sigma(t_k^i)} = M_{\sigma(t_k^i)}E$. Then we have the following main result.

Theorem 1 Assume the undirected adjacency graph of system (2) is switching and connected, and the pair (A, B) is stabilizable. If the following two conditions are satisfied,

(i) $\lim\limits_{t\to\infty}[(I_N \otimes A)h(t) - \dot{h}(t)] = 0$;

(ii) There exists a positive definite matrix $P > 0$ such that

$$A^{\mathrm{T}}P + PA - \alpha PBB^{\mathrm{T}}P + (1 + \beta\rho^2\|\varepsilon E \otimes I_n\|^2)I_n < 0, \tag{12}$$

where

$$\begin{aligned}\alpha = {} & \min\{\lambda_j((EM_{\sigma(t_k^i)})^{\mathrm{T}} + EM_{\sigma(t_k^i)})|\sigma(t_k^i) \in \Lambda, \\ & j = 1, \ldots, N-1\}, \\ \beta = {} & \max\{\lambda_j(M_{\sigma(t_k^i)}^{\mathrm{T}}M_{\sigma(t_k^i)})|\sigma(t_k^i) \in \Lambda, \\ & j = 1, \ldots, N-1\}, \end{aligned}$$

$\rho > 0$ and $\varepsilon > 0$ large enough; then the time-varying formation problem can be solved under the event-triggered control law (7) with $K = B^{\mathrm{T}}P$ and the triggering condition

$$\|\varphi_{\max}PBB^{\mathrm{T}}Pe_i(t)\| \le \rho\|q_i(t)\|, \tag{13}$$

where $\varphi_{\max} = \max\{\lambda_i(E^{\mathrm{T}}E)|\ i = 1, 2, \ldots, N\}$.

Proof To solve the time-varying formation problem, we prove the system (11) is asymptotically stable. If condition (i) holds, we conclude that $\lim\limits_{t\to\infty}[(I_{N-1} \otimes A)H(t) - \dot{H}(t)] = 0$. Then, by Lemma 1 we only need to prove the system

$$\dot{\xi}(t) = (I_{N-1} \otimes A - EM_{\sigma(t_k^i)} \otimes BK)\xi(t) + (I_{N-1} \otimes BK)\hat{e}(t) \tag{14}$$

is asymptotically stable.

Note that the system (14) is switching. Construct the common Lyapunov function candidate

$$V(t) = \xi^{\mathrm{T}}(t)(I_{N-1} \otimes P)\xi(t), \tag{15}$$

where P is a positive definite solution of the inequality (12).

For any $t \in [t_k^i, t_{k+1}^i)$, taking the time derivative of $V(t)$ along the trajectory of the system (14) as

$$\begin{aligned}\dot{V}(t) &= \dot{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes P)\xi(t) + \xi^{\mathrm{T}}(t)(I_{N-1} \otimes P)]\dot{\xi}(t) \\ &= \xi^{\mathrm{T}}(t)[I_{N-1} \otimes (A^{\mathrm{T}}P + PA) - (((EM_{\sigma(t_k^i)})^T + EM_{\sigma(t_k^i)}) \otimes PBB^{\mathrm{T}}P)]\xi(t) + 2\xi^{\mathrm{T}}(t)(I_{N-1} \otimes PBB^{\mathrm{T}}P)\hat{e}(t) \\ &\le \xi^{\mathrm{T}}(t)[I_{N-1} \otimes (A^{\mathrm{T}}P + PA) - (\alpha I_{N-1} \otimes PBB^{\mathrm{T}}P)]\xi(t) + 2\xi^{\mathrm{T}}(t)(I_{N-1} \otimes PBB^{\mathrm{T}}P)\hat{e}(t), \end{aligned} \tag{16}$$

where $\alpha = \min\{\lambda_j((EM_{\sigma(t_k^i)})^{\mathrm{T}} + EM_{\sigma(t_k^i)})|\sigma(t_k^i) \in \Lambda, j = 1, \ldots, N-1\}$.

When $t \in [t_k^i, t_{k+1}^i)$, by enforcing the triggering condition (13), we have

$$\begin{aligned}&\|\varphi_i PBB^{\mathrm{T}}Pe_i(t)\| \le \rho\|q_i(t)\|, \quad \|(E^{\mathrm{T}}E \otimes PBB^{\mathrm{T}}P)e(t)\| \le \rho\|q(t)\|, \\ &\|(E^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)\| \le \rho\|(M_{\sigma(t_k^i)} \otimes I_n)\xi(t)\| \le \rho\|M_{\sigma(t_k^i)} \otimes I_n\|\|\xi(t)\|, \\ &\|(E^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)\|^2 \le \rho^2\|M_{\sigma(t_k^i)} \otimes I_n\|^2 \ \|\ \xi(t)\|^2 \le \beta\rho^2\|\xi(t)\|^2, \end{aligned}$$

where φ_i is the eigenvalue of the matrix $E^{\mathrm{T}}E$, $q(t) = [q_1^{\mathrm{T}}(t), \ldots, q_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\hat{e}(t) = (E \otimes I_n)e(t)$, $M_{\sigma(t_k^i)}$ is one of the solution of the matrix equation $L_{\sigma(t_k^i)} = M_{\sigma(t_k^i)}E$, and $\beta = \max\{\lambda_j(M_{\sigma(t_k^i)}^{\mathrm{T}}M_{\sigma(t_k^i)})|\sigma(t_k^i) \in \Lambda, j = 1, \ldots, N-1\}$.

According to Lemma 4 and the event-triggered condition, we have

$$\begin{aligned}&2\xi^{\mathrm{T}}(t)(I_{N-1} \otimes PBB^{\mathrm{T}}P)\hat{e}(t) \le \xi^{\mathrm{T}}(t)\xi(t) + \hat{e}^{\mathrm{T}}(t)(I_{N-1} \otimes PBB^{\mathrm{T}}P)^2\hat{e}(t) \le \xi^{\mathrm{T}}(t)\xi(t) + \hat{e}^{\mathrm{T}}(t)((\varepsilon EE^{\mathrm{T}})^2 \otimes (PBB^{\mathrm{T}}P)^2)\hat{e}(t) \\ &\quad \le \xi^{\mathrm{T}}(t)\xi(t) + [(\varepsilon EE^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)]^{\mathrm{T}}(\varepsilon EE^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t) = \xi^{\mathrm{T}}(t)\xi(t) + \|(\varepsilon EE^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)\|^2 \\ &\quad = \xi^{\mathrm{T}}(t)\xi(t) + \|(\varepsilon E \otimes I_n)(E^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)\|^2 \le \xi^{\mathrm{T}}(t)\xi(t) + \|\varepsilon E \otimes I_n\|^2\|(E^{\mathrm{T}} \otimes PBB^{\mathrm{T}}P)\hat{e}(t)\|^2 \\ &\quad \le \xi^{\mathrm{T}}(t)\xi(t) + \beta\rho^2\|\varepsilon E \otimes I_n\|^2\|\xi(t)\|^2. \end{aligned} \tag{17}$$

The second inequality above holds because EE^{T} is positive-definite by Lemma 3. We can choose $\varepsilon > 0$ large enough such that all the eigenvalues of $\varepsilon EE^{\mathrm{T}} - I_{N-1}$ are positive.

Then from Eqs. (16) and (17), we have

$$\begin{aligned}\dot{V}(t) &\leq \xi^{\mathrm{T}}(t)[I_{N-1}\otimes(A^{\mathrm{T}}P+PA-\alpha PBB^{\mathrm{T}}P)]\xi(t)+\xi^{\mathrm{T}}(t)\xi(t)+\beta\rho^2\|(\varepsilon E\otimes I_n)\|^2\|\xi(t)\|^2\\ &\leq \xi^{\mathrm{T}}(t)[I_{N-1}\otimes(A^{\mathrm{T}}P+PA-\alpha PBB^{\mathrm{T}}P+(1+\beta\rho^2\|\varepsilon E\otimes I_n)\|^2)I_n]\xi(t). \end{aligned} \tag{18}$$

According to Eq. (12), one can obtain that $\dot{V}(t) < 0$. Thus, the multi-agent system (2) with switching topologies achieves time-varying formation $h(t)$. This completes the proof. □

Remark 1 Condition (i) in Theorem 1 is called the feasibility condition.

Remark 2 Compared with Refs. [11], [13], and [21], the event-triggered control is applied in our work. In Refs. [11], [13], and [21], the controllers are continuous. However, in practice, such control strategies might be inefficient since they can lead to large communication load and waste excessive resources. Therefore, in this paper, we focus on event-triggered control which can reduce the communication and facilitate the efficient usage of the shared resources. Of course, the theoretical deduction becomes more complicated and the methods in Refs. [11], [13], and [21] cannot be used in our problem directly. Furthermore, in Refs. [13] and [21], their controllers are designed using formation compensation signal which depends on time-varying formation. In our work, the formation compensation signal is unnecessary.

3.2. Lower bound for inter-event times

In this section, the feasibility of the proposed event-triggered control strategy will be analyzed by excluding the Zeno behavior. We will prove that in the triggering condition (13) the inter-event times for any agent are lower bounded by a strictly positive constant. The inter-event time means the time interval between any two consecutive triggered instants.

Theorem 2 For the multi-agent system (2) with the control law (7) and the triggering condition (13), no agent will exhibit Zeno behavior, that is, the inter-event times are lower bounded by a positive number $\tau \geq \rho/(a^2+ab\rho) > 0$, where $\rho > 0$, $a = \max\{\|E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P\|, \|I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK\| + W\}$, $b = \max\{1, \|mI_N\otimes P^{-1}\|\}$ and $W > 0$, $m > 0$.

Proof The triggering condition (13) can be transformed into $\|(E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P)e(t)\| \leq \rho\|q(t)\|$. Denote $Q(t) = (E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P)e(t)$, and

$$r(t) = \frac{\|Q(t)\|}{\|q(t)\|}.$$

Then, the range of $r(t)$ between two adjacent events is from 0 to ρ. The time derivative of $r(t)$ over the interval $[t_k^i, t_{k+1}^i)$ is

$$\begin{aligned}\dot{r}(t) &= \left(\frac{\|Q(t)\|}{\|q(t)\|}\right)' = \left(\frac{(Q^{\mathrm{T}}(t)Q(t))^{1/2}}{(q^{\mathrm{T}}(t)q(t))^{1/2}}\right)' = \frac{(Q^{\mathrm{T}}(t)Q(t))^{-1/2}\dot{Q}^{\mathrm{T}}(t)Q(t)\|q(t)\| - (q^{\mathrm{T}}(t)q(t))^{-1/2}\dot{q}^{\mathrm{T}}(t)q(t)\|Q(t)\|}{(\|q(t)\|)^2}\\ &= \frac{\|Q(t)\|^{-1}\dot{Q}^{\mathrm{T}}(t)Q(t)\|q(t)\| - \|q(t)\|^{-1}\dot{q}^{\mathrm{T}}(t)q(t)\|Q(t)\|}{(\|q(t)\|)^2}\\ &= \frac{\dot{Q}^{\mathrm{T}}(t)Q(t)}{\|Q(t)\|\|q(t)\|} + r(t)\frac{\dot{q}(t)q(t)}{(\|q(t)\|)^2} \leq \frac{\|\dot{Q}(t)\|\|Q^{\mathrm{T}}(t)\|}{\|Q(t)\|\|q(t)\|} + r(t)\frac{\|\dot{q}(t)\|\|q^{\mathrm{T}}(t)\|}{(\|q(t)\|)^2} = \frac{\|\dot{Q}(t)\|}{\|q(t)\|} + r(t)\frac{\|\dot{q}(t)\|}{\|q(t)\|}. \end{aligned} \tag{19}$$

Since

$$e_i(t) = \sum_{j\in\mathcal{N}_i(t)} \alpha_{ij}^{\sigma(t_k^i)}(z_j(t_{k'(t)}^j) - z_i(t_k^i)) - q_i(t),\ t\in[t_k^i, t_{k+1}^i),$$

one has $\dot{e}_i(t) = -\dot{q}_i(t)$. Therefore, $\|\dot{e}(t)\| = \|-\dot{q}(t)\| = \|\dot{q}(t)\|$.

Then

$$\begin{aligned}&\|\dot{Q}(t)\| = \|(E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P)\dot{e}(t)\| \leq \|E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P\|\|\dot{e}(t)\| = \|E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P\|\|\dot{q}(t)\|,\\ &\dot{r}(t) \leq (\|E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P\| + r(t))\frac{\|\dot{q}(t)\|}{\|q(t)\|}.\end{aligned}$$

By

$$q_i(t) = \sum_{j\in\mathcal{N}_i(t)} \alpha_{ij}^{\sigma(t_k^i)}(z_j(t) - z_i(t)),$$

we obtain

$$\begin{aligned}\|\dot{q}(t)\| &= \|-(L_{\sigma(t_k^i)}\otimes I_n)\dot{z}(t)\| = \|(L_{\sigma(t_k^i)}\otimes I_N)\dot{z}(t)\| \\ &= \|(L_{\sigma(t_k^i)}\otimes I_n)[(I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK)z(t) + (I_N\otimes BK)e(t) + (I_N\otimes A)h(t) - \dot{h}(t)]\| \\ &\le \|-(I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK)q(t)\| + \|(L_{\sigma(t_k^i)}\otimes BK)e(t)\| + \|(L_{\sigma(t_k^i)}\otimes A)h(t) - (L_{\sigma(t_k^i)}\otimes I_n)\dot{h}(t)\| \\ &\le \|(I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK)q(t)\| + \|(mI_N\otimes P^{-1})(E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P)e(t)\| + \|(L_{\sigma(t_k^i)}\otimes A)h(t) - (L_{\sigma(t_k^i)}\otimes I_n)\dot{h}(t)\| \\ &\le \|(I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK)\|\|q(t)\| + \|mI_N\otimes P^{-1}\|\|(E^{\mathrm{T}}E\otimes PBB^{\mathrm{T}}P)e(t)\| + \|(L_{\sigma(t_k^i)}\otimes A)h(t) - (L_{\sigma(t_k^i)}\otimes I_n)\dot{h}(t)\|, \quad (20)\end{aligned}$$

where $mE^{\mathrm{T}}E > L_{\sigma(t_k^i)}$ for some $m > 0$.

From Expression (20), we can find that unless

$$\|(L_{\sigma(t_k^i)}\otimes A)h(t) - (L_{\sigma(t_k^i)}\otimes I_n)\dot{h}(t)\| \to 0,$$

$\|q(t)\|$ will not approach to zero. Therefore, there exists a positive number $W > 0$ such that

$$\frac{\|(L_{\sigma(t_k^i)}\otimes A)h(t) - (L_{\sigma(t_k^i)}\otimes I_n)\dot{h}(t)\|}{\|q(t)\|} < W.$$

Then we can obtain

$$\begin{aligned}\dot{r}(t) \le\ & (\|E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1}E\otimes PBB^{\mathrm{T}}P\| + r(t)) \\ & \cdot(\|I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK\| + \|mI_N\otimes P^{-1}\|r(t) + W).\end{aligned}$$

Denote

$$\begin{aligned}a = \max\{&\|E^{\mathrm{T}}(EE^{\mathrm{T}})^{-1}E\otimes PBB^{\mathrm{T}}P\|, \\ &\|I_N\otimes A - L_{\sigma(t_k^i)}\otimes BK\| + W\},\end{aligned}$$

and $b = \max\{1, \|mI_N\otimes P^{-1}\|\}$. Then $\dot{r}(t) \le (a + br(t))^2$.

Therefore, the solution of the above differential inequality satisfies $r(t) \le \phi(t,\phi_0)$, where $\phi(t,\phi_0)$ is the solution of $\dot{\phi}(t) = (a + b\phi(t))^2$.

The minimal time τ between two inter-event is the time that makes $r(t)$ evolve from the value 0 to ρ. Then the time τ is no smaller than τ_ϕ for $\phi(t)$ evolving from 0 to ρ. For any finite $\phi(0) = 0$, $\phi(t) = a^2t/(1 - abt)$. Then $\tau_\phi = [\rho/(a^2 + ab\rho)] > 0$. Therefore, $\tau \ge \tau_\phi > 0$, which shows that the time interval between two adjacent events is lower bounded. □

4. Numerical simulations

In this section, a numerical example is provided to demonstrate the effectiveness of the results obtained in the previous section. In this example, the multi-agent system consists of four agents satisfying Eq. (2) with $x_i(t) = [x_{i1}, x_{i2}]^{\mathrm{T}}$,

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

These four agents are required to preserve a time-varying quadrangle formation. And the time-varying formation is specified by

$$h_i(t) = \begin{bmatrix} -10\cos(t + (i-1)\pi/2) \\ 10\sin(t + (i-1)\pi/2) \end{bmatrix}, \quad i = 1,2,3,4.$$

Assume the interaction topology between agents is time-varying. Four possible adjacency graphs, denoted by $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$, are shown in Fig. 1. Clearly, each graph is undirected and connected. The Laplacian matrices of the graphs $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ are

$$L_1 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 2 & 0 & -1 & -1 \\ 2 & 0 & -1 & -1 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}.$$

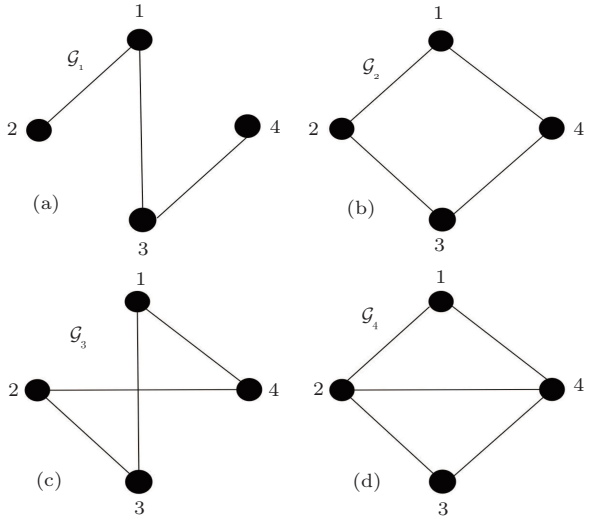


Fig. 1. Switching graphs $\mathcal{G}_{\sigma(t)}$ with $\sigma(t) = 1,2,3,4$.

The topology graphs are switched according to $\mathcal{G}_1 \to \mathcal{G}_3 \to \mathcal{G}_2 \to \mathcal{G}_4 \to \mathcal{G}_3 \to \mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_4 \to \mathcal{G}_1 \to \mathcal{G}_3 \to \mathcal{G}_2 \to \mathcal{G}_4 \to \mathcal{G}_3 \to \mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_4$. The switching signal is shown in Fig. 2, in which the switching duration is arbitrary.

Choose $\rho = 1$, $\alpha = 0.7421$, $\beta = 24.7047$, $\varepsilon = 2$, and $\varphi_{\max} = 3.4142$. By solving the inequality (12), we obtain the positive definite solution

$$P = \begin{bmatrix} 351.0763 & 12.6510 \\ 12.6510 & 13.6451 \end{bmatrix} > 0.$$

Then by Theorem 1, the desired time-varying formation can be achieved under the event-triggered control law (7) with $K = [20.9094, 21.7892]$ and the triggering condition (13).

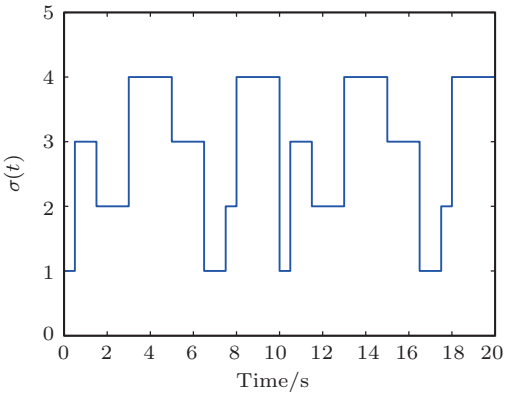


Fig. 2. (color online) Switching signal $\sigma(t)$.

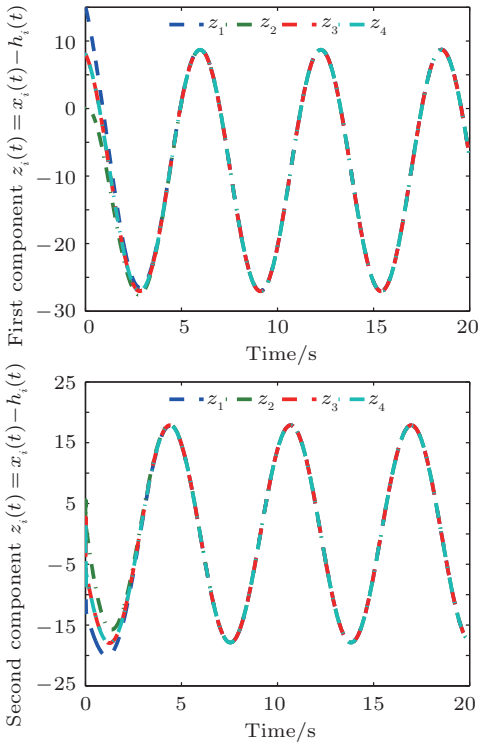


Fig. 3. (color online) The errors $z_i(t)$ of four agents.

With the initial values $x_1(0) = [5,-1]^{\mathrm{T}}$, $x_2(0) = [-0.1,10]^{\mathrm{T}}$, $x_3(0) = [18,3]^{\mathrm{T}}$, $x_4(0) = [8,-15]^{\mathrm{T}}$, the time-varying formation can be achieved. Figure 3 displays the errors $z_i(t)$ relative to the desired positions for all agents. The event-triggering conditions of four agents are given in Fig. 4, where the blue line shows the evolution of $\|\varphi_{\max}PBB^{\mathrm{T}}Pe_i(t)\|$, which stays below the threshold given by the triggering condition (13). The red line represents the function $\rho\|q_i(t)\|$ on the right side of Eq. (13). The controller will be updated when the blue line exceeds the red line. The event-triggering instants $\{t_k^i\}$ of four agents are shown in Fig. 5. One can see that the number of actuator control updates is greatly reduced to reach the desired time-varying formation compared with continuous communication. The position snapshots of four agents at different time are shown in Fig. 6. In Fig. 6(b), four agents reach a quadrangle formation and figures 6(c) and 6(d) show that the quadrangle formation keeps rotation.

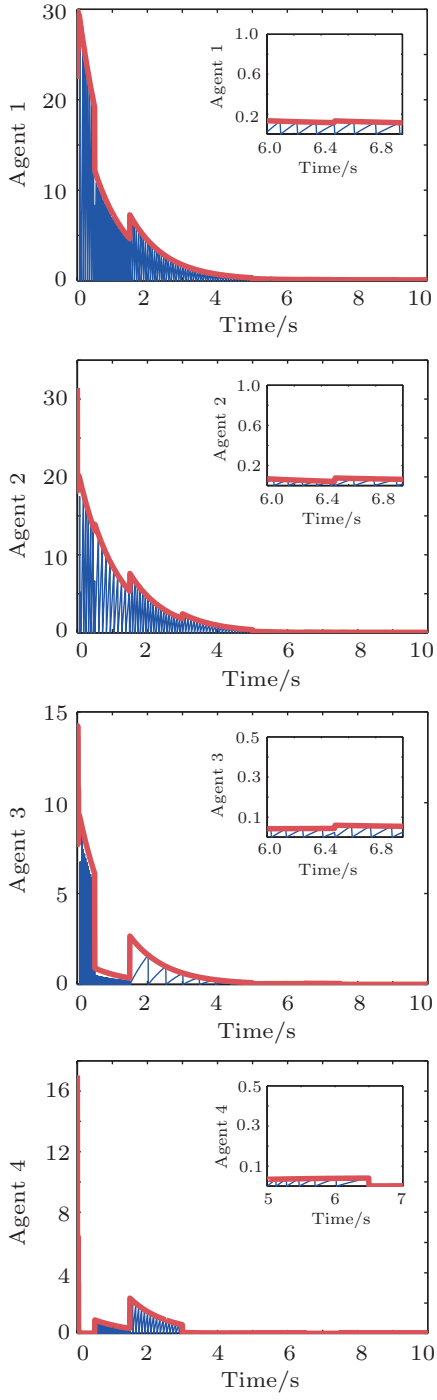


Fig. 4. (color online) Evolution of the event-triggering conditions.

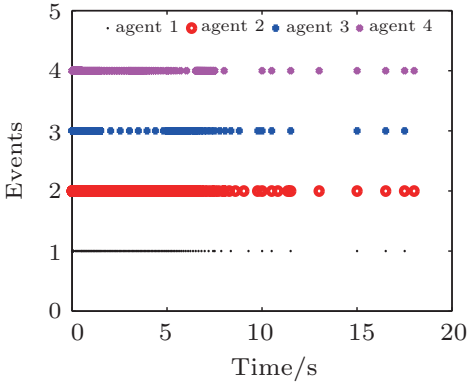


Fig. 5. (color online) The event-triggering instants.

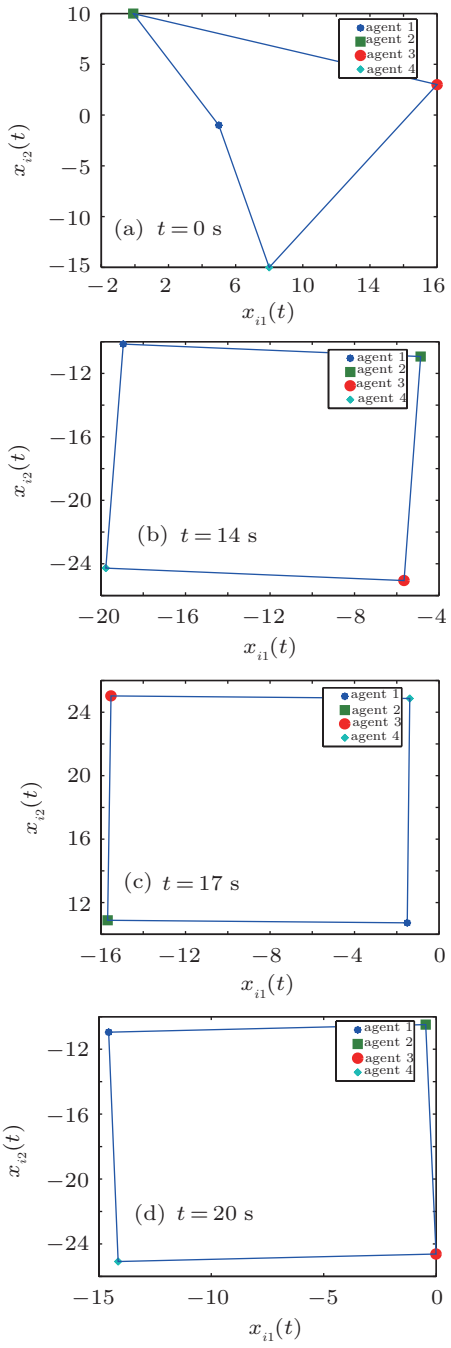


Fig. 6. (color online) Position snapshots of four agents.

5. Conclusion

In this paper, the time-varying formation problem for general linear multi-agent systems under switching topologies was studied. A distributed event-triggered control scheme was designed, in which the controller was updated only when the event condition was driven. Using this kind of controller, one can prove that the desired time-varying formation can be reached if the communication graph is switching and connected. We also showed that the Zeno behavior can be avoided in the event-triggered control scheme. A simulation example was given to illustrate the effectiveness of the proposed control law.

Future research will focus on the formation problem of multi-agent systems subject to unknown external disturbances and communication delays via sliding mode control.

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