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Nucleus-acoustic solitary waves in self-gravitating degenerate quantum plasmas

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Nucleus-acoustic (NA) solitary waves (SWs) propagating in a self-gravitating degenerate quantum plasma (SDQP) system (containing non-relativistically degenerate heavy and light nuclei, and non-/ultra-relativistically degenerate electrons) have been theoretically investigated. The modified Korteweg–de Vries (mK-dV) equation has been derived for both planar and non-planar geometry by employing the reductive perturbation technique. It is shown that the NA SWs exist with positive (negative) electrostatic (self-gravitational) potential. It is also observed that the effects of non-/ultra-relativistically degenerate electron pressure, dynamics of non-relativistically light nuclei, spherical geometry, etc. significantly modify the basic features (e.g., amplitude, width, speed, etc.) of the NA SWs. The applications of our results, which are relevant to astrophysical compact objects, like white dwarfs and neutron stars, are briefly discussed.

Keywords: self-gravitational potential, solitary waves

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1. Introduction

Plasma physics is very often regarded to be an entirely classical field. Plasma may be quantum when the quantum nature of its particles appreciably affects its macroscopic properties. Renewed interest on plasma system has been grown in the last decade when the quantum effects in plasmas are important. Quantum plasmas have a very high electron number density in comparison with classical plasmas. Recently, quantum plasmas have been considerably investigated due to their potential applications in dense astrophysical compact environments (viz white dwarfs and neutron stars), nanowires, quantum dots, semiconductor devices, and high gain free electron lasers.[1–7]

Nowadays, astrophysics deals with massive compact objects (e.g., white dwarfs, neutron stars, other exotic dense stars, cometary tails, black holes, etc.), and also tiny constituents of microscopic world.[8] The density of the degenerate matter in astrophysical compact objects is extremely high (viz. degenerate plasma particle number density is the order of 10^{30} cm^{-3} in white dwarfs, and order of 10^{36} cm^{-3} in neutron stars). These extremely dense compact objects have ceased burning thermonuclear fuel, and are stabilized against gravitational collapse by the degenerate Fermi pressure instead of thermal pressure. Therefore, astrophysical compact objects have gained enormous attention in understanding their constituents and basic properties of extremely dense (degenerate) matter in them.[9–13] Chandrasekhar described in his groundbreaking work that the degenerate pressure of plasma particles creates the outward pressure in astrophysical compact objects, which is counter-balanced by the inward pull of their gravity.[9] Chandrasekhar[14] contributed significantly to the understanding of degenerate electrons and gave two limits, named the non-relativistic limit and the ultra-relativistic limit due to the combination of the ideas of relativity and quantum mechanics. The degenerate pressure P_j associated with plasma species j (where $j = $ e for the electrons, and $j = $ n for nuclei) can be expressed as $P_j = K_j n_j^{\gamma_j}$,[9,10,15–22] where $\gamma_j = 5/3$, and $K_j \simeq 3\Lambda_c\hbar/5$ (with $\Lambda_c = \pi\hbar/m_j c$) for non-relativistic limit,[7,15–20,23,24] whereas $\gamma_j = 4/3$, and $K_j \simeq 3\hbar c/4$ for ultra-relativistic limit.[7,15–18,23–25] It is to be noted that m_e (m_n) is the mass of electrons (nuclei), $\hbar$ is the Planck's constant divided by 2π, and c is the speed of light in vacuum. The subscript j on γ, and K (in ultra-relativistic limit) has been used so that any of the two limits for any degenerate plasma species can be used. However, studies of relativistic degeneracy of plasmas have gained massive attention because of its vital role in different astrophysical situations.[26–29]

Furthermore, white dwarfs contain mainly oxygen, carbon, and helium with a covering of hydrogen gas,[9,12,17] and the electron species are relativistically degenerate not only within the inner core of the white dwarfs but also these are non-relativistically degenerate in their outer mantle.[12,30,31] Garcia-Berro *et al.*[13] studied that the main constituents of compact objects (e.g., white dwarfs, neutron stars, and black holes, etc.) are degenerate electrons, and nuclei of light as well as heavy elements. However, the complete understanding of the formation of astrophysical objects is a big challenge in astrophysics and cosmology. The formation of asteroids, comets, planets, and stars has been revealed from the gravita-

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tional collapse of astrophysical objects, such as nebulae, etc. Jeans explained the gravitational collapse of astrophysical objects for the formation of stars and galaxies in his prominent theory which describes the condition of self-gravitational instability between the neutral fluids.[32] The Jeans instability occurs when internal pressure could not compete with gravitational collapse.

Various authors have studied the gravitational instability of a plasma on various conditions.[33–37] Mamun and Schlickeiser[37,38] studied the propagation of dust-acoustic (DA) solitary (shock) waves in a weakly (strongly) coupled opposite polarity dust-plasma medium by taking the effect of self-gravitational field into account. Mamun *et al.*[36] also studied nucleus–acoustic shock structures in a strongly coupled self-gravitational degenerate quantum plasma consisting of strongly-coupled non-degenerate heavy nuclei, weakly-coupled degenerate non-relativistically light nuclei, and non-/ultra-relativistically degenerate electrons. But both heavy and light nuclei should be either relativistic or non-relativistic in the case of compact objects like white dwarfs or neutron stars. One dimensional (1D) planar geometry cannot be applied in some cases and so non-planar geometry is more appropriate than 1D planar geometry. Mamun *et al.*[39] have another work on heavy nucleus acoustic (HNA) spherical solitons (SSs) associated with HNA waves in self-gravitational degenerate (super-dense) quantum plasmas (SGDQP). They considered non-planar (spherical) geometry instead of 1D planar geometry and that work contained non-degenerate heavy and light nuclear species. But in case of self-gravitational degenerate (super-dense) quantum plasmas system, both heavy and light nuclei may be degenerate.[40–42] Recently Zaman *et al.*[43] have studied the basic features of both planar and non-planar shock structures (SSs) associated with nucleus-acoustic (NA) waves in a strongly coupled self-gravitating degenerate quantum plasma (SCSGDQP) system.

To the best of our knowledge, no theoretical investigation has been made to study the NA SWs in a self-gravitating degenerate quantum plasma (SDQP) system, which contains both non-relativistically degenerate heavy and light nuclei,[12] and non-/ultra-relativistically degenerate electrons[10] for both planar and non-planar (spherical and cylindrical) geometry. However, in our investigation, the nonlinear propagation of NA waves (where the inertia comes mainly from the mass density of the heavy nuclei, and the restoring force is mainly provided by the degenerate pressure of electrons) is considered, and we study 1D planar and non-planar NA SWs in such a realistic astrophysical plasma system.

2. Governing equations

We demonstrate the existence of the NA SWs in our considered SDQP system. The perturbed state of this SDQP system is described by

$$\partial_t n_s + \partial_r u_s + \frac{1}{r^\nu}\partial_r(r^\nu n_s u_s) = 0, \tag{1}$$

$$(\partial_t + u_\mathrm{i}\partial_r)u_\mathrm{i} = -\partial_r\phi - \partial_r\psi - \frac{a'_\mathrm{i}}{1+n_\mathrm{i}}\partial_r(1+n_\mathrm{i})^{\gamma_\mathrm{i}-1}, \tag{2}$$

$$y d_t u_\mathrm{n} + \beta y\partial_r\phi + y\partial_r\psi = -\frac{a'_\mathrm{n}}{1+n_\mathrm{n}}\partial_r(1+n_\mathrm{n})^{\gamma_\mathrm{n}-1}, \tag{3}$$

$$\partial_r\phi = \frac{\alpha'_\mathrm{e}}{1+n_\mathrm{e}}\partial_r(1+n_\mathrm{e})^{\gamma_\mathrm{e}-1}, \tag{4}$$

$$\frac{1}{r^\nu}\partial_r(r^\nu\partial_r\phi) = (1+\mu)n_\mathrm{e} - \mu n_\mathrm{n} - n_\mathrm{i}, \tag{5}$$

$$\frac{1}{r^\nu}\partial_r(r^\nu\partial_r\psi) = \sigma\left(n_\mathrm{i} + \frac{\mu}{\beta}n_\mathrm{n}\right), \tag{6}$$

where $s = \mathrm{n}$ (i) for heavy (light) nuclei, and $y = 1 + n_n$, $\partial_t = \partial/\partial t$; $\nu = 0$ for planar geometry,[36] $\nu = 1$ (2) for non-planar cylindrical (spherical) geometry,[43] $\partial_r = \partial/\partial r$, $d_t = \partial_t + u_n\partial_r$, n_e (n_s) is the perturbed part of electron (nuclear species) number density normalized by its equilibrium value $n_{\mathrm{e}0}$ ($n_{\mathrm{s}0}$), u_s is the fluid speed normalized by $C_\mathrm{i} = (Z_\mathrm{i}m_\mathrm{e}c^2/m_\mathrm{i})^{1/2}$ [where m_e (m_i) is the mass of electron (light nucleus)]; ϕ is the electrostatic wave potential normalized by $m_\mathrm{e}c^2/e$ (with e being the magnitude of an electron charge), ψ is the perturbed part of the self-gravitational potential normalized by C_i^2, r is the space variable normalized by $\lambda_\mathrm{Di} = (m_\mathrm{e}c^2/4\pi n_{\mathrm{i}0}Z_\mathrm{i}e^2)^{1/2}$, t is the time variable normalized by ω_pi^{-1}, $\beta = Z_\mathrm{n}m_\mathrm{i}/Z_\mathrm{i}m_\mathrm{n}$ and $\mu = Z_\mathrm{n}n_{\mathrm{n}0}/Z_\mathrm{i}n_{\mathrm{i}0}$; $\sigma = \omega_\mathrm{Ji}^2/\omega_\mathrm{pi}^2$ [with $\omega_\mathrm{Ji} = (4\pi G n_{\mathrm{p}0}m_\mathrm{i})^{1/2}$ being the Jeans frequency for the light nuclei, and G being the universal gravitational constant]. We have defined $a'_\mathrm{i} = K_\mathrm{i}n_{\mathrm{i}0}^{\gamma_\mathrm{i}-1}/m_\mathrm{e}c^2$, $a'_\mathrm{n} = K_\mathrm{n}n_{\mathrm{n}0}^{\gamma_\mathrm{n}-1}/m_\mathrm{e}c^2$, and $\alpha'_\mathrm{e} = K_\mathrm{e}n_{\mathrm{e}0}^{\gamma_\mathrm{e}-1}/m_\mathrm{e}c^2$. We note that equation (1) represents the continuity equation for light/heavy nuclei; equation (2) [(3)] represents the momentum equation for degenerate light (heavy) nuclei including the effects of electrostatic and self-gravitational forces; equation (4) represents the momentum equation for degenerate electrons where electrostatic and gravitational pressures are balanced by the degenerate pressure of electron species; equation (5) [(6)] represents the Poisson's equation for the electrostatic (self-gravitational) potential.

We further note that (i) the self-gravitational potential depends on the mass density (m_jn_j) of the particles in astrophysical compact objects; (ii) the self-gravitational potential is sometimes so important that in some astrophysical compact objects, the degenerate pressure (which also only depends on the particle density) is balanced by the pressure associated with the self-gravitational force;[9] (iii) however, the degenerate pressure is, in general, balanced by the pressure associated with the combined electrostatic and gravitational forces, and thus in our work, we take the effects of both electrostatic and gravitational forces into account.

3. Derivation of mK-dV equation

We now construct a weakly nonlinear theory for the nonlinear propagation of the NA waves by using the reductive perturbation method.[44] Thus, we first introduce the stretched coordinates[17]

$$\xi = \varepsilon^{1/2}(r - V_{\rm p}t),$$
$$\tau = \varepsilon^{3/2}t, \tag{7}$$

and expand the perturbed quantities (dependent variables, viz n_s, u_s, ϕ, and ψ) in power series of ε:

$$n_s = \varepsilon n_s^{(1)} + \varepsilon^2 n_s^{(2)} + \cdots,$$
$$u_s = \varepsilon u_s^{(1)} + \varepsilon^2 u_s^{(2)} + \cdots,$$
$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots,$$
$$\psi = \varepsilon \psi^{(1)} + \varepsilon^2 \psi^{(2)} + \cdots, \tag{8}$$

where $V_{\rm p}$ is the phase speed of the NA waves normalized by $C_{\rm i}$, ξ (τ) normalized by $\lambda_{\rm Di}$ ($\omega_{\rm pi}^{-1}$), and ε is a smallness parameter measuring the weakness of the dispersion ($0 < \varepsilon < 1$). We now develop equations in various powers of ε by using Eqs. (7) and (8) in Eqs. (1)–(6). To the lowest order in ε [i.e., keeping the terms containing $\varepsilon^{3/2}$ from Eqs. (1)–(4), and ε from Eqs. (5) and (6)], a set of equations are obtained from Eqs. (1)–(6) as

$$n_{\rm i}^{(1)} = \left[\frac{\mu(1-\beta)}{\mu(V_{\rm p}^2 - a_{\rm i}) + \beta(V_{\rm p}^2 - a_{\rm n})}\right]\phi^{(1)}, \tag{9}$$

$$n_{\rm n}^{(1)} = -\left[\frac{\beta(1-\beta)}{\mu(V_{\rm p}^2 - a_{\rm i}) + \beta(V_{\rm p}^2 - a_{\rm n})}\right]\phi^{(1)}, \tag{10}$$

$$\psi^{(1)} = -\left[\frac{\beta(V_{\rm p}^2 - a_{\rm n}) + \beta\mu(V_{\rm p}^2 - a_{\rm i})}{\beta(V_{\rm p}^2 - a_{\rm n}) + \mu(V_{\rm p}^2 - a_{\rm i})}\right]\phi^{(1)}, \tag{11}$$

$$V_{\rm p} = \left[\frac{(1+\mu)(a_{\rm n}\beta + a_{\rm i}\mu) + (1+\beta)^2\mu\alpha_{\rm e}}{(1+\mu)(\mu+\beta)}\right]^{1/2}, \tag{12}$$

where $a_{\rm i} = \gamma_{\rm i}a'_{\rm i}$, $a_{\rm n} = \gamma_{\rm n}a'_{\rm n}$, and $\alpha_{\rm e} = \gamma_{\rm e}\alpha'_{\rm e}$. Equation (12), i.e., $V_{\rm p} = \omega/kC_{\rm i}$, where ω (k) is the wave frequency (propagation constant), represents the linear dispersion relation for the NA SWs propagating in an SDQP system under consideration. We have graphically shown how the NA wave phase speed ($V_{\rm p}$) varies with degenerate plasma particle number densities ($n_{\rm e0}$) for non-relativistically as well as ultra-relativistically degenerate electron species (DES). This is displayed in Fig. 1, which implies that the the phase speed $V_{\rm p}$ of the NA waves is zero for $n_{\rm e0} \leq 0.9 \times 10^{30}$, and then it increases rapidly with $n_{\rm e0}$ upto $n_{\rm e0} < 2 \times 10^{30}$, but decreases with $n_{\rm e0}$ for $n_{\rm e0} > 2 \times 10^{30}$ in both limits of non-relativistically as well as ultra-relativistically DES. The sharp linearly increasing region ($n_{\rm e0} < 2 \times 10^{30}$) represents the linear dispersion relation for the NA waves in which the restoring force (inertia) is mainly provided by the pressure of DES (mass density of heavy nuclear species), while the slightly decreasing region (at $n_{\rm e0} > 2 \times 10^{30}$) is due to the effect of the dynamics and degenerate pressure of light nuclei. We note that the restoring force (which in our case is degenerate pressure), and inertial heavy particles (which in our case is heavy nucleus) make the NA wave mode significantly different from other wave modes like ion-acoustic, electron-acoustic, etc. It is also observed that the effect of ultra-relativistically DES reduces the NA wave phase speed ($V_{\rm p}$). This is due to the fact that the restoring force provided by the ultra-relativistically DES is less than that provided by the non-relativistically DES because of the lower values of both $K_{\rm e}$ and $\gamma_{\rm e}$ in ultra-relativistic limit. We note that $\beta = Z_{\rm n}m_{\rm i}/Z_{\rm i}m_{\rm n} = 0.5$ for nuclei of any heavy element being as heavy nuclear species, and nuclei of hydrogen (protons) being as light nuclear species.

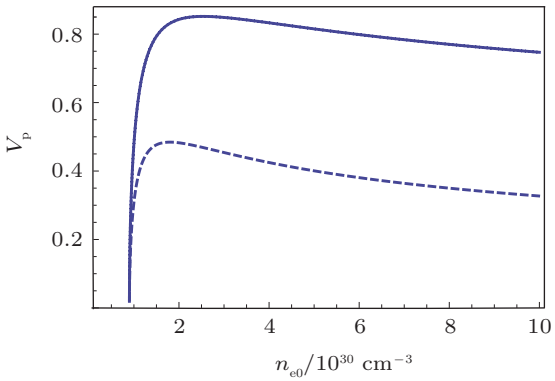


Fig. 1. (color online) The variation of $V_{\rm p}$ with $n_{\rm e0}$ for non-relativistically (solid curve), and ultra-relativistically (dotted curve) DES.

Now, to the next higher order in ε [i.e., keeping the terms containing $\varepsilon^{5/2}$ from Eqs. (1)–(4), and containing ε^2 from Eqs. (5) and (6)], we have a set of equations containing nonlinear terms. The new set of equations along with Eqs. (9)–(12), allow us to eliminate $u_s^{(2)}$, $n_s^{(2)}$, $\phi^{(2)}$, and $\psi^{(2)}$. This elimination finally reduces to the modified Korteweg-de Vries (mK-dV) equation in the form

$$\partial_\tau \phi^{(1)} + \frac{\nu\phi^{(1)}}{2\tau} + A\phi^{(1)}\partial_\xi\phi^{(1)} + B\partial_\xi^3\phi^{(1)} = 0, \tag{13}$$

where A and B denote the nonlinear coefficient, and dispersion coefficient, respectively, given by

$$A = \frac{N(LRG^2 - \mu^2\alpha_{\rm e}^2OF)H + \mu RS_2\alpha_{\rm e}^2FH}{2S_1V_{\rm p}G\alpha_{\rm e}^2[MH - \sigma(\alpha_{\rm n} - \alpha_{\rm i})E]}$$
$$- \frac{ES_2(RF + PF_1)}{2S_1V_{\rm p}G\alpha_{\rm e}^2[MH - \sigma(\alpha_{\rm n} - \alpha_{\rm i})E]}, \tag{14}$$

$$B = \frac{(G_1 - \sigma G)PRH}{2V_{\rm p}S_1\sigma[MH - \sigma(\alpha_{\rm n} - \alpha_{\rm i})E]}, \tag{15}$$

in which

$$E = V_{\rm p}^2(\mu + 1) - (\mu\alpha_{\rm i} + \alpha_{\rm n}),$$
$$M = V_{\rm p}^2(\beta - 1) + (\alpha_{\rm n} - \beta\alpha_{\rm i}),$$
$$F = 3V_{\rm p}^2 + \alpha_{\rm i}(\gamma_{\rm i} - 2),$$
$$F_1 = 3V_{\rm p}^2 + \alpha_{\rm n}(\gamma_{\rm n} - 2),$$

$$G = \beta(V_p^2 - \alpha_n) + \mu(V_p^2 - \alpha_i),$$
$$G_1 = (V_p^2 - \alpha_n) + \mu(V_p^2 - \alpha_i),$$
$$H = V_p^2\sigma(\beta + \mu) - \sigma(\beta\alpha_n + \mu\alpha_i),$$
$$S_1 = \mu(1-\beta), \quad S_2 = \mu(1-\beta)^2,$$
$$P = (V_p^2 - \alpha_i), \quad R = (V_p^2 - \alpha_n),$$
$$L = (1+\mu)(\gamma_e - 2), \quad O = \beta^2(\beta-1)^2.$$

We note that equation (13) represents the mKdV equation for the electrostatic potential ($\phi^{(1)}$). However, it also represents the mKdV equation for the self-gravitational potential ($\psi^{(1)}$) just by substituting Eq. (11) into Eq. (13) and we will gain the opposite polarity.

An exact analytic solution of Eq. (13) is not possible. We now consider two special cases, namely 1D planar ($\nu = 0$), and cylindrically ($\nu = 1$) and spherically ($\nu = 2$) symmetric non-planar NA SWs in the following two subsections.

3.1. 1D planar ($\nu = 0$) NA SWs

We have already mentioned that ($\nu = 0$) corresponds to a 1D planar geometry which reduces Eq. (13) to a standard K-dV equation [44] in the form

$$\partial_\tau \phi^{(1)} + A\phi^{(1)}\partial_\xi \phi^{(1)} + B\partial_\xi^3 \phi^{(1)} = 0. \tag{16}$$

Thus the stationary solitary wave solution of this standard K-dV equation (16) can be expressed as a moving frame $\zeta = \xi - U_0\tau$, where ζ (U_0) is normalized by λ_{Di} (C_i), and introducing the appropriate boundary conditions, viz. $\phi^{(1)} \to 0$, $\partial_\zeta \phi^{(1)} \to 0$, $\partial_\zeta^2 \phi^{(1)} \to 0$ at $\zeta \to \infty$. These allow us to write the stationary solitary wave solution of Eq. (16) as

$$\Phi^{(1)} = \Phi_0^{(1)} \operatorname{sech}^2\left(\frac{\zeta}{\Delta}\right), \tag{17}$$

where $\phi_0^{(1)}$ (height normalized by $m_e c^2/e$), and Δ (thickness normalized by λ_{Di}) are given by

$$\phi_0^{(1)} = \frac{3U_0}{A}, \quad \Delta = \sqrt{\frac{4B}{U_0}}. \tag{18}$$

The solitary waves are steady nonlinear waves,[45,46] and are formed due to the balance between nonlinearity and dispersion. The degree of nonlinearity is proportional to the potential of the plasma system and a highly nonlinear medium causes high electrostatic potential. It is observed from Eq. (18) that the height of the amplitude of the solitary structures is directly proportional to the solitary speed moving with U_0, and inversely proportional to the nonlinear coefficient A. On the other hand, the width of these solitary structures is directly proportional to the dispersive constant B, and inversely proportional to the solitary speed moving with U_0. It is clear from Eqs. (17) and (18) that the NA SWs exist in the SDQP since V_p, A, and B are numerically found (not directly shown here) to be positive for all possible degenerate plasma parameters. It is obvious from Eq. (11) and Eqs. (17), (18) that the NA SWs associated with the electrostatic potential differs from that associated with the gravitational potential only in amplitude, and that the NA SWs exist with positive (negative) electrostatic (self-gravitational) potential. Equation (18) implies that the amplitude (width) of the NA SWs increases (decreases) with the increase of the value of U_0. To have some numerical appreciation of our results, we have shown NA SWs associated not only with the electrostatic potential $\phi^{(1)}$ [as shown in Fig. 2(a)], but also with the self-gravitational potential $\psi^{(1)}$ [as shown in Fig. 2(b)].

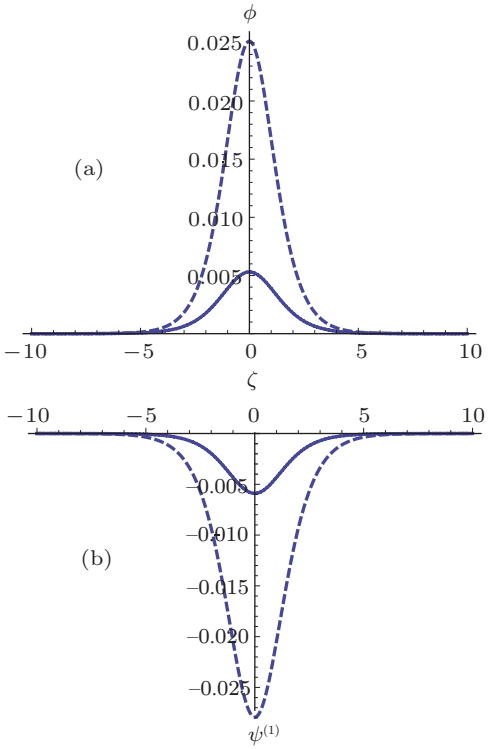


Fig. 2. (color online) The 1D planar NA SWs associated with positive electrostatic (a) and negative self-gravitational (b) potential for both non-relativistically (dotted curve) and ultra-relativistically (solid curve) DES.

We note that in our numerical analysis for NA SWs in SDQP, (i) we have considered both non-relativistically (for which $\gamma_j = 5/3$, and $K_j \simeq 3\Lambda_c\hbar/5$), and ultra-relativistically (for which $\gamma_j = 4/3$ and $K_j \simeq 3\hbar c/4$) DES; (ii) and hydrogen nuclei (H: $Z_i = 1$, and $m_i = m_p$ where m_p is the proton mass) as degenerate light nuclear species, while carbon nuclei (C: $Z_n = 6$ and $m_n \simeq 12m_p$) as non-relativistically heavy nuclear species so that all the parameters except μ are fixed; (iii) we have used $\beta = 0.5$, $\mu = 0.5$, and $U_0 = 0.1$ in Fig. 2. It is observed from Fig. 2 that the effect of ultra-relativistically DES reduces the magnitude of the amplitude as well as the width of the NA SWs associated with both electrostatic and self-gravitational potential. They also indicate that the NA SWs exist with positive electrostatic potential, but with negative self-gravitational potential, and that the magnitude of the amplitude of the electrostatic SWs is significantly higher than that

of self-gravitational SWs. It is well known that as the phase speed (V_p) of the acoustic type of waves increases, the amplitude of the SWs increases. This means that the amplitude of the NA SWs increases (decreases) with n_{e0} for $n_{e0} < 2 \times 10^{30}$ ($n_{e0} > 2 \times 10^{30}$). The effects of n_{e0} and ultra-relativistically DES on the NA SWs can be interpreted as we did it on V_p.

3.2. Non-planar NA SWs

The SWs are illustrated in Figs. 3 and 4, which are modified by the effects of non-planar geometries $\nu = 1$ (2). It is noted that $\nu = 1$ (2) corresponds to cylindrical (spherical) geometry. We have numerically analyzed Eq. (13) for $\nu = 1$ (2), $\sigma = 4$, $\eta = 0.4$, $\mu = 1.4$, and $U_0 = 0.1$. Figures 3 and 4 represent the NA SWs associated with positive electrostatic potential (a) and negative self-gravitational potential (b) for both non-relativistically (upper surface), and ultra-relativistically (lower surface) DES. Figures 3 and 4 also indicate how NA SWs evolve with time (τ).

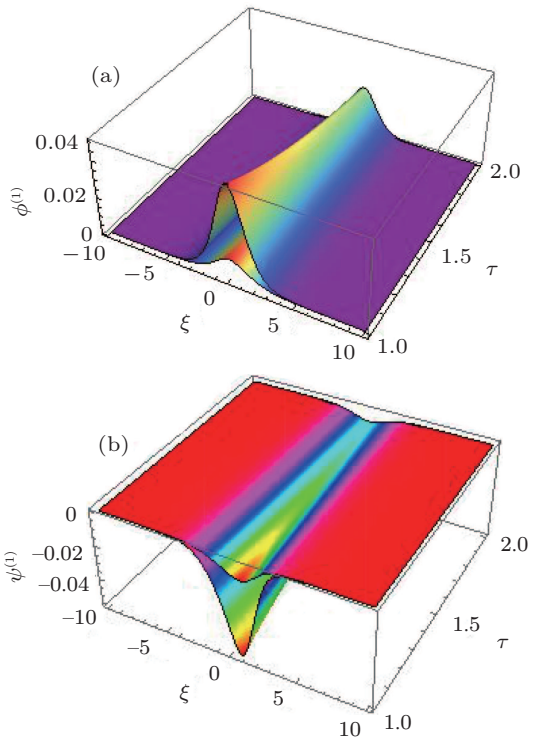


Fig. 3. (color online) The cylindrical ($\nu = 1$) NA SWs associated with positive electrostatic potential (a) and negative self-gravitational potential (b) for both non-relativistically (upper surface), and ultra-relativistically (lower surface) DES.

From Figs. 3 and 4, it is seen that for a very large value of τ, the cylindrical and spherical NA SWs are similar to 1D planar ones. This is due to the fact that for a very large value of τ, the contribution of the term ϕ/τ, which is due to the effect of the non-planar (cylindrical and spherical) geometry, becomes insignificant. However, as the value of τ decreases, the contribution of the term ϕ/τ becomes more and more significant, and the non-planar (cylindrical and spherical) NA SWs differ from 1D planar ones. It is found that with the decrease of the value of τ, the amplitude of the NA SWs increases. It is also found that the amplitude of the non-planar (cylindrical and spherical) NA SWs is larger than that of the 1D planar ones.

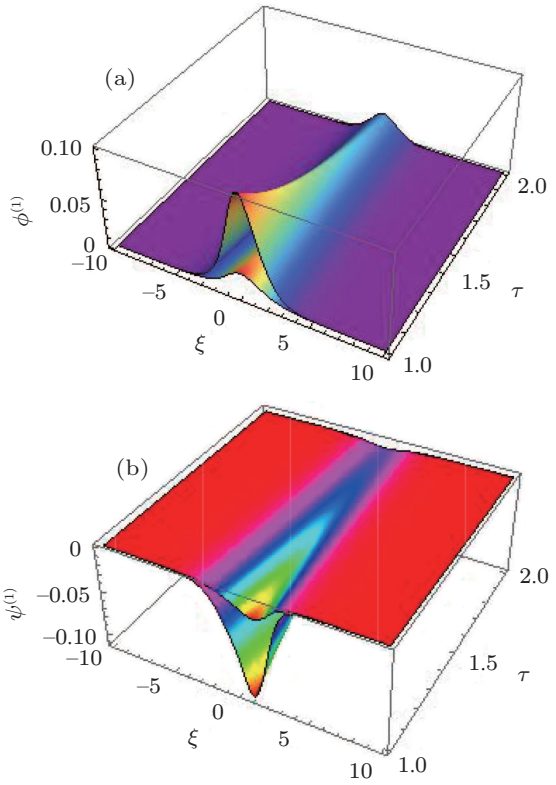


Fig. 4. (color online) The spherical ($\nu = 2$) NA SWs associated with positive electrostatic potential (a) and negative self-gravitational potential (b) for both non-relativistically (upper surface), and ultra-relativistically (lower surface) DES.

4. Discussion

We have considered an SDQP containing non-relativistically degenerate heavy (light) nuclei, and both non-/ultra-relativistically degenerate electrons. Using the reductive perturbation method, the mK-dV equation has been derived, and that equation has been numerically analyzed, and the basic features of the NA SWs have been studied. The results, which have been found from this theoretical investigation, can be summarized as follows:

(i) The phase speed (V_p) of the NA waves is zero for $n_{e0} \leq 0.9 \times 10^{30}$, and linearly increases with n_{e0} for $n_{e0} < 2 \times 10^{30}$, but slightly decreases with n_{e0} for $n_{e0} > 2 \times 10^{30}$ in both non-relativistically and ultra-relativistically DES. The linearly increasing region (at $n_{e0} < 2 \times 10^{30}$) is quite sensible for any kind of acoustic waves, while the slightly decreasing region (at $n_{e0} > 2 \times 10^{30}$) is due to the combined effect of the dynamics and degenerate pressure of light nuclear species.

(ii) The phase speed (V_p) of the NA SWs in the case of ultra-relativistically DES is smaller than that of non-relativistically DES. This occurs due to the fact that the restoring force provided by ultra-relativistically DES is less than that

provided by non-relativistically DES because of the lower values of both γ_e and K_e in the ultra-relativistic limit.

(iii) The NA SWs with positive (negative) electrostatic (self-gravitational) potential is formed due to the strong interaction among heavy nuclei which acts as a source of dispersion.

(iv) The height of the NA SWs is directly proportional to the speed (U_0) and inversely proportional to the nonlinear coefficient A. On the other hand, the thickness of the NA SWs is inversely proportional to the speed (U_0) and directly proportional to the dissipative constant B.

(v) The height and thickness of the NA SWs associated with both electrostatic and self-gravitational potentials are less in the case of ultra-relativistically DES than in the case of non-relativistically DES. The effect of ultra-relativistically DES reduces (enhances) the height (thickness) of the NA SWs since i) the NA wave phase speed (V_p) is decreased by lower values of both γ_e and K_e of ultra-relativistically DES, and ii) the height (thickness) of SWs always increases (decreases) with the increase of the wave phase speed.

(vi) For a very large value of τ, the spherical NA SWs are identical to 1D planar ones. This is because of the very large value of τ, the term ϕ/τ, which is due to the effect of the spherical geometry, is no longer significant. However, as the value of τ decreases, the term ϕ/τ becomes significant, and the non-planar (cylindrical and spherical) NA SWs differ from 1D planar ones.

(vii) The height (thickness) of NA SWs decreases (increases) as the value of τ increases. The height of the cylindrical NA SWs is smaller than that of the spherical ones but larger than that of 1D planar ones. The thickness of the cylindrical NA SWs is always larger than that of the spherical ones but smaller than that of the 1D planar ones.

It may be also noted that our basic equations (1)–(6), which are valid for the perturbed quantities involved, cannot explain the equilibrium state of the phase speed under consideration. To explain the equilibrium state of this phase speed, one has to rewrite the basic equations, which can be valid for $n_{so}(x)$, i.e., for $\phi_0(x)$ and $\psi_0(x)$, where $\phi_0(x)$ and $\psi_0(x)$ are, respectively, the electrostatic and self-gravitational potential at equilibrium. This means that n_{s0}, ϕ_0, and ψ_0 are not constant, but are the function of x so that at equilibrium, the force associated with electrostatic and degenerate pressures are balanced by the self-gravitational force. However, if we neglect the net surface charge density of plasma species, at equilibrium, the force associated with the degenerate pressure is balanced by the gravitational force.[9,10] This is the common scenario for the most astrophysical compact objects, e.g., white dwarfs and neutron stars.[9]

The astrophysical compact objects like black holes emit gravitational waves[47] and this recent discovery lets us dream that in near future an identical or slightly identical waves (like NA waves) and associated nonlinear structures like solitons, shocks, etc., will be discovered from other astrophysical compact objects like white dwarfs and neutron stars. We may hope that this investigation will directly or indirectly be able to verify the future signature of the existence of solitonic signals observed by space experiments as well as to understand the localized electrostatic or gravitational disturbances in astrophysical compact objects like white dwarfs and neutron stars. We finally stress that our present investigation could be very helpful for understanding the nonlinear structures which may be formed in some astrophysical compact objects (e.g., white dwarfs and neutron stars).

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