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Dynamics of entanglement protection of two qubits using a driven laser field and detunings: Independent and common, Markovian and/or non-Markovian regimes

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Preventing quantum entanglement from decoherence effect is of theoretical and practical importance in the quantum information processing technologies. In this regard, we consider the entanglement dynamics of two identical qubits where the qubits which are coupled to two independent (Markovian and/or non-Markovian) as well as a common reservoir at zero temperature are further interacted with a classical driving laser field. Then, we study the preservation of generated two-qubit entanglement in various situations using the concurrence measure. It is shown that by applying the classical driving field and so the possibility of controlling the Rabi frequency, the amount of entanglement of the two-qubit system is improved in the off-resonance condition between the qubit and the central cavity frequencies (central detuning) in both non-Markovian and Markovian reservoirs. While the central detuning has a constructive role, the detuning between the qubit and the classical field (laser detuning) affects negatively on the entanglement protection. The obtained results show that long-living entanglement in the non-Markovian reservoir is more accessible than in the Markovian reservoir. We demonstrate that, in a common reservoir non-zero stationary entanglement is achievable whenever the two-qubit system is coupled to the reservoir with appropriate values of relative coupling strengths.

Keywords: entanglement, Markovian and non-Markovian reservoir, Rabi frequency, concurrence

PACS: 03.67.Mn, 03.65.Yz, 03.67.Bg **DOI:** 10.1088/1674-1056/27/4/040303

1. Introduction

Entanglement as a physical resource for many applications in quantum information,[1] quantum teleportation,[2–4] quantum key distribution,[5,6] quantum cryptography,[7] quantum communication,[8] and entanglement swapping[9–11] has attracted enormous interest in recent decades. On the other hand, entanglement of quantum systems significantly decreases because of the interaction with their surrounding environments.[12] Therefore, various methods have been suggested for generating as well as protecting and enhancing entanglement against induced decoherence.[13–15] As two cases we can refer to quantum Zeno effect[16–18] and decoherence-free subspace method.[19,20] The disadvantage of the first scheme is the required series of particular measurement that should be applied at equal intervals of time to freeze the quantum state for inhibiting the degradation of entanglement. In the second method, the interaction Hamiltonian needs to possess an appropriate symmetry, the fact that might not always be exist. In addition, there are other approaches to achieve the mentioned purpose; for instance, detuning modulation,[21] entanglement distillation,[22,23] quantum interference,[24] weak measurement,[25,26] and coherent combinations of photon subtraction and addition.[27]

On the other hand due to the presence of environment, dissipation is inevitable. The environment can be described as Markovian or non-Markovian. In the Markovian (memoryless) environment the entanglement of qubits decreases exponentially and finally disentanglement occurs after passing the time,[28,29] however, in the non-Markovian (with memory) environment, part of information can return to the system and the revival of entanglement and oscillations can be observed.[30–33] In this line, in Refs. [34] and [35] the authors have demonstrated that the existence of the bound state of the system and its reservoir and the non-Markovian effect are two essential conditions for preserving the entanglement in their considered systems. Also, in Ref. [36] Chen *et al*. have shown that in a common reservoir the entanglement can be preserved due to the decoherence-free state which supplies a useful way to avoid the disadvantage influence of the environment.

Recently, a scheme proposed in Ref. [33] on the entanglement preservation of two two-level atoms that are coupled to two independent non-Markovian environments. Also a scheme contains entanglement protection of two two-level atoms in two independent environments in different situations is introduced in Ref. [13]. Note that, in the two latter Refs. the authors limited their studies to the investigation on the roles of reservoir types as well as detuning, in the absence of any external classical field. In this study, we want to examine the influence of a driving external classical laser field in addition to detunings on the two-qubit entanglement protection in our considered models. In detail, we study the entanglement dynamics of two identical qubits in the presence

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of a classical laser field wherein two independent reservoirs as well as a common reservoir in the Markovian and/or non-Markovian regimes at zero temperature are considered. Therefore, we outline four distinct interaction models and compare them in detail via varying the above-mentioned factors. Our investigations show that, there exist two essential ways to control the amount of entanglement. The first is to adjust the frequency detuning between the atom and the central cavity mode, δ_c (central detuning) and the other is tuning the strength of the driven classical field. With simultaneously adjustment of the these two factors, it is possible to achieve the maximal degree of entanglement. In addition, we observe that in the non-Markovian reservoirs the values of entanglement are much more than in the Markovian reservoirs in a special moment of time, which is a consequence of the memory effect of non-Markovian reservoirs.

This paper is organized as follows. In the next section, we consider two qubits interacting with two independent reservoirs under the impression of the driven classical field in Markovian and/or non-Markovian environments and we study the dynamics of entanglement for different interaction models. In Section 3, we reconsider the same procedure but in a common reservoir. Finally, the paper is ended with a summary and concluding remarks in the last section.

2. Model interaction for two independent reservoirs

Let us study two identical qubits A, B respectively interacting locally with two independent environments a, b at zero temperature. No interaction exists between the two interacting subsystems A–a and B–b. It should be emphasized that we have assumed that the two qubits are far enough apart to avoid their collective behavior. The qubit system in each environment is driven by an external classical laser field with frequency ω_l. We can also consider a situation in which the qubit A is always coupled to its own environment a, while the qubit B can be isolated; so we inserted a parameter κ with $\kappa = 0\,(1)$ denotes to the non-existence (existence) of second environment. The dynamics of the system is governed by the following Hamiltonian in the rotating wave approximation (RWA) with $\hbar = 1$:[37]

$$\hat{H} = \sum_{i=A,B} \frac{\omega_0}{2}\hat{\sigma}_z^{(i)} + \sum_{j=1,2}\sum_k \omega_{k_j}\hat{a}_{k_j}^\dagger \hat{a}_{k_j} + \Big[\Omega \sum_{i=A,B} \hat{\sigma}_+^{(i)} \mathrm{e}^{-\mathrm{i}\omega_l t} + \tilde{g}_1\hat{\sigma}_+^{(A)} + \kappa\tilde{g}_2\hat{\sigma}_+^{(B)} + \mathrm{H.C.}\Big]. \quad (1)$$

In the above Hamiltonian, ω_0 is the transition frequency of the two qubits and $\hat{\sigma}_z^{(i)}, \hat{\sigma}_+^{(i)}$ $(i = A, B)$ are respectively the Pauli population inversion and raising operators of the qubits and Ω is the Rabi frequency of the classical radiation field. The operators $\hat{a}_{k_j}^\dagger$ and $\hat{a}_{k_j}$ are the well-known creation and annihilation operators of the k-th mode of the surrounding j-th environment with frequency ω_k, the index k_j labels the k-th mode of the j-th environment. Also, we defined $\tilde{g}_j = \sum_{k_j} g_{k_j}\hat{a}_{k_j}$, $(j = 1, 2)$, where g_{k_j} is the strength of the qubit-environment associated with the mode k. In the rotating reference frame under the unitary transformation $\hat{U}(t) = \exp\big[-\mathrm{i}\omega_l t\big(\sigma_z^{(A)} + \sigma_z^{(B)}\big)\big]$, the above Hamiltonian can be transformed to an effective Hamiltonian, namely $\hat{H}_{\mathrm{eff}}$, which we divide it in two parts, i.e.,

$$\begin{aligned}
\hat{H}_{\mathrm{eff}} &= \hat{H}_{\mathrm{eff}}^{(1)} + \hat{H}_{\mathrm{eff}}^{(2)},\\
\hat{H}_{\mathrm{eff}}^{(1)} &= \frac{\delta_l}{2}\sum_{i=A,B}\hat{\sigma}_z^{(i)} + \Omega\sum_{i=A,B}\hat{\sigma}_x^{(i)},\\
\hat{H}_{\mathrm{eff}}^{(2)} &= \sum_{j=1,2}\sum_k \omega_{k_j}\hat{a}_{k_j}^\dagger\hat{a}_{k_j}\\
&\quad + \Big[\Big(\tilde{g}_1\hat{\sigma}_+^{(A)} + \kappa\tilde{g}_2\,\hat{\sigma}_+^{(B)}\Big)\mathrm{e}^{\mathrm{i}\omega_l t} + \mathrm{H.C.}\Big]. \quad (2)
\end{aligned}$$

Here, $\delta_l = \omega_0 - \omega_l$ is the detuning between the qubits and the classical driving field (laser detuning) and $\hat{\sigma}_x$ is the Pauli matrix. The Hamiltonian $\hat{H}_{\mathrm{eff}}^{(1)}$ can be diagonalized as $(\omega'/2)\big(\hat{\tilde{\sigma}}_z^A + \hat{\tilde{\sigma}}_z^B\big)$ where $\omega' = \sqrt{4\Omega^2 + \delta^2}$. $\hat{\tilde{\sigma}}_z^{A,B}$ is defined as $\hat{\tilde{\sigma}}_z^A = |1\rangle_1\langle 1| - |0\rangle_1\langle 0|$ and $\hat{\tilde{\sigma}}_z^B = |1\rangle_2\langle 1| - |0\rangle_2\langle 0|$ in the new dressed basis states, i.e.,

$$\begin{aligned}
|1\rangle_j &= \cos\frac{\theta}{2}|e\rangle_j + \sin\frac{\theta}{2}|g\rangle_j,\\
|0\rangle_j &= -\sin\frac{\theta}{2}|e\rangle_j + \cos\frac{\theta}{2}|g\rangle_j, \quad j = 1, 2, \quad (3)
\end{aligned}$$

where $\theta = \arctan(2\Omega/\delta_l)$. By applying RWA in the Hamiltonian $\hat{H}_{\mathrm{eff}}^{(2)}$, the effective Hamiltonian in Eq. (2) can be rewritten in terms of the dressed operators as follows:

$$\hat{H}_d = \frac{\omega'}{2}\sum_{i=A,B}\hat{\tilde{\sigma}}_z^{(i)} + \sum_{j=1,2}\sum_k \omega_{k_j}\hat{a}_{k_j}^\dagger\hat{a}_{k_j} + \cos^2\frac{\theta}{2}\big[(\tilde{g}_1\hat{\tilde{\sigma}}_+^{(A)} + \kappa\tilde{g}_2\,\hat{\tilde{\sigma}}_+^{(B)})\,\mathrm{e}^{\mathrm{i}\omega_l t} + \mathrm{H.C.}\big], \quad (4)$$

in which $\hat{\tilde{\sigma}}_+^A = |1\rangle_1\langle 0|$ and $\hat{\tilde{\sigma}}_+^B = |1\rangle_2\langle 0|$ are defined as the raising operators in the dressed basis states (see Appendix A). Now, it is convenient to rewrite Hamiltonian (4) in the interaction picture which easily arrives one at

$$\hat{H}_{\mathrm{int}} = \cos^2\frac{\theta}{2}\Big[\tilde{g}_1\hat{\tilde{\sigma}}_+^{(A)}\,\mathrm{e}^{-\mathrm{i}(\omega_{k_1}-\omega_l-\omega')t} + \kappa\,\tilde{g}_2\hat{\tilde{\sigma}}_+^{(B)}\,\mathrm{e}^{-\mathrm{i}(\omega_{k_2}-\omega_l-\omega')t} + \mathrm{H.C.}\Big]. \quad (5)$$

To find the state vector of the considered system at any time t, we assume the initial state of system possess only one excitation in the total system as follows:

$$|\psi(0)\rangle = \big(c_1(0)\,|1\rangle_1|0\rangle_2 + c_2(0)\,|0\rangle_1|1\rangle_2\big)|0_{k_1}\rangle_{\mathrm{R}_1}|0_{k_2}\rangle_{\mathrm{R}_2}. \quad (6)$$

In this relation $c_1(0)$ and $c_2(0)$ are complex numbers and $|0_{k_j}\rangle_{\mathrm{R}_j}$ is the j-th reservoir containing no photon in the mode k. Accordingly, the quantum state of total system at any time $t > 0$ can be written as

$$|\psi(t)\rangle = \big(c_1(t)\,|1\rangle_1|0\rangle_2 + c_2(t)\,|0\rangle_1|1\rangle_2\big)|0_{k_1}\rangle_{R_1}|0_{k_2}\rangle_{\mathrm{R}_2}$$

$$+\sum_{k_1} c_{k_1}(t)\,|0\rangle_1|0\rangle_2|1_{k_1}\rangle_{\mathrm{R}_1}|0_{k_2}\rangle_{\mathrm{R}_2}$$
$$+\sum_{k_2} c_{k_2}(t)|0\rangle_1|0\rangle_2|0_{k_1}\rangle_{\mathrm{R}_1}|1_{k_2}\rangle_{\mathrm{R}_2} \quad (7)$$

with $|1_{k_j}\rangle_{\mathrm{R}_j}$ as the j-th reservoir containing one photon in mode k. Using the time-dependent Schrödinger equation, $(\hat{H}_{\mathrm{int}}|\psi(t)\rangle = \mathrm{i}(\partial/\partial t)|\psi(t)\rangle)$ and after some manipulations the following integro-differential equations for $c_1(t)$ and $c_2(t)$ can be obtained

$$\dot{c}_1(t) = -\cos^4\frac{\theta}{2}\int_0^t \mathrm{d}t_1\, f(t-t_1)c_1(t_1), \quad (8)$$
$$\dot{c}_2(t) = -\kappa^2\cos^4\frac{\theta}{2}\int_0^t \mathrm{d}t_1\, f(t-t_1)c_2(t_1), \quad (9)$$

in which the correlation function $f(t-t_1)$ is defined as

$$f(t-t_1) = \int \mathrm{d}\omega J(\omega)\,\exp[\mathrm{i}(\omega_l+\omega'-\omega)(t-t_1)]. \quad (10)$$

The solutions of $c_1(t)$ and $c_2(t)$ depend on the reservoir structure specified by spectral density $J(\omega)$. Here, we focus on the case in which the reservoir is an electromagnetic field inside a lossy cavity. In this case, the spectral density obeys the Lorentzian distribution that implies the imperfect reflectivity of the cavity mirrors and interaction between an atom and an imperfect cavity. So, the considered distribution of the field inside the cavity can be modeled as[38]

$$J(\omega) = \frac{1}{2\pi}\frac{\Gamma\lambda^2}{(\omega_0-\omega-\delta_{\mathrm{c}})^2+\lambda^2}, \quad (11)$$

where ω_0 denotes the transition frequency of atom, δ_{c} is the detuning between atom and the central cavity frequencies (central detuning). The parameter λ explains the width of Lorentzian distribution and is related to the reservoir correlation time τ_{R} by the relationship $\tau_{\mathrm{R}} \approx \lambda^{-1}$. Also, the parameter Γ describes the decay rate for the excited atom which is related to the relaxation time $\tau_{\mathrm{s}} \approx \Gamma^{-1}$.[39] By inserting the Lorentzian spectral density (11) into Eq. (10), the correlation function $f(t-t_1)$ can be obtained as follows:

$$f(t-t_1) = \frac{\Gamma\lambda}{2}\exp[-(\lambda+\mathrm{i}\delta_l-\mathrm{i}\delta_{\mathrm{c}}-\mathrm{i}\omega')(t-t_1)]. \quad (12)$$

Using this correlation function and applying the Laplace transform technique, the analytical form of $c_1(t)$ and $c_2(t)$ are finally deduced as:

$$c_1(t) = c_1(0)\exp\left[-\frac{(\mu-\mathrm{i}\delta_{\mathrm{c}})t}{2}\right]\cdot\left[\cosh\left(\frac{\varepsilon_1 t}{2}\right)+\frac{\mu-\mathrm{i}\delta_{\mathrm{c}}}{\varepsilon_1}\sinh\left(\frac{\varepsilon_1 t}{2}\right)\right], \quad (13)$$
$$c_2(t) = c_2(0)\exp\left[-\frac{(\mu-\mathrm{i}\delta_{\mathrm{c}})t}{2}\right]\cdot\left[\cosh\left(\frac{\varepsilon_2 t}{2}\right)+\frac{\mu-\mathrm{i}\delta_{\mathrm{c}}}{\varepsilon_2}\sinh\left(\frac{\varepsilon_2 t}{2}\right)\right], \quad (14)$$

where

$$\varepsilon_1 = \sqrt{(\mu-\mathrm{i}\delta_{\mathrm{c}})^2-\frac{\Gamma\lambda}{2}(1+\cos\theta)^2},$$
$$\varepsilon_2 = \sqrt{(\mu-\mathrm{i}\delta_{\mathrm{c}})^2-\frac{\Gamma\lambda\kappa^2}{2}(1+\cos\theta)^2} \quad (15)$$

with $\mu = \lambda+\mathrm{i}\delta_l-\mathrm{i}\omega'$. The behavior of $c_1(t)$ and $c_2(t)$ are distinguished in two regimes,[40,41] i.e., weak and strong coupling regimes. By the weak regime we mean that $\Gamma < \lambda/2$, that is, $\tau_{\mathrm{s}} > 2\tau_{\mathrm{R}}$. In this regime, the relaxation time is greater than the reservoir correlation time, where the behavior of qubit–reservoir system which is essentially Markovian exhibits an exponential decay in the time. In the strong regime, that is, for $\Gamma > \lambda/2$ or $\tau_{\mathrm{s}} < 2\tau_{\mathrm{R}}$, in which the reservoir correlation time is greater than the relaxation time, exhibits non-Markovian dynamics with an oscillatory behavior in the time. It seems that, the latter is responsible for the revival phenomenon of entanglement. In the continuation of paper we consider different interaction models in Markovian and/or non-Markovian regimes. The first is "Markovian, Markovian (MM)" in which the qubits A and B are distinctly coupled to the Markovian reservoir. The second is "Non-Markovian, Markovian (NM)" in which the qubit A is coupled to the non-Markovian reservoir and the qubit B is coupled to the Markovian reservoir. Thirdly, "Non-Markovian, Non-Markovian (NN)" where the qubits A and B are distinctly coupled to the non-Markovian reservoir and finally "Non-Markovian, Isolate (NI)", in which the qubit A is coupled to the non-Markovian reservoir but the qubit B is isolated (without any environment).

Using the obtained state vector (7) with its explicit probability amplitudes, the reduced density matrix of the two-qubit system in the atomic basis $\{|1\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |0\rangle_1|0\rangle_2\}$ at time t reads

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |c_1(t)|^2 & c_1(t)c_2^*(t) & 0 \\ 0 & c_1^*(t)c_2(t) & |c_2(t)|^2 & 0 \\ 0 & 0 & 0 & 1-|c_1(t)|^2-|c_2(t)|^2 \end{pmatrix}. \quad (16)$$

In order to measure the amount of entanglement between the qubits A and B, we choose the concurrence measure $C(t)$ according to the definition[42]

$$C(t) = \max\left\{0, \sqrt{h_1}-\sqrt{h_2}-\sqrt{h_3}-\sqrt{h_4}\right\}, \quad (17)$$

in which h_i $(i=1,\ldots,4)$ are the eigenvalues of Hermitian matrix $M = \rho(\hat{\sigma}_y^A\otimes\hat{\sigma}_y^B)\rho^*(\hat{\sigma}_y^A\otimes\hat{\sigma}_y^B)$. In our case, ρ is the reduced density matrix of the pair of qubits presented in Eq. (16), ρ^* is its complex conjugate in the mentioned atomic basis and $\hat{\sigma}_y^A$, $\hat{\sigma}_y^B$ are the well-known Pauli matrices for the qubits A and B. In this way, the concurrence for the two qubits can be analytically obtained as:

$$C(t) = 2|c_1(t)\,c_2^*(t)|, \quad (18)$$

where $c_1(t)$ and $c_2(t)$ have been introduced respectively in Eqs. (13) and (14). The quantity $C(t)$ ranges from 0 to 1; the case $C(t) = 0$ corresponds to separable states but if $C(t) = 1$ the state is maximally entangled.

Now, our aim is to obtain the dynamics of two-qubit system where each qubit interacts only and independently with its own reservoir and driven by an external classical laser field. For the above-defined interactions models, we want to compare the effects of Markovian and non-Markovian reservoirs and generate the maximum entanglement by choosing the appropriate parameters. In Fig. 1 we display the temporal behavior of concurrence for qubits A and B under resonance and off-resonance in the non-Markovian ($\lambda = 0.1\Gamma$) and Markovian ($\lambda = 10\Gamma$) reservoirs. Here, we assume that the initial state of qubits A and B reads as $c_1(0) = c_2(0) = 1/\sqrt{2}$. This figure is depicted for the four mentioned interaction models. As is observed in the four plots, Figs. 1(a)–1(d) in the MM and NM reservoirs the entanglement suddenly decays, i.e., death of entanglement occurs, even though for the NM reservoirs the process of decay takes place at later times, however, the revival of entanglement does not still occur. In the NN reservoirs the entanglement exhibits a few oscillations and revival of entanglement occurs which is due to the memory effect of this type of reservoir. Also, for the NI reservoirs, revival and oscillatory behavior of entanglement can be observed, moreover, comparing with NN reservoirs the revival is more visible in the NI reservoirs. Over all, our results show that, the entanglement cannot be well protected whenever at least one of the two reservoirs being a Markovian, nevertheless the presence of classical field and central detuning have still constructive effect on the concurrence in such cases, too. In this way, the most suitable preservation of entanglement occurs for the NN and NI reservoirs. We also imply that the oscillations of concurrence for NN and NI are more visible in the absence of classical field and central detuning, while in the presence of one of these two factors the oscillations become small or even invisible. In detail, figure 1(a) which is plotted in the absence of the Rabi frequency and central detuning establishes the above statements. In Fig. 1(b) we understand that by entering the driven classical field, the degree of entanglement for each of the reservoir combinations is improved in the resonance condition, even though this influence is more visible for the NN and NI reservoirs. Also, in Fig. 1(c) which is plotted in the off-resonance condition ($\delta_c \neq 0$) with no classical field, it is found that the values of concurrence can be amended in comparison with Fig. 1(a). At last, figure 1(d) displayed the impression of the classical field and central detuning parameter on the amount of entanglement. From this figure we see that the entanglement can be appropriately protected. Summing up, driving classical field as well as off-resonance condition can properly lead to slow down the decay process of entanglement and the preservation of entanglement is accessible by either of these two factors.

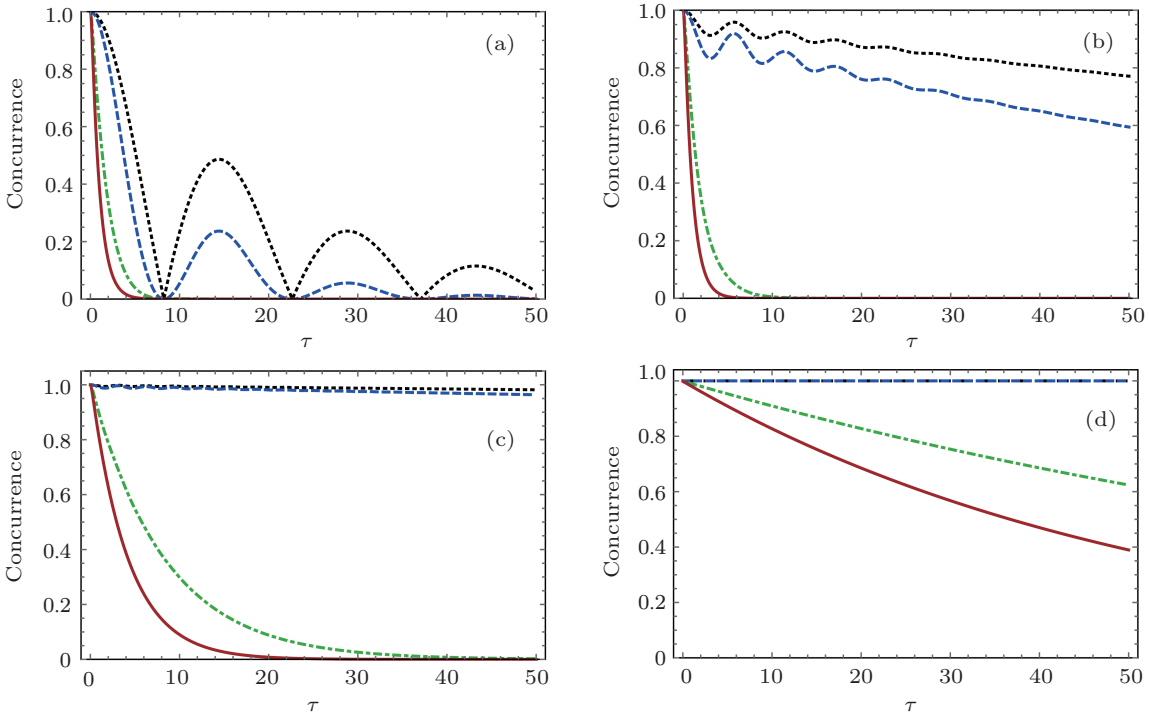


Fig. 1. (color online) Concurrence of the two-qubit system in the independent reservoirs in terms of the dimensionless time $\tau = \Gamma t$ for various values of Rabi frequency and central detuning. For (a) $\delta_c = 0$, $\Omega = 0$, (b) $\delta_c = \Gamma$, $\Omega = 0$, (c) $\delta_c = 0$, $\Omega = \Gamma$, and (d) $\delta_c = 5\Gamma$, $\Omega = 15\Gamma$. Continuous red line (MM reservoirs); dot–dashed green line (NM reservoirs); dashed blue line (NN reservoirs), and dotted black line (NI reservoirs).

Figure 2 is plotted for NN reservoirs. From this figure we understand that with increasing the laser detuning (δ_l), the amount of concurrence decreases to zero and the entanglement between the qubits vanishes as time goes on and clearly entanglement

is not protected. The presented results show that the negative effect of the laser detuning can be compensated with increasing Rabi frequency. Also, the destructive effect of laser detuning on the entanglement and its modification with increasing the Rabi frequency occurs for MM reservoirs.

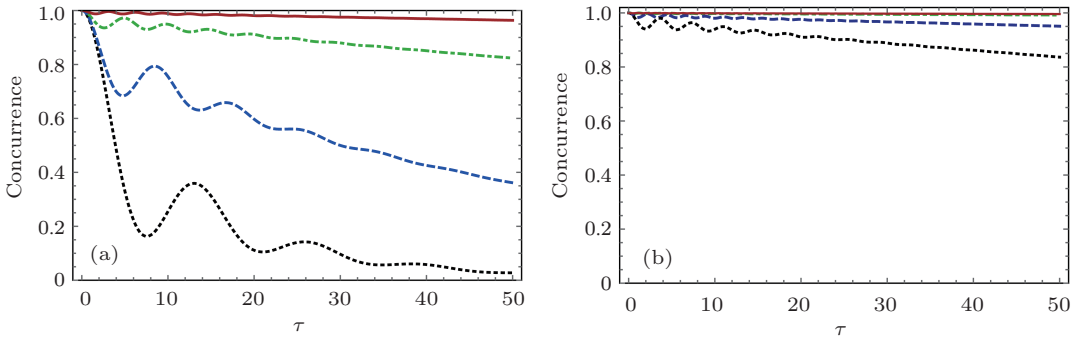


Fig. 2. (color online) Concurrence of the two-qubit system in the independent reservoirs in terms of the dimensionless time $\tau = \Gamma t$ with $\delta_c = 0$ in NN reservoirs. (a) $\Omega = \Gamma$, (b) $\Omega = 3\Gamma$ with different values of the laser detuning (δ_l). Continuous red line ($\delta_l = 0$), dot–dashed green line ($\delta_l = \Gamma$), dashed blue line ($\delta_l = 3\Gamma$) and dotted black line ($\delta_l = 10\Gamma$).

In the following, three-dimensional plot of the concurrence in terms of Ω/Γ, δ_c/Γ is depicted, while figure 3(a) is displayed for non-Markovian regime (NN reservoirs) and figure 3(b) is shown for Markovian regime (MM reservoirs). It is observed that for non-Markovian and Markovian regimes, the maximum value of entanglement increases as the Rabi frequency and central detuning (δ_c) grow up. Moreover, the entanglement induced by non-Markovian regime is much greater than that of induced by Markovian regime.

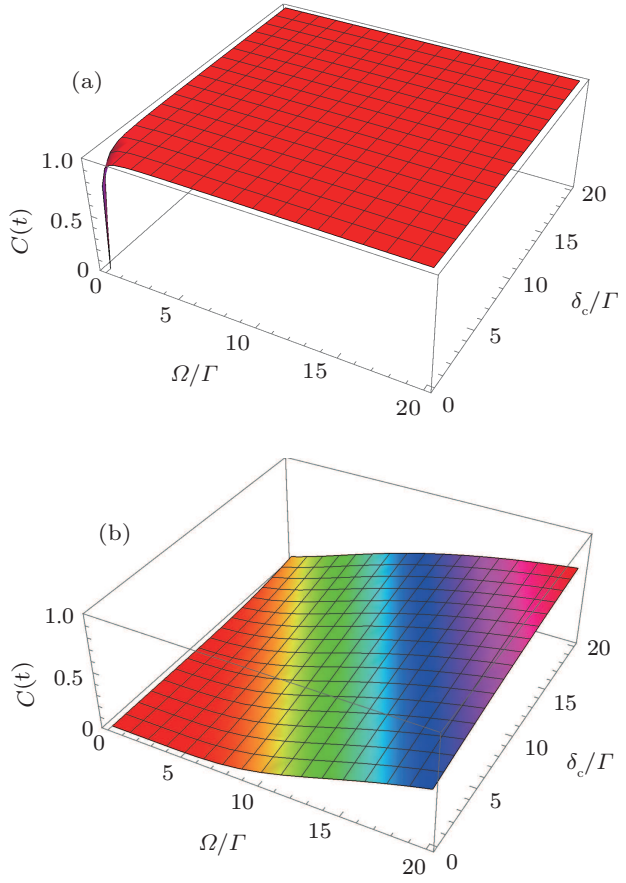


Fig. 3. (color online) Three-dimensional plot of concurrence in the independent reservoirs versus Ω/Γ and δ_c/Γ at $\tau = 50$: (a) in NN reservoirs with $\lambda = 0.1\Gamma$, (b) in MM reservoirs with $\lambda = 10\Gamma$.

3. Model interaction for a common reservoir

Now, we consider the two qubits being in a common reservoir at zero temperature which is driven by an external classical field with frequency ω_l. Similar to the independent reservoirs we have again assumed that the two qubits are far enough apart such that no collective behavior has to be encountered. Henceforth, the total Hamiltonian $\hat{H}$ under RWA can be described as[17]

$$\hat{H} = \frac{\omega_0}{2}(\hat{\sigma}_z^{(A)} + \hat{\sigma}_z^{(B)}) + \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k + \Big[\Omega(\hat{\sigma}_+^{(A)} + \hat{\sigma}_+^{(B)})\, \mathrm{e}^{-\mathrm{i}\omega_l t} + \sum_k g_k \hat{a}_k (\alpha_1 \hat{\sigma}_+^{(A)} + \alpha_2 \hat{\sigma}_+^{(B)}) + \mathrm{H.C.}\Big], \tag{19}$$

in which ω_0 is the transition frequency of two qubits, $\hat{\sigma}_z^{A,B}$, $\hat{\sigma}_+^{A,B}$, $\hat{\sigma}_-^{A,B}$ represent atomic inversion, raising, and lowering operators, respectively and Ω is the Rabi frequency of the classical field. The operators $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the well-known bosonic creation and annihilation operators of the k-th mode of the field of the reservoir with frequency ω_k and g_k are the coupling constant between the qubit and the k-th mode of the cavity field. Since the qubits A and B are at different positions relative to the cavity center, they experience different values of the cavity field, so we introduce the dimensionless real constants α_1, α_2 that measure the interaction strength of each qubit with cavity mode.[17] Following the same procedure as has been done in Section 2, we are readily led to the Hamiltonian in the dressed basis states introduced in Eq. (3) as:

$$\hat{H}_\mathrm{d} = \frac{\omega'}{2}(\hat{\tilde{\sigma}}_z^{(A)} + \hat{\tilde{\sigma}}_z^{(B)}) + \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k + \cos^2\frac{\theta}{2}\Big[\sum_k g_k \hat{a}_k (\alpha_1 \hat{\tilde{\sigma}}_+^{(A)} + \alpha_2 \hat{\tilde{\sigma}}_+^{(B)})\, \mathrm{e}^{\mathrm{i}\omega_l t} + \mathrm{H.C.}\Big]. \tag{20}$$

After some manipulations we arrive at the interaction picture Hamiltonian,

$$\hat{H}_\mathrm{int} = \cos^2\frac{\theta}{2}$$

$$\cdot\Big[\sum_k g_k \hat{a}_k(\alpha_1\hat{\tilde{\sigma}}_+^{(A)}+\alpha_2\hat{\tilde{\sigma}}_+^{(B)})\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_l-\omega')t}+\mathrm{H.C.}\Big]. \quad (21)$$

We consider the initial condition such that only one excitation exists in the total system as follows:

$$|\psi(0)\rangle = \big(c_1(0)\,|1\rangle_1|0\rangle_2 + c_2(0)\,|0\rangle_1|1\rangle_2\big)|0_k\rangle_{\mathrm{R}}. \quad (22)$$

So, at any time $t>0$ the state of the total system can be written as

$$|\psi(t)\rangle = \big(c_1(t)\,|1\rangle_1|0\rangle_2 + c_2(t)\,|0\rangle_1|1\rangle_2\big)|0_k\rangle_{\mathrm{R}} + \sum_k c_k(t)\,|0\rangle_1|0\rangle_2|1_k\rangle_{\mathrm{R}}, \quad (23)$$

where we denote $|0_k\rangle_{\mathrm{R}}$ as the vacuum state of reservoir and $|1_k\rangle_{\mathrm{R}}$ as its state where only one excitation exists in the k-th field mode. By substituting Eqs. (21) and (23) into the time-dependent Schrödinger equation $\hat{H}_{\mathrm{int}}|\psi(t)\rangle = \mathrm{i}(\partial/\partial t)|\psi(t)\rangle$, the following coupled integro-differential equations for $c_1(t)$ and $c_2(t)$ are deduced:

$$\dot{c_1}(t) = -\cos^4\frac{\theta}{2}\int_0^t \mathrm{d}t_1 f(t-t_1)\big[\alpha_1^2 c_1(t_1)+\alpha_1\alpha_2 c_2(t_1)\big], \quad (24)$$

$$\dot{c_2}(t) = -\cos^4\frac{\theta}{2}\int_0^t \mathrm{d}t_1 f(t-t_1)\big[\alpha_1\alpha_2 c_1(t_1)+\alpha_2^2 c_2(t_1)\big], \quad (25)$$

in which the correlation function $f(t-t_1)$ has been defined in Eq. (10). Now, using the spectral density $J(\omega)$ in Eq. (11) the explicit forms of $c_1(t)$ and $c_2(t)$ can be obtained by utilizing the Laplace transform technique which results in

$$c_1(t) = r_2^2 c_1(0) - r_1 r_2 c_2(0) + \big[r_1^2 c_1(0) + r_1 r_2 c_2(0)\big]\xi(t), \quad (26)$$

$$c_2(t) = r_1^2 c_2(0) - r_1 r_2 c_1(0) + \big[r_2^2 c_2(0) + r_1 r_2 c_1(0)\big]\xi(t), \quad (27)$$

where

$$\xi(t) = \exp\Big[-\frac{(\mu-\mathrm{i}\delta_{\mathrm{c}})t}{2}\Big] \cdot\Big[\cosh\Big(\frac{\varepsilon\, t}{2}\Big)+\frac{\mu-\mathrm{i}\delta_{\mathrm{c}}}{\varepsilon}\sinh\Big(\frac{\varepsilon\, t}{2}\Big)\Big],$$
$$\varepsilon = \sqrt{(\mu-\mathrm{i}\delta_{\mathrm{c}})^2 - \frac{\Gamma\lambda\ \alpha_T^2}{2}(1+\cos\theta)^2},$$
$$\mu = \lambda + \mathrm{i}\delta_l - \mathrm{i}\omega'. \quad (28)$$

Here, we introduced the collective coupling constant $\alpha_T = \sqrt{\alpha_1^2+\alpha_2^2}$ and relative coupling strengths as $r_j = \alpha_j/\alpha_T$, $j=1,2$ with $r_1^2+r_2^2=1$. Similar to the previous section, the dynamics of two-qubit entanglement for common reservoir is completely determined by $c_1(t)$ and $c_2(t)$ using Eq. (18). We can acquire the temporal behavior of concurrence by considering $r_1 = r_2 = 1/\sqrt{2}$ with different values of the Rabi frequency and central detuning (δ_{c}) in the non-Markovian as well as Markovian reservoir.

From Figs. 4 and 5 it can be observed that, applying the driven classical field in the presence of central detuning enhances the degree of entanglement of qubits. In addition to the above conditions, in Fig. 4, due to the memory effect of non-Markovian reservoir, the concurrence increases up to its maximum value, while in Fig. 5 in the Markovian reservoir the concurrence decays to zero, i.e, finally the two qubits are completely separable. Meanwhile, with considering the laser detuning ($\delta_l \neq 0$), similar to Fig. 2 for the independent reservoir, we have the same results for the common reservoir, i.e., concurrence decreases.

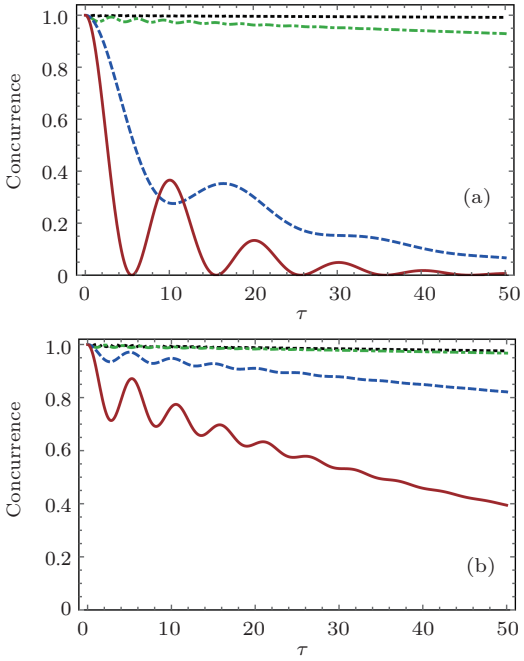


Fig. 4. (color online) Concurrence of the two-qubit system in a common non-Markovian ($\lambda = 0.1\Gamma$) reservoir in terms of the dimensionless time $\tau = \Gamma t$; (a) $\delta_{\mathrm{c}} = 0$, (b) $\delta_{\mathrm{c}} = \Gamma$. Continuous red line ($\Omega = 0$), dashed blue line ($\Omega = 0.1\Gamma$), dot–dashed green line ($\Omega = \Gamma$) and dotted black line ($\Omega = 3\Gamma$).

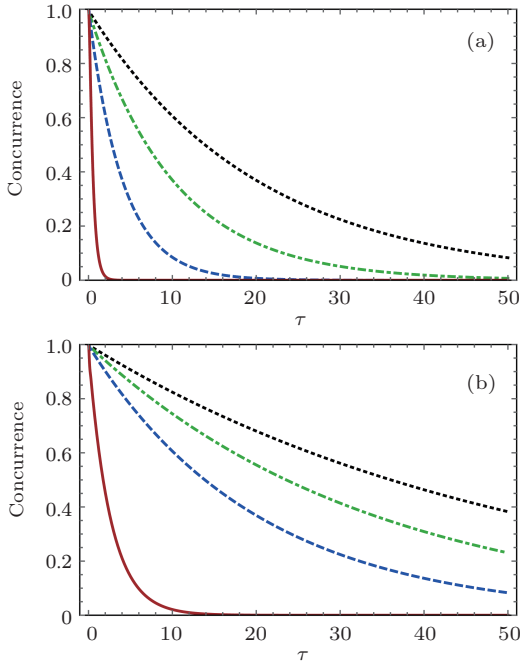


Fig. 5. (color online) Concurrence of the two-qubit system in a common Markovian ($\lambda = 10\Gamma$) reservoir in terms of the dimensionless time $\tau = \Gamma t$; (a) $\delta_{\mathrm{c}} = 0$, (b) $\delta_{\mathrm{c}} = 20\Gamma$. Continuous red line ($\Omega = 0$), dashed blue line ($\Omega = 5\Gamma$), dot–dashed green line ($\Omega = 10\Gamma$) and dotted black line ($\Omega = 15\Gamma$).

In the above discussion we assumed that the two qubits are coupled to the reservoir equivalently ($r_1 = r_2 = 1/\sqrt{2}$). Moreover, figure 6 displays the temporal behavior of concurrence in the Markovian ($\lambda = 10\Gamma$) and non-Markovian ($\lambda = 0.1\Gamma$) reservoirs with considering different values of relative coupling strengths, $r_1 \neq r_2$. In this way, $r_1 = 0$ and $r_1 = 1$ both describe the case in which only one qubit is coupled to the reservoir, so the corresponding two plots are overlapped. Other values of r_1 can be included $r_1 = 1/\sqrt{2}$ in which the two qubits are symmetrically coupled to reservoir and $r_1 = \sqrt{3}/2$ wherein the two qubits are coupled to the reservoir with asymmetric values of r_1 and r_2. From Fig. 6 one observes that with choosing the coupling parameter r_1 as $0, 1, 1/\sqrt{2}$ the concurrence decreases after passing the time and finally tends to zero, i.e., entanglement vanishes. On the other hand we find that for $r_1 = \sqrt{3}/2$ the non-zero stationary value of entanglement occurs. In detail, the concurrence monotonically decreases down to its stationary value at $C(t) \simeq 0.058$.

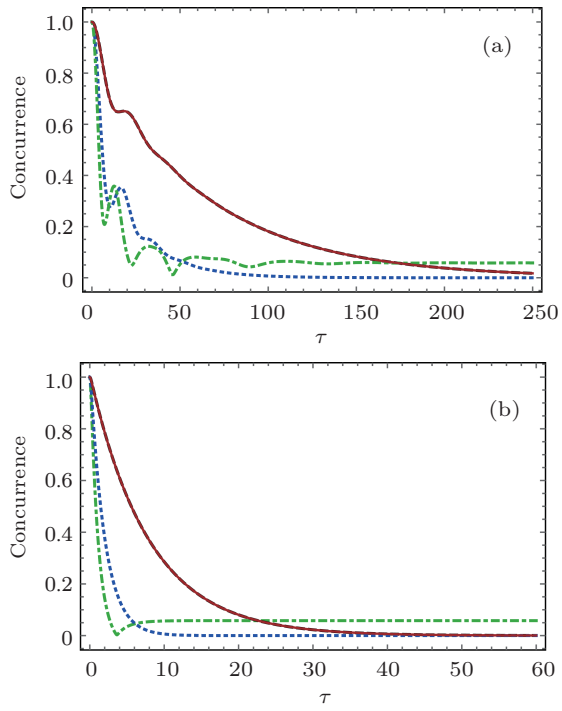


Fig. 6. (color online) Concurrence of the two-qubit system in terms of the dimensionless time $\tau = \Gamma t$ for $\Omega = 0.1\Gamma$ and $\delta_c = 0$: (a) in a common non-Markovian reservoir and (b) in a common Markovian reservoir. With only one coupled qubit $r_1 = 0$ (continuous red line) and $r_1 = 1$ (dashed black line), asymmetric coupling $r_1 = \sqrt{3}/2$ (dot–dashed green line), and symmetric coupling $r_1 = 1/\sqrt{2}$ (dotted blue line).

4. Summary and concluding remarks

In the present paper we explored entanglement preservation of a two-qubit system where the qubits are driven by an external classical field and interact independently with their own reservoirs. We first derived the effective Hamiltonian of the system and rewrote it in the interaction picture. Then, we obtained the explicit form of the state vector associated with total system. Next, by using the concurrence measure, entanglement dynamics of the two qubits analytically obtained when the Markovian or non-Markovian environment considered. We illustrated that the entanglement can be protected, not only by detuning between the atoms and the central cavity mode frequencies, δ_c (central detuning) but also by increasing and controlling the strength of driven classical field. In addition, with introducing four distinct interaction models we observed that the entanglement and its preservation in non-Markovian reservoirs are more sensitive to the increase in central detuning and Rabi frequency than Markovian reservoirs. In this respect, due to memory effect of non-Markovian environments, the maximum accessible entanglement takes place in non-Markovian reservoirs. Such a result is consistent with the ones reported in Refs. [30], [31], and [32], however, it ought to be emphasized that in the latter Refs. entanglement (preservation) is studied without the influence of classical field. We also found that, in the presence of detuning between frequencies of each qubit and the classical field, δ_l (laser detuning) the entanglement of qubits decreases and so the decoherence phenomenon occurs in the Markovian and non-Markovian reservoirs. Moreover, it is worthwhile to notice that such a disadvantage of laser detuning can be compensated with increasing Rabi frequency of classical field, i.e., strengthening the amplitude of classical field. We also presented the dynamics of the two-qubit system in the common reservoir, where we generally obtained the same results as in the independent reservoir. Moreover, we observed that the degree of entanglement of qubits in the independent reservoirs may be improved a bit more comparing with the common reservoirs.

Appendix A

In this appendix we want to obtain the effective Hamiltonian in terms of the dressed operators that is introduced in Eq. (4). By using the unitary transformation $\hat{U}(t) = \exp\big[-\mathrm{i}\omega_l t(\hat{\sigma}_z^{(A)} + \hat{\sigma}_z^{(B)})\big]$ the Hamiltonian (1) can be transformed to[43–45]

$$\begin{aligned}\hat{H}_{\rm eff} &= \hat{U}^{\dagger}(t)\,\hat{H}\,\hat{U}(t) - \mathrm{i}\hat{U}^{\dagger}(t)\frac{\partial \hat{U}(t)}{\partial t}\\ &= \hat{H}_{\rm eff}^{(1)} + \hat{H}_{\rm eff}^{(2)},\end{aligned} \tag{A1}$$

with

$$\begin{aligned}\hat{H}_{\rm eff}^{(1)} &= \frac{\delta_l}{2}\sum_{i=A,B}\hat{\sigma}_z^{(i)} + \Omega\sum_{i=A,B}\hat{\sigma}_x^{(i)},\\ \hat{H}_{\rm eff}^{(2)} &= \sum_{j=1,2}\sum_{k}\omega_{k_j}\hat{a}_{k_j}^{\dagger}\hat{a}_{k_j}\\ &\quad + \big[(\tilde{g_1}\hat{\sigma}_+^{(A)} + \kappa\tilde{g_2}\,\hat{\sigma}_+^{(B)})\,\mathrm{e}^{\mathrm{i}\omega_l t} + \mathrm{H.C.}\big],\end{aligned} \tag{A2}$$

in which equation (A2) is Hamiltonian (2), where we have used the following definitions as Pauli operators,

$$\hat{\sigma}_z^{A} = |e\rangle_1\langle e| - |g\rangle_1\langle g|,\ \hat{\sigma}_x^{A} = |e\rangle_1\langle g| + |g\rangle_1\langle e|,$$

$$\hat{\sigma}_z^B = |e\rangle_2\langle e| - |g\rangle_2\langle g|,\ \hat{\sigma}_x^B = |e\rangle_2\langle g| + |g\rangle_2\langle e|$$
$$\hat{\sigma}_+^A = |e\rangle_1\langle g|,\ \hat{\sigma}_+^B = |e\rangle_2\langle g|. \tag{A3}$$

The Hamiltonian $\hat{H}_{\rm eff}^{(1)}$ can be diagonalized as the following form

$$\hat{H}_{\rm eff}^{(1)} = \frac{\omega'}{2}(\hat{\tilde{\sigma}}_z^A + \hat{\tilde{\sigma}}_z^B). \tag{A4}$$

Here, we defined $\hat{\tilde{\sigma}}_z^A = |1\rangle_1\langle 1| - |0\rangle_1\langle 0|$ and $\hat{\tilde{\sigma}}_z^B = |1\rangle_2\langle 1| - |0\rangle_2\langle 0|$ in the new dressed basis states as Eq. (3). Also, the Hamiltonian $\hat{H}_{\rm eff}^{(2)}$ can be recasted in this new dressed states as follows:

$$\begin{aligned}\hat{H}_{\rm eff}^{(2)} = & \sum_{j=1,2}\sum_k \omega_{k_j}\hat{a}_{k_j}^\dagger \hat{a}_{k_j} \\ & + \Big[\mathrm{e}^{\mathrm{i}\omega_l t}\sum_{k_1} g_{k_1}\Big(\frac{1}{2}\sin\theta\ \hat{a}_{k_1}\hat{\tilde{\sigma}}_z^{(A)} + \cos^2\frac{\theta}{2}\ \hat{a}_{k_1}\hat{\tilde{\sigma}}_+^{(A)} - \sin^2\frac{\theta}{2}\ \hat{a}_{k_1}\hat{\tilde{\sigma}}_-^{(A)}\Big) \\ & + \mathrm{e}^{\mathrm{i}\omega_l t}\sum_{k_2} \kappa\, g_{k_2}\Big(\frac{1}{2}\sin\theta\ \hat{a}_{k_2}\hat{\tilde{\sigma}}_z^{(B)} + \cos^2\frac{\theta}{2}\ \hat{a}_{k_2}\hat{\tilde{\sigma}}_+^{(B)} - \sin^2\frac{\theta}{2}\ \hat{a}_{k_2}\ \hat{\tilde{\sigma}}_-^{(B)}\Big) + \text{H.C.}\Big].\end{aligned} \tag{A5}$$

With $\hat{\tilde{\sigma}}_+^A = |1\rangle_1\langle 0|$, $\hat{\tilde{\sigma}}_-^A = |0\rangle_1\langle 1|$, $\hat{\tilde{\sigma}}_+^B = |1\rangle_2\langle 0|$, and $\hat{\tilde{\sigma}}_-^B = |0\rangle_2\langle 1|$, where $|0\rangle_j$, $|1\rangle_j$ $(j = 1,2)$ are respectively the ground and excited states of the qubits in the dressed states. In fact, the terms $\sin\theta\ \hat{a}_{k_{1,2}}\hat{\tilde{\sigma}}_z^{A,B}\mathrm{e}^{\mathrm{i}\omega_l t}$ and $\sin^2(\theta/2)\,\hat{a}_{k_{1,2}}\hat{\tilde{\sigma}}_-^{A,B}\mathrm{e}^{\mathrm{i}\omega_l t}$ and their complex conjugates cannot conserve energy; so, by applying rotating wave approximation, the following expression can be overlooked,

$$\hat{H}_{\rm eff}^{(2)} = \sum_{j=1,2}\sum_k \omega_{k_j}\hat{a}_{k_j}^\dagger\hat{a}_{k_j} + \cos^2\frac{\theta}{2}\Big[(\tilde{g}_1\hat{\tilde{\sigma}}_+^{(A)} + \kappa\,\tilde{g}_2\hat{\tilde{\sigma}}_+^{(B)})\,\mathrm{e}^{\mathrm{i}\omega_l t} + \text{H.C.}\Big]. \tag{A6}$$

By Eqs. (A4) and (A6), the Hamiltonian (4) is readily obtained.

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