ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理 Chinese Physics B

Multi-window transparency and fast-slow light switching in a quadratically coupled optomechanical system assisted with three-level atoms

Wan-Ying Wei(魏晚迎), Ya-Fei Yu(於亚飞), Zhi-Ming Zhang(张智明)
Citation: Chin. Phys. B . 2018, 27(3): 034204. **doi:** 10.1088/1674-1056/27/3/034204

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Parallel generation of 31 tripartite entangled states based on optical frequency combs

Jing Zhang(张静), Yan-Fang Wang(王艳芳), Xiao-Yu Liu(刘晓宇), Rong-Guo Yang(杨荣国)
Chin. Phys. B . 2017, 26(12): 124205. **doi:** 10.1088/1674-1056/26/12/124205

Generation of squeezed vacuum on cesium D2 line down to kilohertz range

Jian-Feng Tian(田剑锋), Guan-Hua Zuo(左冠华), Yu-Chi Zhang(张玉驰), Gang Li(李刚), Peng-Fei Zhang(张鹏飞), Tian-Cai Zhang(张天才)
Chin. Phys. B . 2017, 26(12): 124206. **doi:** 10.1088/1674-1056/26/12/124206

Quantum feedback cooling of two trapped ions

Shuo Zhang(张硕), Wei Wu(吴伟), Chun-Wang Wu(吴春旺), Feng-Guang Li(李风光), Tan Li(李坦), Xiang Wang(汪翔), Wan-Su Bao(鲍皖苏)
Chin. Phys. B . 2017, 26(7): 074205. **doi:** 10.1088/1674-1056/26/7/074205

Single-photon scattering by two separated atoms in a supercavity

Wei Zhu(朱伟), Xiao Xiao(肖骁), Duan-Lu Zhou(周端陆), Peng Zhang(张芃)
Chin. Phys. B . 2016, 25(6): 064203. **doi:** 10.1088/1674-1056/25/6/064203

Quadripartite entangled state from cascaded second-harmonic generation

Yang Rui, Zhai Shu-Qin, Yang Rong-Guo
Chin. Phys. B . 2014, 23(4): 044202. **doi:** 10.1088/1674-1056/23/4/044202

Multi-window transparency and fast–slow light switching in a quadratically coupled optomechanical system assisted with three-level atoms*

Wan-Ying Wei(魏晚迎), Ya-Fei Yu(於亚飞), and Zhi-Ming Zhang(张智明)†

Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices (SIPSE), Guangdong Provincial Key Laboratory of Quantum Engineering Quantum Materials, South China Normal University, Guangzhou 510006, *China*

(Received 16 September 2017; revised manuscript received 11 October 2017; published online 11 February 2018)

We study theoretically the features of the output field of a quadratically coupled optomechanical system assisted with three-level atoms. In this system, the atoms interact with the cavity field and are driven by a classical field, and the cavity is driven by a strong coupling field and a weak signal field. We find that there exists a multi-window transparency phenomenon. The width of the transparent windows can be adjusted by controlling the system parameters, including the number of the atoms, the powers of the lasers driving the atoms and driving the cavity, and the environment temperature. We also find that a tunable switch from fast light to slow light can be realized in this system.

Keywords: quantum optics, optomechanics, optomechanically induced transparency, fast and slow light

PACS: 42.50.–p, 42.50.Ct, 42.50.Pq **DOI:** 10.1088/1674-1056/27/3/034204

1. Introduction

Optomechanical systems have attracted a great deal of attention these days since they offer devices extending quantum mechanics to macroscopic objects.[1] With the development of theory and experiment, many interesting phenomena have been confirmed, for example, the optomechanical entanglement,[2–5] the optomechanically induced transparency (OMIT),[6–11] the quantum ground state cooling,[12–14] the quantum squeezing,[15–19] and so on.[20,21] These phenomena rely on the light–mechanical interaction in the coupled optomechanical system. The radiation pressure plays an important role in optomechanical systems. The radiation pressure coupling between the light fields and oscillating mirrors can depend on the location of the mirror linearly or quadratically.[22] The quadratically coupled optomechanical systems (QC-OMSs),[23–26] in which the optical field is coupled to the square of the position of a mechanical oscillator, have attracted a great deal of attention.[27,28] Many effects have been devoted to the QC-OMSs, for example, the two-phonon OMIT,[23,29] the mechanical cooling,[30] and the mechanical squeezing.[31] Hybrid linearly coupled optomechanical systems containing atomic ensembles have been investigated.[32–40] It is interesting to extend studies to the QC-OMSs assisted with atoms. In a QC-OMS filled with two-level atoms,[29] the output field exhibits an analogous electromagnetically induced transparency (EIT) phenomenon. Although a QC-OMS with a two-level atomic ensemble has been studied,[29] there are few works to explore what will happen in a QC-OMS with a three-level atomic ensemble. In this paper, we report a theoretical study on this system. We find that there exists a multi-window OMIT phenomenon and a fast–slow light switching effect, and these effects can be adjusted by controlling the systemic parameters. The results of our system have distinct differences with those of the QC-OMS without atom[23] and the QC-OMS with two-level atoms,[29] in which only one transparent window can occur. In Ref. [41], the authors studied an optomechanical system containing a single atom, and found that the multi-window transparency and the fast light can be realized. Compared with that work, the multi-window transparency in our system can be adjusted more easily and the fast–slow light switch can occur. Compared with Ref. [37], the position of the fast–slow light switch can be controlled in our system.

The organization of this paper is as follows. In Section 2, we describe our systemic model and present the analytic calculation. In Section 3, we study the multi-window OMIT, and show how to adjust the width of the OMIT windows. In Section 4, we study the fast–slow light switch in detail. Finally in Section 5, we present a brief summary.

2. Model and analytic calculation

Our system model is shown in Fig. 1. A membrane as a quantum-mechanical harmonic oscillator with finite reflectivity R is placed at the center of the optical cavity formed by two fixed mirrors separated from each other by a distance L. The three-level atomic ensemble trapped inside the cavity is driven

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61775062, 11574092, 61378012, 91121023, and 60978009) and the National Basic Research Program of China (Grant No. 2013CB921804).

†Corresponding author. E-mail: zhangzhiming@m.scnu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

by a strong classical light field of frequency ω_c, and the cavity mode of frequency ω_0 is driven simultaneously by a strong coupling field of frequency ω_c and a weak probe field of frequency ω_p. The Hamiltonian of the total system can be written as

$$\begin{aligned} H &= \hbar\omega_0 c^\dagger c + \left(\frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2\right) \\ &+ \hbar\sum_{i=1}^{N}(\omega_{21}\sigma_{22}^{(i)} + \omega_{31}\sigma_{33}^{(i)}) \\ &+ \hbar g_0 c^\dagger c q^2 + \hbar g\sum_{i=1}^{N}(c^\dagger\sigma_{13}^{(i)} + \text{H.c.}) \\ &+ \hbar\sum_{i=1}^{N}(\Omega\, e^{-i\omega_c t}\sigma_{32}^{(i)} + \text{H.c.}) + i\hbar\varepsilon_c(c^\dagger e^{-i\omega_c t} - \text{H.c.}) \\ &+ i\hbar\left(\varepsilon_p c^\dagger e^{-i\omega_p t} - \text{H.c.}\right), \end{aligned} \tag{1}$$

where the first term describes the free Hamiltonian of the cavity field in which c ($c^\dagger$) is the annihilation (creation) operator of the cavity field. The second term represents the free energy of the membrane with frequency ω_m, mass m, momentum p, and displacement q. The third term is the free Hamiltonian of the three-level atomic ensemble with the operators $\sigma_{\alpha\alpha}^{(i)} = |\alpha\rangle^{(i)}\langle\alpha|$ ($\alpha = 2,3$); here we assume the ground state $|1\rangle^{(i)}$ as the energy reference point, and frequency ω_{21} (ω_{31}) is the energy-level spacing between $|2\rangle^{(i)}$ and $|1\rangle^{(i)}$ ($|3\rangle^{(i)}$ and $|1\rangle^{(i)}$) for the i-th atom. The fourth term is the interaction Hamiltonian between the cavity field and the membrane with the quadratically coupling constant g_0, and $g_0 = (8c\pi^2/\lambda^2 L)\sqrt{R/(1-R)}$,[16] where c is the speed of light in the vacuum and λ is the wavelength of the pump field. The fifth term denotes the interaction between the atom ensemble and the cavity field with coupling g, and the sixth term is the interaction between the atom ensemble and the classical field, where Ω is the Rabi frequency associated with the coupling between the classical field and the three-level atom. The last two terms describe the interactions of the cavity field with the coupling field and the probe field, respectively, with the amplitudes $\varepsilon_c = \sqrt{2\kappa P_c/\hbar\omega_c}$ and $\varepsilon_p = \sqrt{2\kappa P_p/\hbar\omega_p}$, where κ is the decay rate of the cavity field, and P_c and P_p are the laser powers.

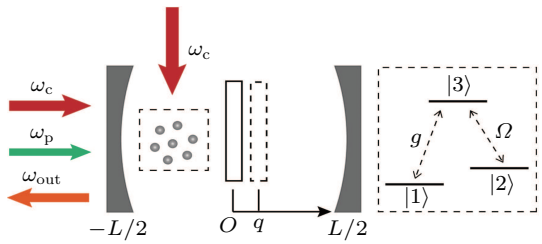


Fig. 1. (color online) Sketch of the system. A membrane is at the center position of the cavity and a three-level atomic ensemble driven by an external classical field is trapped inside the cavity. The cavity is driven by a coupling laser and a probe laser. The energy-level configuration of the three-level atom is shown on the right.

In the rotating frame with the coupling field frequency ω_c, Hamiltonian (1) can be rewritten as

$$\begin{aligned} H &= \hbar\Delta_0 c^\dagger c + \left(\frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2\right) \\ &+ \hbar\sum_{i=1}^{N}(\omega_{21}\sigma_{22}^{(i)} + \Delta_1\sigma_{33}^{(i)}) + \hbar g_0 c^\dagger c q^2 \\ &+ \hbar g\sum_{i=1}^{N}(c^\dagger\sigma_{13}^{(i)} + \text{H.c.}) + \hbar\sum_{i=1}^{N}(\Omega\sigma_{32}^{(i)} + \text{H.c.}) \\ &+ i\hbar\varepsilon_c(c^\dagger - \text{H.c.}) + i\hbar\left(\varepsilon_p c^\dagger e^{-i\delta t} - \text{H.c.}\right), \end{aligned} \tag{2}$$

where $\Delta_0 = \omega_0 - \omega_c$, $\Delta_1 = \omega_{31} - \omega_c$, and $\delta = \omega_p - \omega_c$ are the detunings. We define the collective operators of the atomic ensemble as[33,35]

$$A^\dagger = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sigma_{31}^{(i)}, \quad B^\dagger = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sigma_{21}^{(i)}. \tag{3}$$

We suppose that the number of the atoms is large enough, and most of the atoms are in the ground state, i.e., the atomic system is in very low excitation.[38] In this situation, $\langle\sigma_{11}^{(i)}\rangle \approx 1$, $\langle\sigma_{22}^{(i)}\rangle \approx 0$, $\langle\sigma_{33}^{(i)}\rangle \approx 0$, and $\langle\sigma_{23}^{(i)}\rangle \approx 0$, then the above collective operators satisfy the commutation relations

$$\begin{aligned} [A, A^\dagger] &= \frac{1}{N}\sum_{i=1}^{N}(\sigma_{11}^{(i)} - \sigma_{33}^{(i)}) \approx 1, \\ [B, B^\dagger] &= \frac{1}{N}\sum_{i=1}^{N}(\sigma_{11}^{(i)} - \sigma_{22}^{(i)}) \approx 1, \\ [A, B^\dagger] &= \frac{1}{N}\sum_{i=1}^{N}\sigma_{23}^{(i)} \approx 0. \end{aligned} \tag{4}$$

The quantum Langevin equations for the atom–field–membrane system are given as follows:

$$\begin{aligned} \frac{dp}{dt} &= -m\omega_m^2 q - 2g_0 c^\dagger c q - \gamma_m p + \xi, \\ \frac{dq}{dt} &= \frac{p}{m}, \\ \frac{dc}{dt} &= -[i(\Delta_0 + g_0 q^2) + \kappa]c - iG_A A \\ &\quad + \varepsilon_c + \varepsilon_p e^{-i\delta t} + \sqrt{2\kappa}c_{in}, \\ \frac{dA}{dt} &= [-\gamma_1 - i\Delta_1]A - iG_A c - i\Omega B + \sqrt{\gamma_1}A_{in}, \\ \frac{dB}{dt} &= [-\gamma_2 - i\omega_{21}]B - i\Omega A + \sqrt{\gamma_2}B_{in}, \end{aligned} \tag{5}$$

in which the commutation relations in Eq. (4) have been used. In Eq. (5), $G_A = \sqrt{N}g$ is the effective coupling strength between the cavity field and the atomic ensemble, κ is the cavity decay rate, γ_1 (γ_2) is the decay rate of the atomic transition $|1\rangle \leftrightarrow |3\rangle$ ($|1\rangle \leftrightarrow |2\rangle$), γ_m is the damping rate of the membrane; ξ, c_{in}, A_{in}, and B_{in} are the input vacuum noise operators,[42] and satisfy $\langle\xi\rangle = \langle c_{in}\rangle = \langle A_{in}\rangle = \langle B_{in}\rangle = 0$. We are interested in the mean response of the system. By using the mean-field

assumption $\langle abc\rangle = \langle a\rangle\langle b\rangle\langle c\rangle$, we write the Langevin equations for the mean values as

$$\begin{aligned}
\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t} &= -m\omega_{\mathrm{m}}^2\langle q\rangle - 2g_0\langle c^{\dagger}\rangle\langle c\rangle\langle q\rangle - \gamma_{\mathrm{m}}\langle p\rangle,\\
\frac{\mathrm{d}\langle q\rangle}{\mathrm{d}t} &= \frac{\langle p\rangle}{m},\\
\frac{\mathrm{d}\langle c\rangle}{\mathrm{d}t} &= -[\mathrm{i}(\Delta_0 + g_0\langle q^2\rangle) + \kappa]\langle c\rangle - \mathrm{i}G_{\mathrm{A}}\langle A\rangle + \varepsilon_{\mathrm{c}} + \varepsilon_{\mathrm{p}}\mathrm{e}^{-\mathrm{i}\delta t},\\
\frac{\mathrm{d}\langle A\rangle}{\mathrm{d}t} &= [-\gamma_1 - \mathrm{i}\Delta_1]\langle A\rangle - \mathrm{i}G_{\mathrm{A}}\langle c\rangle - \mathrm{i}\Omega\langle B\rangle,\\
\frac{\mathrm{d}\langle B\rangle}{\mathrm{d}t} &= [-\gamma_2 - \mathrm{i}\omega_{21}]\langle B\rangle - \mathrm{i}\Omega\langle A\rangle. \qquad (6)
\end{aligned}$$

In order to solve the above set of equations, we need the following equations:

$$\begin{aligned}
\frac{\mathrm{d}\langle q^2\rangle}{\mathrm{d}t} &= \frac{1}{m}\langle pq+qp\rangle,\\
\frac{\mathrm{d}\langle pq+qp\rangle}{\mathrm{d}t} &= \frac{2\langle p^2\rangle}{m} - 2(m\omega_{\mathrm{m}}^2 + 2\hbar\langle c^{\dagger}\rangle\langle c\rangle)\langle q^2\rangle\\
&\quad - \gamma_{\mathrm{m}}\langle pq+qp\rangle,\\
\frac{\mathrm{d}\langle p^2\rangle}{\mathrm{d}t} &= -(m\omega_{\mathrm{m}}^2 + 2\hbar\langle c^{\dagger}\rangle\langle c\rangle)\langle pq+qp\rangle\\
&\quad - 2\gamma_{\mathrm{m}}\langle p^2\rangle + 2\gamma_{\mathrm{m}}(1+2n)m\hbar\omega_{\mathrm{m}}, \qquad (7)
\end{aligned}$$

in which $2\gamma_{\mathrm{m}}(1+2n_{\mathrm{th}})m\hbar\omega_{\mathrm{m}}$ is the coupling constant from the membrane and the thermal environment,[18] where $n_{\mathrm{th}} = [\exp(\hbar\omega_{\mathrm{m}}/k_{\mathrm{B}}T) - 1]^{-1}$ is the mean phonon occupation number at temperature T, and k_{B} is the Boltzmann's constant. For convenience, we define operators $\langle E\rangle = \langle q^2\rangle$, $\langle F\rangle = \langle pq+qp\rangle$, and $\langle D\rangle = \langle p^2\rangle$. For obtaining the steady-state solution, we make the ansatz

$$\langle x\rangle = x_0 + x_+\mathrm{e}^{-\mathrm{i}\delta t} + x_-\mathrm{e}^{\mathrm{i}\delta t}, \qquad (8)$$

where x can be any one of the quantifies c, A, B, E, D, F. By substituting Eq. (8) into Eqs. (6) and (7), and letting the time derivatives be equal to zero, we can obtain

$$\begin{aligned}
D_0 &= \frac{(1+2n)m\hbar\omega_{\mathrm{m}}}{2}, \quad E_0 = \frac{D_0}{m^2\omega_{\mathrm{m}}^2(1+2\alpha)},\\
F_0 &= 0, \quad c_0 = \frac{\varepsilon_{\mathrm{c}}}{\kappa + \mathrm{i}\Delta + \dfrac{G_{\mathrm{A}}^2(\gamma_2 + \mathrm{i}\omega_{21})}{(\gamma_1 + \mathrm{i}\Delta)(\gamma_2 + \mathrm{i}\omega_{21}) + \Omega^2}},\\
c_+ &= \frac{S_1WS_3 - 4\mathrm{i}\omega_{\mathrm{m}}^3\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)}{S_1S_2WS_3 + 4\mathrm{i}\omega_{\mathrm{m}}^2\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)X}\varepsilon_{\mathrm{P}}, \qquad (9)
\end{aligned}$$

where

$$\begin{aligned}
\alpha &= \frac{\hbar g_0|c_0|^2}{m\omega_{\mathrm{m}}^2}, \quad \beta = \frac{g_0E_0}{\omega_{\mathrm{m}}}, \quad \Delta = \Delta_0 + \beta\omega_{\mathrm{m}},\\
L &= \frac{[\gamma_2 - \mathrm{i}(\omega_{21} + \delta)]}{[\gamma_1 - \mathrm{i}(\Delta_1 + \delta)][\gamma_2 - \mathrm{i}(\omega_{21} + \delta)] + \Omega^2},\\
M &= \frac{[\gamma_2 + \mathrm{i}(\omega_{21} - \delta)]}{[\gamma_1 + \mathrm{i}(\Delta_1 - \delta)][\gamma_2 + \mathrm{i}(\omega_{21} - \delta)] + \Omega^2},\\
S_1 &= \kappa - \mathrm{i}(\Delta + \delta) + G_{\mathrm{A}}^2L,\\
S_2 &= \kappa + \mathrm{i}(\Delta - \delta) + G_{\mathrm{A}}^2M, \quad S_3 = (\gamma_{\mathrm{m}} - \mathrm{i}\delta),\\
W &= \delta^2 - 4\omega_{\mathrm{m}}^2 - 8\omega_{\mathrm{m}}^2\alpha + 2\mathrm{i}\gamma_{\mathrm{m}}\delta,\\
X &= \left[-2\mathrm{i}\Delta + G_{\mathrm{A}}^2(L-M)\right], \qquad (10)
\end{aligned}$$

in which α is the ratio of the radiation pressure energy to the potential energy of the mechanical oscillator and Δ is the effective cavity detuning.[43] Using the input–output relation $c_{\mathrm{out}}(t) = \varepsilon_{\mathrm{c}} + \varepsilon_{\mathrm{p}}\mathrm{e}^{-\mathrm{i}\delta t} - 2\kappa c(t)$,[21,38] we obtain

$$\begin{aligned}
c_{\mathrm{out}}(t) &= c_{\mathrm{out}_0} + c_{\mathrm{out}_+}\mathrm{e}^{-\mathrm{i}\delta t} + c_{\mathrm{out}_-}\mathrm{e}^{\mathrm{i}\delta t}\\
&= (\varepsilon_{\mathrm{c}} - 2\kappa c_0) + (\varepsilon_{\mathrm{p}} - 2\kappa c_+)\mathrm{e}^{-\mathrm{i}\delta t} - 2\kappa c_-\mathrm{e}^{\mathrm{i}\delta t}. \qquad (11)
\end{aligned}$$

It is the second term on the right-hand side that corresponds to the response of the system to the weak probe field at frequency ω_{p}. The transmission of the probe field, which is the ratio of the returned probe field from the coupling system divided by the input probe field, can be written as

$$\begin{aligned}
t_{\mathrm{p}}(\omega_{\mathrm{p}}) &= \frac{\varepsilon_{\mathrm{p}} - 2\kappa c_+}{\varepsilon_{\mathrm{p}}}\\
&= 1 - 2\kappa\frac{S_1WS_3 - 4\mathrm{i}\omega_{\mathrm{m}}^3\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)}{S_1S_2WS_3 + 4\mathrm{i}\omega_{\mathrm{m}}^2\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)X}. \qquad (12)
\end{aligned}$$

In order to obtain the accurate transmission, we introduce the normalized transmission[44] $T_{\mathrm{p}}(\omega_{\mathrm{p}}) = (t_{\mathrm{p}} - t_{\mathrm{r}})/(1 - t_{\mathrm{r}})$, where t_{r} is the resonance transmission in the absence of the coupling laser $t_{\mathrm{r}} = t_{\mathrm{p}}(\delta = 2\omega_{\mathrm{m}}, \varepsilon_{\mathrm{c}} = 0)$. The optomechanically induced transparency is then described by the normalized transmission coefficient

$$|T_{\mathrm{p}}|^2 = \left|1 - \kappa\frac{S_1(\gamma_{\mathrm{m}} - \mathrm{i}\delta)W - 4\mathrm{i}\omega_{\mathrm{m}}^3\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)}{S_1S_2W(\gamma_{\mathrm{m}} - \mathrm{i}\delta) + 4\mathrm{i}\omega_{\mathrm{m}}^2\alpha\beta(2\gamma_{\mathrm{m}} - \mathrm{i}\delta)X}\right|^2. \qquad (13)$$

What is more, in the resonant region of the transparency window, the probe field has a phase dispersion $\phi_{\mathrm{t}}(\omega_{\mathrm{p}}) = \arg[T_{\mathrm{p}}(\omega_{\mathrm{p}})]$, which can cause the transmission group delay $\tau_{\mathrm{g}} = \mathrm{d}\phi_{\mathrm{t}}(\omega_{\mathrm{p}})/\mathrm{d}\omega_{\mathrm{p}} = \mathrm{d}\{\arg[T_{\mathrm{p}}(\omega_{\mathrm{p}})]\}/\mathrm{d}\omega_{\mathrm{p}}$. The group delay $\tau_{\mathrm{g}} > 0$ ($\tau_{\mathrm{g}} < 0$) corresponds to the slow (fast) light propagation of the probe field. In the following two sections, we will discuss the optomechanically induced transparency and the fast–slow light switching, respectively.

3. Adjustable multi-window transparency

In this section, we first explain why there are multiple transparency windows, and then discuss how to control the width of the transparency windows. The parameters used in our numerical simulation are taken from Refs. [23], [43], and [45]. The wavelength of the laser $\lambda_{\mathrm{c}} = 2\pi c/\omega_{\mathrm{c}} = 532$ nm, the cavity length $L = 6.7$ cm, the cavity decay rate $\kappa = 2\pi\times10^3$ Hz, the frequency of the moving membrane $\omega_{\mathrm{m}} = 2\pi\times10^4$ Hz, the decay rate of the membrane $\gamma_{\mathrm{m}} = 20$ Hz, the mechanical quality factor $Q = \omega_{\mathrm{m}}/\gamma_{\mathrm{m}} = 3.14\times10^3$, the membrane's reflectivity $R = 0.45$, the effective coupling strength

$G_A = \pi \times 10^4$ Hz, the Rabi frequency $\Omega = \pi \times 10^4$ Hz, the decay rate of the atomic transition $\gamma_1 = \gamma_2 = \pi \times 100$ Hz, and the mass of the oscillating membrane is $m = 2 \times 10^{-11}$ g. Considering the two-phonon process (an intracavity phonon needs absorption of two phonons to be converted into an anti-Stokes photon) in the QC-OMS,[23] we take the detunings $\Delta_c = 2\omega_m$ and $\Delta_1 = \omega_{21} = 2\omega_m$. We also consider the sideband-resolved situation, i.e., $\omega_m \gg \kappa$.

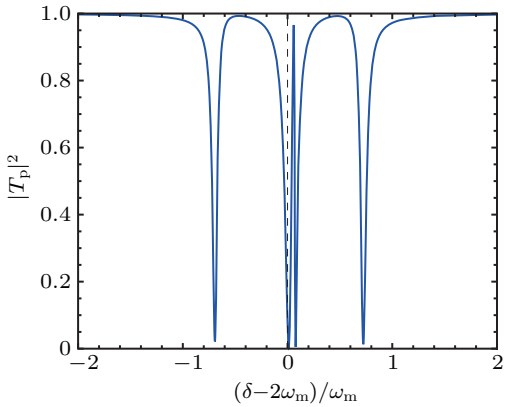


Fig. 2. (color online) The transmission coefficient is plotted as a function of $(\delta - 2\omega_m)/\omega_m$, which shows multiple transparency windows. The corresponding parameters are $G_A = \pi \times 10^4$ Hz, $\Omega = \pi \times 10^4$ Hz, $P_c = 10$ μW, $T = 20$ K, $\gamma_1 = \gamma_2 = 100\pi$ Hz, and $\kappa = 2\pi \times 10^3$ Hz.

In Fig. 2, we show the transmission coefficient as a function of $(\delta - 2\omega_m)/\omega_m$. It can be seen that there are three OMIT windows, the middle one is narrower, and the two side ones are wider. It should be pointed out that there is no atomic ensemble and the laser driving it in Ref. [23], and there is only one OMIT window about $\delta = 2\omega_m$. In our system, we introduce the atomic ensemble and the laser driving it, and this leads to the appearing of the two additional OMIT windows in our system.

3.1. The reason for multi-window transparency

Now we explain the reason of the multi-window transparency by changing the systemic models and parameters. Figure 3(a) corresponds to the case in which we only keep the coupling field and the probe field to the cavity. In this case, there is one OMIT window near $\delta = 2\omega_m$, this result is the same as that in Ref. [23]. We know that there is a two-phonon process involved in the QC-OMS, and the nonlinear coherence effects between the probe field and the anti-Stokes field generated by the quadratic coupling lead to the OMIT window about $\delta = 2\omega_m$. Figure 3(b) corresponds to the situation in which we turn off the coupling field driving the cavity but introduce the driven atomic ensemble. In this situation, two OMIT windows occur at the symmetrical places. Due to $\Delta_0 = \Delta_1 = \omega_{21} = 2\omega_m$ at the resonances, the single transparency window splits into two transparency windows. The process can be treated as a single-phonon up-conversion,[26] in which an intracavity phonon can convert to an anti-Stokes photon via the absorption of a phonon. In the presence of both the coupling field to the cavity and the driven atomic ensemble, we obtain Fig. 2, in which there are three OMIT windows. By comparing these figures, we can conclude that the middle OMIT window is due to the presence of the coupling field driving the cavity, and the other two windows are due to the driven atomic ensemble.

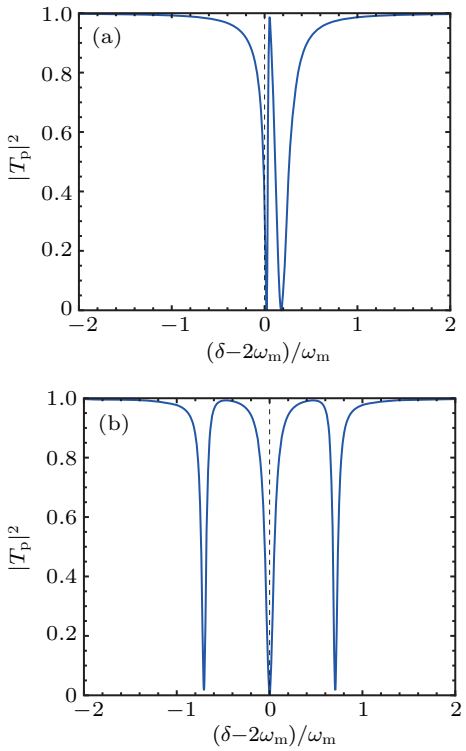


Fig. 3. (color online) The transmission coefficient of the probe field as a function of $(\delta - 2\omega_m)/\omega_m$: (a) $\varepsilon_c \neq 0$, $G_A = 0$, and $\Omega = 0$; (b) $\varepsilon_c = 0$, $G_A \neq 0$, and $\Omega \neq 0$. The other parameters are the same as those in Fig. 2.

3.2. Adjusting the width of the OMIT windows

Next we show that the width of the OMIT windows can be adjusted by controlling the systemic parameters, for example, the number of atoms, the power of the laser fields driving the atoms and driving the cavity, and the environment temperature.

3.2.1. Effects of the number of atoms

We now discuss the influence of the number of atoms in the atomic ensemble on the width of the OMIT windows. In Figs. 4(a) and 4(b), the different curves correspond to different numbers of atoms. From Fig. 4(a), we can see clearly that the width of the two side windows increases with the increase of the number of atoms. The reason for this is that the effective coupling strength between the cavity field and the atomic ensemble $G_A = \sqrt{N}g$ increases with increasing the number of atoms and this leads to broader OMIT windows. So we can adjust the width of the OMIT windows by changing the number of atoms. In Fig. 4(b), we enlarge the central transparency

window of Fig. 4(a). It can be seen that we can obtain a higher transmission coefficient at $\delta \approx 2.06\omega_{\rm m}$ for a larger number of atoms. This result is similar to that in Ref. [29]. However, in Ref. [25], a two-level atomic ensemble was used in the QC-OMS, while in our system, a three-level atomic ensemble was used in the QC-OMS. From Ref. [29], we know that the fluctuation in the displacement increases with the increase of the atomic number. The fluctuation in the displacement plays the role of the atomic coherence in the QC-OMS.[23] The fact that the central peak increases with the number of atoms shows that the increase of the number of three-level atoms can increase the fluctuation in the displacement and benefit the two-phonon process in the QC-OMS.

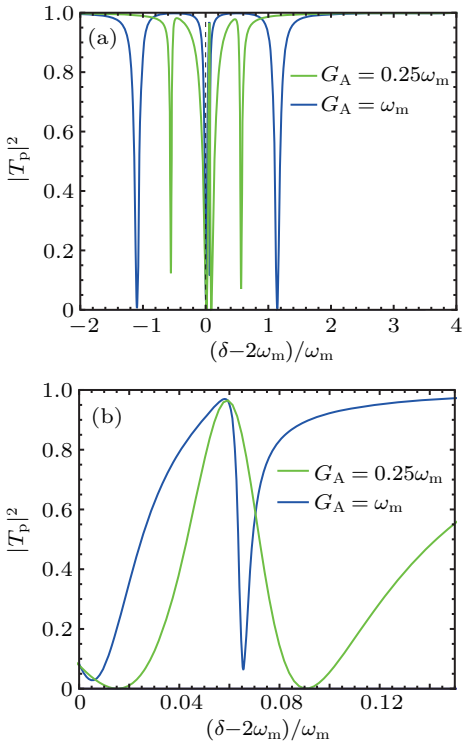


Fig. 4. (color online) The transmission coefficient of the probe field as a function of $(\delta - 2\omega_{\rm m})/\omega_{\rm m}$. (a) For different numbers of three-level atoms. (b) Enlargement of the central OMIT window in panel (a). The other parameters are the same as those in Fig. 2.

3.2.2. Effects of the power of the laser driving the atoms

Figure 5(a) shows that the width of the two side OMIT windows increases with the increase of the power of the laser driving the atoms. This is due to the increase of the power of the laser driving the atomic ensemble that can increase the mean photon number of the cavity field and the width of the OMIT windows.[44] Thus we can adjust the power of the laser driving the atoms to control the two side OMIT windows. From Fig. 5(b), we can see that the central OMIT window for a smaller $\Omega = 0.25\omega_{\rm m}$ has a higher transmission coefficient than that for a larger $\Omega = 1.25\omega_{\rm m}$ at $\delta \approx 2.06\omega_{\rm m}$. The physical reason can be understood as follows. When the environment temperature is fixed, the mean phonon number $n_{\rm th} = [\exp(\hbar\omega_{\rm m}/k_{\rm B}T) - 1]^{-1}$ is also fixed, therefore, with increasing the power of the laser driving the atomic ensemble, the single-phonon up-conversion process is enhanced, and then the two-phonon process is reduced. So the central OMIT window has a lower transmission coefficient with the increase of the power of the laser driving the atoms.

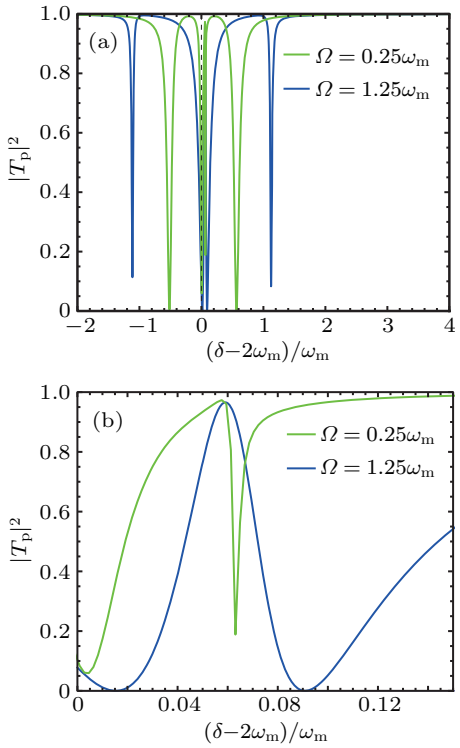


Fig. 5. (color online) The transmission coefficient of the probe field as a function of $(\delta - 2\omega_{\rm m})/\omega_{\rm m}$. (a) For different driving powers on the atoms. (b) Enlargement of the central OMIT window in panel (a). The other parameters are the same as those in Fig. 2.

3.2.3. Effects of the power of the laser driving the cavity

Figure 6(a) shows that the change of the power of the laser driving the cavity (the coupling field) almost does not influence the two side OMIT windows. This is consistent with Fig. 3, where we have pointed out that the appearance of the two side OMIT windows is mainly due to the driven atoms. In Fig. 6(b), the width of the central window becomes wider with the increase of the power of the pump field. This is due to that the ratio of the radiation pressure energy to the potential energy of the mechanical oscillator $\alpha = \hbar g_0 |c_0|^2 / m\omega_{\rm m}^2$ [37] increases with the increase of the power of the pump field, i.e., the radiation pressure enhances with the increase of the power of the pump field. Besides, the central peak appears at a larger detuning. The reason is that the effective cavity detuning $\Delta = \Delta_0 + \beta\omega_{\rm m}$ increases when the power of coupling light increases.

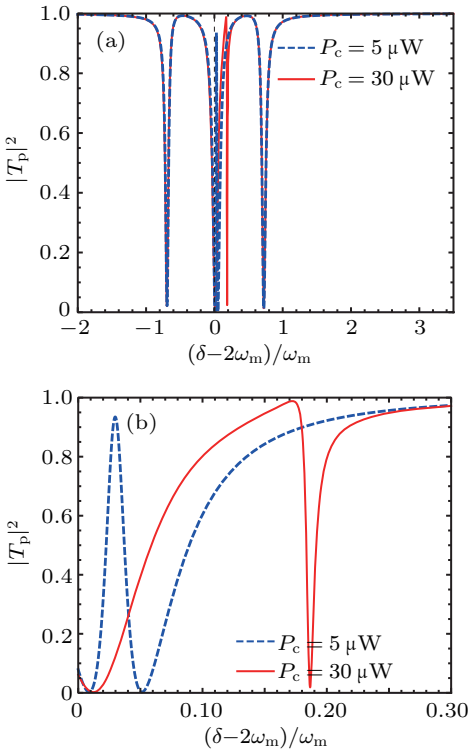


Fig. 6. (color online) The transmission coefficient of the probe field as a function of $(\delta-2\omega_m)/\omega_m$. (a) For different powers of the laser driving the cavity. (b) Enlargement of the central OMIT window in panel (a). The other parameters are the same as those in Fig. 2.

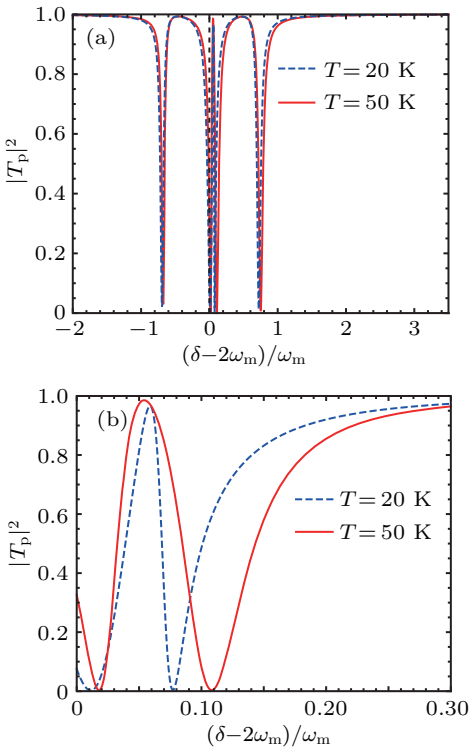


Fig. 7. (color online) The transmission coefficient of the probe field as a function of $(\delta-2\omega_m)/\omega_m$. (a) For different environment temperatures. (b) Enlargement of the central OMIT window in panel (a). The other parameters are the same as those in Fig. 2.

From Fig. 7(a), we can see clearly that the change of the environment temperature has almost no influence on the two side OMIT windows. The reason is that the environment temperature has no contribution to nonlinear coherence during the single-phonon up-conversion process.[26] Figure 7(b) shows that both the width and the peak value of the central OMIT window increase with the increase of the environment temperature. This is due to that, in the two-phonon process, the displacement fluctuation of the mechanical oscillator comes from the environment, and the increase of the environment temperature can increase the mean phonon-number and enhance the two-phonon process.

4. Fast light and slow light

In this section, we analyze the fast and slow light effects by using the group delay $\tau_g = d\{\arg[T_p(\omega_p)]\}/d\omega_p$. In Fig. 8, the group delay of the output field at the frequency of the probe field is plotted as a function $(\delta-2\omega_m)/\omega_m$. Figure 8(a) shows that one can obtain the fast light when the coupling field is absent ($P_c = 0$). This is due to the quantum interference effect between the probe field and the anti-Stokes field generated by three-level atoms in the absence of the coupling light. From Fig. 8(b), we can see clearly that a fast–slow light switching is obtained near $\delta \approx 2\omega_m$ when the coupling light is present ($P_c = 1\ \mu W$). So the fast–slow light switching results from the quantum interference effect between the coupling field and the anti-Stokes field generated by the quadratically coupled cavity. Figure 8(c) shows an enlarged part of Fig. 8(b) near $\delta \approx 2\omega_m$, the maximum and minimum valves of the group delay near $\delta = 2\omega_m$ are $\tau_{max} = 14\ \mu s$ and $\tau_{min} = -19\ \mu s$, respectively. When we increase the power of the pump laser, the position of the extreme valves appear at a larger detuning, and the abstract values of the extreme valves become larger. Figure 8(d) is an example, in which $P_c = 10\ \mu W$, the extreme valves appear near $\delta = 2.01\omega_m$, and the maximum and minimum valves of the group delay are $\tau_{max} = 50\ \mu s$ and $\tau_{min} = -95\ \mu s$, respectively. So we can control the fast–slow light switching by changing the power of the coupling field.

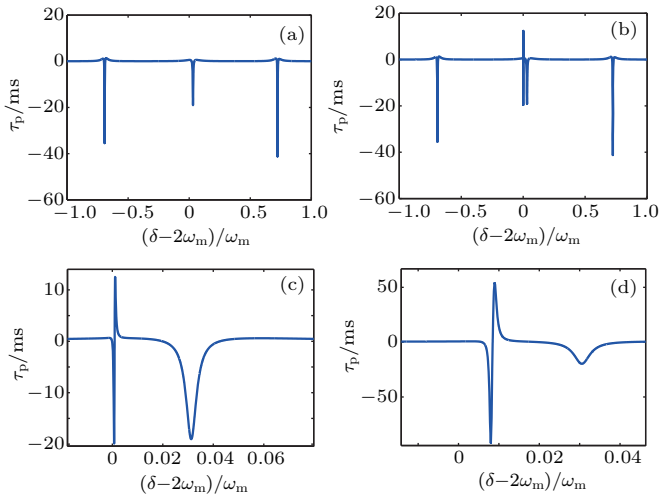


Fig. 8. (color online) The group delay as a function of $(\delta-2\omega_m)/\omega_m$: (a) $P_c = 0$, (b) $P_c = 1\ \mu W$, (c) enlargement of panel (b) near $\delta \approx 2\omega_m$, (d) $P_c = 10\ \mu W$, enlargement near $\delta \approx 2\omega_m$. The other parameters are the same as those in Fig. 2.

5. Conclusion

We have studied a quadratically coupled optomechanical system assisted with a three-level atomic ensemble. We have examined the optical properties of the output field with experimentally accessible parameters, and found that a phenomenon with three OMIT windows can appear in this system. Our analysis shows that the middle OMIT window mainly comes from the quadratic coupling, while the two side OMIT windows are due to the driven atomic ensemble. We have analyzed the effects of some systemic parameters on the width of the OMIT windows. These parameters include the number of atoms in the atomic ensemble, the powers of the lasers driving the atoms and driving the cavity, and the environment temperature. Our analysis shows that the width of the OMIT windows can be adjusted by controlling these systemic parameters. We have also studied the fast and slow light effects in this system, and found that one can achieve a fast–slow light switching in our model. We have also shown that the fast–slow light switching can be adjusted by the power of the laser driving the cavity.

References

[1] Kleckner D and Bouwmeester D 2006 *Nature* **444** 75
[2] Huang S and Agarwal G S 2009 *New J. Phys* **11** 103044
[3] Palomaki T, Teufel J, Simmonds R and Lehnert K 2013 *Science* **342** 710
[4] Zhou L, Han Y, Jing J T and Zhang W P 2011 *Phys. Rev. A* **83** 052117
[5] Liao J Q, Wu Q Q and Nori F 2014 *Phys. Rev. A* **89** 014302
[6] Xia W Q, Yu Y F and Zhang Z M 2017 *Chin. Phys. B* **26** 054210
[7] Zou B C and Zhu Y F 2013 *Phys. Rev. A* **87** 053802
[8] Agarwal G S and Huang S 2010 *Phys. Rev. A* **81** 041803
[9] Wang W X, Zhang Y H, Zhang J P and Zhu Y F 2011 *Phys. Rev. A* **84** 045806
[10] Safavi-Naeini A H, Alegre T P M, Chan J, Eichenfield M, Winger M, Lin Q, Hill J T, Chang D E and Painter O 2011 *Nature* **472** 69
[11] Han Y, Cheng J and Zhou L 2011 *J. Mod. Opt.* **59** 1336
[12] Stadler P, Belzig W and Rastelli G 2016 *Phys. Rev. Lett.* **117** 197202
[13] Sarma B and Sarma A K 2016 *Phys. Rev. A* **93** 033845
[14] Chan J, Alegre T P, Safavinaeini A H, Hill J T, Krause A, Aspelmeyer M and Painter O 2011 *Nature* **478** 89
[15] Xu X X and Yuan H C 2012 *Acta Phys. Sin.* **61** 064205 (in Chinese)
[16] WollmanE E, Lei C U, Weinstein A J, Suh J, Kronwald A and Marquardt 2015 *Science* **349** 952
[17] Cai Q H, Xiao Y, Yu Y F and Zhang Z M 2016 *Opt. Express* **24** 20036
[18] Yu L X, Li C F, Fan J T, Chen G, Zhang T C and Jia S T 2016 *Chin. Phys. B* **25** 050301
[19] Purdy T P, Yu P L, Peterson R W, Kampel N S and Regal C A 2013 *Phys. Rev. X* **3** 031012
[20] Liu G Y and Malinovskaya S A 2015 *J. Phys. B: At. Mol. Opt.* **48** 065502
[21] Wang Q, He Z, Yao C M and Li L 2016 *Chin. Phys. B* **25** 080304
[22] Bhattacharya M, Uys H and Meystre P 2008 *Phys. Rev. A* **77** 033819
[23] Agarwal G S and Huang S 2011 *Phys. Rev. A* **83** 023823
[24] Karuza M, Biancofiore C, Bawaj M, Molineli C, Galassi M, Natali R, Tombesi P, Giuseppe G D and Vitali D 2013 *Phys. Rev. A* **88** 013804
[25] Huang Y X, Zhou X F, Guo G C and Zhang Y S 2015 *Phys. Rev. A* **92** 013829
[26] Zhan X. G, Si L G, Zheng A S and Yang X X 2013 *J. Phys. B: At. Mol. Opt. Phys.* **46** 025501
[27] Sankey J C, Yang C, Zwickl B M, Jayich A M and Harris J G E 2010 *Nat. Phys.* **6** 707
[28] Thompson J D, Zwickl B M, Jayich A M, Marquardt F ,Girvin S M and H arris J E H 2008 *Nature* **452** 72
[29] Han Y, Cheng J and Zhou L 2012 *Journal of Modern Optics* **59** 1
[30] Gu W J, Yi Z, Sun L H and Xu D H 2015 *Phys. Rev. A* **92** 023811
[31] Chauhan A K and Biswas A 2016 *Phys. Rev. A* **94** 023831
[32] Agarwal G S, Di K, Wang L Y and Zhu Y F 2016 *Phys. Rev. A* **93** 063805
[33] Huang Y X, Zhou X F, Guo G C and Zhang Y S 2015 *Phys. Rev. A* **92** 013829
[34] Yang G Q, Gu W J, Li G X, Zou B C and Zhu Y F 2015 *Phys. Rev. A* **92** 033822
[35] Fu C B, Gu K H, Yan X B, Yang X, Cui C L and Wu J H 2013 *Phys. Rev. A* **87** 053841
[36] Wei X G, Wang Y H, Zhang J P and Zhu Y F 2011 *Phys. Rev. A* **84** 045806
[37] Li L, Nie W J and Chen A X 2016 *Sci. Rep.* **6** 35090
[38] Li Y and Sun C P 2004 *Phys. Rev. A* **69** 051802
[39] Sawant R and Rangwala S A 2016 *Phys. Rev. A* **93** 023806
[40] Chang Y, Shi T, Liu Y X, Sun C P and Nori F 2011 *Phys. Rev. A* **83** 063826
[41] Akram M J, Ghafoor F and Saif F *arxiv* 1512.07297
[42] Gardiner C W and Zoller P 2004 *Quantum Noise* (Berlin: Springer) p. 152
[43] Si L G, Hao X, Zubairy M S and Wu Y 2017 *Phys. Rev. A* **95** 033803
[44] Xiao Y, Yu Y F and Zhang Z M 2014 *Opt. Express* **22** 17979
[45] Thompson J D, Zwickl B M, Jayich A M, Marquardt F, Girvin S M and Harris 2008 *Nature* **452** 72