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Effects of intrinsic and extrinsic noises on transposons kinetics*

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The absolute concentration robustness (ACR) steady state of a biochemical system can protect against changing a large concentration of the system's components. In this paper, a minimal model of autonomous–nonautonomous transposons driven by intrinsic and extrinsic noises is investigated. The effects of intrinsic and extrinsic noises on ACR steady state of the transposons kinetics are studied by numerical simulations. It is found that the predator-prey-like oscillations around the ACR steady state are induced by the intrinsic or extrinsic noises. Comparing with the case of intrinsic noises, the extrinsic noises can inhibit the amplitude of oscillations of transposon kinetics. To characterize the predator-prey-like oscillations, we calculate the probability distributions and the normalized correlation functions of a system in the stability domain. With the increasing of noise intensity, the peak of the probability distribution is shifted from the ACR steady state to the trivial steady state. The normalized autocorrelation and cross-correlation functions indicate that the state of the predator–prey oscillator is transmitted to 50 successive generations at least.

Keywords: stability, noises, oscillation, transposon kinetics

PACS: 05.45.-a, 87.17.Aa, 87.18.Vf **DOI:** 10.1088/1674-1056/27/3/030501

1. Introduction

Nonlinear phenomenon is ubiquitous in various biological systems,[1–6] existing over multiple scales of biological systems from population level to molecular level, which has been widely studied in the context of coupled limit cycle oscillator systems, for instance, the population systems (such as the predator–prey model[7–10]), the excitable cells (such as the neuron model[11–17]), and the intracellular signaling systems (such as the calcium oscillation model[18–21]).

The transposable elements (TEs) are the DNA sequences which can change its position within a genome. In a eukaryotic cell, the TEs play a critical role in genome function and evolution. The edit operation of TEs on the host genome is separated into two types:[22] one is the "copy and paste", and the other is the "cut and paste". Some transposons (such as the autonomous long interspersed nuclear elements, LINEs) encode the enzymes which perform their excision, while others are parasitic (such as the nonautonomous short interspersed nuclear elements, SINEs), relying on the enzymes produced by the regular TEs.

Some mathematical models have been proposed to study the kinetics of transposons. In order to describe the equilibrium distribution of transposons in a population, for instance, Le Rouzic *et al.*[23,24] developed the population genetics models, and a mean-field predator–prey type model was proposed to describe the interactions between nonautonomous transposons and autonomous ones.[25,26] More recently, a minimal model of interactions between LINEs and SINEs was proposed by Xue and Goldenfeld,[27] and it was found that the internal fluctuation governed by the size of the system can induce predator–prey oscillations with a characteristic time scale which is much longer than the cell replication time, and the state of the predator–prey oscillator is stored in the genome and transmitted to successive generations.

On the one hand, the intracellular biochemical interactions occur far from thermodynamic equilibrium and in complicated environments, and there are various intrinsic and extrinsic noises in channels of biochemical interaction networks. In gene networks, the intrinsic noise originates from the random births and deaths of individual molecules, and the extrinsic noise comes from the fluctuations in reaction rates.[28] Noise can modify the copy number or the concentrations of molecular species, and the variations of concentrations in turn propagate along networks of chemical reactions. On the other hand, although the variations in the concentrations of biomolecular species are inevitable, the biological systems require robustness, that is, the capacity for sustained and precise function even in the presence of structural or environmental disruption. Shinar and Feinberg[29] demonstrated that there is an absolute concentration robustness (ACR) for an active molecular species if the concentration of that species is identical in every positive steady state, and the steady state is called an ACR steady state. The ACR steady state of a biochemical system can protect against a large change of the system's components.

Interesting questions now arise. How do the intrinsic and extrinsic noises affect the ACR steady state of the transposon system? What are the effects of these noises on the transposon kinetics? In this paper, based on the model of autonomous–

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nonautonomous transposons, the intrinsic and extrinsic noises are respectively introduced in the model, and the effects of these noises on the system are investigated by using numerical simulations. Compared with the stochastic model of Ref. [27] in which the internal noises come from the small copy numbers of the active transposons in a cell, our stochastic transposon kinetic model is driven by the intrinsic noises which originate from the random births and deaths of individual molecules, and by the extrinsic noises which come from the fluctuations in reaction rates, respectively.

In this paper, the stability of steady states of a minimal model is analyzed by using the Routh–Hurwitz conditions, and then the stochastic transposon kinetic models driven by intrinsic and extrinsic noises are respectively proposed. The effects of intrinsic and extrinsic noises on transposon kinetics are studied. In the case of stability domains of steady state, the probability distribution and power spectrum of stochastic transposon models are calculated, and the normalized autocorrelation and cross-correlation functions are discussed.

2. Mathematical model of transposon kinetics

The scheme of biochemical interactions between LINEs and SINEs is described as[27] L$\rightarrow$L+R$_{\rm L}$, R$_{\rm L}$ $\rightarrow$L, S+R$_{\rm L}$ $\rightarrow$2S, L$\rightarrow\phi$, R$_{\rm L}$ $\rightarrow\phi$, and S$\rightarrow\phi$, where L, S, and R$_{\rm L}$ represent the active LINEs, SINEs, and ribosome/L-mRNA/protein complexes. According to the law of mass action of biochemical reactions, three variables in the basic scheme of transposon kinetics are governed by the ordinary differential equations

$$\begin{cases} \dfrac{{\rm d}L}{{\rm d}t} = b_{\rm L}R_{\rm L} - d_{\rm L}L, \\ \dfrac{{\rm d}S}{{\rm d}t} = b_{\rm S}R_{\rm L}S - d_{\rm S}S, \\ \dfrac{{\rm d}R_{\rm L}}{{\rm d}t} = b_{\rm R}L - b_{\rm L}R_{\rm L} - d_{\rm R}R_{\rm L} - b_{\rm S}R_{\rm L}S, \end{cases} \tag{1}$$

where L, S, and $R_{\rm L}$ represent the copy numbers or the concentrations of active LINEs, SINEs, and ribosome/L-mRNA/protein complexes, respectively. The parameters $b_{\rm L}$, $b_{\rm S}$, and $b_{\rm R}$ are the activation rate constants of three molecular species, and $d_{\rm S}$, $d_{\rm L}$, and $d_{\rm R}$ are the degradation rate constants. The time is in the unit of a cell generation. It should be pointed out that all the parameters in Eq. (1) must be positive in the real biological system.

Under the condition of L, S, $R_{\rm L} \geq 0$, there are two steady states (L^*, S^*, $R_{\rm L}^*$) for the biochemical reaction system (1):

$$L^* = 0, \quad S^* = 0, \quad R_{\rm L}^* = 0, \tag{2}$$

and

$$L^* = \frac{b_{\rm L}d_{\rm S}}{d_{\rm L}b_{\rm S}}, \quad S^* = \frac{b_{\rm L}b_{\rm R}}{d_{\rm L}b_{\rm S}} - \frac{b_{\rm L}}{b_{\rm S}} - \frac{d_{\rm R}}{b_{\rm S}}, \quad R_{\rm L}^* = \frac{d_{\rm S}}{b_{\rm S}}. \tag{3}$$

The first steady state (2) is a trivial solution since the copy number or the concentrations L, S, and $R_{\rm L}$ are not zero in a real cell. The second steady state (3) is ACR steady state[29] for the three molecular species. It can be seen that the second steady state is determined by the rate constants (i.e., the structural attributes) of biochemical reaction systems, and independent of the copy number or the concentration of any species. In a real biological system, the function of an ACR possessing system is protected even against large changes in the overall supply of the system's components.[29,30] The stability of the steady states is determined by the community matrix

$$\boldsymbol{A} = \begin{bmatrix} -d_{\rm L} & 0 & b_{\rm L} \\ 0 & b_{\rm S}R_{\rm L}^* - d_{\rm S} & b_{\rm S}S^* \\ b_{\rm R} & -b_{\rm S}R_{\rm L}^* & -b_{\rm L} - d_{\rm R} - b_{\rm S}S^* \end{bmatrix}. \tag{4}$$

For the first steady state (2), the eigenvalue λ of $\boldsymbol{A}$ are given by

$$\begin{aligned} |\boldsymbol{A} - \lambda \boldsymbol{I}| &= \begin{vmatrix} -d_{\rm L} - \lambda & 0 & b_{\rm L} \\ 0 & -d_{\rm S} - \lambda & 0 \\ b_{\rm R} & 0 & -b_{\rm L} - d_{\rm R} - \lambda \end{vmatrix} \\ &= 0 \Rightarrow \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \end{aligned}$$

where

$$\begin{aligned} a_1 &= b_{\rm L} + d_{\rm R} + d_{\rm L} + d_{\rm S}, \\ a_2 &= (d_{\rm L} + d_{\rm S})(b_{\rm L} + d_{\rm R}) + d_{\rm L}d_{\rm S} - b_{\rm L}b_{\rm R}, \\ a_3 &= d_{\rm L}d_{\rm S}(b_{\rm L} + d_{\rm R}) - b_{\rm L}b_{\rm R}d_{\rm S}. \end{aligned}$$

The first steady state is stable when ${\rm Re}\,\lambda < 0$, i.e., the conditions are

$$a_1 > 0, \quad a_3 > 0, \quad a_1a_2 - a_3 > 0. \tag{5}$$

For the second steady state (3), the eigenvalue λ of $\boldsymbol{A}$ are given by

$$\begin{aligned} |\boldsymbol{A} - \lambda \boldsymbol{I}| &= \begin{vmatrix} -d_{\rm L} - \lambda & 0 & b_{\rm L} \\ 0 & -\lambda & b_{\rm R}b_{\rm L}/d_{\rm L} - b_{\rm L} - d_{\rm R} \\ b_{\rm R} & d_{\rm S} & -b_{\rm R}b_{\rm L}/d_{\rm L} - \lambda \end{vmatrix} \\ &= 0 \Rightarrow \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \end{aligned}$$

where

$$\begin{aligned} &a_1 = \frac{b_{\rm R}b_{\rm L}}{d_{\rm L}} + d_{\rm L}, \quad a_2 = \frac{b_{\rm R}b_{\rm L}d_{\rm S}}{d_{\rm L}} - d_{\rm S}(b_{\rm L} + d_{\rm R}), \\ &a_3 = b_{\rm R}b_{\rm L}d_{\rm S} - d_{\rm L}d_{\rm S}b_{\rm L} - d_{\rm L}d_{\rm S}d_{\rm R}. \end{aligned}$$

The second steady state is stable when ${\rm Re}\,\lambda < 0$, so the steady state

$$L^* = \frac{b_{\rm L}d_{\rm S}}{d_{\rm L}b_{\rm S}}, \quad S^* = \frac{b_{\rm L}b_{\rm R}}{d_{\rm L}b_{\rm S}} - \frac{b_{\rm L}}{b_{\rm S}} - \frac{d_{\rm R}}{b_{\rm S}}, \quad R_{\rm L}^* = \frac{d_{\rm S}}{b_{\rm S}}$$

is

$$\begin{cases} \text{stable}, \\ \text{unstable}, \end{cases} \quad \text{if} \quad \begin{cases} b_{\rm R}b_{\rm L} > d_{\rm L}b_{\rm L} + d_{\rm L}d_{\rm R}, \\ b_{\rm R}b_{\rm L} < d_{\rm L}b_{\rm L} + d_{\rm L}d_{\rm R}. \end{cases} \tag{6}$$

It is interesting that the conditions (6) of stability for the second steady state do not depend on the reaction rates ($b_{\rm S}$ and $d_{\rm S}$) of nonautonomous transposons.

2.1. The model with intrinsic noises

The intrinsic noise can originate from the random births and deaths of individual molecules in the interaction mechanism between a pair of autonomous–nonautonomous transposons. Thus, the intrinsic noises are considered as additive fluctuations in the model (1). Then, the three variables of transposon kinetics are governed by the stochastic differential equations

$$\begin{cases} \dfrac{\mathrm{d}L}{\mathrm{d}t} = b_{\mathrm{L}}R_{\mathrm{L}} - d_{\mathrm{L}}L + \xi_1(t), \\ \dfrac{\mathrm{d}S}{\mathrm{d}t} = b_{\mathrm{S}}R_{\mathrm{L}}S - d_{\mathrm{S}}S + \xi_2(t), \\ \dfrac{\mathrm{d}R_{\mathrm{L}}}{\mathrm{d}t} = b_{\mathrm{R}}L - b_{\mathrm{L}}R_{\mathrm{L}} - d_{\mathrm{R}}R_{\mathrm{L}} - b_{\mathrm{S}}R_{\mathrm{L}}S + \xi_3(t), \end{cases} \tag{7}$$

where $\xi_i(t)$ $(i = 1,2,3)$ are the Gaussian white noises with zero mean, and the auto-correlation functions are

$$\begin{cases} \langle \xi_1(t)\xi_1(s)\rangle = 2D_1\delta(t-s), \\ \langle \xi_2(t)\xi_2(s)\rangle = 2D_2\delta(t-s), \\ \langle \xi_3(t)\xi_3(s)\rangle = 2D_3\delta(t-s), \end{cases} \tag{8}$$

with the noise intensities D_i $(i = 1,2,3)$. Here it is assumed that the biochemical reaction system is homogenous and the noises have different origins, treated as independent random variables. Thus $D_1 = D_2 = D_3 = D$.

2.2. The model with extrinsic noises

The extrinsic noise can come from the fluctuations in reaction rates in the interaction mechanism between a pair of autonomous–nonautonomous transposons. Then, the extrinsic noises are expected as multiplicative fluctuations through the control parameter of the transposon model. We assumed that the extrinsic noises are introduced through the activation rate parameters of the biochemical reactions

$$b_{\mathrm{R}} \to b_{\mathrm{R}} + \xi_1(t), \quad b_{\mathrm{S}} \to b_{\mathrm{S}} + \xi_2(t), \quad b_{\mathrm{L}} \to b_{\mathrm{L}} + \xi_3(t).$$

Thus, the three variables of transposon kinetics are governed by the stochastic differential equations

$$\begin{cases} \dfrac{\mathrm{d}L}{\mathrm{d}t} = b_{\mathrm{L}}R_{\mathrm{L}} - d_{\mathrm{L}}L + R_{\mathrm{L}}\xi_1(t), \\ \dfrac{\mathrm{d}S}{\mathrm{d}t} = b_{\mathrm{S}}R_{\mathrm{L}}S - d_{\mathrm{S}}S + R_{\mathrm{L}}S\xi_2(t), \\ \dfrac{\mathrm{d}R_{\mathrm{L}}}{\mathrm{d}t} = b_{\mathrm{R}}L - b_{\mathrm{L}}R_{\mathrm{L}} - d_{\mathrm{R}}R_{\mathrm{L}} - b_{\mathrm{S}}R_{\mathrm{L}}S - R_{\mathrm{L}}\xi_1(t) \\ \qquad\qquad - R_{\mathrm{L}}S\xi_2(t) + L\xi_3(t), \end{cases} \tag{9}$$

where the Gaussian white noises $\xi_1(t)$, $\xi_2(t)$, and $\xi_3(t)$ have the same statistical properties given by Eq. (8), and the system is also assumed as being homogenous and no cross-correlation between noises.

3. Effects of intrinsic and extrinsic noises on transposon models

The effects of intrinsic and extrinsic noises on the transposons kinetics are studied by using numerical simulations, where the algorithm of stochastic differential equation (7) with additive noises is followed from Ref. [31], the algorithm of stochastic differential equation (9) with multiplicative noises is followed from Ref. [32], the time step is $\Delta t = 0.001$, and the parameters of transposon models are set as in Ref. [27]: $d_{\mathrm{R}} = 2.0$, $d_{\mathrm{L}} = 0.5$, $d_{\mathrm{S}} = 0.5$, and $b_{\mathrm{S}} = 1.0$. All the biochemical reaction rates are positive.

For the second steady state, figure 1 illustrates the stable and unstable domains separated by a bifurcation line in the $(b_{\mathrm{R}}, b_{\mathrm{L}})$ parameters space from Eq. (6). For example, the second steady state is stable at point B (2, 1.2), and unstable at point A (1, 1).

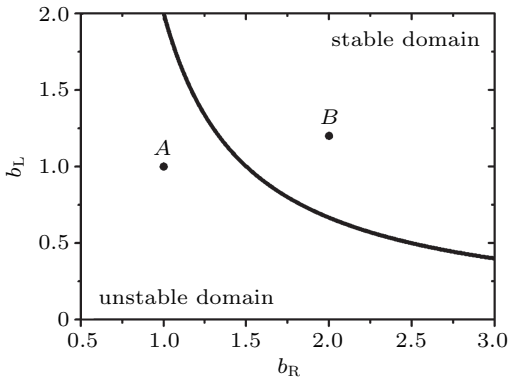


Fig. 1. Stability bifurcation curve for the transposon model in $(b_{\mathrm{R}}, b_{\mathrm{L}})$ parameters plane.

3.1. Additive noise-induced predator–prey oscillations in transposon kinetics

In the case of stability domains (e.g., at the point B in Fig. 1), the effects of systemic intrinsic noises on the transposon model are studied by using Eq. (7) with Eq. (8). Figure 2 shows that the additive noises can induce predator-prey-like oscillations around the second steady state. The time evolutions of $L(t)$ and $S(t)$ quickly tend to the second steady state under any initial perturbation of $L(0)$ and $S(0)$ in the absence of noises. However, the time evolutions of the solutions $L(t)$ and $S(t)$ are the predator-prey-like oscillations around the second steady state in the presence of noises.

The predator-prey-like oscillations induced by noises are those that copy numbers of active LINEs and SINEs exhibit quasi-periodic (or quasi-cycles) behavior. In a mechanical oscillator, it is well known that the driving frequency must be tuned to achieve resonance. In the absence of noises, the copy numbers of active LINEs and SINEs fail to quasi-predict cycles. However, in the presence of noise, the white noise covers all frequencies, and the resonant frequency of the autonomous–nonautonomous transposons system is excited

without tuning. Thus, in a stochastic kinetic model of active LINEs and SINEs no tuning is necessary. From a statistical physics viewpoint, the deterministic model of autonomous–nonautonomous transposons is a mean field theory, whereas the stochastic model discussed here includes statistical fluctuations.

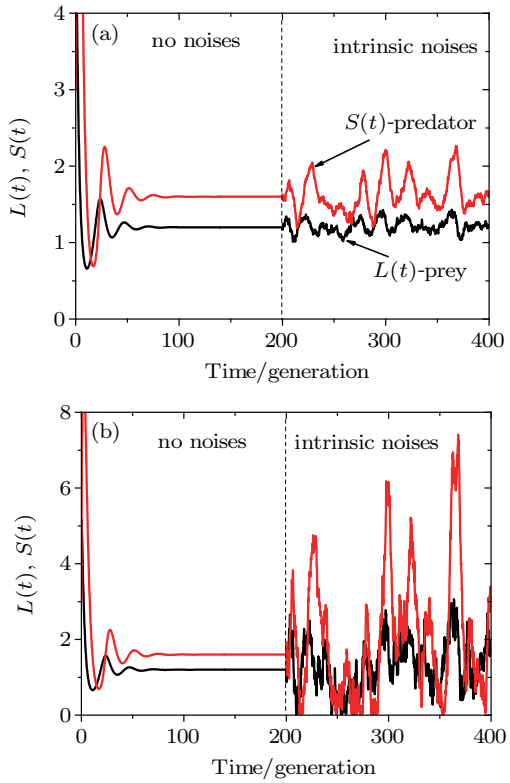


Fig. 2. (color online) Time evolution of the solutions $L(t)$ and $S(t)$ without and with additive noises in stability domains at point B in Fig. 1. (a) $D = 0.001$. (b) $D = 0.1$.

In the case of instability domains (e.g., at the point A in Fig. 1), the second steady state is instability, and the time evolutions of the solutions $L(t)$ and $S(t)$ quickly decay to zero (the first steady state) with any initial perturbation of $L(0)$ and $S(0)$. Figure 3 shows that the additive noises cannot induce the predator-prey-like oscillations around the first steady state, and the additive noises just lead to random oscillations of the system for different values $(b_{\mathrm{L}}, b_{\mathrm{R}})$.

3.2. Multiplicative noise-induced predator–prey oscillations in transposon kinetics

In the case of stability domains (e.g., at the point B in Fig. 1), the effects of systemic extrinsic noises on the transposon model are studied by using Eq. (9) with Eq. (8). Figure 4 shows that the multiplicative noises can also induce predator-prey-like oscillations around the second steady state. The time evolutions of the solutions $L(t)$ and $S(t)$ quickly tend to the second steady state under any initial perturbation of $L(0)$ and $S(0)$ in the absence of noises. However, the time evolutions of the solutions $L(t)$ and $S(t)$ become the predator-prey-like oscillations in the presence of noises.

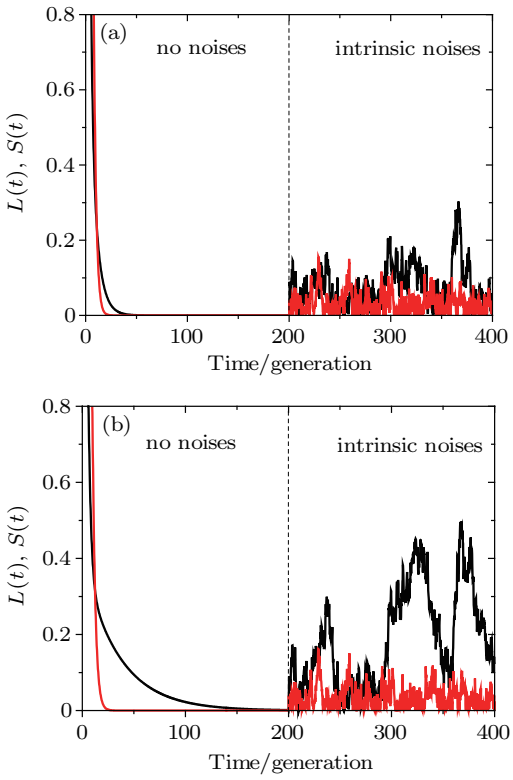


Fig. 3. (color online) Time evolution of the solutions $L(t)$ and $S(t)$ without and with additive noises ($D = 0.001$) in instability domains in Fig. 1. (a) $b_{\mathrm{L}} = 1$, $b_{\mathrm{R}} = 1$. (b) $b_{\mathrm{L}} = 1$, $b_{\mathrm{R}} = 1.4$.

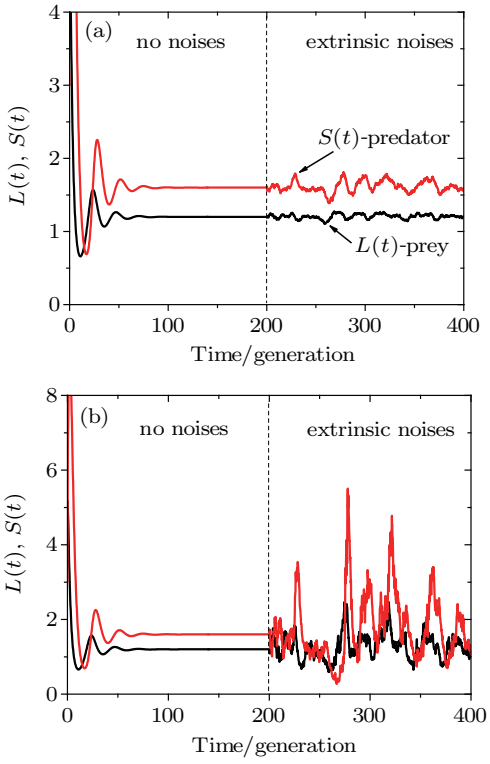


Fig. 4. (color online) Time evolution of the solutions $L(t)$ and $S(t)$ without and with multiplicative noises in stability domains at point B in Fig. 1. (a) $D = 0.001$. (b) $D = 0.1$.

Under the same value of noise intensity, comparing Fig. 2 with Fig. 4, it is found that the amplitude of oscillations induced by multiplicative noises is smaller than that by additive noises, which means that the extrinsic noises can inhibit the amplitude of oscillations of transposon kinetics.

In the case of instability domains (e.g., at the point A in Fig. 1), the second steady state is instability, and the time evolutions of the solutions $L(t)$ and $S(t)$ of transposon kinetics quickly decay to zero (the first steady state) with any initial perturbation of $L(0)$ and $S(0)$. Figure 5 shows that the multiplicative noises with the intensity $D = 0.001$ cannot induce any oscillations around the first steady state.

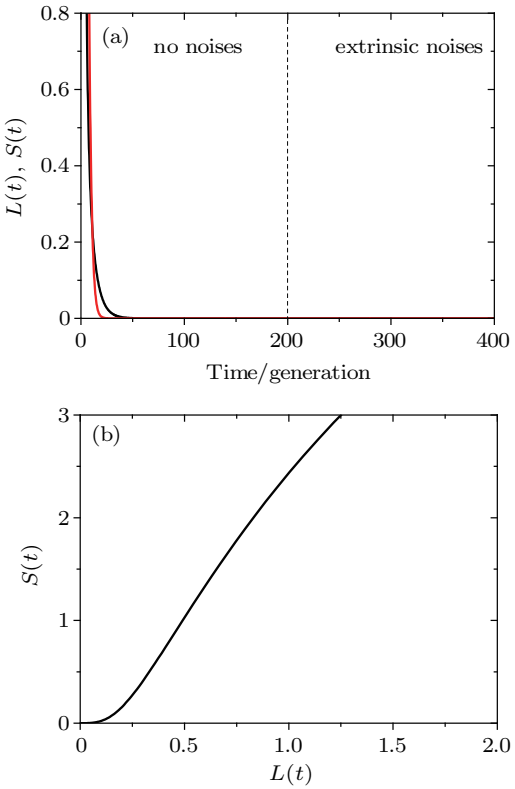


Fig. 5. (color online) (a) Time evolution of the solutions $L(t)$ and $S(t)$ without and with multiplicative noises ($D = 0.001$) in instability domains (at point A in Fig. 1). (b) Phase plane plot of the data represented in panel (a).

3.3. The power spectrum of $L(t)$ and $S(t)$ oscillations

For the transposons model driven by the intrinsic or extrinsic noises, the above results show that the predator-prey-like oscillations around the ACR steady state are induced by the intrinsic or extrinsic noises. We calculate the averaged power spectrum of the time series of $L(t)$ and $S(t)$ in the case of stability domains (e.g., the point B in Fig. 1) by using the fast Fourier transformation, and each plot is provided by the average of 500 runs.

The negative feedback of SINE on the LINE transposition rate causes the predator-prey-like oscillations, and the quasi-cycles are induced by the intrinsic or extrinsic noises. Figure 6 shows that the peak angular frequency of power spectra of the LINE and SINE copy numbers or concentration fluctuations is equal to 0.1376 generation^{-1}, corresponding to a period of 45 generations, which differs from that of Ref. [27] where the period is 27 generations. Although the period of quasi-cycles provided here is larger than that of Ref. [27], the resultant quasi-cycle period should be roughly 1000 generations in a real biological cell.

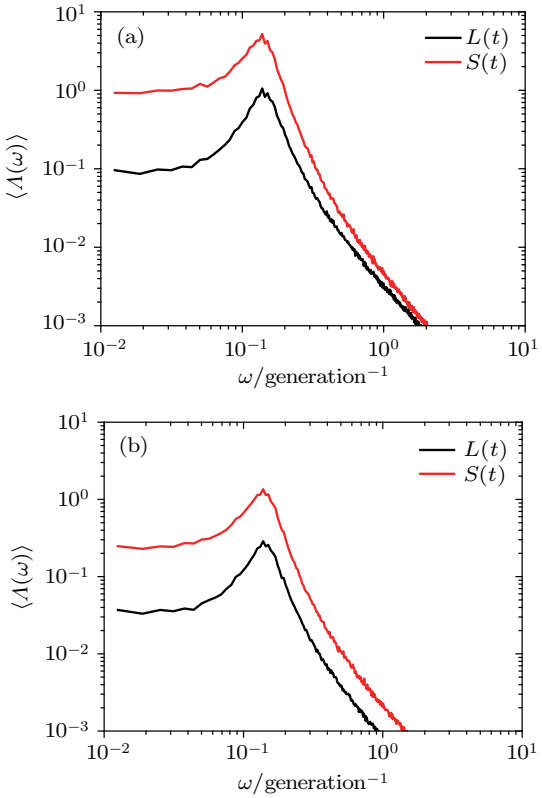


Fig. 6. (color online) The power spectrum of $L(t)$ and $S(t)$ oscillations of the transposons model driven by the intrinsic noises (a) and the extrinsic noises (b) in the case of stability domains (at the point B in Fig. 1) with the noises intensity $D = 0.001$.

3.4. The probability distributions of $L(t)$ and $S(t)$

In the case of stability domains, the probability distribution of the stochastic transposon model is discussed under different noise intensities. Figure 7 shows that, with the increasing of noise intensity D, the peak of the probability distribution of a system driven by additive noises or multiplicative noises is shifted from the second steady state to the first steady state, and the noises enlarge the width of the probability distribution.

In the stochastic kinetics of autonomous–nonautonomous transposons, our numerical simulation results indicate that the probability distribution of active LINEs and SINEs concentrations has a peak, and the peak of probability distribution is decreased with the increasing of noise intensity.

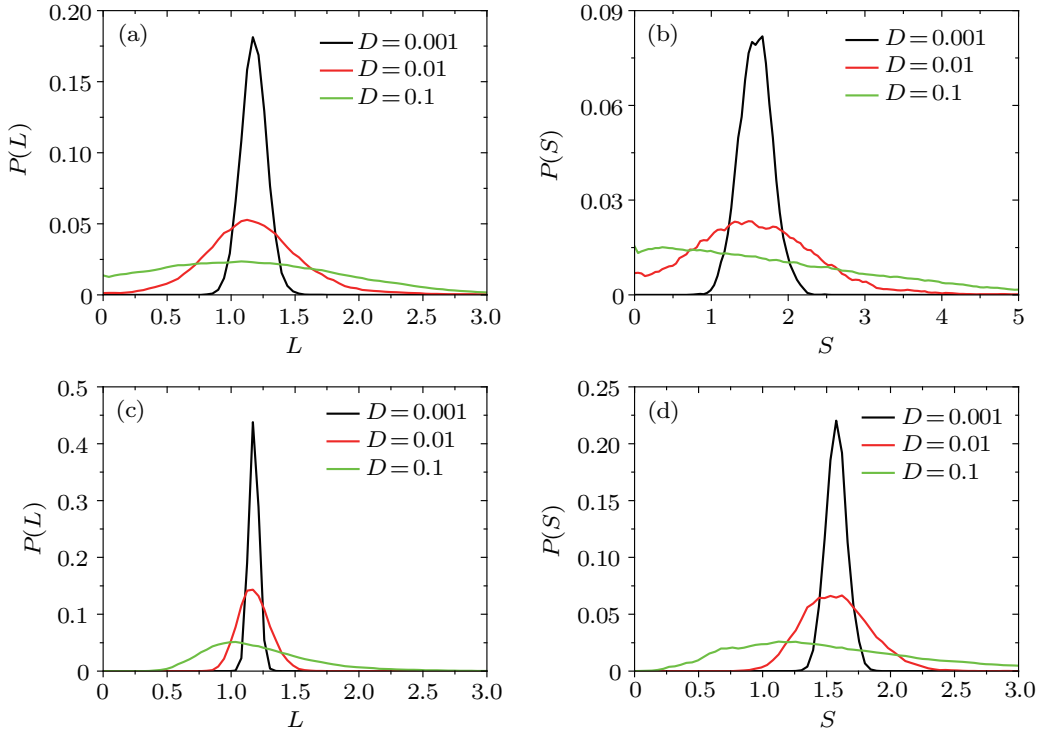


Fig. 7. (color online) Probability distributions of a system driven by additive noises ((a), (b)) and multiplicative noises ((c), (d)) with different noise intensities in stability domains (at point *B* in Fig. 1).

3.5. The normalized correlation function of $L(t)$ and $S(t)$

How does the value of a stochastic variable at a time t influence the value of the same stochastic variable (or another stochastic variable) at a later time $t+\tau$? We define the normalized autocorrelation function of $L(t)$ and $S(t)$ as

$$C_{\mathrm{L}}(\tau)=\frac{\langle\tilde{L}(t)\tilde{L}(t+\tau)\rangle}{\langle\tilde{L}^2\rangle},\quad C_{\mathrm{S}}(\tau)=\frac{\langle\tilde{S}(t)\tilde{S}(t+\tau)\rangle}{\langle\tilde{S}^2\rangle},\quad (10)$$

and the normalized cross-correlation function

$$C_{\mathrm{LS}}(\tau)=\frac{\langle\tilde{L}(t)\tilde{S}(t+\tau)\rangle}{\sqrt{\langle\tilde{L}^2\rangle\langle\tilde{S}^2\rangle}},\quad (11)$$

where $\tilde{L}=L(t)-\langle L\rangle$, $\tilde{S}=S(t)-\langle S\rangle$, τ is the correlation time in the unit of a cell generation. All the correlation functions range from -1 to 1. The correlation function is a measure of the order in a system. The autocorrelation function describes how a stochastic variable at different times are related, and the cross-correlation function quantifies how a stochastic variable co-varies with the other stochastic variable on the across time τ.

Figures 8 and 9 show that the predator-prey-like oscillations of the transposon model are induced by intrinsic noises or extrinsic ones since both autocorrelation and cross-correlation functions of active $L(t)$ and $S(t)$ are oscillated by the noises. The oscillations of correlation functions mean that the kinetics of autonomous–nonautonomous transposons is the long-time order (that is, the predator–prey oscillator). However, all oscillations of correlation functions gradually decay as the correlation time τ increases. For both additive and multiplicative noises cases, the larger the noise intensity is, the faster the attenuation of oscillations of autocorrelation and cross-correlation functions will be.

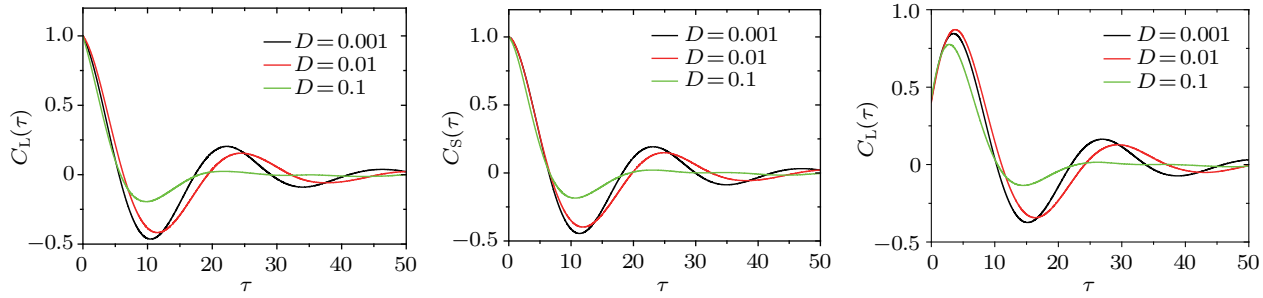


Fig. 8. (color online) The normalized autocorrelation and cross-correlation functions of $L(t)$ and $S(t)$ driven by additive noises under different noise intensities.

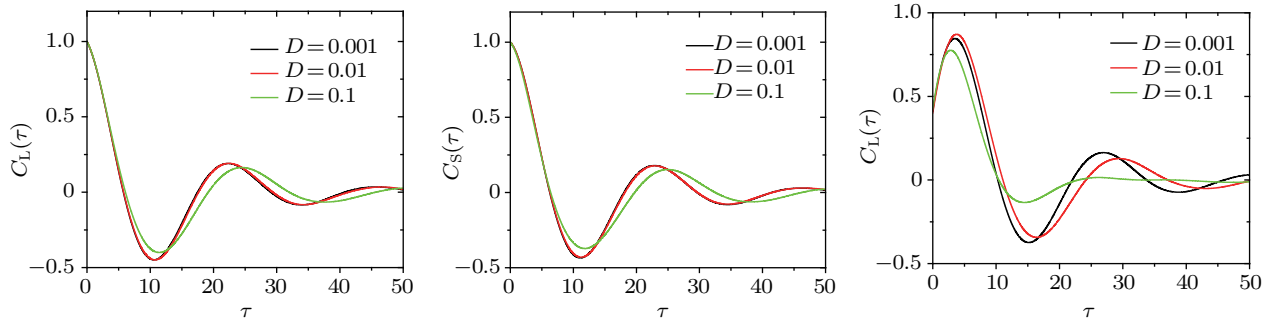


Fig. 9. (color online) The normalized autocorrelation and cross-correlation functions of $L(t)$ and $S(t)$ driven by multiplicative noises under different noise intensities.

In the cases of intrinsic and extrinsic noises, the autocorrelation function of active $L(t)$ or $S(t)$ is the largest at $\tau = 0$, which indicates that $L(t)$ or $S(t)$ has the strongest influence on itself when the correlation time is zero. The correlation time τ corresponding to the first peak of autocorrelation function represents the period of predator-prey-like oscillations of $L(t)$ or $S(t)$. However, the cross-correlation function between autonomous and nonautonomous transposons is not the strongest at $\tau = 0$, which is firstly increased with the increasing of correlation time, and then arrives at the largest value (i.e., the first peak of $C_{\rm LS}$). The correlation time τ corresponding to the first peak of the cross-correlation function represents the time delay of predator-prey-like oscillations between autonomous and nonautonomous transposons.

Both autocorrelation and cross-correlation functions indicate that the state of the predator–prey oscillator is stored in the genome and transmitted to successive generations. The state of the predator–prey oscillator of active autonomous–nonautonomous transposons is transmitted to successive 50 generations at least.

4. Conclusion

The effects of the intrinsic and extrinsic noises on the transposon kinetics are investigated by using numerical simulations in this paper. Our results showed that the complex nonlinear phenomena (i.e., the predator-prey-like oscillations) of a biological system might be induced by the disturbed environments where the organism lives.

Based on the minimal model of the interactions between LINEs and SINEs, the effects of the intrinsic and extrinsic noises on the system are investigated by using numerical simulations. It is found that the predator-prey-like oscillations around the ACR steady state are induced by the intrinsic and extrinsic noises, respectively. Under the same noise intensity, the amplitude of oscillations induced by intrinsic noises is larger than that by extrinsic noises. With the increasing of noise intensity, the peak of the probability distribution of the system is shifted from the second steady state to the first steady state, and both extrinsic and intrinsic noises can enlarge the width of the probability distribution. For both the additive and multiplicative noises cases, the larger the noise intensity is, the faster the attenuation of oscillations of autocorrelation and cross-correlation functions will be. The correlation time corresponding to the first peak of the cross-correlation function can represent the time delay of predator-prey-like oscillations between nonautonomous transposons with autonomous ones. The state of the predator–prey oscillator induced by noises is stored in the genome and transmitted to 50 successive generations at least.

In this paper, both intrinsic and extrinsic noises are assumed to have different origins, treated as independent random variables. However, in certain situations the noises may have a common origin and thus may be correlated with each other as well in biological systems.[33–38] In addition, the ideal of ACR is unlikely to be attained exactly in vivo experimental systems,[29] and complete reaction models for real biological systems should not be expected to exhibit ACR exactly. Therefore, there are still some open questions. For example, what are the effects of correlation between noises on the transposon kinetic model; how does the biological organism utilize the disturbed environments for various biological functions if there is not the ACR steady state?

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