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On fairness, full cooperation, and quantum game with incomplete information*

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Quantum entanglement has emerged as a new resource to enhance cooperation and remove dilemmas. This paper aims to explore conditions under which full cooperation is achievable even when the information of payoff is incomplete. Based on the quantum version of the extended classical cash in a hat game, we demonstrate that quantum entanglement may be used for achieving full cooperation or avoiding moral hazards with the reasonable profit distribution policies even when the profit is uncertain to a certain degree. This research further suggests that the fairness of profit distribution should play an important role in promoting full cooperation. It is hopeful that quantum entanglement and fairness will promote full cooperation among distant people from various interest groups when quantum networks and quantum entanglement are accessible to the public.

Keywords: quantum game, fairness, cooperation

PACS: 02.50.Le, 03.67.–a, 03.65.–w **DOI:** 10.1088/1674-1056/27/3/030202

1. Introduction

With the development of quantum technology, we would like to ask the following questions: What will quantum information technology bring to daily life in the future? What will happen when quantum networks and quantum entanglements are not only available for scientists, but also accessible to the general public? In this paper, we aim to investigate the conditions on which the quantization of classical games can bring full cooperation even when the information of payoff is incomplete.

Classical game theory concerns the study of multi-person decision problems and plays an important role in economics, social sciences, communication, and biology. The work of Neumann and Morgenstern[1] laid the foundation of classical game theory, but a better comprehension was achieved mainly due to the contribution of John Nash.[2] In 1999, Meyer[3] proposed the first quantized game. It has been demonstrated that a quantum game can offer new ways to enhance cooperation, to remove dilemma, and to change equilibrium.[4,5] Eisert *et al.*[6] first introduced the quantum prisoners' dilemma game, in which the players succeeded in escaping the dilemma. Researchers also studied quantum auctions,[7] the quantum battle of sexes,[8,9] quantum gambling,[10] quantum duel,[11,12] quantum Russian roulette,[13,14] quantum Hawk–Dove game,[15] quantum Parrondo game,[16] quantum magic squares game,[17,18] etc.

Marinatto and Weber[19] took inspiration from Ref. [6] and introduced another interesting and acceptable scheme for the analysis of nonzero sum games by means of quantum strategies. Since then many researchers followed the scheme to analyze many specific games in the quantum domain, and Nawaz and Toor[20] generalized the MW quantization scheme[19] for two-person nonzero sum games.

Besides the aforementioned researches, Pappa and his coauthors[21] presented the first conflicting interest Bayesian game in which quantum mechanics leads to a higher total payoff. More conflicting interest games were further discussed by Situ[22] and other researchers.

This paper will focus on games of players with different but common interests, and we aim to explore the conditions under which full cooperation is achievable even when the information of payoff is incomplete. Recently, our research group[23] has explored the quantum "cash in a hat" game, whose classical version is a famous simplified model between a borrower and a lender in economics. It has been demonstrated in Ref. [23] that entanglement resources can be helpful in avoiding moral hazards. Working on the extended cash in a hat game with different profit distribution policies, we show that the fairness of profit distribution policies plays an important role in promoting the cooperation or avoiding moral hazards.

2. Main results

First, let us consider the extended classical cash in a hat game with different distribution policies under different investments. There are two players, Alice and Bob, each with two

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choices. Alice can put a small (K_S) or a large amount of coins (K_L) into a hat according to her wishes. Then she gives the hat with her investment to Bob. Bob can add the same amount of coins or simply take all the cash away. The payoff of this game is shown in Table 1. Suppose that the total profits for small and large investments are T_S and T_L, respectively. Let α_i and β_i be the distribution ratio of total profits (T_i) for Alice and Bob, where $\alpha_i + \beta_i = 1, i = S, L$. If Alice puts K_S (or K_L) in and Bob matches, the payoff for Alice is $T_S\alpha_S$ (or $T_L\alpha_L$) and Bob's payoff is $T_S\beta_S$) (or $T_L\beta_L$), i.e., Alice obtains $T_S \cdot \alpha_S + K_S$ (or $T_L \cdot \alpha_L + K_L$) back and Bob obtains $T_S \cdot \beta_S + K_S$ (or $T_L \cdot \beta_L + K_L$) finally. This is a simple version of economics behaviors involving a lender and a borrower in economics.[24] For example, Alice is an investment banker, Bob is a budding entrepreneur, and Alice will invest in his project. She can decide the amount of the investment, large (strategy C) or small (strategy D). With this money, Bob could work hard (strategy C) or could just disappear (strategy D).

Table 1. Extended cash in a hat game with different investments (K_S, K_L), different total profits (T_S, T_L), and different distribution policies (α_i, β_i), where $i = S, L$, $0 < \alpha_i, \beta_i < 1$, $\alpha_i + \beta_i = 1$, $T_S\beta_S > K_S$, and $K_L > T_L\beta_L$.

Alice/Bob	D	C
D	$(-K_S, K_S)$	$(T_S\alpha_S, T_S\beta_S)$
C	$(-K_L, K_L)$	$(T_L\alpha_L, T_L\beta_L)$

The classical cash in a hat game is a sequential game, and we plot a decision tree in Fig. 1 to represent the sequence of moves and countermoves.

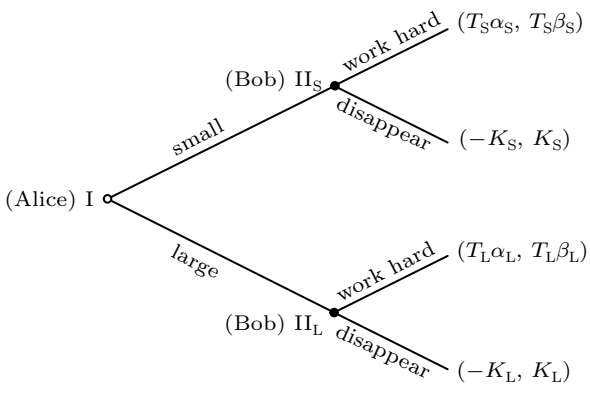


Fig. 1. A decision tree for the classical quantum-like cash in a hat game.

Now, let us briefly analyze the aforementioned extended cash in a hat game. When Alice puts small investment K_S into the hat (strategy D), Bob will match the cash (strategy C) in order to maximize his own profit if $T_S \cdot \beta_S > K_S$. If $K_L > T_L \cdot \beta_L$, Bob may take away all the investment K_L of Alice to realize his maximal benefit. This situation is called moral hazard by economists.[25,26]

However, if Bob does not know the previous moves of Alice, the extended classical cash in a hat game can be considered as a simultaneous game. Therefore, we represent the extended classical cash in a hat game in the so-called normal form as shown in Table 1.

Here, the moral hazard of this classical game is the risk that the borrower may evade his repayment. We have no way to avoid the problem of moral hazard in the classical cash in a hat game. The equilibrium can be obtained through the method called backward induction. Moral hazard forces the investor to reduce her investment because Alice may wonder whether or not Bob will take the cash away if she chooses strategy C, that is to say, the investor prefers strategy D. Thus, we can easily find that the equilibrium solution of this game ends up in DC. However, we prefer that the game ends up in CC obviously, since CC is not only Pareto optimal, but also maximizes the welfare of the society.

It should be underlined that we can avoid moral hazard if $K_L < T_L \cdot \beta_L$. In this situation, we just assume that Bob will work hard to earn the profit $T_L \cdot \beta_L$ instead of taking away the large investment K_L of Alice. Interestingly, it should be emphasized that Bob will prefer to accept the small investment K_S of Alice if Bob's profit of small investment is larger than that of large investment $T_S \cdot \beta_S > T_L \cdot \beta_L$. On the other hand, Alice will choose the small investment K_S to maximize her profit if $T_S \cdot \alpha_S > T_L \cdot \alpha_L$.

Based on the aforementioned discussion, we recognize that the moral hazard issue should be explored only when the following payoff conditions are satisfied:

$$K_S < T_S \cdot \beta_S < T_L \cdot \beta_L < K_L, \tag{1}$$

$$T_S \cdot \alpha_S < T_L \cdot \alpha_L. \tag{2}$$

However, the information of T_L may be incomplete and we only know $T_L \in [T_L^0, T_L^1]$. The aforementioned payoff conditions should be modified as

$$K_S < T_S \cdot \beta_S < T_L^0 \cdot \beta_L < T_L^1 \cdot \beta_L < K_L, \tag{3}$$

$$T_S \cdot \alpha_S < T_L^0 \cdot \alpha_L. \tag{4}$$

Subsequently, we will further explore the sufficient conditions under which the moral hazard can be solved and players can reach the Pareto optimal solution. For simplicity, we borrow the idea from Refs. [6] and [27] and plot the physical model of the quantum-like cash in a hat game in Fig. 2. The model consists of three ingredients: an entangling source of two qubits which provides the initial state, a set of physical instruments that enables the player to manipulate his/her own qubit in a strategic manner, and a physical measurement device which determines the payoffs of the players from the outstate of the two qubits. It is assumed that the information of all three ingredients is perfectly available for both players.

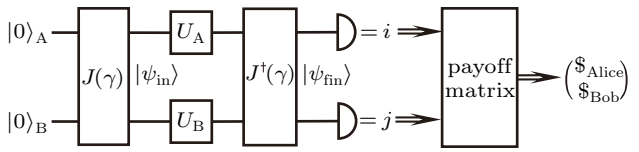


Fig. 2. The physical model of the quantum-like cash in a hat game. $\hat{T}$ is a unitary evolution operation that yields the entanglement. $\hat{U}_A$ and $\hat{U}_B$ represent the strategic operations of Alice and Bob, respectively. $\hat{T}^\dagger$ is the operation to remove the entanglement, followed by the measurement device.

The state for the quantum-like version of the extended cash in a hat game is described by a vector in a Hilbert space spanned by the basics $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, where the former and latter entries represent Alice's and Bob's qubits, respectively. They all have classical interpretation: $|00\rangle$ denotes $\hat{U}_A = C$, $\hat{U}_B = C$, i.e., Alice chooses strategy C and Bob also chooses strategy C; $|01\rangle$ denotes $\hat{U}_A = C$, $\hat{U}_B = D$; $|10\rangle$ denotes $\hat{U}_A = D$, $\hat{U}_B = C$; and $|11\rangle$ denotes $\hat{U}_A = D$, $\hat{U}_B = D$

$$|00\rangle = \begin{pmatrix}1&0&0&0\end{pmatrix}^T, \quad |01\rangle = \begin{pmatrix}0&1&0&0\end{pmatrix}^T,$$
$$|10\rangle = \begin{pmatrix}0&0&1&0\end{pmatrix}^T, \quad |11\rangle = \begin{pmatrix}0&0&0&1\end{pmatrix}^T. \tag{5}$$

The initial state of the quantum-like game is given by

$$|\psi_0\rangle = J|00\rangle, \tag{6}$$

where J is a unitary evolution operation with the entanglement parameter γ,

$$J = \begin{pmatrix} \cos\frac{\gamma}{2} I_2 & -\sin\frac{\gamma}{2}\sigma_y \\ \sin\frac{\gamma}{2}\sigma_y & \cos\frac{\gamma}{2} I_2 \end{pmatrix}. \tag{7}$$

Here, $\gamma \in [0, \pi/2]$ is a real parameter and γ can be considered as a measure of entanglement.[6] There is no entanglement for $\gamma = 0$, and $\gamma = \pi/2$ corresponds to the maximal entanglement. The initial state has the form

$$|\psi_0\rangle = \cos\frac{\gamma}{2}|00\rangle + \mathrm{i}\sin\frac{\gamma}{2}|11\rangle. \tag{8}$$

Unitary operations $\hat{U}_A(\theta_1, \phi_1)$ and $\hat{U}_B(\theta_2, \phi_2)$ for Alice and Bob are respectively given by

$$\hat{U}_A(\theta_1, \phi_1) = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_1}\cos\frac{\theta_1}{2} & \sin\frac{\theta_1}{2} \\ -\sin\frac{\theta_1}{2} & \mathrm{e}^{-\mathrm{i}\phi_1}\cos\frac{\theta_1}{2} \end{pmatrix}, \tag{9}$$

$$\hat{U}_B(\theta_2, \phi_2) = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_2}\cos\frac{\theta_2}{2} & \sin\frac{\theta_2}{2} \\ -\sin\frac{\theta_2}{2} & \mathrm{e}^{-\mathrm{i}\phi_2}\cos\frac{\theta_2}{2} \end{pmatrix}. \tag{10}$$

Here $\theta_1, \theta_2 \in [0, \pi]$ and $\phi_1, \phi_2 \in [0, \pi/2]$.

We respectively associate Alice's and Bob's classical strategies with operations

$$C = \hat{U}_A(0,0) \otimes I_2, \quad D = \hat{U}_A(\pi, 0) \otimes I_2; \tag{11}$$
$$C = I_2 \otimes \hat{U}_B(0,0), \quad D = I_2 \otimes \hat{U}_B(\pi, 0). \tag{12}$$

The strategic moves of Alice and Bob are associated with unitary operations $\hat{U}_A \otimes I_2$ and $I_2 \otimes \hat{U}_B$, respectively. After their moves, the state of the quantum cash in a hat game becomes $(I_2 \otimes \hat{U}_B)(\hat{U}_A \otimes I_2)J|00\rangle$, then they send their qubits to the final measurement device which determines their payoffs. The final state of the quantum game is given by

$$|\psi_f\rangle = J^\dagger(I_2 \otimes \hat{U}_B)(\hat{U}_A \otimes I_2)J|00\rangle. \tag{13}$$

Eventually, the payoffs are obtained from the final state. Alice's and Bob's expected payoffs are given by

$$\begin{cases} \$_A = -K_S P_{DD} + T_S \alpha_s P_{DC} - K_L P_{CD} + T_L \alpha_L P_{CC}, \\ \$_B = K_S P_{DD} + T_S \beta_s P_{DC} + K_L P_{CD} + T_L \beta_L P_{CC}, \end{cases} \tag{14}$$

where $P_{ee'} = |\langle ee'|\psi_f\rangle|^2$ is the probability, the subscript ee' is an element of the set $\{CC, CD, DC, DD\}$.

It can be proved that this quantum-like game is reduced to the classical game when $\gamma = 0$. In case DD, the final state is given by

$$\begin{aligned} |\psi_f\rangle &= J^\dagger_{\gamma=0}(I_2 \otimes \hat{U}_B(\pi, 0))(\hat{U}_A(\pi, 0) \otimes I_2)J_{\gamma=0}|00\rangle \\ &= (0\,0\,0\,1)^T. \end{aligned} \tag{15}$$

After that, by using Eq. (14), we can obtain $\$_A = -K_S, \$_B = K_S$ which is the same as the payoffs of DD in Fig. 2. In the same way, we can find that the cases DC, CD, and CC also have classical correspondence.

Here, only a single new quantized strategy (strategy Q) is introduced in our quantum-like version of the extended cash in a hat game. For Alice,

$$Q = \hat{U}_A\left(0, \frac{\pi}{2}\right) \otimes I_2, \tag{16}$$

and for Bob

$$Q = I_2 \otimes \hat{U}_B\left(0, \frac{\pi}{2}\right). \tag{17}$$

Tables 2 and 3 show Alice's and Bob's expected payoffs for the quantum-like game when Alice's or Bob's strategy is Q.

For the aforementioned quantum-like game, a novel Nash equilibrium QQ arises with payoffs $(\$_A, \$_B) = (T_L\alpha_L, T_L\beta_L)$, which is also the Pareto optimal for this game. The moral hazard in the extended classical cash in a hat game is overcome if the following entanglement condition is satisfied:

$$\gamma > \gamma_{thQ} = \arcsin\sqrt{\frac{K_L - T_L\beta_L}{K_L - T_S\beta_S}}, \tag{18}$$

i.e., the entanglement exceeds the quantum threshold γ_{thQ}. Since $T_L \in [T_L^0, T_L^0]$, the aforementioned entanglement condition (18) is specified by

$$\gamma > \gamma^0_{thQ} = \arcsin\sqrt{\frac{K_L - T_L^0\beta_L}{K_L - T_S\beta_S}}. \tag{19}$$

So far, we can conclude that the aforementioned entanglement condition given by Eq. (18) will enable the entanglement resource to play an important role in overcoming moral hazard only when the payoff conditions given by Eqs. (3) and (4) are satisfied.

Table 2. Alice's and Bob's expected payoffs for the quantum-like game when Alice's strategy is Q.

Bob	($\$_A$, $\$_B$)
D	$(T_S\alpha_S\sin^2\gamma - K_L\cos^2\gamma, T_S\beta_S\sin^2\gamma + K_L\cos^2\gamma)$
C	$(T_L\alpha_L\cos^2\gamma - K_S\sin^2\gamma, T_L\beta_L\cos^2\gamma + K_S\sin^2\gamma)$
Q	$(T_L\alpha_L, T_L\beta_L)$

Table 3. Alice's and Bob's expected payoffs for the quantum-like game when Bob's strategy is Q.

Alice	($\$_A$, $\$_B$)
D	$(T_S\alpha_S\cos^2\gamma - K_L\sin^2\gamma, T_S\beta_S\cos^2\gamma + K_L\sin^2\gamma)$
C	$(T_L\alpha_L\cos^2\gamma - K_S\sin^2\gamma, T_L\beta_L\cos^2\gamma + K_S\sin^2\gamma)$
Q	$(T_L\alpha_L, T_L\beta_L)$

For example, let $K_S = 1$, $K_L = 3$, $T_S = 3$, and $T_L \in [5,6]$. To satisfy the payoff conditions given by Eqs. (3) and (4), we have the following inequalities:

$$1 < 3\beta_S < 5\beta_L < 6\beta_L < 3, \quad 3\alpha_S < 5\alpha_L. \tag{20}$$

One can choose $\beta_S = 1/2$ and $\beta_L = 1/3$. The moral hazard in the extended classical cash in a hat game is overcome if the following entanglement condition is satisfied:

$$\gamma > \gamma_{thQ}^0 = \arcsin\sqrt{\frac{3-5/3}{3-3/2}} = \arcsin\frac{2\sqrt{2}}{3}. \tag{21}$$

We can conclude from Table 4 that a Nash equilibrium QQ with payoffs ($\$_A = 2T_L/3, \$_B = T_L/3$) is the Pareto optimal. This choice also maximizes the welfare of the whole society. It may be underlined from Table 5 that quantum entanglement cannot bring full cooperation when $\sin^2\gamma = 3/4 < 8/9$.

Table 4. Example 1: payoffs of a quantum-like game with $K_S = 1$, $K_L = 3$, $T_S = 3$, $T_L \in [5,6]$, $\alpha_S = \beta_S = 1/2$, $\alpha_L = 2/3$, $\beta_L = 1/3$, and $\sin^2\gamma = 1$.

Alice/Bob	D	C	Q
D	$(-1,1)$	$(1.5,1.5)$	$(-3,3)$
C	$(-3,3)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$	$(-1,1)$
Q	$(1.5,1.5)$	$(-1,1)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$

Table 5. Example 2: payoffs of a quantum-like game with $K_S = 1$, $K_L = 3$, $T_S = 3$, $T_L \in [5,6]$, $\alpha_S = \beta_S = 1/2$, $\alpha_L = 2/3$, $\beta_L = 1/3$, and $\sin^2\gamma = 3/4$.

Alice/Bob	D	C	Q
D	$(-1,1)$	$(1.5,1.5)$	$\left(-\frac{15}{8}, \frac{21}{8}\right)$
C	$(-3,3)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$	$\left(\frac{T_L}{6} - \frac{3}{4}, \frac{T_L}{6} + \frac{3}{4}\right)$
Q	$\left(\frac{3}{8}, \frac{15}{8}\right)$	$\left(\frac{T_L}{6} - \frac{3}{4}, \frac{T_L}{6} + \frac{3}{4}\right)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$

One can also choose $\beta_S = 2/3$ and $\beta_L = 1/2$. The moral hazard in the extended classical cash in a hat game is overcome if the following entanglement condition is satisfied:

$$\gamma > \gamma_{thQ}^0 = \arcsin\sqrt{\frac{3-2.5}{3-2}} = \arcsin\frac{\sqrt{2}}{2}. \tag{22}$$

We can conclude from Table 6 that the Nash equilibrium QQ with payoffs ($\$_A = T_L/2, \$_B = T_L/2$) is the Pareto optimal. This choice also maximizes the welfare of the whole society. From Table 7, we find that quantum entanglement fails to promote the cooperation of Alice and Bob because the payoff conditions given by Eqs. (3) and (4) are not satisfied.

Table 6. Example 3: payoffs of a quantum-like game with $K_S = 1$, $K_L = 3$, $T_S = 3$, $T_L \in [5,6]$, $\alpha_S = 1/3$, $\beta_S = 2/3$, $\alpha_L = \beta_L = 1/2$, and $\sin^2\gamma = 3/4$.

Alice/Bob	D	C	Q
D	$(-1,1)$	$(1,2)$	$\left(-2, \frac{11}{4}\right)$
C	$(-3,3)$	$\left(\frac{T_L}{2}, \frac{T_L}{2}\right)$	$\left(\frac{T_L}{8} - \frac{3}{4}, \frac{T_L}{8} + \frac{3}{4}\right)$
Q	$(0,2.25)$	$\left(\frac{T_L}{8} - \frac{3}{4}, \frac{T_L}{8} + \frac{3}{4}\right)$	$\left(\frac{T_L}{2}, \frac{T_L}{2}\right)$

Table 7. Example 4: payoffs of a quantum-like game with $K_S = 1$, $K_L = 3$, $T_S = 3$, $T_L \in [5,6]$, $\alpha_S = 1/3$, $\beta_S = 2/3$, $\alpha_L = 2/3$, $\beta_L = 1/3$, and $\sin^2\gamma = 1$.

Alice/Bob	D	C	Q
D	$(-1,1)$	$(1,2)$	$(-3,3)$
C	$(-3,3)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$	$(-1,1)$
Q	$(1,2)$	$(-1,1)$	$\left(\frac{2T_L}{3}, \frac{T_L}{3}\right)$

The aforementioned analysis and examples suggest that even the information of payoff is incomplete, the quantum physical resources including quantum entanglement provide us with an alternative scheme to enhance the cooperation and overcome the constraints of social morals if the players are assumed to be ideally rational and the payoff conditions given by Eqs. (3) and (4) are satisfied. On the other hand, quantum entanglement cannot bring full cooperation between Alice and Bob if equations (3) and (4) are not satisfied.

3. Discussion and conclusion

In summary, quantum entanglement, with equitable profit distribution policies, can be used for promoting full cooperation or avoiding moral hazards even when the information of payoff is incomplete, but quantum entanglement cannot work without reasonable or fair profit distributions policies. In other words, entanglement cannot enhance the cooperation if the profit distribution polices are not reasonable or fair. Our recent research[28] also indicates that entanglement can also be utilized to enhance the cooperation of distant players with the help of quantum networks. We come

to realize that neither quantum entanglement nor quantum network can offer full cooperation without fair profit distribution policies. Full cooperation relies on not only the degree of quantum entanglement but also the degree of fairness. We will further explore full cooperation problems for hybrid noncooperative–cooperative games[29,30] and repeated games with incomplete information.[31] In our opinion, it is a worthy research topic to understand the conditions under which full cooperation is achievable among distant people from different interest groups.

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