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# Sub-external cavity effect and elimination method in laser self-mixing interference wave plate measurement system

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Laser self-mixing interference (SMI) wave plate measurement method is a burgeoning technique for its simplicity and efficiency. But for the non-coated sample, the reflected light from the surface can seriously affect the measurement results. To analyze the reason theoretically, a self-consistent model for laser operation with a sub-external and an external cavity is established, and the sub-external cavity formed by the sample and a cavity mirror is proved to be the main error source. A synchronous tuning method is proposed to eliminate the sub-external cavity effect. Experiments are carried out on the synchronously tuning double external cavities self-mixing interference system, and the error of the system is in the range of $-0.435^\circ \sim 0.387^\circ$ compared with the ellipsometer. The research plays an important role in improving the performance and enlarging the application range of the laser self-mixing interference system.

**Keywords:** self-mixing interference, non-coated wave plate measurement, self-consistent model

**PACS:** 42.25.Hz, 42.30.Lr, 42.25.Lc, 06.30.Ka **DOI:** 10.1088/1674-1056/27/2/024201

## 1. Introduction

Wave plates are important optical elements that can change the polarization states of incident lights,[1,2] and are widely used in polarization related systems such as optical isolators,[3,4] optical attenuators,[5] and polarization rotators.[6] The retardation accuracy of the wave plate has a direct effect on the performance of the system.[7] Therefore, accurate measurement of retardation in wave plates plays an important role in processing high quality optical elements and improving the performances of optical systems.

Wave plate measurement based on the laser self-mixing interference (SMI) effect,[8,9] is a burgeoning technique for its advantages such as simplicity, low-cost, and high precision.[10] However, for non-coated wave plates, the repeatability of the method is too poor to meet the measurement application. In order to reduce the influence of the reflected light on the surface of sample, Chen[11] has proposed a refractive index matching method and achieved good results under laboratory conditions. The method requires the upper and lower surfaces of the wave plate being coated with a refractive index-matching fluid, and sandwiched between a glass substrate and a cover sheet that coated with the antireflective film. The need to align the instrument for each piece of wave plate and the need to clean off the index-matching liquid from the sample make these measurements complex and inefficient, so refractive index matching method is not suitable for large-scale industrial production applications.

We calculate the laser output characteristics with double external cavities and propose a synchronous tuning laser SMI system in this paper. The system eliminates the sub-external cavity effect by tuning the external and sub-external cavity at the same frequency. The measurement errors of non-coated wave plates have been proved to be reduced at the same time. The research plays an important role in improving the performance and enlarging application range of the SMI system.

## 2. Sub-external cavity effect in SMI system

As shown in Fig. 1, $M_1$ and $M_2$ constitute the resonator of the laser, whose length is $L$. WP is the non-coated wave plate, and its surface is perpendicular to the laser axis. $M_3$ is the reflective mirror with reflectivity of 10%. $M_2$ and $M_3$ constitute the laser external cavity, whose length is $l_2$. WP and $M_2$ constitute the sub-external cavity, whose length is $l_1$. As the reflectivities of $M_3$ and WP are low, only one round-trip of the beam is considered.

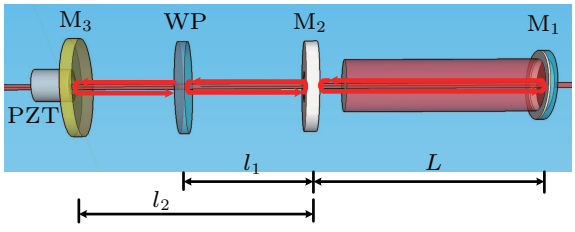



**Fig. 1.** (color online) Sub-external cavity effect.

After one round-trip in the laser cavity, the laser needs to have the same amplitude and phase to be excited, the condition is still tenable for SMI system. Assuming that the starting point of the light wave is $M_2$, and the initial electric field can

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be expressed as

$$E_1 = E_0 \exp[\mathrm{i}(\omega t + \varphi_0)], \tag{1}$$

where $\omega$ is the angular frequency of the laser, $E_0$ and $\varphi$ is the amplitude and initial phase of the light-wave, respectively. After a round-trip in the laser cavity, the electric field of light is denoted as $E_2$. After passing through $M_2$, the light is reflected by non-coated wave plate and coupled into the inner cavity, and is denoted as $E_3$. The light coupled into inner cavity by $M_3$ is denoted as $E_4$. According to the condition of laser excitation, when the system is stable, superposition of the three electric fields should have the same amplitude and phase with the initial electric field, which can be expressed as

$$E_1 = E_2 + E_3 + E_4, \tag{2}$$

$E_2$, $E_3$, and $E_4$ are expressed as follows:

$$E_2 = r_1 r_2 E_1 \mathrm{e}^{\mathrm{i}2kL} \mathrm{e}^{2gL}, \tag{3}$$

$$E_3 = t_2^2 r_{\mathrm{wp}} r_1 E_1 \mathrm{e}^{\mathrm{i}2k(L+l_1)} \mathrm{e}^{2gL}, \tag{4}$$

$$E_4 = t_2^2 t_{\mathrm{wp}}^2 r_1 r_3 E_1 \mathrm{e}^{\mathrm{i}2k(L+l_2)} \mathrm{e}^{2gL}; \quad k = 2\pi/\lambda, \tag{5}$$

where $r_1$, $r_2$, $r_3$, and $r_{\mathrm{wp}}$ are the reflectivities of $M_1$, $M_2$, $M_3$, and the wave plate, respectively; $t_1$, $t_2$, $t_3$, and $t_{\mathrm{wp}}$ are the corresponding transmittances. Substituting $E_2$, $E_3$, and $E_4$ into Eq. (2), the expression can be written as

$$\begin{aligned} 1 \;=\; & r_1 r_2 \mathrm{e}^{\mathrm{i}2kL} \mathrm{e}^{2gL} + t_2^2 r_{\mathrm{wp}} r_1 \mathrm{e}^{\mathrm{i}2k(L+l_1)} \mathrm{e}^{2gL} \\ & + t_2^2 t_{\mathrm{wp}}^2 r_1 r_3 \mathrm{e}^{\mathrm{i}2k(L+l_2)} \mathrm{e}^{2gL}. \end{aligned} \tag{6}$$

After simplification, equation (6) can be expressed as

$$\mathrm{e}^{\mathrm{i}2kL} \mathrm{e}^{2gL} (r_1 r_2 + r_{\mathrm{wp}} r_1 t_2^2 \mathrm{e}^{\mathrm{i}2kl_1} + r_1 r_3 t_2^2 t_{\mathrm{wp}}^2 \mathrm{e}^{\mathrm{i}2kl_2}) = 1. \tag{7}$$

Separating the imaginary and real part of Eq. (7), the following expression can be gained

$$\begin{aligned} & \arg\left[ r_1 r_2 + r_{\mathrm{wp}} r_1 t_2^2 \mathrm{e}^{\mathrm{i}2kl_1} + r_1 r_3 t_2^2 t_{\mathrm{wp}}^2 \mathrm{e}^{\mathrm{i}2kl_2} \right] \\ & + 2kL - 2m\pi = 0, \end{aligned} \tag{8}$$

where $m$ is the longitudinal mode order of the laser resonator. According to Eq. (7) and Eq. (8), the gain of the laser can be obtained as

$$gL = -\frac{1}{2} \ln \left| r_1 r_2 + r_{\mathrm{wp}} r_1 t_2^2 \mathrm{e}^{\mathrm{i}2kl_1} + r_1 r_3 t_2^2 t_{\mathrm{wp}}^2 \mathrm{e}^{\mathrm{i}2kl_2} \right|. \tag{9}$$

Assuming $A = r_{\mathrm{wp}} r_1 t_2^2$, $B = r_1 r_3 t_2^2 t_{\mathrm{wp}}^2$, $r_1 r_2 \approx 1$, equation (9) can be expressed as

$$gL = -\frac{1}{2} \ln \left| r_1 r_2 + \mathrm{A} \mathrm{e}^{\mathrm{i}2kl_1} + B \mathrm{e}^{\mathrm{i}2kl_2} \right|. \tag{10}$$

Ignoring the high-order terms, the expression can be reduced as

$$gL = -\frac{1}{2} \ln |r_1 r_2| - (A' \cos 2kl_1 + B' \cos 2kl_2),$$

$$A' = \frac{A}{r_1 r_2}, \quad B' = \frac{B}{r_1 r_2}. \tag{11}$$

The gain of the laser without the external cavity is

$$g_0 L = -\frac{1}{2} \ln |r_1 r_2|. \tag{12}$$

The change of intensity induced by the feedback and sub-external cavity is

$$\frac{I - I_0}{I_0} = \frac{gL - g_0 L}{g_0 L} = \frac{A' \cos(2kl_1) + B' \cos(2kl_2)}{\ln |r_1 r_2|}. \tag{13}$$

From Eq. (13) we can find that, both the length change of external and sub-external cavities will cause the fluctuation of the laser intensity. Assuming the tuning frequency of the external cavity is $\varsigma$, the tuning frequency of the sub-external cavity is $10\varsigma$, and the theoretical intensity tuning curve is shown in the following Fig. 2.

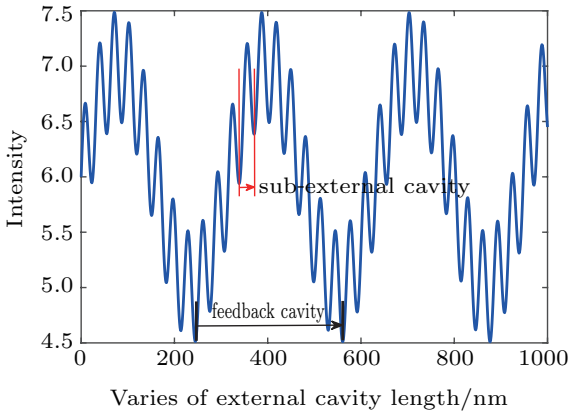



**Fig. 2.** (color online) Theoretical output intensity of laser with external and sub-external cavity.

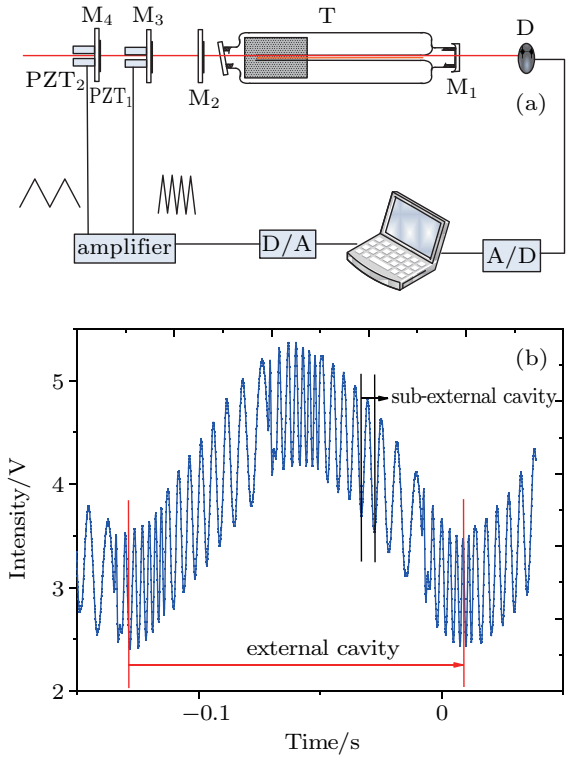



**Fig. 3.** (color online) Experimental tuning curve.

A laser SMI experiment with double external cavities is carried out to verify the theoretical analysis. As shown in Fig. 3(a), $M_1$ and $M_2$ is the laser cavity, $M_3$ and $M_4$ are reflecting mirrors. $PZT_1$ and $PZT_2$ are connected to $M_3$ and $M_4$, respectively. Triangular voltages with different frequency are applied to $PZT_1$ and $PZT_2$ to tune the external and sub-external cavity at the same time, the output of the laser is shown in Fig. 3(b).

The envelope of the curve is generated by the tuning of the external cavity, and the movement of the sub-external cavity mirror is superimposed on the external cavity tuning curve. The experimental result is consistent with theoretical calculations.

In the laser SMI wave plate measurement system, when the length of the sub-external cavity is changed by the environmental disturbance, the generated harmonic would cause the distortion of the measuring signal, which directly leads to unreliable precision of measurement results.

## 3. Synchronous tuning laser SMI system

It can be found from Eq. (13) that, if the external and sub-external cavity can be tuned at the same frequency, the sub-external cavity is modulated to the amplitude and phase of the measurement signal rather than distortion.

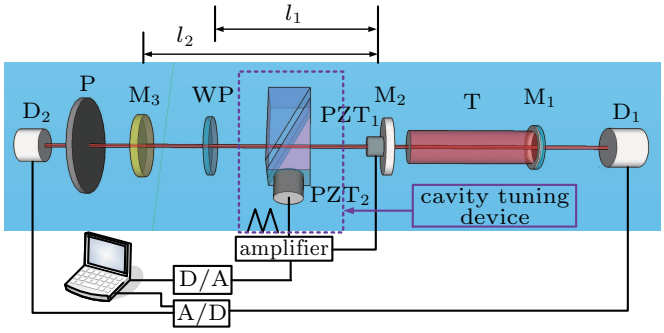



**Fig. 4.** (color online) Structure of the improved system.

In order to achieve synchronous tuning, a cavity tuning device is designed to be placed in the sub-external cavity. The synchronous tuning system is shown in Fig. 4. The light source is a half-external cavity, single-mode, linearly polarized He–Ne laser, and the working wavelength is 632.8 nm. The ratio of gaseous pressure in the laser is He:Ne = 9:1 and $Ne^{20}$:$Ne^{22}$ = 1:1 to eliminate lamb dip.[12] The laser cavity is made up of mirrors $M_1$ and $M_2$ with reflectivity of 99.8% and 98.8% respectively and the length is 150 mm. The initial polarization state of the laser is parallel to the $Y$ axis. $PZT_1$ is connected to $M_2$ to make the longitudinal mode work in the center of the bandwidth. P is the polarizer whose polarization direction is in the $X$ axis. WP is the non-coated wave plate, $D_1$ and $D_2$ are photodetectors. $D_1$ is used to detect the light intensity, and $D_2$ is used to detect the laser intensity behind the polarizer. Because the polarization direction of P is perpendicular to the initial polarization state of laser, the light cannot reach $D_2$ at the beginning. The lengths of external and sub-external cavity are $l_2$ and $l_1$, respectively.

The cavity tuning device is composed of a pair of glass wedges and a piezoelectric ceramics $PZT_2$, and the surface of the glass wedges where light passes through are coated with antireflection film for 632.8-nm wavelength. As shown in Fig. 5, $\theta$, $n_g$, $n_a$, and $\alpha$ represent the wedge angle, the refraction index of the optical wedge, the refractive index of air, and the refraction angle, respectively. When triangular voltage is applied on $PZT_2$ and optical wedge 2 is pushed at displacement of $S$, the change of optical length of the laser in the two optical wedges and the air gap between them can be expressed as follows:

$$S_1 = S \times \frac{\sin\theta}{\cos\alpha} \cdot n_a \cdot \left[\cos(\alpha-\theta)\cdot n_g - 1\right],$$
$$\alpha = \left[\arcsin\left(\frac{n_g}{n_a}\cdot\sin\theta\right)\right]. \tag{14}$$

The displacement of the light spot on optical wedge 2 is $S_2$, and $S_2$ can be expressed as follows:

$$S_2 = S \times \frac{\sin\theta}{\cos\alpha} \cdot \sin(\alpha-\theta),$$
$$\alpha = \left[\arcsin\left(\frac{n_g}{n_a}\cdot\sin\theta\right)\right]. \tag{15}$$

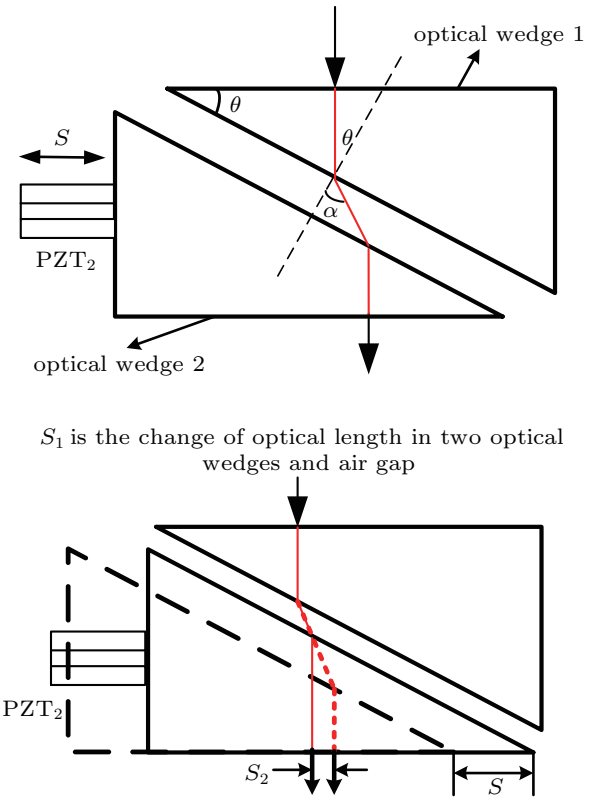



**Fig. 5.** (color online) Operational principle of cavity tuning device.

The material of the optical wedge is K9 glass, the refractive index of which is 1.514 at the wavelength of 632.8 nm. The refractive index of air is between $1+2.5938\times10^{-3}$ and $1+2.6825\times10^{-3}$ under the temperature of 25 °C.[13] We take 1.000 in the calculation because the fractional part has little effect on the result. The wedge angle $\theta$ is 30°. Substituting the above parameter into Eqs. (14) and (15), we obtain the following relationship

$$S_1 = 0.329S, \quad S_2 = 0.252S. \tag{16}$$

It can be seen from Eq. (16) that $S_1$ and $S_2$ are proportional to the displacement of $PZT_2$. When triangular voltage is applied on $PZT_2$, the length of both external and sub-external cavities are linearly tuned by $S_1$ synchronously. If $S$ is set to 0 μm–3 μm, $S_1$ and $S_2$ are 0 nm–987 nm, 0 nm–756 nm, respectively. The light spot of the laser is about 1.1 mm, $S_2$ is quite small in comparison with the size of the light spot and it has no influence on the measurement.

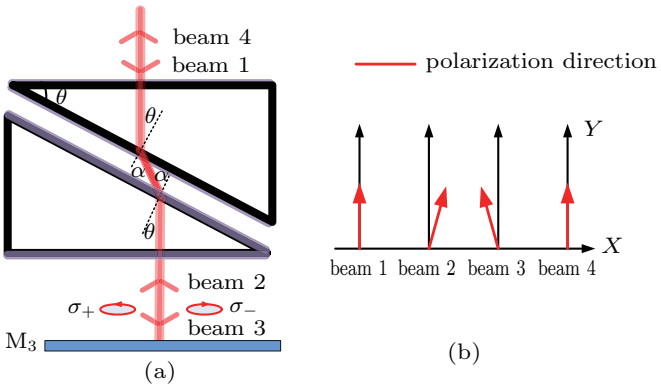



**Fig. 6.** (color online) Spin-dependent splitting phenomena and polarization states.

According to Luo's research,[14,15] the photonic spin-dependent splitting occurs when the refractive index gradient exists for spin Hall effect, so the transmitted beam splits by a fraction of wavelength into its two left- and right-handed circularly polarized components ($\sigma_+$ and $\sigma_-$) after passing through the cavity tuning device. The spin-dependent splitting is restricted to a few tens of nanometers,[16,17] and its scale is much smaller than the radius of laser spot. The circularly polarized components have opposite rotating direction and the superposition of them is linearly polarized, but the polarization direction is deflected by 2° compared with the initial state through our experimental test.

As shown in Fig. 6(a), assuming that the incident light is beam 1, the light passing through the cavity tuning device is beam 2, the light reflected by the feedback mirror is beam 3, and the feedback light emitted from the cavity tuning device is beam 4, the polarization directions of them can be indicated in Fig. 6(b). Above all, the light re-injected into the laser cavity has the same polarization direction with the intra-cavity light field. Therefore, the impact of photonic spin-dependent splitting can be ignored in the SMI system.

## 4. Experiment results

Figures 7(a)–7(c) show the tuning curves when triangular voltage is applied on $PZT_2$. Curves 2 are laser output intensities detected by $D_1$, while curves 3 are polarization states curves detected by $D_2$. Figure 7(a) shows that when there is no wave plate, the output intensity of the laser is modulated in the form of cosine, and each modulation period corresponds to 1/2 wavelength change of the external cavity length; when the wave plate is inserted in the external cavity, the polarization state of the laser flips from the initial direction to its orthogonal direction, as shown in Figs. 7(b) and 7(c). When the retardation changes, the flipping point changes accordingly, as shown in curves 3. *AC* and *ac* are modulation periods; *B* and *b* are polarization flipping points for different wave plates.

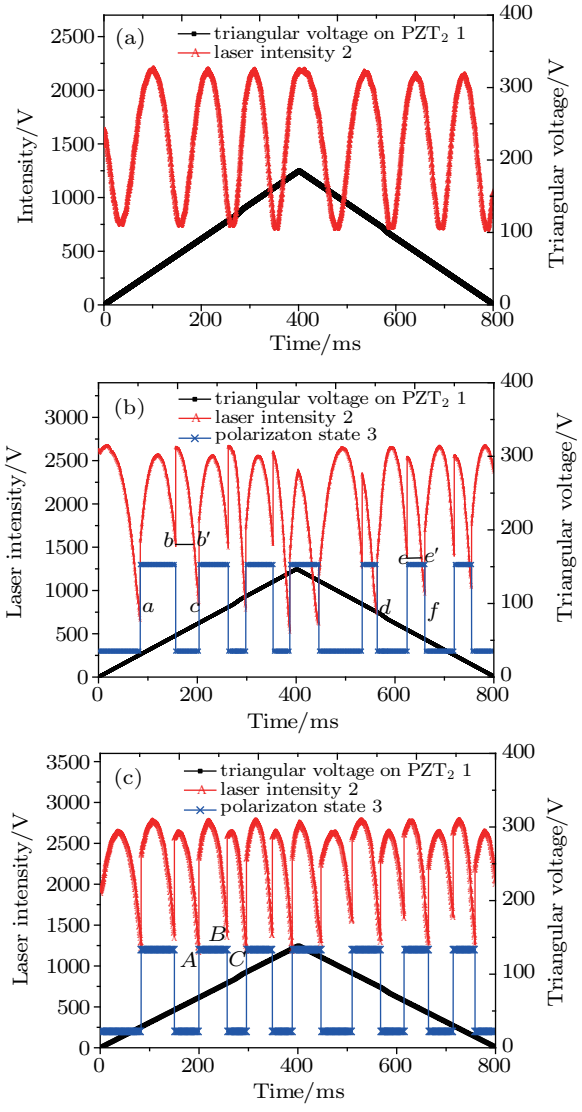



**Fig. 7.** (color online) Intensity tuning curves of the laser SMI system.

According to the equivalent cavity of a Fabry–Perot interferometer, reflectivities of laser in $X$ and $Y$ directions can be expressed as

$$R_{\text{eff}}^{1} = R_2 + 2R_2(1-R_2)(1-R_3)^{1/2}\cos(2kl),$$
$$R_{\text{eff}}^{2} = R_2 + 2R_2(1-R_2)(1-R_3)^{1/2}\cos(2kl+2\delta). \quad (17)$$

As effective gain of the laser is proportional to the equivalent reflectivity, the intensity of the laser is sinusoidally modulated. The phase retardation of the wave plate is half the phase difference of the two polarization states. The phase retardation $\sigma$ can be obtained by the equation

$$\delta = \frac{\varphi}{2} = \frac{bb'}{ac} \times \pi, \quad (18)$$

where points $b$ and $b'$ are a pair of isocandela points and $bb'$ denotes the time interval between the isocandela points, so as $ac$.

$S_1$ displays nonlinearity for the nonlinearity of $PZT_2$ and non-parallelism of the glass wedges, which is the major error source. We measure the nonlinearity of $S_1$ by a heterodyne interferometer with accuracy of sub-nanometer.[18] The measuring principle diagram is shown in Fig. 8. The output of the laser is an orthogonal polarized laser with the frequency of $f_1$ and $f_2$, light is split into two parts by a beam splitter (BS). One part passes through the polarizer (P) and generates reference signal $\Delta f$, which is detected by detector $D_1$. The other part is split into $f_1$ and $f_2$ by polarizing beam splitter (PBS). $f_1$ is reflected by measuring corner prism($CCR_2$), $f_2$ is reflected by reference corner prism($CCR_1$) and then merges in PBS with $f_1$. When $PZT_2$ is moving with the speed of $v$, Doppler frequency shift $\pm f_D$ is generated in the measuring optical path. Measuring signal $\Delta f \pm f_D$ is detected by detector $D_2$. By the comparison of reference signal and measuring signal, the change of optical path can be obtained.

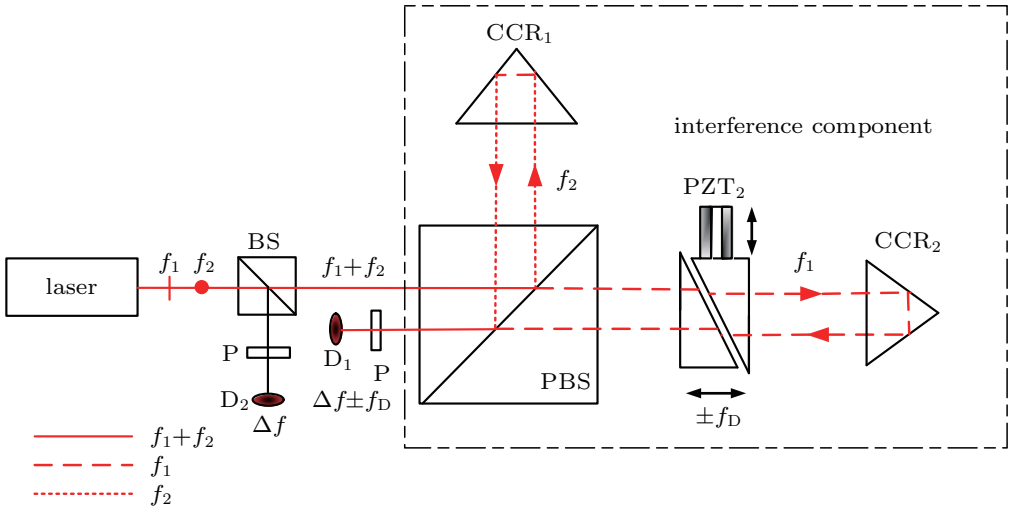



**Fig. 8.** (color online) Measurement diagram.

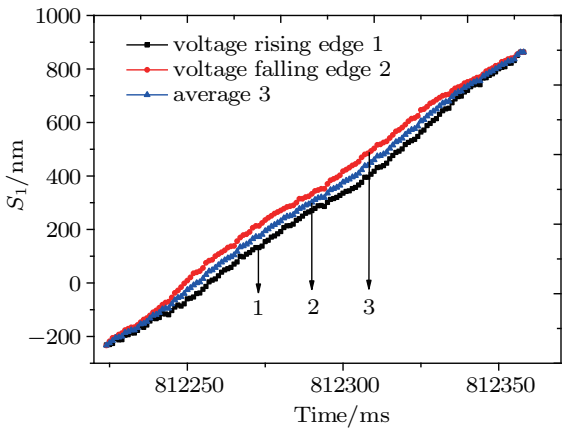



**Fig. 9.** (color online) Displacement nonlinearity of $S_1$.

The measurement result is shown in Fig. 9. It can be found that the displacement nonlinearity of $S_1$ differs from the rising edge and falling edge of the driving voltage. If the average of the displacement–voltage curve is taken for the rising and falling edges, the nonlinearity is reduced.

Referring to Fig. 7, equation (18) can thus be written as follows:

$$\delta = \frac{\varphi}{2} = \left(\frac{bb'}{ac} + \frac{ee'}{df}\right) \times \frac{\pi}{2}. \quad (19)$$

The phase retardation of a quartz wave plate is measured repeatedly and the results are shown in Table 1. Table 1 shows that the repeatability of the system is 0.16°.

**Table 1.** Repeatability of improved system (deg).

| Ordinal | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Result/(°) | 36.202 | 36.141 | 36.193 | 36.097 | 36.042 |

Five non-coated wave plates with different orders and phase retardations are measured with the SMI system and el-

lipsometer for comparison, the measurement accuracy of the ellipsometer was better than 0.1°, and the error of laser SMI system is in the range of $-0.435° \sim 0.387°$, as shown in Table 2.

**Table 2.** Comparison result (deg).

| Number | Order | SMI/(°) | Ellipsometer/(°) | Error/(°) |
|---|---|---|---|---|
| 1 | 16 | 36.135 | 36.397 | 0.262 |
| 2 | 14 | 48.952 | 49.321 | 0.369 |
| 3 | 5 | 62.481 | 62.868 | 0.387 |
| 4 | 16 | 105.071 | 104.823 | –0.248 |
| 5 | 13 | 110.541 | 110.106 | –0.435 |

## 5. Conclusion

In this paper, a self-consistent model for the laser operation with double external cavities is established for a self-mixing interference laser wave plate measurement system. According to the conclusion, a synchronous tuning method of external cavity and sub-external cavity is proposed to eliminate the disturbance of the sub-external cavity. Experiments are carried out on the synchronously tuning feedback system. Compared with the ellipsometer, the error of the system is in the range of $-0.435° \sim 0.387°$ with repeatability of 0.16°.

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