ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

Destroying MTZ black holes with test particles

Yu Song(宋宇), Hao Tang(唐浩), De-Cheng Zou(邹德成), Cheng-Yi Sun(孙成一), Rui-Hong Yue(岳瑞宏)
Citation: Chin. Phys. B . 2018, 27(2): 020401. **doi:** 10.1088/1674-1056/27/2/020401

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Gravitational quasi-normal modes of static R^2 Anti-de Sitter black holes

Hong Ma(马洪), Jin Li(李瑾)
Chin. Phys. B . 2017, 26(6): 060401. **doi:** 10.1088/1674-1056/26/6/060401

Singularities of noncompact charged objects

M. Sharif, G. Abbas
Chin. Phys. B . 2013, 22(3): 030401. **doi:** 10.1088/1674-1056/22/3/030401

with electric charge and magnetic charge

Xie Zhi-Kun, Pan Wei-Zhen, Yang Xue-Jun
Chin. Phys. B . 2013, 22(3): 039701. **doi:** 10.1088/1674-1056/22/3/039701

Scattering and absorption of particles by a black hole involving a global monopole

Shao Jian-Zhou,Wang Yong-Jiu
Chin. Phys. B . 2012, 21(4): 040404. **doi:** 10.1088/1674-1056/21/4/040404

Destroying MTZ black holes with test particles*

Yu Song(宋宇)[1,2], Hao Tang(唐浩)[1,2], De-Cheng Zou(邹德成)[3], Cheng-Yi Sun(孙成一)[1,2], and Rui-Hong Yue(岳瑞宏)[1,3,†]

[1] *Institute of Modern Physics, Northwest University, Xi'an* 710069, *China*
[2] *Shaanxi Key Laboratory for Theoretical Physics Frontiers, Northwest University, Xi'an* 710069, *China*
[3] *College of Physics and Technology, Yangzhou University, Yangzhou* 225009, *China*

(Received 19 April 2017; revised manuscript received 23 October 2017; published online 5 January 2018)

Neglecting the self-force and radiative effects, we follow the spirit of Wald's gedanken experiment and discuss whether a (2+1)-dimensional Martinez–Teitelboim–Zanelli (MTZ) black hole can turn into a naked singularity by capturing a charged and massive particle. We find that after capturing a charged and massive test particle, an extremal or near-extremal MTZ black hole could turn into naked singularity, leading to a possible violation of the cosmic censorship. There exist ranges of the test particles' energies ΔE which allow the appearance of naked singularities from both extremal and near extremal MTZ black holes.

Keywords: cosmic censorship, MTZ black hole, singularity

PACS: 04.20.Dw, 04.70.Bw, 97.60.Lf **DOI:** 10.1088/1674-1056/27/2/020401

1. Introduction

As demonstrated by the elegant theorems of Hawking and Penrose,[1] spacetime singularities are inevitable in gravitational collapse. These results do not rely on specific solutions of Einstein's equations or special symmetry requirements. The utility of general relativity in describing gravitational phenomena is maintained by cosmic censorship which is first proposed by Penrose.[2] According to the hypothesis, all physical singularities due to gravitational collapse are hidden behind an event horizon. This hypothesis, which implies that no naked singularity occurs in our universe. One way of testing the cosmic censorship is to throw a test particle into a black hole, if the particle is captured by the black hole and changes the parameters of the black hole such that the horizon disappears and naked singularity appears, then the black hole is destroyed by the test particles. In 1974, Wald showed that it is impossible to destroy the horizon of an extremal Kerr–Newman black hole by capturing a test particle.[3] However, in Ref. [4], Gao and Zhang showed that, if the linear approximation of the particle's energy is not taken into account, it is possible to destroy an extremal Kerr–Newman black hole with a test particle. In Ref. [5] Saa *et al.* showed it is also possible to destroy the event horizon if we push a test particle with electric charge and angular momentum towards the near-extremal Kerr–Newman black hole. This issue has been widely studied in recent years.[6–19]

The research mentioned above are all focused on the (3+1)-dimensional or higher dimensional black hole, we have many (2+1)-dimensional black hole solutions of different gravity models since Banãdos, Teitelboim and Zanelli (BTZ) discovered a black hole solution to the Einstein–Maxwell equation in (2+1)-dimensions in 1992,[20] e.g, Peldan electrostatic solution and MTZ charged rotating black hole in three spacetime dimensions.[21]

In the present paper, by neglecting the self-force and radiative effects, we investigated whether an MTZ black hole can turn into a naked singularity or not when it captures a charged particle. We find that the black holes could be destroyed by capturing a test particle both in extremal and near-extremal cases. In Section 4, similar discussions will be extended to a near-extremal MTZ black hole. Finally, we give a discussion and our conclusion in Section 5. In this paper we use the unit system where $c = G = \hbar = 1$.

2. Martinez–Teitelboim–Zanelli black hole and charged test particle

Martinez–Teitelboim–Zanelli solution,[22,23] is defined by the metric

$$\begin{aligned} &\mathrm{d}s^2 = -N(\rho)^2\mathrm{d}t^2 + \frac{1}{L(\rho)^2}\mathrm{d}\rho^2 + K(\rho)^2[\mathrm{d}\phi + W(\rho)\mathrm{d}t]^2, \\ &N(\rho) = \rho\frac{L(\rho)}{\sqrt{H(\rho)}}, \quad K(\rho) = \sqrt{H(\rho)}, \end{aligned} \tag{1}$$

where the functions $H(\rho)$, $L(\rho)$, and $W(\rho)$ are given by

$$\begin{aligned} H(\rho) &= \rho^2 + \frac{l^2\omega^2}{l^2-\omega^2}\left(M + \frac{Q^2}{4}\ln\rho^2\right), \\ L(\rho) &= \sqrt{\frac{\rho^2}{l^2} - M - \frac{Q^2}{4}\ln\rho^2}, \\ W(\rho) &= -\frac{\omega l^2}{l^2-\omega^2}\frac{1}{H(\rho)}\left(M + \frac{Q^2}{4}\ln\rho^2\right), \end{aligned} \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11275099, 11435006, and 11405130) and the Double First-Class University Construction Project of Northwest University.

†Corresponding author. E-mail: rhyue@yzu.edu.cn

© 2018 Chinese Physical Society and IOP Publishing Ltd

where M and Q are the mass and charge of the black hole, respectively, l is related to the cosmological constant, and ω is the angular velocity of the boost. This solution can also be derived from the rotating under $SL(2,R)$ transformations-Peldan solution with[22]

$$t=\frac{1}{\sqrt{1-\omega^2/l^2}}(t-\omega\phi),\quad \phi=\frac{1}{\sqrt{1-\omega^2/l^2}}\left(\phi-\frac{\omega}{l^2}t\right). \tag{3}$$

The electromagnetic field of the MTZ black hole has the vector potential

$$A=-\frac{Q}{2\sqrt{1-\omega^2/l^2}}\ln\rho\left(\mathrm{d}t-\omega l^2\,\mathrm{d}\phi\right). \tag{4}$$

In a general curved background, the Lagrangian of a massive and charged particle can be expressed as

$$\mathcal{L}=\frac{1}{2}mg_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta+qA_\mu\dot{x}^\mu, \tag{5}$$

which yield the equation of motion of a particle[24]

$$\ddot{x}^\mu+\Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta=\frac{q}{m}F^{\mu\nu}\dot{x}_\nu, \tag{6}$$

where the "dot" stands for the derivative with respect to some affine parameter s, q, and m are the electric charge and mass of the particle respectively.

Accordingly, in a stationary background, the energy E and angular momentum L as the constants of a particle's motion are given by

$$\begin{aligned}E&=-\frac{\partial\mathcal{L}}{\partial\dot{t}}=-m\left(g_{tt}\dot{t}+g_{t\phi}\dot{\phi}\right)-qA_t,\\ L&=\frac{\partial\mathcal{L}}{\partial\dot{\phi}}=m\left(g_{t\phi}\dot{t}+g_{\phi\phi}\dot{\phi}\right)+qA_\phi.\end{aligned} \tag{7}$$

From the two equations above, using timelike condition $\dot{x}_\mu\dot{x}^\mu=-1$ for the particle, the conserved energy of the test particle is given as[3]

$$E=\frac{g_{t\phi}}{g_{\phi\phi}}\left(qA_\phi-L\right)-qA_t\pm\sqrt{\left(\frac{g_{t\phi}^2-g_{\phi\phi}g_{tt}}{g_{\phi\phi}^2}\right)\left(\left(L-qA_\phi\right)^2+m^2g_{\phi\phi}\left(1+g_{\rho\rho}\dot{\rho}^2\right)\right)}. \tag{8}$$

The above conserved energy is defined at infinity, which assures the observer at the event horizon and infinity is equivalent. Equation (8) is dictated by the requirement that the $\dot{t}>0$ for all the future directed timelike motions outside the horizon. Therefore, we should take the plus sign in front of the square root of Eq. (8). Substituting the corresponding components of $g_{\mu\nu}$ of Eq. (1) in the square root of Eq. (8), we can see the expression in the square root of Eq. (8) will equal to zero when the extremal condition is satisfied (i.e. $L(\rho)^2=\rho^2/l^2-M-Q^2/4\ln\rho^2=0$), consequently, we obtain the minimum energy which allows the test particle to reach the event horizon.

$$E_{\min}=\frac{g_{t\phi}}{g_{\phi\phi}}qA_\phi-qA_t. \tag{9}$$

Substituting the corresponding components of $g_{\mu\nu}$ of Eq. (1) into Eq. (9) which is followed by $\rho=\rho_{\mathrm{h}}$, we gain the minimum energy that allows the test particle to reach the event horizon.

$$E_{\min}=\frac{qQ\rho_{\mathrm{h}}(4l^4M\omega^2+4\rho_{\mathrm{h}}^2\omega^2-4l^2\omega^2-4M\omega^2l^2+l^2(l^2-1)Q^2\omega^2\ln\rho_{\mathrm{h}}^2)}{2\sqrt{1-\omega^2/l^2}(4l^2\rho_{\mathrm{h}}^2+4Ml^2\omega^2-4\rho_{\mathrm{h}}^2\omega^2+l^2Q^2\omega^2\ln\rho_{\mathrm{h}}^2)}, \tag{10}$$

where ρ_{h} is the radius of the event horizon, so we could see there exist a lower bound for the particle's energy which could reach the black hole's horizon, that is $E\geq E_{\min}$. In the next two sections, we will examine the two cases for the extremal and near-extremal black holes.

3. The extremal case

From Eq. (1), an MTZ black hole becomes extremal if the roots of $L(\rho)^2$ are equal. For the sake of simplicity, we define

$$F(\rho)=L(\rho)^2=\frac{\rho^2}{l^2}-M-\frac{Q^2}{4}\ln\rho^2. \tag{11}$$

From the above function, we can see if $\rho\to+\infty$, then $F(\rho)\to+\infty$, and if $\rho\to0$, then $F(\rho)\to+\infty$. By using $\partial F(\rho)/\partial\rho_{\mathrm{h}}=0$, we obtain a unique extremal point. Combining with the extremal condition $F(\rho)=0$, where the outer horizon and inner horizon coincide with each other, we obtain the event horizon and extremal mass of the MTZ black hole

$$\rho_{\mathrm{h}}=\frac{lQ}{2},\quad M_{\mathrm{ex}}=\frac{Q^2}{4}-\frac{Q^2}{2}\ln\frac{lQ}{2}. \tag{12}$$

Then we can obtain the minimum energy for a particle to pass through the event horizon of an extremal MTZ black hole

$$E_{\min-\mathrm{ex}}=\frac{qQ^3(\omega^2-l^2)\ln lQ/2}{2\sqrt{1-\omega^2/l^2}[l^2Q^2+4M\omega^2-Q^2\omega^2+2Q^2\omega^2\ln lQ/2]}. \tag{13}$$

For clarification, in the extremal case, we define Eq. (11) as

$$F_{\rm ex}(\rho)=\frac{\rho^2}{l^2}-M_{\rm ex}-\frac{Q^2}{4}\ln\rho^2. \tag{14}$$

Now we consider a test particle with energy E, electric charge q captured by an MTZ black hole. After capturing a test particle, the black hole can be described by the changed parameters.[24]

$$M'=M+E\,,\quad Q'=Q+q\,,\quad L'=L_{\rm b}, \tag{15}$$

where the prime $'$ is used to describe the black hole parameters after capturing a test particle, M and Q are the physical mass, physical charge, and $L_{\rm b}$ is the angular momentum after the rotation boost, respectively. The parameter can be expressed by the following expression

$$\begin{aligned}M&=\frac{1}{1-\omega^2}\left((1+\omega^2)\widetilde{M}-\frac{\omega^2\widetilde{Q}^2}{2}\right),\\ Q&=\frac{\widetilde{Q}}{\sqrt{1-\omega^2}}\,,\quad L_{\rm b}=\frac{2\omega}{1-\omega^2}\left(\widetilde{M}-\frac{\widetilde{Q}^2}{4}\right).\end{aligned} \tag{16}$$

where $\widetilde{M}$ and $\widetilde{Q}$ are "static mass" and "static charge" without the rotation boost. After capturing a test particle, the new extremal condition can be written formally as

$$F'_{\rm ex}(\rho)=\frac{\rho^2}{l^2}-M'-\frac{Q'^2}{4}\ln\rho^2=0\,. \tag{17}$$

If the extremal MTZ black hole turns into a naked singularity after absorbing a test particle, $F'_{\rm ex}(\rho)$ will increase everywhere and the degenerate root will disappear, and the condition $F'_{\rm ex}(\rho)>0$ must be satisfied, so we have

$$M'<\frac{\rho^2}{l^2}-\frac{Q'^2}{4}\ln\rho^2. \tag{18}$$

Using Eq. (15) and Eq. (18), we obtain the following equation

$$E<\frac{\rho^2}{l^2}-\frac{Q'^2}{4}\ln\rho^2-M_{\rm ex}. \tag{19}$$

Obviously, it shows that if an MTZ black hole is destroyed, there is an upper bound for the test particle's energy, which could destroy the black hole's horizon. By using Eqs. (12) and (15), we obtain the maximum energy of a test particle to destroy an extremal MTZ black hole as

$$E_{\rm max-ex}=\frac{Q'^2}{4}-\frac{Q'^2}{4}\ln\frac{l^2Q'^2}{4}-M_{\rm ex}. \tag{20}$$

As a result, if the test particle's energy is between $E_{\rm min-ex}$ and $E_{\rm max-ex}$, i.e.

$$E_{\rm min-ex}<E<E_{\rm max-ex}. \tag{21}$$

The black hole can be destroyed by capturing the test particle. For clarification, we define

$$\Delta E=E_{\rm max-ex}-E_{\rm min-ex}. \tag{22}$$

Notice that the test particle approximation requires $E/M\ll 1$, and $e/Q\ll 1$, assuring, in this way, that backreaction effects are negligible. From Eq. (13), we can see that the two conditions must be satisfied, i.e., $\omega^2<l^2$, and $0<lQ<2$. Here we assume l, Q, and ω are positive. In order to find the range of the test particle's energy to destroy the black hole, we choose static charge $\widetilde{Q}=1.5$, with $l=1$, $\omega=0.05$, then according to Eq. (12), we gain $\rho_{\rm h}=0.750939$, $M_{\rm ex}=0.8869517$, where $\rho_{\rm h}$ and $M_{\rm ex}$ are the event horizon and the mass of the extremal MTZ black hole respectively. Based on the choosing parameters and Eqs. (13) and (20), we obtain the results as shown in Fig. 1.

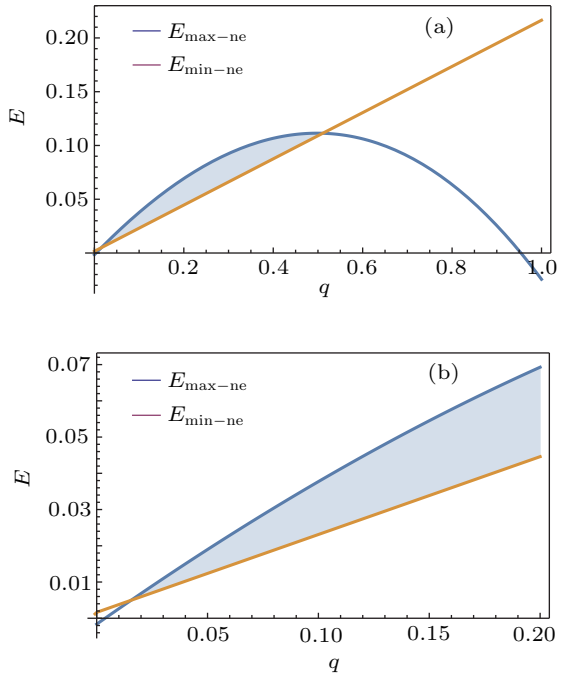


Fig. 1. (color online) Plot of E versus q. (a) Plot of E–q, where $\omega=0.05$, $0\leqslant q\leqslant 1$. (b) Plot of E–q, where $\omega=0.05$, $0\leqslant q\leqslant 0.2$. The shaded area is the intersection of $E_{\rm min-ex}<E$ and $E<E_{\rm max-ex}$.

In Fig. 1, the shaded area is the intersection of $E_{\rm min-ex}<E$ and $E<E_{\rm max-ex}$. If the particle's energy is in the shaded area, the black hole can be destroyed by the particle, e.g., if we choose $q=0.03\ll Q$, then we obtain $E_{\rm min-ex}=0.00807564$, $E_{\rm max-ex}=0.0109551$, and $\Delta E=0.00287946$. To prove the event horizon disappeared, we choose $E=0.009$, using the same parameters as mentioned above and Eq. (17), we plot the curve of $F'(\rho)$–ρ in Fig. 2. As is shown in Fig. 2, the solution of $F'(\rho)=0$ does not exist anymore. This implies that the horizon of the black hole vanishes and the naked singularity appears. On the other hand, if we choose $E=0.012$, which is larger than $E_{\rm max-ex}$, using the same parameters as in Fig. 2, we obtain the rsults drawn in Fig. 3. We can see there are two roots of $F'(\rho)=0$. This means that after capturing a test particle, the extremal black hole will have inner and outer horizons, thus it turns into a common black hole. Figures 2 and 3 show that the MTZ black hole will be destroyed by absorbing a test particle whose energy E and electric charge q are located

in the shaded area shown in Fig. 1. Next, we will show that a particle from far away, whose energy between $E_{\min-\mathrm{ex}}$ and $E_{\max-\mathrm{ex}}$ could really fall into the black hole. Since the metric is axisymmetric, there exist orbits lying entirely in the plane $\theta=\pi/2$, so the test particle can fall along the orbit. For such an orbit, one can solve Eq. (8) for $\dot{\rho}^2$,[4] and obtain

$$\dot{\rho}^2=-V(\rho), \tag{23}$$

where the effective potential $V(\rho)$ is given by

$$\begin{aligned}V(\rho)=&\frac{1}{4l^2m^2\rho^2(l^2-\omega^2)\sqrt{1-\omega^2/l^2}}\\&\times(4l^2qQ\rho^2(\omega^2-l^2)E\ln\rho\\&+l^2q^2Q^2\rho^2(\omega^2-l^2)\sqrt{1-\omega^2/l^2}\ln^2\rho\\&-\sqrt{1-\omega^2/l^2}(4m^2\rho^4\omega^2-4l^2m^2\rho^4-4l^2\rho^2m^2M\omega^2\\&-4l^2r^2\omega^2E^2+4l^4m^2M\rho^2+4l^4\rho^2E^2\\&+4l^4M\omega^2E^2+(l^4Q^2m^2\rho^2+l^4Q^2\omega^2E^2\\&-m^2l^2Q^2\rho^2\omega^2)\ln\rho^2)).\end{aligned} \tag{24}$$

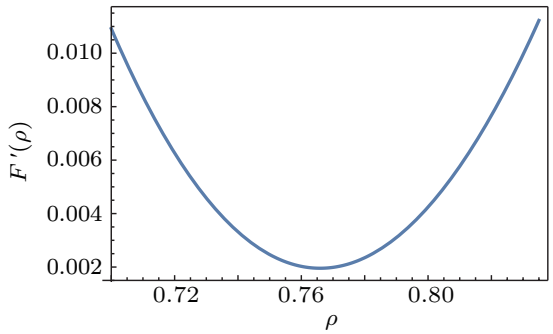


Fig. 2. (color online) Plot of $F'_{\mathrm{ex}}(\rho)$. There is no point of the intersection on the horizontal axis, which means that there is no horizon anymore.

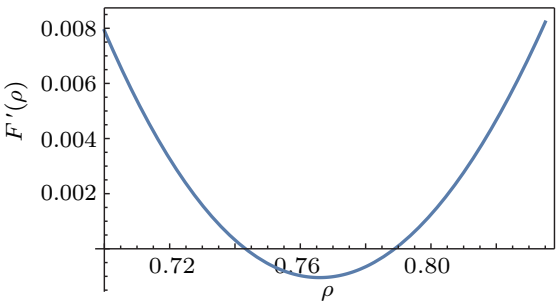


Fig. 3. (color online) Plot of $F'_{\mathrm{ex}}(\rho)$. There are two points of intersection on the horizontal axis. The left point stands for the inner horizon and the right one stands for the outer horizon. The extreme black hole becomes a common black hole.

We still choose $\widetilde{Q}=1.5$, $l=1$, $\omega=0.05$ as mentioned above, and $m=E=0.009$ such that E is in the allowed range, the results are shown in Fig. 4. Figure 4 shows that the curve is descending and the particle is easy to fall into an MTZ black hole. The choice $m=E$ indicates that the particle stays at rest relative to a stationary observer at infinity, so it is realizable in practice.

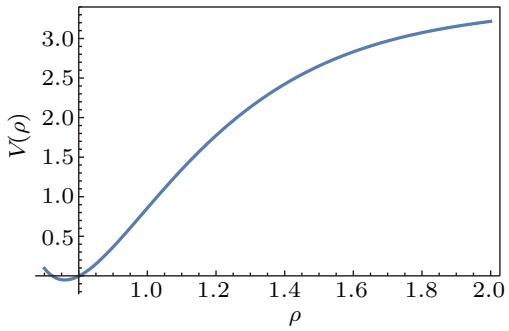


Fig. 4. (color online) The effective potential $V(\rho)$ of a test particle outside the event horizon with $0.7<\rho<2$. test particle's energy $E_{\min}<E<E_{\max}$.

4. Near-extremal case

As discussed in Section 3, if an MTZ black hole is an extremal black hole, equation (14) should be satisfied. Now we consider a near-extremal condition, indicated by

$$\frac{\rho^2}{l^2}-M_{\mathrm{ex}}-\frac{Q^2}{4}\ln\rho^2=\delta. \tag{25}$$

We rewrite the above equation as

$$F_{\mathrm{ne}}(\rho)=\frac{\rho^2}{l^2}-M_{\mathrm{ex}}-\frac{Q^2}{4}\ln\rho^2-\delta=F_{\mathrm{ex}}(\rho)-\delta, \tag{26}$$

where the function $F_{\mathrm{ne}}(\rho)$ is defined to describe the near extremal case. consequently, we have

$$M_{\mathrm{ne}}=M_{\mathrm{ex}}+\delta=\frac{\rho^2}{l^2}-\frac{Q^2}{4}\ln\rho^2, \tag{27}$$

where M_{ne} is the mass of a near-extremal MTZ black hole, and δ is positive and much smaller than the mass of extremal MTZ black hole, i.e., $\delta\ll M_{\mathrm{ex}}$. From Eq. (26), we can easily see that if $\delta>0$, $F_{\mathrm{ne}}(\rho)$ would have two different roots, the two roots correspond to the outer and inner horizon of the black hole. If we choose the same parameters as mentioned above, i.e., $\widetilde{Q}=1.5$, $l=1$, $\omega=0.05$, $M_{\mathrm{ex}}=0.8869517$, and $\delta=0.001$. By solving the equation $F_{\mathrm{ne}}(\rho)=0$, we gain the outer horizon of a near extremal black hole as

$$\rho_+=0.759628. \tag{28}$$

Putting ρ_+ into Eq. (10), we obtain $E_{\min}=0.00085872$. According to Eqs. (15), (26), if a near-extremal MTZ black hole captures a test particle, we have

$$\begin{aligned}F'_{\mathrm{ne}}(\rho)&=\frac{\rho^2}{l^2}-\frac{(q+Q)^2}{4}\ln\rho^2-M'_{\mathrm{ne}}\\&=\frac{\rho^2}{l^2}-\frac{(q+Q)^2}{4}\ln\rho^2-M_{\mathrm{ex}}-E-\delta.\end{aligned} \tag{29}$$

If the black hole is destroyed after capturing a test particle, as in the extremal black hole case, the following condition must be satisfied

$$F'_{\mathrm{ne}}(\rho)>0. \tag{30}$$

Using the above equation and Eqs. (26), (29), and (30), we have

$$E < \frac{\rho^2}{l^2} - \frac{(Q+q)^2}{4}\ln\rho^2 - M_{\rm ex} - \delta = E_{\rm max-ex} - \delta. \quad (31)$$

As a result, we obtain the upper bound on the test particle's energy to destroy the black hole

$$E_{\rm max-ne} = E_{\rm max-ex} - \delta. \quad (32)$$

By using the same parameters as mentioned in the extremal case, i.e., $\widetilde{Q} = 1.5, l = 1, \omega = 0.05$, with δ chosen to be 0.001, we provide Fig. 5, which shows there is a range of q that allows the test particle's energy between $E_{\rm min-ne}$ and $E_{\rm max-ne}$ to have the ability to destroy the black hole. If we chose $q = 0.03 \ll Q$, then we get $E_{\rm min-ne} = 0.00917139$, $E_{\rm max-ne} = 0.0099551$ and $\Delta E = 0.00078371$.

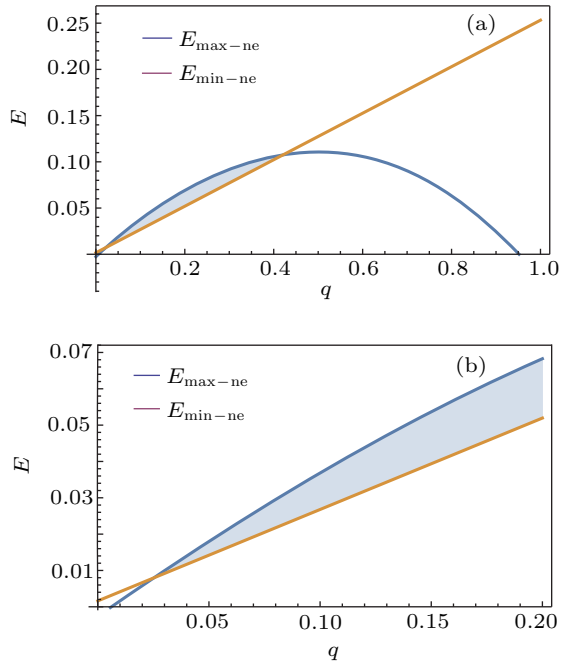


Fig. 5. (color online) Plot of E versus q, in the near-extremal case: (a) $\omega = 0.05$, $\delta = 0.001$, $0 \leqslant q \leqslant 2$; (b) $\omega = 0.05$, $\delta = 0.001$, $0 \leqslant q \leqslant 0.001$. The shaded area is the intersection of $E_{\rm min-e} < E$ and $E < E_{\rm max-e}$.

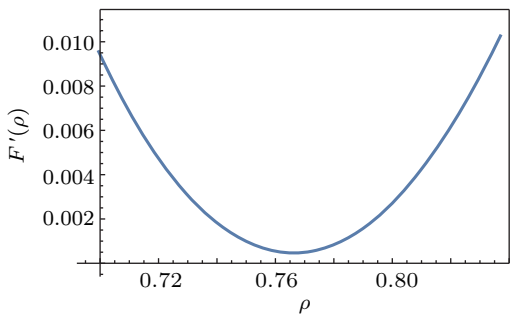


Fig. 6. (color online) Plot of $F'_{\rm ne}(\rho)$ versus ρ with $\delta = 0.001$ and $q = 0.03$. There is no point of the intersection on the horizontal axis, which means that there is no horizon anymore.

To prove the black hole has been destroyed, we chose $E = 0.0095$ with $q = 0.03$, which is just between $E_{\rm min-ne}$ and $E_{\rm max-ne}$. Using Eq. (29) and the same parameters as mentioned in Fig. 5, we draw Fig. 6. From Fig. 6, we can see the solution of $F'(\rho) = 0$ does not exist anymore. This implies that the horizon of the black hole vanishes and the naked singularity appears. We should notice that δ will shift the curve graph, that is, if we choose $E = 0.0095$, we obtain $F'_{\rm ne}(\rho) = 0.00271068 - \delta$. If the black hole is destroyed by the test particle, the condition $F'_{\rm ne}(\rho) > 0$ must be satisfied, we need $\delta < 0.00271068$. So the value of $\delta = 0.001$ we choose is appropriate.

To show there is a range of test particle's energy, ΔE, which allows the disappearances of event horizons from near-extremal MTZ black hole, we provide Fig. 7, which shows the plots of $E_{\rm min}$ and $E_{\rm max}$ where the choices of numerical values are $l = 1$, $\widetilde{Q} = 1.5$, $M = 0.8869517$, $\omega = 0.05$, $q = 0.03$, with δ chosen to be 0.001. We find that the shaded area in Fig. 7, which represents the dependency of ΔE with respect to the parameter of near-extremality δ, becomes narrower as δ increases. This means the black hole becomes harder to destroy when it moves away from the extremality.

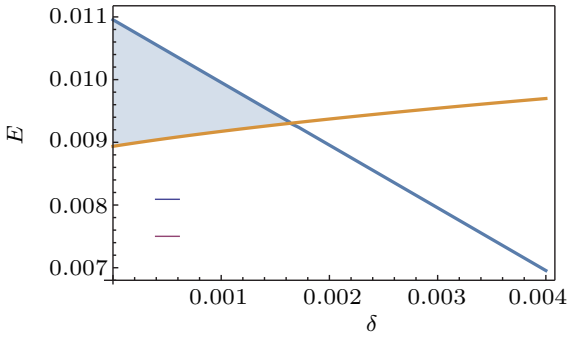


Fig. 7. (color online) Plot of E versus δ, where $\omega = 0.05$, $0 \leqslant \delta \leqslant 0.004$. The shaded area is the intersection $E_{\rm min-e} < E$ and $E < E_{\rm max-e}$.

By using the same method as in the extremal case, we obtain the corresponding effective potential $V(\rho)$ for the test particle in the near-extremal case

$$\begin{aligned} V(\rho) = & \frac{1}{4l^2m^2\rho^2(l^2-\omega^2)\sqrt{1-\omega^2/l^2}} \\ & \times (4l^2qQ\rho^2(\omega^2-l^2)E\ln\rho + l^2q^2Q^2\rho^2(\omega^2-l^2) \\ & \times \sqrt{1-\omega^2/l^2}\ln^2\rho - \sqrt{1-\omega^2/l^2}(4m^2\rho^4\omega^2 \\ & - 4l^2m^2\rho^4 - 4l^2\rho^2m^2(M+\delta)\omega^2 - 4l^2r^2\omega^2E^2 \\ & + 4l^4m^2(M+\delta)\rho^2 + 4l^4\rho^2E^2 \\ & + 4l^4(M+\delta)\omega^2E^2 + (l^4Q^2m^2\rho^2 + l^4Q^2\omega^2E^2 \\ & - m^2l^2Q^2\rho^2\omega^2)\ln\rho^2)). \end{aligned} \quad (33)$$

We still choose $l = 1$, $\widetilde{Q} = 1.5$, $M = 0.8869517$, $\omega = 0.05$, $q = 0.03$, $\delta = 0.001$ as mentioned above, and $m = E = 0.0095$ such that E is between $E_{\rm min-ne}$ and $E_{\rm max-ne}$. Using the above

equation, we plot the effective potential $V(\rho)$ in Fig. 8. Figure 8 shows that the curve is also descending as ρ decreases, this means that the particle is easy to fall into an MTZ black hole from infinity.

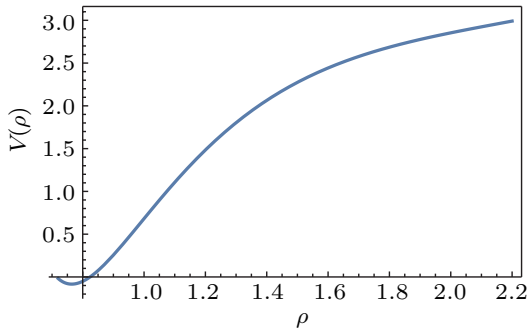


Fig. 8. (color online) The effective potential $V(\rho)$ of a test particle outside an event horizon with $0.7 < \rho < 2$, for test particle energy $E_{\min} < E < E_{\max}$.

5. Conclusions and discussions

We have shown that, without taking into account the radiative and self-force effects, a test particle may destroy the horizon of an MTZ black hole in both extremal and near-extremal cases, resulting in an apparent violation of the cosmic censorship. However, in our work, the ranges of the allowed energies to destroy the black holes are slightly larger, which are different from other works. This means that the MTZ black hole is easier to destroy. In some other works, they take analyticity and contain some high-order terms, while we sacrifice the analyticity and use a numerical method. It should be noted that all of those results come from the precondition that we ignore the self-force and radiative effects.

Acknowledgments

We are extremely grateful to Bin Wu and Ming Zhang for useful discussions.

References

[1] Hawking S W and Penrose R 1970 *Proc. R. Soc. Lond. A* **314** 529
[2] Penrose R 1969 *Revistas del Nuovo Cimento* **1** 252
[3] Wald R M 1974 *Ann. Phys.* **82** 548
[4] Gao S and Zhang Y 2013 *Phys. Rev. D* **87** 044028
[5] Saa A and Santarelli R 2011 *Phys. Rev. D* **84** 027501
[6] Hubeny V E 1999 *Phys. Rev. D* **59** 064013
[7] Hod S 2002 *Phys. Rev. D* **66** 024016
[8] Jacobson T and Sotiriou T P 2009 *Phys. Rev. Lett* **103** 141101
[9] Chirco G, Liberati S and Sotiriou T P 2010 *Phys. Rev. D* **82** 104015
[10] Gwak B and Lee B H 2012 *Class. Quantum Grav.* **29** 175011
[11] Toth G Z 2012 *Gen. Rel. Grav.* **44** 2019
[12] Bouhmadi-Lopez M, Cardoso V, Nerozzi A and Rocha J V 2010 *Phys. Rev. D* **81** 084051
[13] Rocha J V and Cardoso V 2011 *Phys. Rev. D* **83** 104037
[14] Virbhadra K S and Ellis G F R 2002 *Phys. Rev. D* **65** 103004
[15] Li Z and Bambi C 2013 *Phys. Rev. D* **87** 124022
[16] Isoyama S, Sago N and Tanaka T 2011 *Phys. Rev. D* **84** 124024
[17] Bouhmadi-Lopez M, Cardoso V, Nerozzi A and Rocha J V 2010 *Phys. Rev. D* **81** 084051
[18] Semiz I 2011 *Gen. Rel. Grav.* **43** 833
[19] Haryanto and Siahaan M 2016 *Phys. Rev. D* **93** 064028
[20] Banados M, Teitelboim C and Zanelli J 1992 *Phys. Rev. Lett.* **69** 1849
[21] Martinez C, Teitelboim C and Zanelli J 2000 *Phys. Rev. D* **61** 104013
[22] Garcia A A 2009 *Ann. Phys.* **324** 2004
[23] Alberto A Garcia-Diazm 2013 *Physics* arxiv: 1307.6652 [gr-qc]
[24] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Francisco: W H Freeman and Company San Francisco) p. 1279