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A local energy-preserving scheme for Zakharov system*

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In this paper, we propose a local conservation law for the Zakharov system. The property is held in any local time-space region which is independent of the boundary condition and more essential than the global energy conservation law. Based on the rule that the numerical methods should preserve the intrinsic properties as much as possible, we propose a local energy-preserving (LEP) scheme for the system. The merit of the proposed scheme is that the local energy conservation law can be conserved exactly in any time-space region. With homogeneous Dirchlet boundary conditions, the proposed LEP scheme also possesses the discrete global mass and energy conservation laws. The theoretical properties are verified by numerical results.

Keywords: Zakharov system, local energy-preserving scheme, global mass and energy conservation laws

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1. Introduction

In this paper, we consider the following one-dimensional Zakharov system (ZS)[1]

$$\begin{cases} \mathrm{i}\phi_t + \phi_{xx} - \phi\psi = 0, \\ \psi_{tt} - \psi_{xx} - |\phi|^2_{xx} = 0, \end{cases} \tag{1}$$

which describes the interaction between high-frequency Langmuir and ion acoustic waves in a plasma. Here, $\mathrm{i} = \sqrt{-1}$, the unknown complex function ϕ denotes the slowly varying envelope of the highly oscillatory electric field, and the unknown real function ψ represents the fluctuation of the ion density about its equilibrium value. With initial and Dirichlet boundary conditions

$$\begin{aligned} &\phi(x,0) = \phi_0(x),\ \psi(x,0) = \psi_0(x),\ \psi_t(x,0) = \psi_1(x), \\ &\phi(a,t) = \phi(b,t) = 0,\ \psi(a,t) = \psi(b,t) = 0, \end{aligned} \tag{2}$$

the Dirichlet initial-value Zakharov equations (1) possess mass and global energy conservation laws (GECL):

$$\begin{aligned} &\mathscr{M}(t) = \int_a^b |\phi(x,t)|^2 = \int_a^b |\phi(x,0)|^2 = \mathscr{M}(0), \\ &\mathscr{E}(t) = \int_{x_L}^{x_R} \left(\frac{1}{2} f_x^2 + \frac{1}{2}\psi^2 + |\phi_x|^2 + |\phi|^2\psi \right) = \mathscr{E}(0), \end{aligned} \tag{3}$$

where $f_{xx} = \psi_t$.

Extensive mathematical and numerical studies have been developed for the Zakharov system during recent decades. Mathematically, the existence and uniqueness, the wellposedness and the regularity of smooth solutions, and the collision of solitons were established by Hadouaj, Bourgain, Colliander, and Fang.[2–6] Numerically, such as the spectral method,[7–11] the symplectic and multi-symplectic method,[12,13] the finite element method,[14] and the finite difference method[15–19] have been carried out for the Zakharov system. Nowadays, structure-preserving methods have been more and more popular and successfully applied to various PDEs.[20–23] To the best of our knowledge, there have been few approaches which can preserve the local energy of the Zakharov system in the literature. This motivates our study to introduce energy-preserving methods for solving the Zakharov system.

As we all know, the GECL (3) depends on the suitable boundary conditions, such as the periodic or homogeneous boundary conditions. However, in practice, the boundary conditions may not be always suitable in many cases. Actually, the Zakharov system admits a local energy conservation law, which is more essential than the global energy conservation law (3) in physics. Let $\phi(x,t) = u(x,t) + \mathrm{i}v(x,t)$, where $u(x,t)$ and $v(x,t)$ are real-valued functions. Introducing some intermediate variables, the Zakharov system (1) can be reformed into a combination of ordinary differential equations

$$\begin{cases} v_t = \alpha - \psi u, \\ u_t = -\beta + \psi v, \end{cases} \tag{4a}$$

$$\begin{cases} f_t = \psi + (u^2 + v^2), \\ \psi_t = m, \end{cases} \tag{4b}$$

$$\begin{cases} p_x = \alpha, \\ u_x = p, \end{cases} \quad \begin{cases} q_x = \beta, \\ v_x = q, \end{cases} \quad \begin{cases} g_x = m, \\ f_x = g. \end{cases} \tag{4c}$$

Proposition 1 The system (4) possesses a local energy conservation law (LECL)

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$$\partial_t\left(|\phi|^2\psi+|\phi_x|^2+\frac{1}{2}f_x^2+\frac{1}{2}\psi^2\right)+\partial_x(-2pu_t-2qv_t-f_tg)=0. \tag{5}$$

Proof In Eq. (4a), multiplying the first and second line by u_t and $-v_t$, respectively, we deduce

$$0=\alpha u_t-\psi uu_t+\beta v_t-\psi vv_t=\partial_x(pu_t+qv_t)-\frac{1}{2}\partial_t|\phi_x|^2-\frac{1}{2}\psi\partial_t|\phi|^2. \tag{6}$$

Next, multiplying the first line of Eq. (4b) by ψ_t yields

$$0=f_t\psi_t-\psi\psi_t-(u^2+v^2)\psi_t=\partial_x(f_tg)-\frac{1}{2}\partial_t f_x^2-\frac{1}{2}\partial_t\psi^2-|\phi|^2\psi_t. \tag{7}$$

Adding Eqs. (6) and (7) leads to Eq. (5).

The LECL (5) is a local property, which is independent of the boundary conditions and more essential than the GECL in physics. Naturally, when constructing numerical algorithms for the Zakharov system (1), we expect that the LECL can still be preserved. Hereafter, we call these numerical methods the local energy-preserving (LEP) algorithms. Note that when the boundary conditions are suitable, such as the periodic or homogeneous boundary conditions, summing the LECL over the space index, the LEP algorithms will become global energy-preserving algorithms. For the concept of the LEP algorithm and its applications, please refer to Refs. [24]– [27] and the references therein. The aim of this paper is to construct an LEP algorithm for the Zakharov system, and then check the performance of the proposed scheme. In order to achieve this goal, we use the mid-point rule and leap-frog rule to discrete the temporal and spatial direction, respectively.

The paper is organized as follows. In Section 2, we construct the LEP algorithm for the Zakharov system. Moreover, we prove that the proposed scheme conserves the discrete local energy, discrete global energy, and mass conservation laws exactly. Numerical simulations of the single solitary propagation and solitons collision are presented in Section 3. Finally, we finish the paper with conclusions in Section 4.

2. The local energy-preserving algorithm (LEP)

Let N be a positive even integer. The domain $\Omega=[a,b]$ is uniformly divided into N subdomains $\Omega_h=\{x_j|x_j=a+jh, 0\le j\le N-1\}$, where $h=(b-a)/N$. Let $V_h=\{u_j|x_j\in\Omega_h\}$ be the space of grid functions on Ω_h. For a positive integer N_t, we denote time step $\tau=T/N_t$, $t_n=n\tau$, $0\le n\le N_t$. We define

$$\delta_t u_j^n=(u_j^{n+1}-u_j^n)/\tau,\quad \delta_x u_j^n=(u_{j+1}^n-u_j^n)/h,$$
$$A_t u_j^n=(u_j^{n+1}+u_j^n)/2,\quad A_x u_j^n=(u_{j+1}^n+u_j^n)/2.$$

According to the discrete Leibnitz rule,[28] we have

$$\delta_x(u\cdot v)_i^j=\left(\varepsilon u_{i+1}^j+(1-\varepsilon)u_i^j\right)\cdot\delta_x v_i^j+\delta_x u_i^j\cdot\left((1-\varepsilon)v_{i+1}^j+\varepsilon v_i^j\right),\quad\forall\ 0\le\varepsilon\le 1.$$

In system (4), discretizing the time and space derivatives by the mid-point rule and leap-frog rule, respectively, we obtain

$$\begin{cases}\delta_t v_j^n=A_t\alpha_j^n-A_t\psi_j^nA_tu_j^n,\\ \delta_t u_j^n=-A_t\beta_j^n+A_t\psi_j^nA_tv_j^n,\end{cases} \tag{8a}$$

$$\begin{cases}\delta_t f_j^n=A_t\psi_j^n+A_t(|\phi_j^n|^2),\\ \delta_t\psi_j^n=A_tm_j^n,\end{cases} \tag{8b}$$

$$\begin{cases}\delta_x p_j^n=\alpha_j^n,\\ \delta_x u_j^n=p_{j+1}^n,\end{cases}\quad\begin{cases}\delta_x q_j^n=\beta_j^n,\\ \delta_x v_j^n=q_{j+1}^n,\end{cases}\quad\begin{cases}\delta_x g_j^n=m_j^n,\\ \delta_x f_j^n=g_{j+1}^n,\end{cases} \tag{8c}$$

where $|\phi_j^n|^2=(u_j^n)^2+(v_j^n)^2$. Eliminating the auxiliary variables, we obtain a full discrete scheme

$$\begin{cases}\mathrm{i}\delta_t\phi_j^n+A_t\delta_x^2\phi_{j-1}^n-A_t\psi_j^nA_t\phi_j^n=0,\\ \delta_t^2\psi_j^n-A_t^2\delta_x^2\psi_{j-1}^n-A_t^2\delta_x^2|\phi_{j-1}^n|^2=0.\end{cases} \tag{9}$$

Theorem 1 The algorithm (8) or (9) satisfies the following discrete local energy conservation law

$$\mathscr{E}(x_j,t_n)=\delta_t\left(|\phi_j^n|^2\psi_j^n+|\delta_x\phi_j^n|^2+\frac{1}{2}(\delta_x f_j^n)^2+\frac{1}{2}(\psi_j^n)^2\right)+\delta_x\left(-2A_tp_j^n\delta_tu_j^n-2A_tq_j^n\delta_tv_j^n-\delta_tf_j^nA_tg_j^n\right)=0. \tag{10}$$

Proof In Eq. (8a), multiplying the first and second line by $\delta_t u_j^n$ and $-\delta_t v_j^n$, respectively, and then adding them together yields

$$\begin{aligned}0&=A_t\delta_xp_j^n\cdot\delta_tu_j^n+A_t\delta_xq_j^n\cdot\delta_tv_j^n-\frac{1}{2}A_t\psi_j^n\cdot\delta_t|\phi_j^n|^2\\&=\delta_x\left(A_tp_j^n\cdot\delta_tu_j^n+A_tq_j^n\cdot\delta_tv_j^n\right)-\frac{1}{2}\delta_t(p_{j+1}^n)^2-\frac{1}{2}\delta_t(q_{j+1}^n)^2-\frac{1}{2}A_t\psi_j^n\cdot\delta_t|\phi_j^n|^2.\end{aligned} \tag{11}$$

Analogously, we obtain

$$0=\delta_x(\delta_t f_j^n\cdot A_t g_j^n)-A_t|\phi_j^n|^2\cdot\delta_t\psi_j^n-\delta_t\left(\frac{1}{2}(\delta_x f_j^n)^2+\frac{1}{2}(\psi_j^n)^2\right). \tag{12}$$

Noticing $\delta_x u_j^n=p_{j+1}^n$, $\delta_x v_j^n=q_{j+1}^n$, we sum Eqs. (11) and (12), and then gain Eq. (10), which completes the proof.

Remark 1 In the above derivation, the discrete Leibnitz rule has been used. The discrete LECL is consistent with the continuous LECL (5) and independent of the boundary conditions.

Theorem 2 Under the zero boundary condition, the LEP algorithm possesses the global energy law

$$\mathscr{E}^{n+1}=\mathscr{E}^n=\cdots=\mathscr{E}^1=\mathscr{E}^0, \tag{13}$$

where

$$\mathscr{E}^n=h\sum_{j=0}^{N-1}|\phi_j^n|^2\psi_j^n+\frac{1}{2}\|\psi^n\|^2+\|\delta_x\phi^n\|^2+\frac{1}{2}\|\delta_x f^n\|^2.$$

The proof is straightforward and is thus omitted.

As we all know, the mass conservation law (3) plays an important part in quantum physics. Therefore, it is necessary to discuss whether it can be conserved exactly.

Theorem 3 Under the zero boundary condition, the LEP algorithm preserves the mass exactly, that is

$$M^{n+1}=\|\phi^{n+1}\|^2=\cdots=\|\phi^0\|^2=M^0. \tag{14}$$

Proof Multiplying Eq. (9) by $2hA_t\bar{\phi}_j^n$ and summing it over space index j, we have

$$\mathrm{i}\sum_{j=1}^{N-1}\delta_t\phi_j^n\cdot 2hA_t\bar{\phi}_j^n+\sum_{j=1}^{N-1}A_t\delta_x^2\phi_{j-1}^n\cdot 2hA_t\bar{\phi}_j^n-\sum_{j=1}^{N-1}A_t\psi_j^n\cdot A_t\phi_j^n\cdot 2hA_t\bar{\phi}_j^n=0. \tag{15}$$

By some simple calculations, the first term of Eq. (15) becomes

$$\mathrm{i}\sum_{j=1}^{N-1}\delta_t\phi_j^n\cdot 2hA_t\bar{\phi}_j^n=\frac{\mathrm{i}}{\tau}\left(\|\phi^{n+1}\|^2-\|\phi^n\|^2\right)-\frac{2h}{\tau}\sum_{j=1}^{N-1}\mathrm{Im}\,(\phi_j^{n+1}\bar{\phi}_j^n), \tag{16}$$

where 'Im' denotes the imaginary part of the complex function. The second term becomes

$$\begin{aligned}\sum_{j=1}^{N-1}A_t\delta_x^2\phi_{j-1}^n\cdot 2hA_t\bar{\phi}_j^n&=\frac{2}{h}\sum_{j=1}^{N-1}\left(A_t\phi_{j-1}^n\cdot A_t\bar{\phi}_j^n-2A_t\phi_j^n\cdot A_t\bar{\phi}_j^n+A_t\phi_{j+1}^n\cdot A_t\bar{\phi}_j^n\right)\\&=\frac{2}{h}\left(2\mathrm{Re}\,(\sum_{j=1}^{N-1}A_t\phi_{j-1}^n\cdot A_t\bar{\phi}_j^n)-2\sum_{j=1}^{N-1}|A_t\phi_j^n|^2\right),\end{aligned} \tag{17}$$

where 'Re' denotes the real part of the complex function. The last term becomes

$$\sum_{j=1}^{N-1}A_t\psi_j^n\cdot A_t\phi_j^n\cdot 2hA_t\bar{\phi}_j^n=2h\sum_{j=1}^{N-1}A_t\psi_j^n\cdot|A_t\phi_j^n|^2\in\mathbb{R}^1. \tag{18}$$

Therefore, taking the imaginary part of Eq. (16), we can deduce

$$\|\phi^{n+1}\|^2=\|\phi^n\|^2,$$

which completes the proof.

3. Numerical experiments

In this section, some numerical experiments are carried out to show the performance of the proposed LEP scheme. Our computations will work on the spatial domain $[-L/2,L/2]$ over the time interval $0\le t\le T$ with time step length τ and spatial step length $h=L/N$. Here N is the number of grid points for the spatial domain. Furthermore, errors between numerical solutions and analytic solutions in L^2 and L^∞ norms are, respectively, defined as

$$\begin{aligned}\|e(\chi)\|_2&=\left(h\sum_{j=1}^{N-1}|\chi(x_j,t_n)-\chi_j^n|^2\right)^{1/2},\\\|e(\chi)\|_\infty&=\max_{1\le j\le N-1}|\chi(x_j,t_n)-\chi_j^n|,\\\|\tilde{e}(\phi)\|_\infty&=\max_{1\le j\le N-1}|\tilde{\phi}_j^n-\phi_j^n|,\\\|\tilde{e}(\psi)\|_\infty&=\max_{1\le j\le N-1}|\tilde{\psi}_j^n-\psi_j^n|,\end{aligned}$$

where $\chi\in\{\phi,\psi\}$ and $\tilde{\phi}$ denotes the numerical solution as the initial value is disturbed, etc. The convergence order is calculated by the following formula

$$\mathrm{Order}=\frac{\log(error_1/error_2)}{\log(\delta_1/\delta_2)},$$

where δ_i and $error_i$ denote step size and the corresponding discrete error, respectively.

Example 1 (single soliton)

The well-known solitary-wave solution of the Zakharov system is given in Ref. [8]

$$\begin{aligned}\phi(x,t) &= \sqrt{2B^2(1-\varepsilon^2C^2)}\,\mathrm{sech}(B(x-Ct))\\ &\quad\cdot\exp\left(\mathrm{i}(C/2)x-\left((C/2)^2-B^2\right)t\right),\\ \psi(x,t) &= -2B^2\,\mathrm{sech}^2(B(x-Ct)),\end{aligned}\tag{19}$$

where $\varepsilon=1$, $B=1$, and $C=0.5$. To investigate the stability of our scheme, we slightly perturb the initial functions as follows:

$$\tilde{\phi}(x,0)=\phi(x,0)+\lambda\sin(x),$$
$$\tilde{\psi}(x,0)=\psi(x,0)+\lambda\sin(x).$$

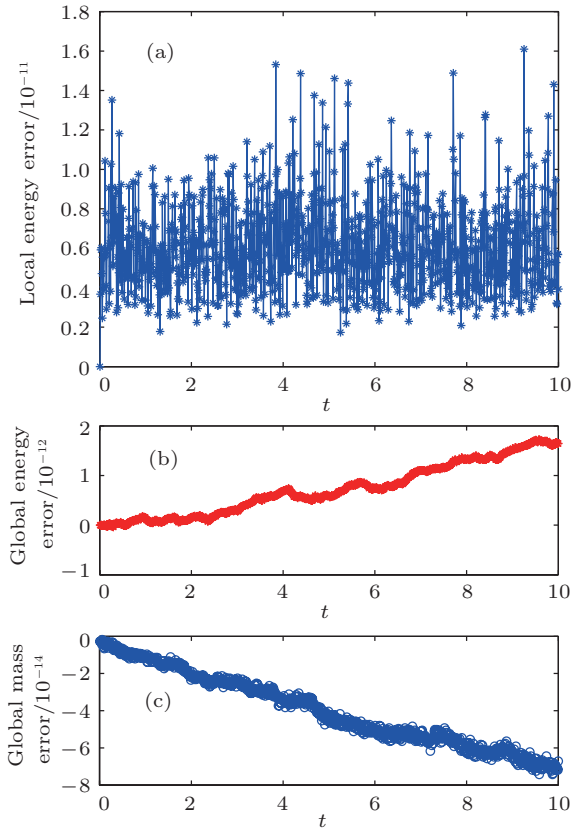


Fig. 1. (color online) (a) The variation of the local energy from $t=0$ to $t=10$; (b) the variation of the errors in the global energy, and (c) the global mass from $t=0$ to $t=10$, respectively.

The invariants-preserving results are shown in Fig. 1. It is easy to find that the local energy is conserved by the proposed LEP scheme throughout the computations. The errors in global mass and global energy, which oscillate near zero in the scale of 10^{-12}–10^{-14}, indicate the two global invariants are also captured. In order to test the convergence order of the scheme, we solve the problem on the interval $[-32,32]$ with $T=4$. The numerical results are given in Table 1, where a second order convergence of the proposed LEP scheme can be explicitly observed. Table 2 indicates the proposed LEP scheme is stable for different initial values.

Table 1. Convergence order of the proposed LEP scheme ($T=4$, $\tau=h$).

$h=\tau$	0.4	0.2	0.1	0.05
$\|e(\phi)\|_2$	8.960×10^{-2}	2.112×10^{-2}	5.231×10^{-3}	1.309×10^{-3}
Order	–	2.085	2.014	1.998
$\|e(\phi)\|_\infty$	6.130×10^{-2}	1.464×10^{-2}	3.587×10^{-3}	8.963×10^{-4}
Order	–	2.066	2.029	2.001
$\|e(\psi)\|_2$	0.206	4.451×10^{-2}	1.082×10^{-2}	2.693×10^{-3}
Order	–	2.208	2.040	2.007
$\|e(\psi)\|_\infty$	0.176	3.611×10^{-2}	8.714×10^{-3}	2.167×10^{-3}
Order	–	2.287	2.051	2.008

Table 2. Stability test of the proposed LEP scheme.

		$\lambda=0.001$		$\lambda=0.0001$	
h	τ	$\|\tilde{e}(\phi)\|_\infty$	$\|\tilde{e}(\psi)\|_\infty$	$\|\tilde{e}(\phi)\|_\infty$	$\|\tilde{e}(\psi)\|_\infty$
0.4	0.4	1.276×10^{-3}	1.735×10^{-3}	1.276×10^{-4}	1.736×10^{-4}
0.2	0.2	1.205×10^{-3}	2.157×10^{-3}	1.205×10^{-4}	2.158×10^{-4}
0.1	0.1	1.173×10^{-3}	2.301×10^{-3}	1.173×10^{-4}	2.032×10^{-4}

Example 2 (soliton–soliton collisions)

In this example, we will study soliton–soliton collisions using the proposed LEP scheme. The initial-boundary conditions are taken as[8]

$$\begin{aligned}&\phi(x,0)=\phi_s(x+p,0,v_1,E^1_{\max})+\phi_s(x-p,0,v_2,E^2_{\max}),\\ &\psi(x,0)=\psi_s(x+p,0,v_1,E^1_{\max})+\psi_s(x-p,0,v_1,E^1_{\max}),\\ &\psi_t(x,0)=\frac{\partial\psi_s(x+p,0,v_1,E^1_{\max})}{\partial t}+\frac{\partial\psi_s(x-p,0,v_2,E^2_{\max})}{\partial t},\\ &\phi(x_l,0)=\phi(x_r,0)=0,\ \psi(x_l,0)=\psi(x_r,0)=0,\ t\in[0,T],\end{aligned}$$

where

$$\begin{aligned}&\phi_s(x,t;v,E_{\max})=F(x-vt)\exp[\mathrm{i}(x-ut)\rho],\\ &\psi_s(x,t)=G(x-vt),\quad \rho=\frac{v}{2},\\ &u=\frac{v}{2}+\frac{2N_0}{v}-\frac{(E_{\max})^2+(E_{\min})^2}{v(1-v^2)},\\ &F(x-vt)=E_{\max}\cdot\mathrm{dn}(w,q),\quad G(x-vt)=\frac{F(x-vt)^2}{v^2-1}+N_0,\\ &w=\frac{E_{\max}}{\sqrt{2(1-v^2)}}\cdot(x-vt),\quad q=\frac{\sqrt{(E_{\max})^2-(E_{\min})^2}}{E_{\max}}.\end{aligned}$$

Here, $\mathrm{dn}(w,q)$ is a Jacobian function and $x=\pm p$ are initial locations of the two solitons. In this example, we choose $p=10$, $E_{\min}=0$, $N_0=0$, $x_l=-60$, $x_r=60$, and $T=30$. We present computations for the following two cases.

Case I Collision of two solutions with equal amplitude and opposite velocity

$$E^1_{\max}=E^2_{\max}=1,\quad v_1=-v_2=0.628319.$$

Case II Collision of two solutions with different amplitudes and opposite velocity

$$E^1_{\max}=0.5,\quad v_1=0.628319,\quad E^2_{\max}=1,\quad v_2=-0.628319.$$

Figure 2 displays the evolution of $|\phi|$ and $-\psi$ at different time with $\tau = 0.04$ and $h = 0.2$ for Cases I and II. These results coincide with the theoretical analysis. Figure 3 shows the errors of global energy and local energy for Cases I and II. It can be seen from the pictures that the proposed LEP scheme conserves the local and global energy conservation laws simultaneously.

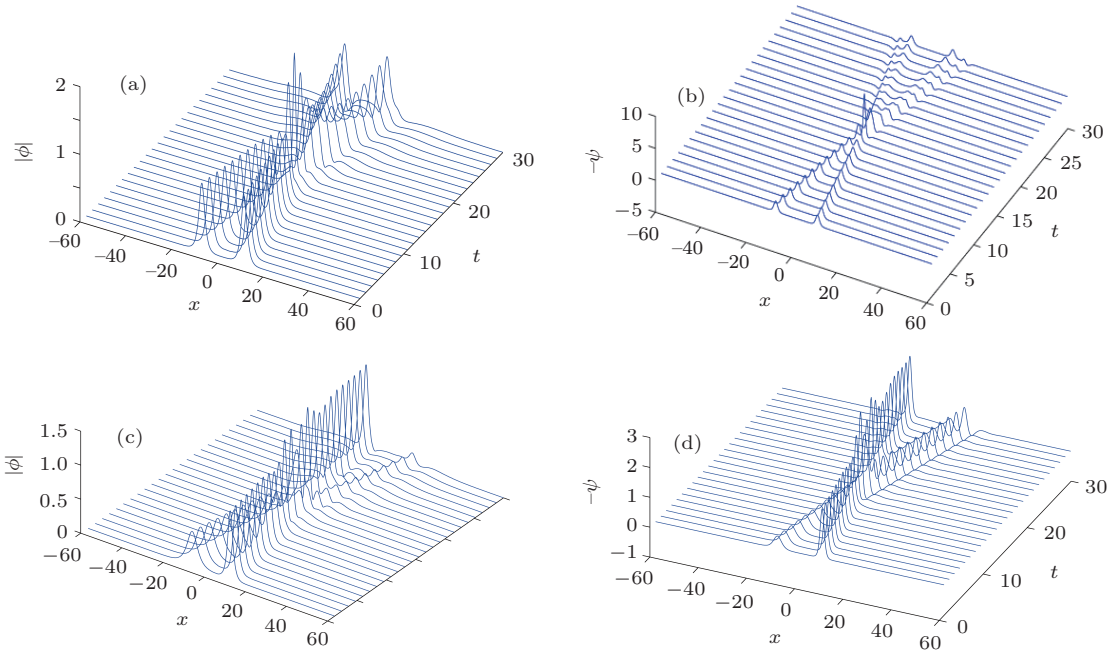


Fig. 2. (color online) The values of $|\phi|$ and $-\psi$ at different times for Case I [panels (a) and (b)] and Case II [panes (c) and (d)].

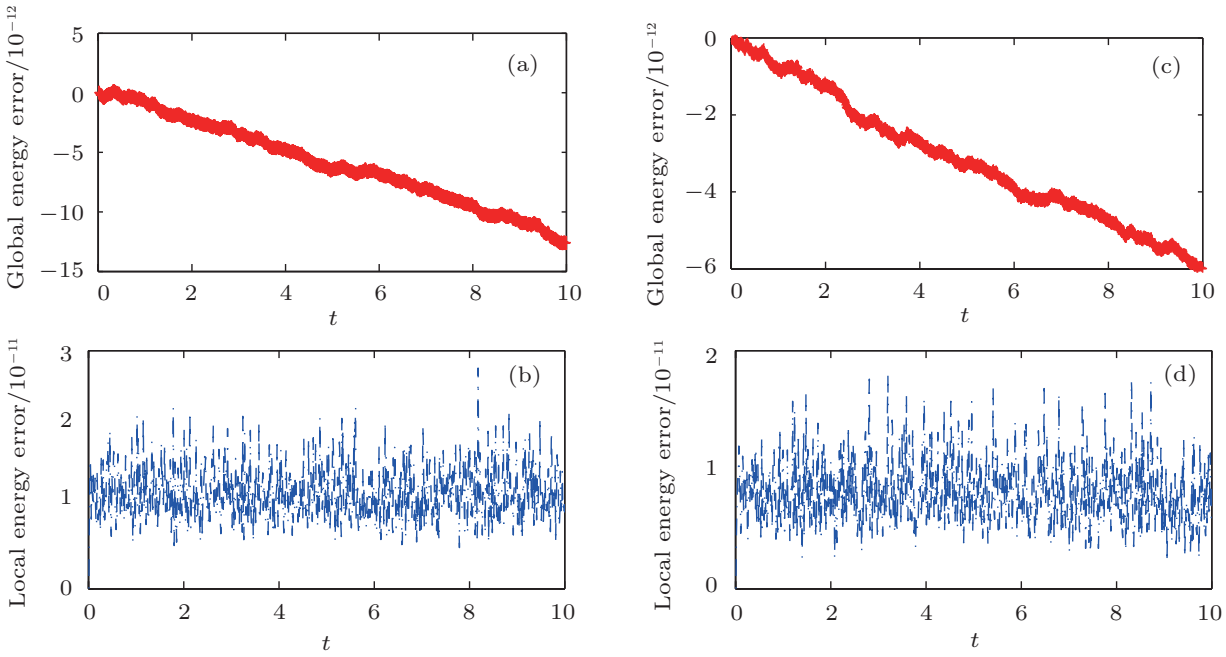


Fig. 3. (color online) The errors of global energy and local energy for Case I [panels (a) and (b)] and Case II [panels (c) and (d)].

4. Conclusions

In the present paper, we proposed a local energy-preserving (LEP) scheme to simulate the Zakharov system. The scheme preserves the discrete LECL in any time–space region, which is independent of the boundary conditions and is more essential than the discrete GECL. If the boundary conditions are suitable, summing the discrete LECLs over all space index leads to the GECLs. The scheme proposed in this paper extends the applying scope of the traditional global energy-preserving algorithm. Numerical results show the excellent performance in conserving local energy, global energy, and mass invariants. What is more, we find that the LEP scheme is stability and a second order convergence can be explicitly observed.

For the local energy conservation law of the Zakharov system (1), we can also use other useful methods to construct

local energy-preserving schemes. Actually, we can even propose a series of local momentum-preserving schemes for the Zakharov system, which conserve discrete momentum conservation laws in any time–space region.

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