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Numerical simulation of the multiple reversed shear Alfvén eigenmodes associated with the triangularity Alfvén gap*

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It was found that there are multiplicity of low shear toroidicity-induced Alfvén eigenmodes in a zero beta limit if the inverse aspect ratio is larger than the magnetic shear at the mode location (Candy 1996 *Phys. Lett. A* **215** 299). Because the reversed shear Alfvén eigenmode (RSAE) and even the RSAE associated with the non-circular triangularity-induced Alfvén eigenmode (NAE) gap (NAE–RSAE) usually reside near the shear-reversal point, the condition that the inverse aspect ratio is larger than the magnetic shear is naturally satisfied. For this reason, we numerically investigate the existence of multiplicity of core-localized NAE–RSAEs and mode characteristics in the present work. We firstly verify the existence of the multiplicity for zero beta plasma by using a D-shaped equilibrium. It is pointed out that, for a given toroidal mode number, the Alfvén cascade spectrum accommodates down-sweeping and up-sweeping modes above and below the NAE range of frequencies. An analytical model for the existence of multiple RSAE modes is in good agreement with the simulation results. One notices that the triangularity has a greater effect on the odd-type modes than that on the even-type modes: the odd-type modes come into existence because of the plasma triangularity.

Keywords: multiple Alfvén eigenmodes, negative magnetic shear, simulation

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1. Introduction

The discrete shear Alfvén eigenmodes (AEs) excited by the alpha particles[1] in tokamak plasmas can be readily destabilized by fast particles with velocities comparable to the Alfvén velocity and can therefore degrade the confinement.[2,3] Wang *et al.*[4] have studied the discrete Alfvén eigenmodes in steady-state operation scenarios with negative magnetic shear in the international thermonuclear experimental reactor. Toroidicity-induced Alfvén eigenmodes (TAEs) and a particular class of shear Alfvén waves known as the reversed shear Alfvén eigenmode (sometimes called Alfvén cascades), are such modes which are driven unstably by energetic ions in toroidal plasmas. RSAEs which consist of one dominant poloidal harmonic usually reside near the shear-reversal point and has a frequency that is above the continuum frequency, thus avoiding strong continuum damping, in recent years, have been intensively studied in experiments[5–10] and explained theoretically.[11–17] The hallmarks of these modes are their frequencies sweep up or down when $q_{\min}$ drops in time. It also serves as a useful plasma diagnostic for the measurement of safety-factor profiles.[6,18] Therefore, the study of the RSAEs and their stability characteristics has attracted considerable attention.

In the early studies, it was found that the RSAEs are associated with the TAE gap. But, Kramer[9,19] has found that in the higher frequency gaps such as the ellipticity-induced Alfvén eigenmode (EAE) gap,[20] formed by the coupling of poloidal harmonics m and $m+2$, the NAE gap,[21] formed by the coupling of poloidal harmonics m and $m+3$, there are also similar RSAEs near the shear-reversal point. The theory[19] points out that, even if the plasma has a circular cross-section, a high-order gap will exist with the change of $q_{\min}$, and high-order RSAEs exist, which has been confirmed by simulation and experiments.[22] It was found that the RSAEs in the gaps above the TAE gap are not like the conventional RSAEs in the TAE gap, which can exist even at zero pressure gradient and only exist above a critical pressure gradient.[19] A late work by Kramer[9] indicated that at large values of ellipticity, κ, the NAE–RSAE can exist even when the pressure gradient, α is zero.

Candy *et al.*[23] found that there are multiplicity of low-shear toroidal Alfvénic eigenmodes in a zero beta limit if the inverse aspect ratio is larger than the magnetic shear at the mode location. A recent work by Marchenko[24] has studied the plasma pressure effect on the multiple low-shear toroidal Alfvén eigenmodes. Recently, we have done performed some

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simulations of multiple TAEs.[25] Because the RSAE and even the NAE–RSAE usually reside near the shear-reversal point, the condition of $\varepsilon/s > 1$ (where $s = (r/q)\mathrm{d}q/\mathrm{d}r$ the magnetic shear, q the safety factor, $\varepsilon = r/R$ the inverse aspect ratio, and $r(R)$ the minor (major) radius of the torus) is naturally satisfied. In this sense, it is also normal that there exists multiplicity of the NAE–RSAE modes at the minimum of the magnetic safety factor or q profile. In modern tokamaks, the realistic plasma cross section is not circular but highly shaped and this shaping often opens up higher frequency gaps such as the NAE gap in the Alfvén spectrum. If there are multiple RSAEs in the NAE gap, how will these modes behave? Will they have some new features? However, thus far, no numerical simulation or analytical work has been taken in the studies of the multiplicity of the multiple reversed shear Alfvén eigenmodes associated with the triangularity Alfvén gap for tokamaks.

In this work, we study the multiple RSAEs in the triangularity Alfvén gap through numerical simulations. We use the non-variational magnetohydrodynamic (MHD) code (NOVA[26]), which solves ideal MHD eigenmode equations in general MHD equilibrium and has been widely used in experimental analysis and simulations.[15,27] We focus on the realistic D-shaped plasma (elongation and triangularity are often non-zero). For D-shaped plasmas, the TAE, EAE and NAE gaps are all open. The remainder of this paper is organized as follows. In Section 2, the existence of the multiple NAE–RSAEs is verified numerically. In Section 3, we present the frequency behavior of the multiple NAE–RSAEs when the minimum of the q profile decreases. In Section 4, we give an analytical model for the multiple RSAEs. In Section 5, the triangularity effect on the multiple NAE–RSAEs is described. A summary and conclusion of this work are given in Section 6.

2. The existence of multiple NAE–RSAES

Here, we discuss the existence and mode structure of multiple NAE–RSAE modes in the ideal MHD limit using the NOVA code. Theory predicts that at large values of κ, the NAE–RSAE can exist even when α is zero. First, we consider a D-shaped equilibrium ($\kappa = 1.5$ and triangularity, $\delta = 0.4$) with zero pressure gradient and the profile

$$q(y) = q(0) + y\{q(1) - q(0) + \left[(y-1)\left[q'(1) - q(1) + q(0)\right](1-y_s)\right]/(y-y_s)\},$$

where

$$y_s = \left[q'(1) - q(1) + q(0)\right]/\left\{q'(0) + q'(1) - 2[q(1) - q(0)]\right\},$$

in this paper, all the equilibria are chosen with the same density profiles, which is given by $\rho = \left(1.0001 - 0.9y^{1.1}\right)^{0.1}$, where y is the normalized poloidal magnetic flux. The parameters are $R = 2.6228$ m, $R/a = 3.9776$, the toroidal magnetic field $B_0 = 1$ T, $q(0) = 1.90$, $q(1) = 1.95$, $q'(0) = -0.5$, and $q'(1) = 0.5$. The q-profile and density profile are shown in Fig. 1.

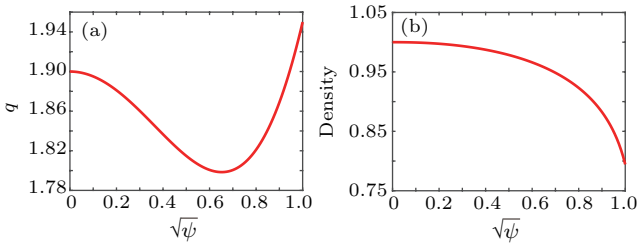


Fig. 1. (color online) (a) The q-profile with parameters: $q(0) = 1.90$, $q(1) = 1.95$, $q'(0) = -0.5$, and $q'(1) = 0.5$. (b) The density profile is chosen to be $\rho = (1.00001 - \alpha y^{p\rho})^{a\rho}$, where $\alpha = 0.9$, $p\rho = 1.1$, $a\rho = 0.1$.

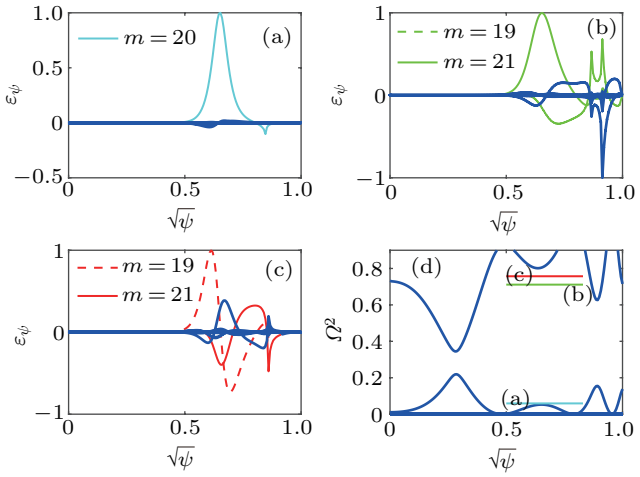


Fig. 2. (color online) The $n = 11$ model in the TAE gap. (a) The radial ξ_ψ of an $l = 0$ mode as a function of plasma minor radius; (b) ξ_ψ of an $l = 0$ odd-type mode; (c) ξ_ψ of an $l = 1$ odd-type mode; (d) the corresponding eigenfrequencies of the multiple TAE–RSAEs with $\Omega = q(1)\,\omega R/VA$.

For the above equilibrium, if we take $n = 11$, we will obtain the continuous spectrum shown in Fig. 4. From it, one notices that the TAE gap and NAE gap open at the same time near the minimum value of q. For this reason, we might as well search for multiple RSAE modes in the TAE gap before studying the multiplicity of Alfvén cascades modes of the NAE gap. We have found three RSAE modes as presented in Fig. 2. Similar to the theory of multiple TAE waves, multiple RSAEs also have the $l = 0$ modes (Figs. 2(a) and 2(b)) and the $l = 1$ mode (Fig. 2(c)), where l is the radial mode number. Following the multiple TAEs theory, we define the modes with their frequencies near the upper tip of the continuum as the odd-type modes and the mode with its frequency near the lower tip of the continuum as the even-type mode. However, as a particular class of shear Alfvén waves, multiple RSAE modes also have the unique property: As a single-poloidal harmonic dominant wave, the dominant poloidal mode number of the odd-type RSAE modes and the even-type mode are different. We find that the dominant poloidal mode number of

the odd-type mode is $m_1 = 19$ and of the even-type mode is $m_2 = 20$. Interestingly, one notice that m_1 and m_2 just satisfy the condition of $m_2 - m_1 = 1$ and $m_2 \sim nq_{\min}$ which means the even-type mode is the standard Alfvén cascades mode.

Kramer[9] confirmed that the m-number of the NAE–RSAE is one higher than that of the RSAE associated with the TAE gap (TAE–RSAE). Since our simulation results show that for TAE–RSAEs, the dominant poloidal mode number of the odd-type modes and the even-type modes are different. Then, will there also be multiple RSAE modes with different mode numbers in the NAE gap? With this question, we decided to simulate the existence of the multiple RSAE modes in the NAE gap. And Figs 3 and 4 confirms this conjecture.

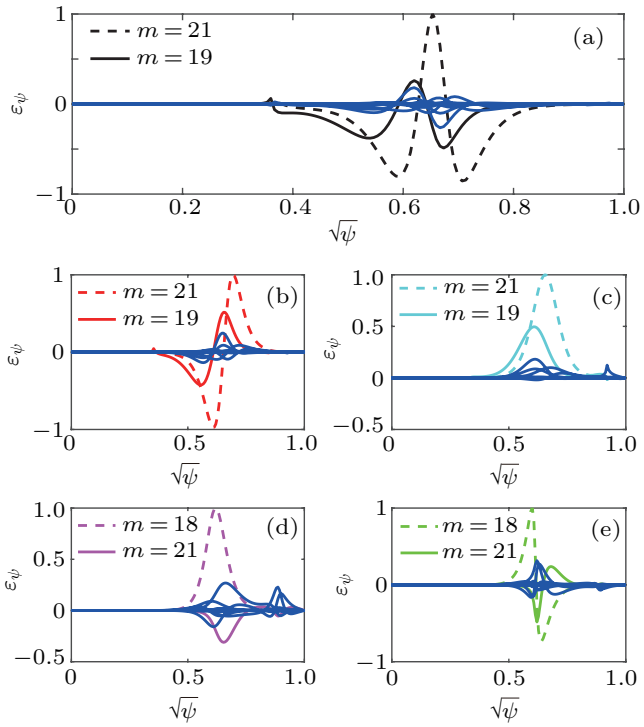


Fig. 3. (color online) The $n = 11$ model in the NAE gap. (a) The radial ξ_ψ of an $l = 2$ even-type mode as a function of plasma minor radius; (b) ξ_ψ of an $l = 1$ even-type mode; (c) ξ_ψ of an $l = 0$ even-type mode; (d) ξ_ψ of an $l = 0$ odd-type mode; (e) ξ_ψ of an $l = 1$ odd-type mode.

As can be seen from Fig. 3, multiple RSAEs in the NAE gap have richer results than those in the TAE gap. One notices that, in the NAE gap, the RSAE modes with their frequencies near the lower tip of the continuum are the even-type modes and one of them is the multi-node mode of $l = 2$. One can also see from Fig. 3 that the shapes of the $l = 1$ odd and even modes are mirror symmetric. For the multiple TAE, the $l = 0$ even mode is usually in the 'forbidden region' for the zero pressure gradient case, but from Figs. 2 and 4 we can see that there is no mode present in the 'forbidden zone', either in the TAE gap or in the NAE gap. Comparing Figs. 2 and 3, we find that the m-number of the even-type NAE–RSAEs is one higher than that of the TAE–RSAE with its frequency close to the lower tip of the continuum, which is in agreement with theory. However, for the odd-type modes, the m-number of the NAE–RSAEs is one smaller than that of the TAE–RSAEs, but we think this is reasonable for it just ensures that the dominant poloidal mode number, m_2 of the even-type NAE–RSAE modes and the dominant poloidal mode number, m_1 of the odd-type NAE–RSAEs satisfy the condition of $m_2 = m_1 + 3$, which is just the reason for the triangularity Alfvén gap formation that exists in the multiple RSAEs. In general, high frequency modes are less important than low frequency gap modes because they are more difficult to excite. Based on this, one decided to use the non-variational kinetic-MHD stability code (NOVA-K) to estimate their damping ratio, γ_{damp} (As shown in Table 1), and found that the damping ratio of the TAE–RSAE modes is positive, while that of the NAE–RSAE modes is negative. This means that the background plasma itself may excite low-frequency modes, while the emergence of high-frequency modes requires other factors such as energy particles to excite. It is not difficult to find from Table 1 that the more nodes, the greater the damping. After all, the more nodes, the closer the frequency is to the continuous spectrum.

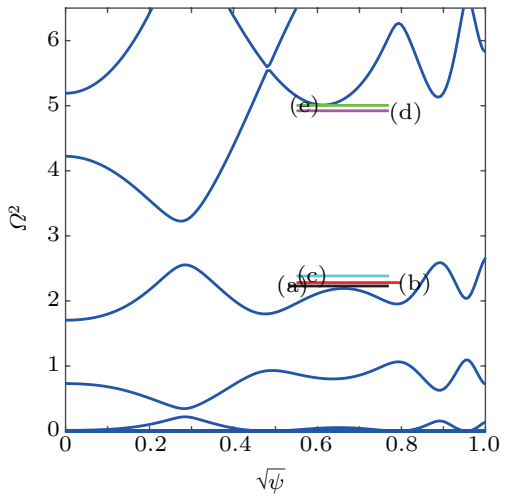


Fig. 4. (color online) Zero β approximation of $n = 11$ continua, and the corresponding eigenfrequencies of the multiple NAE–RSAEs.

Table 1. The damping ratio of each mode, where E-TAE represents the even-type TAE–RSAE, O-NAE represents the odd-type NAE–RSAE.

	E-TAE $l = 0$	O-TAE $l = 0$	O-TAE $l = 1$	E-NAE $l = 2$	E-NAE $l = 1$	E-NAE $l = 0$	O-NAE $l = 0$	O-NAE $l = 1$
γ_{damp}	0.4835	0.0033	0.0011	–0.0040	–0.0039	–0.0020	–0.0012	–0.0016

The above equilibrium is arbitrarily constructed by the NOVA code. In order to better understand the experiment, we decided to use the real parameters of the existing device, such as the Experimental Advanced Superconducting Tokamak (EAST). The simulation results show that when a new equilibrium is adopted, multiple RSAE modes as shown in Fig. 3 will still appear in the high-order gaps near the minimum value of the safety factor, the relative position of their frequencies is shown in Fig. 4. Of course, this result is not accidental, because in the process of simulation, all physical quantities are normalized, which means that our simulation results can be generalized without relying on a specific device.

3. Frequency behavior

In tokamak experiments with nonmonotonic q profiles, where RSAEs are observed, due to the minimum value of q, $q_{\min}$ decreases in time, we have simulated the frequency behavior of the multiple Alfvén cascade modes in the NAE gap. In this simulation we shift the q profile downward gradually while keeping the other parameters unchanged. We scan $q_{\min}$ in the range from 1.7984 to 1.4984. The frequency behavior of the $n = 11$ multiple Alfvén cascade modes in the NAE gap as the function of the minimum value of q are shown in Fig. 5. From Fig. 5 one notices that the even-type modes are the up sweeping RSAEs and the odd-type modes are the down sweeping RSAEs when $q_{\min}$ decreases, and when $q_{\min}$ satisfies the condition of $(m_1 + m_2)/2 \sim nq_{\min}$, the even-type modes will become the even NAEs, the odd-type NAE–RSAEs will become the odd NAEs with their dominant poloidal mode numbers are m_1 and m_2, where m_1 is the dominant poloidal mode number of the odd-type mode and m_2 is the dominant poloidal mode number of the odd-type mode. And all of the multiple NAE–RSAEs exhibit a quasiperiodic pattern of frequency sweeping, as shown in Fig. 5. In this sense, one can see that the so-called even-type Alfvén cascade mode we define is one that changes with the safety factor into the even mode. Similarly, the so-called odd-type mode is the wave that can be converted into odd mode with the change of safety factor.

Figure 6 shows the NAE modes transformed by the reverse shear Alfvén waves when $nq_{\min} \sim 16.5$ (in this case $m_1 = 15$ and $m_2 = 18$). And it just proves that there exists multiplicity of low shear triangularity-induced Alfvénic eigenmodes if the inverse aspect ratio is larger than the magnetic shear at the mode location. As for the properties of the multiple NAEs, we will be in the future work to study, not to discuss in detail here. Examining Fig. 6(e), we can find that the mode structure has singularities at the radial location $\sqrt{\psi} = 0.57$. And it is just where the frequency of the mode intersects the continua, as is shown in Fig. 7. This indicates that the singularities are due to the continuum resonance.[28]

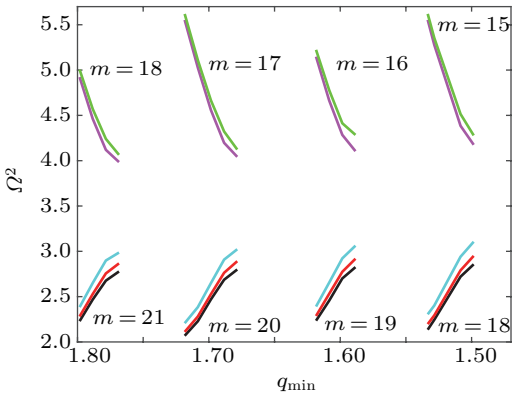


Fig. 5. (color online) The frequency behavior of $n = 11$ multiple RSAEs in the NAE gap as function of $q_{\min}$. The dominant poloidal mode number, m, is indicated for different branches.

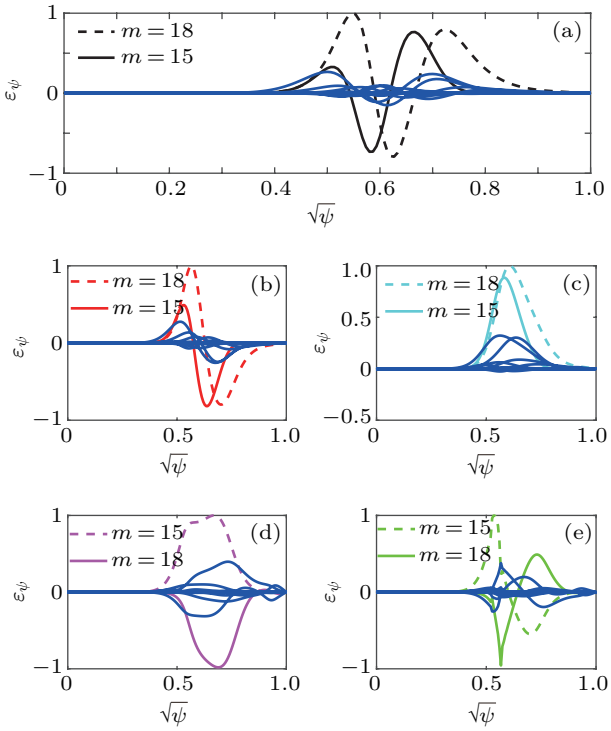


Fig. 6. (color online) Five NAE modes within the central continuum gap of Fig. 7 with $nq_{\min} \sim 16.5$.

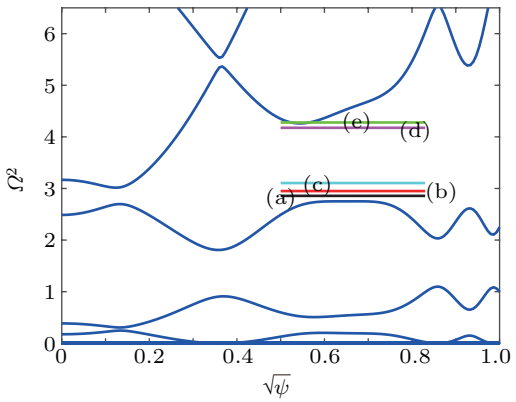


Fig. 7. (color online) Zero β approximation of $n = 11$ continua, and the corresponding eigenfrequencies of the multiple NAEs.

In Fig. 8, we also show that there is a continuous transformation of the Alfvén cascade modes into the NAE modes as

the safety factor changes in time ($nq_{\min}$ varies from 16.87 to 16.48) at the shear reversal point by taking the odd-type modes and the $l=1$ and $l=0$ even-type modes as examples. One can notice that starting from $nq_{\min}<16.6$, all the modes gradually show the characteristics of the NAE modes. It is noted from Fig. 8 that the $l=1$ modes are more likely to have singularities, which is of course normal, after all, they are closer to the continuous spectrum.

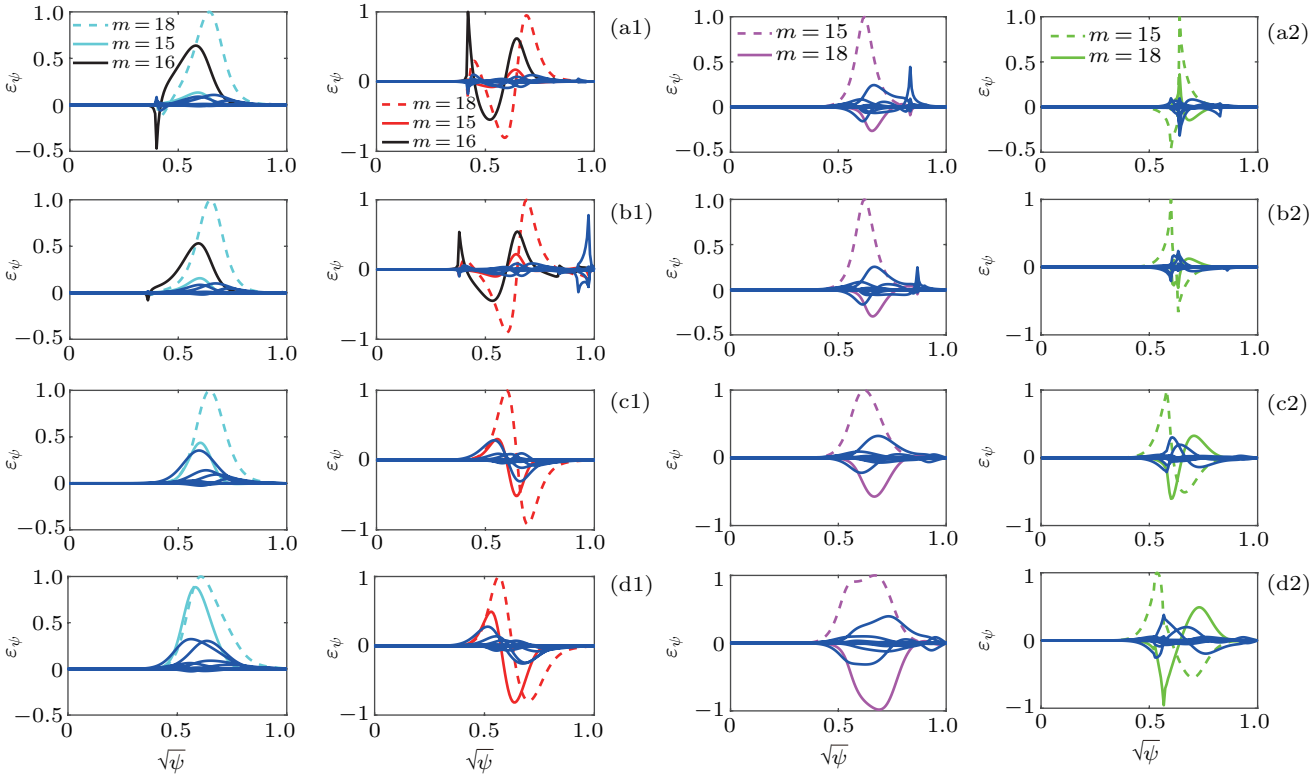


Fig. 8. (color online) Snapshots of the mode structures for the multiple NAE–RSAE modes with $n=11$ and $m=(18;15)$ during the transition from the cascade modes to NAEs: from left to right are $l=0$ even-type mode, $l=1$ even-type modes, $l=0$ odd-type mode, $l=1$ odd-type modes. The values of $nq_{\min}$ for the snapshots are [(a1) and (a2)] $nq_{\min}\approx 16.87$; [(b1) and (b2)] $nq_{\min}\approx 16.81$; [(c1) and (c2)] $nq_{\min}\approx 16.59$; [(d1) and (d2)] $nq_{\min}\approx 16.48$.

4. Analytical model

In order to analyze the existence of multiple RSAEs associated with the NAE gap and their frequency behavior with q changes, in this section we will give an analytical treatment to the multiple RSAEs associated with the TAE gap firstly. Under the assumption of radial localization of the mode structure near $r_{\min}$ (in the limit of high m) the set of two coupled eigenmode equations become[12]

$$\frac{\partial}{\partial x}D_m\frac{\partial}{\partial x}\Phi_m - D_m\Phi_m + L_-\Phi_{m-1}=0, \tag{1}$$

$$\frac{\partial}{\partial x}D_{m-1}\frac{\partial}{\partial x}\Phi_{m-1} - D_{m-1}\Phi_{m-1} + L_+\Phi_m=0, \tag{2}$$

wherein $x=(r-r_0)m/r_0$, and the quantities D_m, D_{m-1}, L_+, and L_- are given by

$$D_m=\varpi^2-(qn-m)^2, \tag{3}$$

$$D_{m-1}=\varpi^2-(qn-m+1)^2, \tag{4}$$

$$L_\pm=\frac{\varepsilon+\Delta'}{2}\frac{\partial^2}{\partial x^2}\pm\frac{\varepsilon+2\Delta'}{2}\frac{\partial}{\partial x}+\frac{\Delta'}{2}, \tag{5}$$

and $\varpi=\Omega q_{\min}/q(1)$, ε the inverse aspect ratio, Δ' the Shafranov shift parameter. It can already be seen that at $q=q_{\min}=m/n$ and $r=r_0$, if the mode with poloidal harmonic $m-1$ dominates because $D_{m-1}=\varpi^2-1$ is much smaller than the absolute value of $D_m=\varpi^2$. Like[16] it is instructive to use a WKB analysis to demonstrate the existence of the eigenmode solution:

$$\Phi_{m-1}=\exp\left(-\mathrm{i}\int k(x)\,\mathrm{d}x\right). \tag{6}$$

With the radial wave number, k, much larger than the spatial variation of D_{m-1}. Using Eq. (1) we can express Φ_m in terms of Φ_{m-1} at the mode location:

$$\Phi_m=\frac{L_-}{k^2+1}\Phi_{m-1}. \tag{7}$$

Substituting Eq. (7) into Eq. (2) we obtain the (WKB) dispersion relation:

$$\varpi^2=\eta^2-\frac{\eta^2-(\Delta'/2)^2}{k^2+1}+(qn-m+1)^2. \tag{8}$$

With $\eta=(\varepsilon+\Delta')/2$. Similarly, if the mode with poloidal harmonic m dominates we obtain the (WKB) dispersion relation:

$$\varpi^2=\frac{\eta^2-(\Delta'/2)^2}{k^2+1}-\eta^2+(qn-m)^2. \tag{9}$$

The above solutions imply the appropriate quantization condition:

$$\int_{x_1}^{x_2} k(x)\,\mathrm{d}x = (l+1)\,\pi. \tag{10}$$

where the integration boundaries (x_1, x_2) limit the domain to where k^2 is positive. We can estimate the value of k qualitatively:

$$k \cong \frac{(l+1)\,\pi}{x_2 - x_1}. \tag{11}$$

In conjunction with Eq. (9), we can see from Eqs. (8) and (9) that for odd-type modes, the larger l, the larger the frequency of the mode, and the opposite for even-type modes. These means that the larger l, the closer the mode distance to the continuous spectrum, and it is consistent with our simulation results. As for the frequency scanning behavior of modes, $nq_{\min}$ varies from m to $m-1/2$, we know that the dominant poloidal mode number of the odd-type RSAE modes is $m-1$ and that of the even-type modes is m. As can be seen from Eq. (8), ϖ^2 decreases with the change of nq, while equation (9) shows that ϖ^2 increases with the change of nq. For the multiple RSAE modes associated with the NAE gap, the dominant poloidal mode number of the odd-type modes becomes $m-2$ and that of the even-type modes becomes $m+1$. These mean that in the NAE gap the even-type RSAEs are also the up-sweeping modes, and the odd-type RSAEs are the down sweeping modes. And it is consistent with the display of Fig. 5.

5. Triangularity effect on the multiple NAE–RSAEs

In this section, we study the effect of triangularity on mode existence and mode structure of the multiple NAE–RSAE modes. Based on the initial equilibrium, we still choose the safety factor as shown in Fig. 1 and scan the δ in the range from -0.2 to 0.5. Figure 10 shows the mode structures for the $\delta = 0.2$ case. We find that, compared with the $\delta = 0.4$ case, the frequencies of the odd-type modes and the $l = 0$ even-type modes have a significant downward trend and the frequencies of $l > 0$ even modes are not obviously decreased, the change of the even-type mode structures is not obvious (although the mode width of the even-type modes becomes smaller) but the change of the odd-type mode structures is obvious (This, of course, is also well understood: $\delta = 0.2$ is already very close a critical value of δ, below which the odd-type modes disappear). From Fig. 9(b) we note that the upper end of the gap becomes significantly larger as δ increases which can be verified in Fig. 9(a) and the variation of the lower end of the gap with the increase of δ is not significant or monotonous: near $\sqrt{\psi} = 0.63$, the lower end of the gap is increases first and then decreases with the increase of δ.

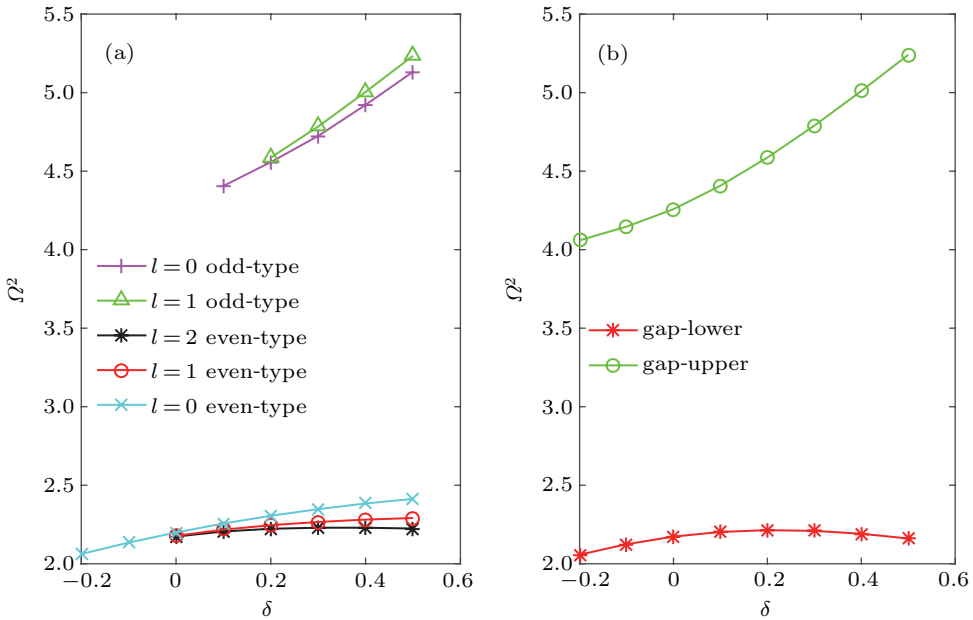


Fig. 9. (color online) (a) The eigenfrequencies of the $n = 11$ multiple NAE–RSAE modes as a function of the triangularity. (b) The frequencies of the continuum of the gap near $\sqrt{\psi} = 0.63$ as a function of the triangularity.

One notices that triangularity has a great effect on the existence of the multiple modes. From Fig. 9(a) we can see that the triangularity has a greater effect on the odd-type modes than that on the even-type modes. The modes will disappear one by one as δ decreases: when $\delta < 0.1$, the odd-type modes will disappear; when $\delta < 0$, *i.e.*, the negative D-shaped plasma, the $l > 0$ even modes will also disappear, even if the pressure gradient is increased, the modes will not appear again and therefore the multiple NAE–RSAEs can be suppressed by selecting the negative D-shaped plasma. However, in this pro-

cess, the $l = 0$ even-type mode always exist.

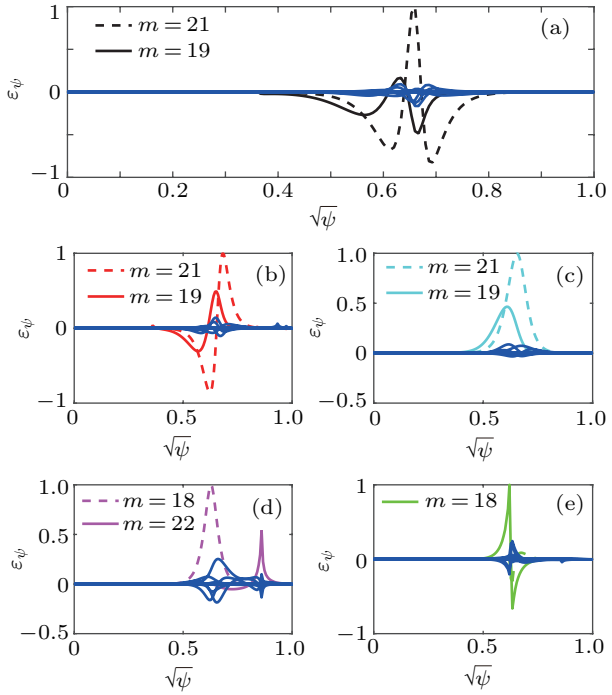


Fig. 10. (color online) Five NAE–RSAE modes within the central continuum gap of the $\delta = 0.2$ case.

6. Summary and conclusion

In this study, we have numerically studied the multiple RSAEs in the triangularity Alfvén gap in a tokamak with a D-shaped plasma using the NOVA code. Previously it was found that at large values of κ, the NAE–RSAE can exist even when α is zero,[9] which is in agreement with our simulations: the existence of the multiple NAE–RSAEs in the case $\kappa = 1.5$ and $\delta = 0.4$ with zero plasma pressure was verified. Like the multiple TAEs, we divided the multiple NAE–RSAEs into the even-type modes which change with the safety factor into the even modes are the up-sweeping modes and the odd-type modes which change with the safety factor into the odd modes are the down sweeping modes. For a given toroidal mode number, the dominant poloidal mode number of the odd-type RSAE modes and the even-type RSAE modes are different. The dominant poloidal mode number, m_2 of the even-type NAE–RSAE modes and the dominant poloidal mode number, m_1 of the odd-type NAE–RSAEs just satisfy the condition of $m_2 = m_1 + 3$. For the odd-type modes, in contrast to the standard RSAEs, the m-number of the NAE–RSAEs is one smaller than that of the TAE–RSAEs. The background plasma itself may excite TAE–RSAE modes, while the emergence of NAE–RSAE modes requires other factors such as energy particles to excite. In this process, we also verified the existence of multiple NAEs. Then we give an analytical model for the multiple RSAEs. Finally, we studied the effect of δ on multiple NAE–RSAE modes. However, one also noticed that when $\delta = 0$, in the vicinity of $q_{\min}$ where only the even-type NAE–RSAEs reside in elongated tokamak plasmas and triangularity contributes favorably to the existence of the odd-type NAE–RSAEs: when δ is greater than zero, the odd-type modes will gradually appear. We also found that although the frequencies of the two type modes increases monotonically with the triangularity.

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