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SPECIAL TOPIC — 60th Anniversary of the Department of Physics at the Nanjing Normal University

# Effect of temporal disorder on wave packet dynamics in one-dimensional kicked lattices*

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Inspired by the recent experimental progress in noisy kicked rotor systems, we investigate the effect of temporal disorder or quasi-periodicity in one-dimensional kicked lattices with pulsed on-site potential. We found that, unlike the spatial disorder or quasi-periodicity which usually leads to localization, the effect of the temporal one is more complex and depends on the spatial configuration. If the kicked on-site potential is periodic in real space, then the wave packet will stay diffusive in the presence of temporal disorder or quasi-periodicity. On the other hand, if the kicked on-site potential is spatially quasi-periodic, then the temporal disorder or quasi-periodicity may lead to a shift of the transition point of the dynamical localization and destroy the dynamical localization in a certain parameter range. The results we obtained can be readily tested by experiments and may help us better understand the dynamical localization.

**Keywords:** temporal disorder, quasi-periodicity, dynamical localization

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## 1. Introduction

Anderson localization is a common phenomenon in quantum systems subject to disorder, which breaks the spatial translational symmetry.[1] It has been widely observed not only in condensed matter physics, but also in other physical systems, including the light waves in photonic lattices and atomic matter waves in ultracold atom systems.[2–5] Beside disorder, spatial quasi-periodicity can also induce localization, which has been intensively investigated in the Aubry–André (AA) and the Harper models, both theoretically and experimentally.[6–19] Furthermore, in the study of the periodically kicked quantum rotors, the notion of dynamical localization has been put forward, where the localization occurs in momentum space, instead of the usual real space.[20–27] Recently, it was shown, in the periodically kicked AA model, the dynamical localization may also occur in real space, which is controlled by both the strength of the quasi-periodic potential and the kicking period.[28] An interesting issue remains unaddressed is, if the kicked AA model is subject to temporal disorder or quasi-periodicity, then what will happen to the real space dynamical localization? Similar issues have been investigated in the kicked quantum rotors, where it has been shown that the temporal disorder can induce decoherence and destroy the momentum space dynamical localization.[29–39] In this paper, based on the periodically kicked AA model, we introduce temporal disorder or quasi-periodicity by suppressing kicks entirely at certain time instants[39] and then study the wave packet dynamics along a one-dimensional chain. The joint effects of the spatial and temporal disorder or quasi-periodicity on the real space localization properties are discussed and thanks to the experimental developments in quantum control, these effects may readily be tested by future experiments and advance our knowledge of dynamical localization to a higher level.

## 2. Method

We consider a one-dimensional kicked AA model whose Hamiltonian is written as

$$\hat{H} = \sum_i \left[ -\left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{H.c.}\right) + \sum_n \delta(t - nT) g_n V_i \hat{c}_i^\dagger \hat{c}_i \right], \quad (1)$$

where $\hat{c}_i^\dagger$ ($\hat{c}_i$) is the creation (annihilation) operator at site $i$ and $V_i = \lambda \cos(2\pi\alpha i)$ is the on-site potential. The form of the on-site potential is that adopted in the AA model[7] and if $\alpha$ is rational/irrational, then the on-site potential will be periodic/quasi-periodic in real space. Furthermore, $g_n$ is a number whose value is either zero or one and $n$ is an integer. If $g_n = 1$ for all $n$, then the on-site potential will be periodically

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added in time, with $T$ being the kicking period and this is the usual periodically kicked AA model considered in Ref. [28]. In our study, following Ref. [39], $g_n$ is randomly or quasi-periodically taken to be zero or one. If $g_n = 0$ for a certain $n$, then the kick is suppressed entirely at this time instant. In this way, temporal disorder or quasi-periodicity is introduced to the kicked AA model. Then we set $\hat{H}^0 = -\sum_i(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{H.c.})$. It is time-independent and in the lattice representation, its nonzero elements are $\hat{H}^0_{i,i+1} = \hat{H}^0_{i+1,i} = -1$, for $i = 1,\ldots,L-1$ (here $L$ is the number of the lattice sites and open boundary condition is adopted). On the other hand, the time-dependent part of $\hat{H}$ can be written as

$$\sum_n \delta(t-nT) g_n \lambda \sum_i \cos(2\pi\alpha i)\hat{c}_i^\dagger \hat{c}_i = \sum_n \delta(t-nT)\lambda_n \hat{M}, \quad (2)$$

where $\lambda_n = \lambda g_n$ and $\hat{M}$ is a diagonal matrix with its $i$-th diagonal element being $M_i = \cos(2\pi i\alpha)$, for $i = 1,\ldots,L$. The time evolution operator over one kicking period can be expressed as

$$U[nT,(n-1)T] = \mathrm{e}^{-\mathrm{i}\hat{H}^0 T}\,\mathrm{e}^{-\mathrm{i}\lambda_{n-1}\hat{M}}. \quad (3)$$

Given an initial state $|\Psi(0)\rangle = \sum_{i=1}^L a_i(0)|i\rangle$ at $t = 0$ (here $|i\rangle$ represents the state with a particle located at the $i$-th site), the state after one kicking period is $|\Psi(T)\rangle = U(T,0)|\Psi(0)\rangle = \sum_{i=1}^L a_i(T)|i\rangle$. Similarly, we have the state after $n$ kicking periods as

$$|\Psi(nT)\rangle = U[nT,(n-1)T]\cdots U(T,0)|\Psi(0)\rangle. \quad (4)$$

The matrix elements of the time evolution operator $\langle i|U[nT,(n-1)T]|j\rangle$ can be calculated as follows. Firstly, suppose the $\mu$-th eigenenergy of $\hat{H}^0$ is $E_\mu$, with the corresponding eigenvector being $|\phi_\mu\rangle$, which can be written as $|\phi_\mu\rangle = \sum_j C_{\mu j}|j\rangle$. Then we have $|j\rangle = \sum_\mu C^*_{\mu j}|\phi_\mu\rangle$ and $\langle i| = \sum_\mu C_{\mu i}\langle\phi_\mu|$, and

$$\begin{aligned}\langle i|U[nT,(n-1)T]|j\rangle &= \mathrm{e}^{-\mathrm{i}\lambda_{n-1}M_j}\langle i|\,\mathrm{e}^{-\mathrm{i}\hat{H}_0 T}|j\rangle \\ &= \sum_\mu C_{\mu i}C^*_{\mu j}\,\mathrm{e}^{-\mathrm{i}(\lambda_{n-1}M_j + E_\mu T)}. \quad (5)\end{aligned}$$

Based on Eq. (5), we have,

$$\begin{aligned}|\Psi(nT)\rangle &= \sum_i a_i(nT)|i\rangle = U[nT,(n-1)T]|\Psi[(n-1)T]\rangle \\ &= U[nT,(n-1)T]\sum_j a_j[(n-1)T]|j\rangle, \\ a_i(nT) &= \sum_j a_j[(n-1)T]\langle i|U[nT,(n-1)T]|j\rangle. \quad (6)\end{aligned}$$

Combining Eqs. (4), (5), and (6), we can obtain the wave function after $n$ kicking periods.

To see how the wave packet spreads with time, we take the initial state as $|\Psi(0)\rangle = |L/2\rangle$ and calculate the mean-square displacement, which is defined as

$$\sigma^2(t) = \sum_{i=1}^L (i - L/2)^2 |a_i(t)|^2. \quad (7)$$

In general, $\sigma^2(t) \sim t^\gamma$ during the expansion process in the long time limit. In the following, we consider different configurations of both the spatial and temporal disorder or quasi-periodicity in the kicked AA model and discuss the wave packet dynamics. Furthermore, throughout this work, we calculate the results only at $t = nT$.

## 3. Results and discussion

We introduce two kinds of temporal disorder or quasi-periodicity. In the first case, each $g_n$ independently takes the value of zero or one, with an even probability, while in the second case, $g_n$ is consecutively extracted from a very long Fibonacci sequence, which is generated by the substitution rule $A \to AB$ and $B \to A$, with $A = 1$ and $B = 0$. Since the Fibonacci sequence is quasi-periodic, therefore it leads to temporal quasi-periodicity. In the following, we denote the first case as randomly kicked and the second one as Fibonacci kicked. All the results shown below have been averaged over 50 different realizations of disorder or quasi-periodicity. For the spatial on-site potential $V_i$, we consider four cases. In the first case, $\alpha$ is a rational number, while in the second case, $\alpha$ is irrational. Therefore the on-site potential is periodic and quasi-periodic in the first and second cases, respectively. For the purpose of comparison, in the third case, we let the on-site potential $V_i$ obey the Thue–Morse (TM) sequence,[40,41] which is generated by the substitution rule $A \to AB$ and $B \to BA$, with $A = +\lambda$ and $B = -\lambda$. Finally, in the fourth case, the on-site potential $V_i$ satisfies the following condition

$$V_{2n-1} = V_{2n} = \lambda\varepsilon_n, \quad (8)$$

where $\varepsilon_n$ are uncorrelated random variables and each $\varepsilon_n$ can be either $+1$ or $-1$, with an even probability. This distribution of the on-site potential is exactly the same as that in the random-dimer (RD) model.[42,43]

### 3.1. In the limit of $T, \lambda \ll 1$

As is well known, in the static AA model where the on-site potential is present all the time (not kicked), the wave packet dynamics is different for rational and irrational $\alpha$. If $\alpha$ is rational, then the on-site potential will be spatially periodic and all the eigenstates will be extended due to the Bloch theorem, leading to ballistic diffusion $\sigma^2(t) \sim t^2$ of the wave packet.[44] In contrast, if $\alpha$ is irrational, then the on-site potential will be spatially quasi-periodic and the wave packet will be extended (localized) for $\lambda < 2$ ($\lambda > 2$). On the other

hand, in the periodically kicked AA model, it has been shown that in the limit of $T, \lambda \ll 1$, this model can be mapped to the static AA model with a rescaled strength of the on-site potential $\lambda' = \lambda/T$. Therefore a dynamical localization occurs across $\lambda/T = 2$ in the periodically kicked AA model when $\alpha$ is irrational.[28] When we further introduce temporal disorder or quasi-periodicity into the kicked AA model, firstly we also concentrate on the $T, \lambda \ll 1$ limit. In this limit, we further take $\alpha = 1$, $\alpha = 1/5$ and $\alpha = (5^{1/2}-1)/2$ to illustrate our qualitative results.

For $\alpha = 1$, in Figs. 1(a) and 1(b), we show the mean-square displacement $\sigma^2(\tau)$ as a function of $\tau$ ($\tau = t/T$) in the randomly kicked and Fibonacci kicked cases, respectively. We can see that, for different kicking periods $T$, the mean-square displacement always evolves as $\sigma^2(\tau) \sim \tau^2$, which means that the wave packet stays ballistically diffusive, with no sign of localization characterized by $\sigma^2(\tau) \sim \tau^0$. We have also verified that, for arbitrary values of $T$ and $\lambda$, as long as $\alpha = 1$, then $\sigma^2(\tau)$ will always increase as $\tau^2$, no matter the model is randomly kicked or Fibonacci kicked. The reason is that, for $\alpha = 1$, $\hat{M}$ is a unit matrix which commutes with $\hat{H}^0$, therefore equation (4) can be written as

$$|\Psi(nT)\rangle = \mathrm{e}^{-\mathrm{i}(\lambda_0+\lambda_1+\cdots+\lambda_{n-1})}\,\mathrm{e}^{-\mathrm{i}\hat{H}^0 nT}|\Psi(0)\rangle. \tag{9}$$

The phase factor $\mathrm{e}^{-\mathrm{i}(\lambda_0+\lambda_1+\cdots+\lambda_{n-1})}$ does not contribute to $\sigma^2(t)$, due to the $|a_i(t)|^2$ term in Eq. (7), which means that the temporal disorder or quasi-periodicity has no effect on $\sigma^2(t)$ and it is only determined by $\hat{H}^0$. Since $\hat{H}^0$ is time-independent and is periodic in real space, it is well known that the wave packet shows ballistic diffusion in this case.

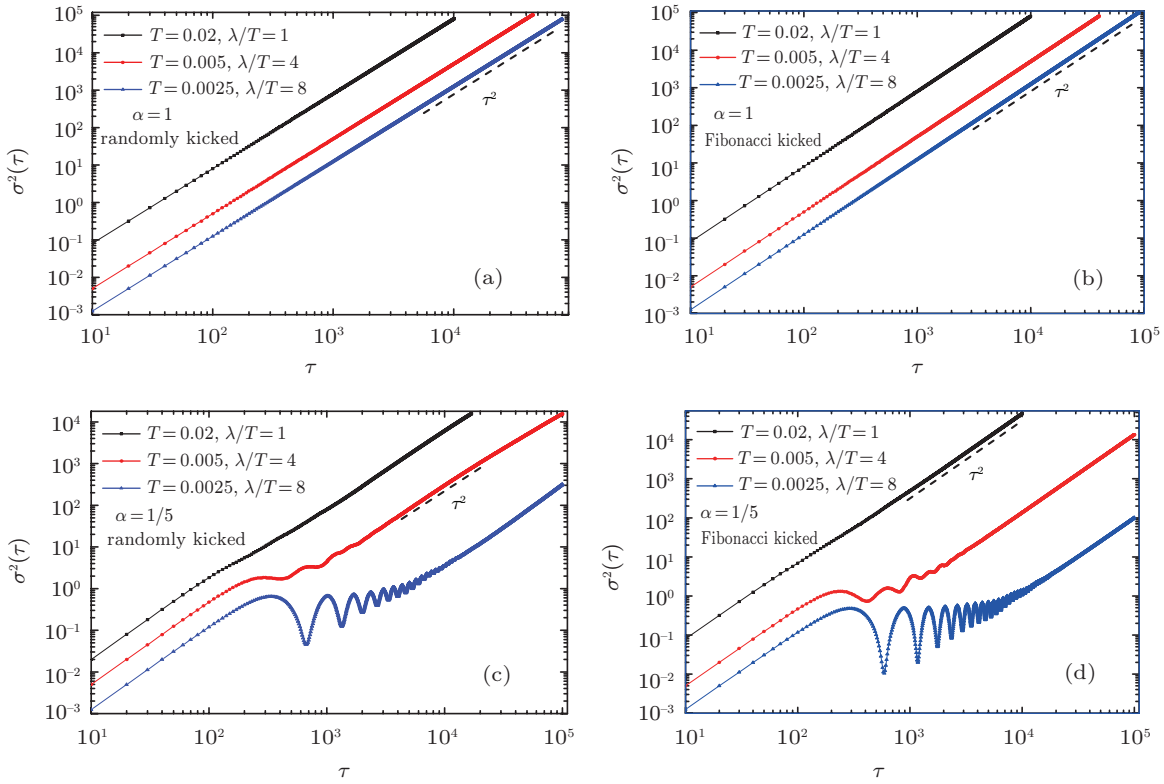



**Fig. 1.** (color online) (a) Time dependence of $\sigma^2(\tau)$ in the randomly kicked case, with $\alpha = 1$, $\lambda = 0.02$, and $L = 900$. The dashed line represents a power-law fitting. Panel (b) is similar to panel (a), but for the Fibonacci kicked case. Panels (c) and (d) are similar to panels (a) and (b), respectively, but for $\alpha = 1/5$.

For $\alpha = 1/5$, as we can see from Figs. 1(c) and 1(d), $\sigma^2(\tau)$ also shows a $\tau^2$ dependence as $\tau \to \infty$, similar to the $\alpha = 1$ case. Here, unlike the $\alpha = 1$ case, $\hat{M}$ in the $\alpha = 1/5$ case is not a unit matrix and generally it does not commute with $H^0$. However, according to the Baker–Campbell–Hausdorff (BCH) formula,[45] equation (3) can also be expressed as

$$U[nT,(n-1)T] = \mathrm{e}^{-\mathrm{i}H_n^{\mathrm{eff}}T},$$

$$H_n^{\mathrm{eff}} = \hat{H}^0 + \frac{\lambda_{n-1}}{T}\hat{M} - \frac{\mathrm{i}\lambda_{n-1}}{2}[\hat{H}^0,\hat{M}]$$

$$- \frac{\lambda_{n-1}T}{12}\left[\hat{H}^0,[\hat{H}^0,\hat{M}]\right]$$

$$- \frac{\lambda_{n-1}^2}{12}\left[\left[\hat{H}^0,\hat{M}\right],\hat{M}\right] + \cdots. \tag{10}$$

In the limit of $T, \lambda \ll 1$, we have

$$H_n^{\mathrm{eff}} \approx \hat{H}^0 + \frac{\lambda_{n-1}}{T}\hat{M}$$

and

$$\exp\left(-\mathrm{i}\hat{H}_2^{\mathrm{eff}}T\right)\exp\left(-\mathrm{i}\hat{H}_1^{\mathrm{eff}}T\right)=\exp\left\{-\mathrm{i}T\left(\hat{H}_2^{\mathrm{eff}}+\hat{H}_1^{\mathrm{eff}}-\frac{\mathrm{i}T}{2}\left[\hat{H}_2^{\mathrm{eff}},\hat{H}_1^{\mathrm{eff}}\right]+\cdots\right)\right\}\approx\exp\left[-\mathrm{i}T\left(\hat{H}_2^{\mathrm{eff}}+\hat{H}_1^{\mathrm{eff}}\right)\right]. \quad (11)$$

In this limit, equation (4) can be approximated as

$$\begin{aligned}\left|\Psi(nT)\right\rangle&=\exp\left(-\mathrm{i}\hat{H}_n^{\mathrm{eff}}T\right)\exp\left(-\mathrm{i}\hat{H}_{n-1}^{\mathrm{eff}}T\right)\cdots\exp\left(-\mathrm{i}\hat{H}_1^{\mathrm{eff}}T\right)\left|\Psi(0)\right\rangle\approx\exp\left[-\mathrm{i}T(\hat{H}_n^{\mathrm{eff}}+\hat{H}_{n-1}^{\mathrm{eff}}+\cdots+\hat{H}_1^{\mathrm{eff}})\right]\left|\Psi(0)\right\rangle\\&=\exp\left\{-\mathrm{i}nT\left[\hat{H}_0+\frac{(\lambda_n+\lambda_{n-1}+\cdots+\lambda_1)}{nT}\hat{M}\right]\right\}\left|\Psi(0)\right\rangle=\exp\left\{-\mathrm{i}nT[\hat{H}_0+\frac{\lambda\bar{g}}{T}\hat{M}]\right\}\left|\Psi(0)\right\rangle,\end{aligned} \quad (12)$$

where $\bar{g}=(g_1+\cdots+g_n)/n$ is the mean value of $g_n$. From Eq. (12) we can see, in the limit of $T,\lambda\ll 1$, the time evolution of the wave packet is governed by a time-independent effective Hamiltonian $\hat{H}_0+(\lambda\bar{g}/T)\hat{M}$. Since both $\hat{H}_0$ and $\hat{M}$ are periodic in real space, therefore the wave packet shows ballistic diffusion such that $\sigma^2(\tau)\sim\tau^2$ as $\tau\to\infty$. However, if the condition $T,\lambda\ll 1$ is not satisfied, then equation (12) will not hold anymore and $\sigma^2(\tau)$ will no longer evolve as $\tau^2$, as we will show later.

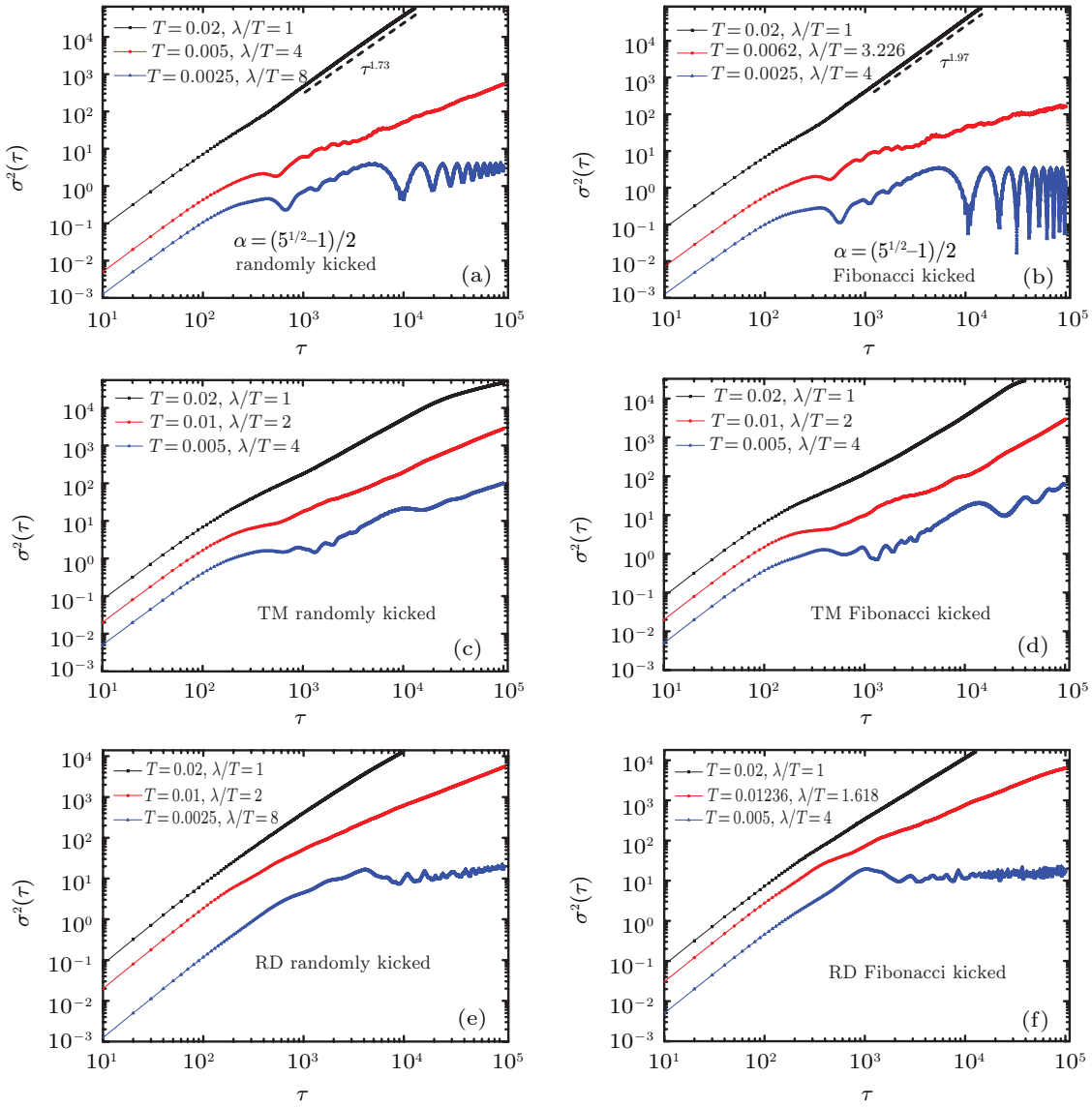



**Fig. 2.** (color online) Similar to Fig. 1. Panels (a) and (b) are for $\alpha=(5^{1/2}-1)/2$. Panels (c) and (d) are for the kicked TM model, while panels (e) and (f) are for the kicked RD model.

For $\alpha=(5^{1/2}-1)/2$, the mean-square displacement is shown in Figs. 2(a) and 2(b), for the randomly kicked and Fibonacci kicked cases, respectively. Compared to the cases with periodic on-site potential [see Figs. 1(a) to 1(d)] which show no sign of localization, the cases with quasi-periodic on-site potential exhibit clear signature of localization manifested

by $\sigma^2(\tau) \sim \tau^0$. For example, in the randomly kicked case, for $\lambda = 0.02$, we find that the wave packet is diffusive at $T = 0.02$ while it is localized at $T = 0.0025$. The localization transition occurs at $T \approx 0.005$, where $\lambda/T \approx 4$. On the other hand, in the Fibonacci kicked case, for the same value of $\lambda$, the wave packet is diffusive/localized at $T = 0.02/0.005$, where the localization transition occurs at $T \approx 0.0062$ ($\lambda/T \approx 3.2$). We have also verified that, for other values of $\lambda$, as long as the condition $\lambda \ll 1$ is satisfied, then the transition will always occur at $\lambda/T \approx 4/3.2$ in the randomly/Fibonacci kicked case. The origin of the transition can also be understood from Eq. (12), where in the limit of $T, \lambda \ll 1$, the wave packet dynamics is determined by a time-independent effective Hamiltonian $\hat{H}_0 + (\lambda\bar{g}/T)\hat{M}$. This Hamiltonian is equivalent to that of the static AA model, with a rescaled strength of the on-site potential $\lambda' = \lambda\bar{g}/T$. In the randomly kicked case, since $g_n$ is uniformly taken to be zero or one, then $\bar{g}$ should be 0.5, leading to the transition point at $0.5\lambda/T = 2$ ($\lambda/T = 4$). On the other hand, in the Fibonacci kicked case, $\bar{g}$ is 0.618, therefore the transition point is at $0.618\lambda/T = 2$ ($\lambda/T \approx 3.2$). Furthermore, if $g_n = 1$ for all $n$, then our model can be reduced to the periodically kicked AA model and the transition point should be at $\lambda/T = 2$ ($\bar{g} = 1$), consistent with that found in Ref. [28]. From above we can see, compared to the periodically kicked AA model, the temporal disorder or quasi-periodicity can lead to a shift of the dynamical localization point. Furthermore, in some parameter range, the dynamical localization may be suppressed. For example, at $\lambda/T = 3$, the wave packet is localized in the periodically kicked AA model, while it is diffusive in both the randomly kicked and Fibonacci kicked cases.

We then come to the case where the on-site potential $V_i$ obeys the TM sequence. It has been shown that, if this kind of on-site potential is static, then the wave packet will spread superdiffusively where $\sigma^2(t) \sim t^{1.65}$.[41] On the other hand, if the on-site potential is kicked, first of all, we denote this model as kicked TM model. Then, if temporal disorder or quasi-periodicity is further introduced, we show the results in Figs. 2(c) and 2(d). We can see that the wave packet stays diffusive, with no sign of localization.

Finally we consider the case where the on-site potential $V_i$ obeys Eq. (8). It was suggested that, in the static RD model, the wave packet is localized when $\lambda > 1$, otherwise it is diffusive.[42] In contrast, for the kicked RD model, we show the results in Figs. 2(e) and 2(f). Now the localization transition point is at $\lambda\bar{g}/T = 1$ due to Eq. (12).

### 3.2. Beyond the $T, \lambda \ll 1$ limit

If the condition $T, \lambda \ll 1$ is not satisfied, then equation (12) will not hold and we show our numerical results in Figs. 3 and 4, where we take $\lambda = 8$ and $T = 1$ as an example.

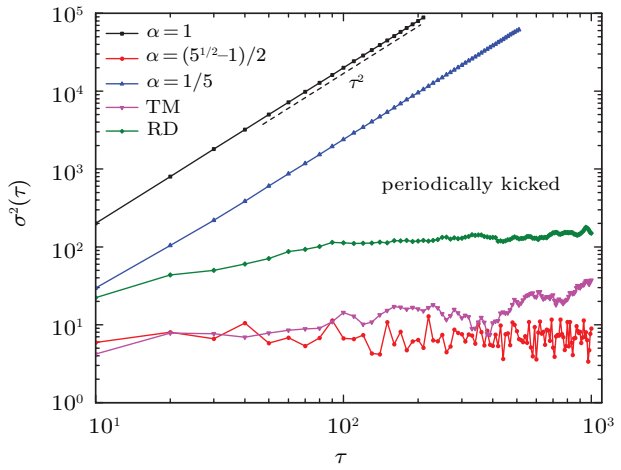



**Fig. 3.** (color online) The time dependence of $\sigma^2(\tau)$ in the periodically kicked AA model, as well as TM and RD models, with $\lambda = 8$, $T = 1$, and $L = 900$.

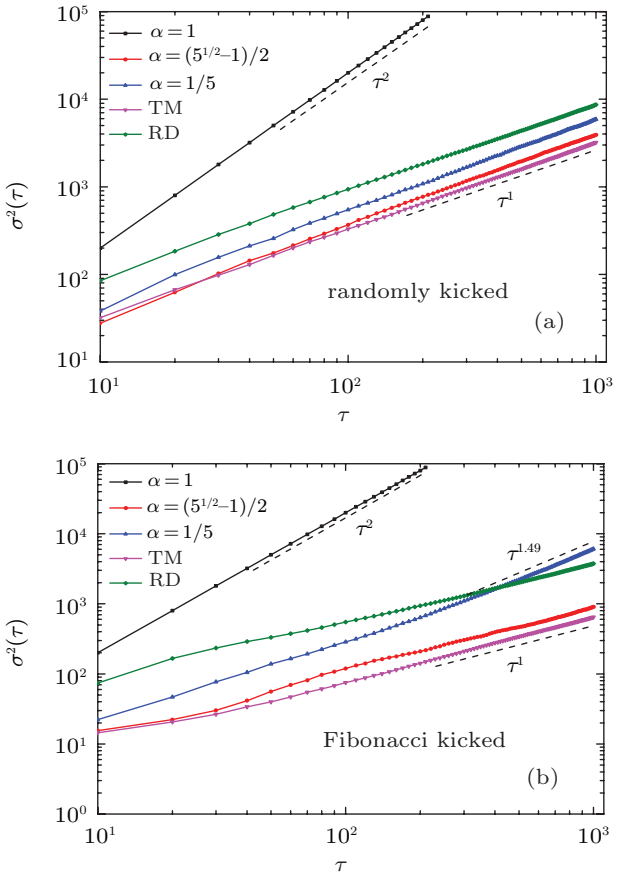



**Fig. 4.** (color online) Similar to Fig. 3, but for the randomly kicked (a) and Fibonacci kicked (b) cases.

In the periodically kicked case without temporal disorder or quasi-periodicity (as shown in Fig. 3), the wave packet is ballistically diffusive if the on-site potential is periodic in real space ($\alpha = 1$ and $\alpha = 1/5$), while it is localized if the system is spatially quasi-periodic ($\alpha = (5^{1/2} - 1)/2$). For the periodically kicked TM and RD models, the wave packet is diffusive and localized for the former and latter cases, respectively. Once the temporal disorder is introduced [see Fig. 4(a)],

if $\alpha = 1$, then the wave packet will still spread ballistically due to Eq. (9). In comparison, for rational $\alpha$ other than one or irrational $\alpha$, it seems that the temporal disorder destroys the quantum coherence completely and the wave packet shows normal diffusion denoted by $\sigma^2(t) \sim t$. We have verified that, for other values of $T, \lambda \geq 1$, the results remain qualitatively the same, that is, the wave packet is ballistically diffusive for $\alpha = 1$ while it shows normal diffusion for $\alpha$ other than one, no matter $\alpha$ is rational or irrational. Similar behaviors exist for the TM and RD models.

In contrast, in the Fibonacci kicked case [see Fig. 4(b)], for $\alpha = 1$, the wave packet shows similar behavior as that in the periodically kicked and randomly kicked cases. For other values of $\alpha$, however, the quantum coherence is not suppressed completely and the mean-square displacement evolves as $\sigma^2(t) \sim t^\gamma$, where the index $\gamma$ can vary from one to two, depending on the values of both $T$ and $\lambda$, instead of $\lambda/T$. That is, the wave packet spreads superdiffusively for $\alpha \neq 1$. Nevertheless, we can see that the temporal disorder or quasi-periodicity can also destroy dynamical localization for the parameters we choose (see the $\alpha = (5^{1/2}-1)/2$ case in Figs. 3 and 4).

Furthermore, beyond the $T, \lambda \ll 1$ limit, an analytical expression for $\sigma^2(t)$ is difficult to derive, for the following reasons. First, following the similar method in Ref. [44], we have

$$a_{i_n}(nT) = \exp\left[\mathrm{i}\frac{\pi}{2}(i_n - i_0)\right] \exp[-\mathrm{i}\lambda_0 M_{i_0}] \sum_{i_1} \cdots \sum_{i_{n-1}} \exp(-\mathrm{i}\lambda_1 M_{i_1}) \cdots \exp(-\mathrm{i}\lambda_{n-1} M_{i_{n-1}}) \mathrm{J}_{i_1 - i_0}(2T) \cdots \mathrm{J}_{i_n - i_{n-1}}(2T). \tag{13}$$

Here $i_0 = L/2$ and $\mathrm{J}_n(x)$ is the Bessel function of the first kind. If $\alpha = 1$, then $M_i = 1$ for all $i$ and we have

$$a_{i_n}(nT) = \exp\left[\mathrm{i}\frac{\pi}{2}(i_n - i_0)\right] \exp[-\mathrm{i}(\lambda_0 + \cdots + \lambda_{n-1})] \sum_{i_1} \cdots \sum_{i_{n-1}} \mathrm{J}_{i_1 - i_0}(2T) \cdots \mathrm{J}_{i_n - i_{n-1}}(2T). \tag{14}$$

By repeatedly using the property of the Bessel function $\sum_m \mathrm{J}_m(x)\mathrm{J}_{n-m}(z) = \mathrm{J}_n(x+z)$, we obtain

$$\begin{aligned} a_{i_n}(nT) &= \exp\left[\mathrm{i}\frac{\pi}{2}(i_n - i_0)\right] \exp[-\mathrm{i}(\lambda_0 + \cdots + \lambda_{n-1})]\, \mathrm{J}_{i_n - i_0}(2nT), \\ \sigma^2(nT) &= \sum_{i_n} (i_n - i_0)^2 \mathrm{J}^2_{i_n - i_0}(2nT) = 2(nT)^2. \end{aligned} \tag{15}$$

Therefore we have $\sigma^2(t) \sim t^2$ for $\alpha = 1$. However, if $\alpha \neq 1$, then the property $\sum_m \mathrm{J}_m(x)\mathrm{J}_{n-m}(z) = \mathrm{J}_n(x+z)$ cannot be used anymore, since $M_i$ now depends on $i$. Thus currently we do not have an analytical expression of $\sigma^2(t)$ for $\alpha \neq 1$ and beyond the $T, \lambda \ll 1$ limit.

## 4. Conclusion

In summary, we have investigated the effect of temporal disorder or quasi-periodicity on the wave packet dynamics in the kicked AA model. We found that, if the kicked on-site potential is periodic in real space, then the wave packet will stay diffusive no matter the system is kicked randomly or quasi-periodically. On the other hand, if the kicked on-site potential is spatially quasi-periodic, then the introduction of temporal disorder or quasi-periodicity may shift the dynamical localization point and even destroy the dynamical localization in some certain parameter range. Since the spatial disorder or quasi-periodicity usually leads to localization, therefore, the effect of temporal disorder or quasi-periodicity is quite different from that of the spatial one. Furthermore, at $T, \lambda \ll 1$, the behavior of the wave packet can be fully understood based on Eq. (12), while at $T, \lambda \geq 1$, an analytical understanding of the wave packet dynamics will constitute our future study. Finally, whether such temporal disorder or quasi-periodicity can lead to localization in time in the recently proposed time crystal also needs our further investigation.

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