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# Possible generation of $\pi$-condensation in a free space by collisions between photons and protons

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A sharply peaked structure is found in the angular distribution of emitted $\pi^+$ mesons from photon–proton collisions. This offers a possible way to generate a $\pi^+$-condensation in free space. To make the stimulated emission of $\pi^+$-mesons efficient, a ring resonator is designed.

**Keywords:** hadron quantum electrodynamics, stimulated $\pi^+$-emission, $\pi$-condensation, ring resonator

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## 1. Introduction

Possible $\pi$-condensation in nuclear matter was first considered in the 1970s.[1,2] To put the work on a better mathematic foundation, we have exactly solved the Dirac equation for the nucleon in a classical $\pi$-field.[3] A theory of nuclear matter was therefore proposed in the form of a relativistic mean-field theory, in which a mean $\pi$-field is taken into account.[4–7] To minimize the energy per-nucleon, a nonzero value of mean $\pi$-field appears. This shows the possible existence of the $\pi$-condensation in nuclear matter. The $\pi$-condensation is also considered in quark–gluon plasma.[8–10] However, until now, we have not seen any direct experimental evidence for the existence of a $\pi$-condensation in either nuclear matter or in free space. Consequently, we aim to generate a $\pi$-condensation experimentally, in free space first. Of course, the generation of a $\pi$-condensation in free space itself is interesting.

We designed a method to generate a $\gamma$-ray laser using photon–electron collisions.[11,12] We see that the angular distribution of the outgoing $\gamma$-photons is sharply peaked. Because the energy of an emitted $\gamma$-photon is a definite function of its outgoing direction, the $\gamma$-photons are almost emitted into one state. In this paper, we shall show that a similar situation appears for the $\pi^+$-meson emissions in the photon–proton collisions. The angular distribution of the emitted $\pi^+$-mesons is also sharply peaked. Therefore, we will design a similar way to generate a $\pi^+$-condensation in free space, by use of photon–proton collisions.

## 2. The angular distribution of the emitted $\pi^+$-mesons in a head on photon–proton collision

Consider the reaction

$$\gamma + \mathrm{p} \to \pi^+ + \mathrm{n}, \tag{1}$$

in which a $\gamma$-photon and a proton p collide head on, and then transit into a $\pi^+$-meson and a neutron n. When the energy of the photon is not too high, the color degrees of freedom in hadrons are not important. Therefore, this problem may be handled by hadron quantum electrodynamics. We denote the proton field, neutron field, charged meson field, and photon field by $\Psi_\mathrm{p}$, $\Psi_\mathrm{n}$, $\Phi$, and $(A_\mu)$, respectively, the Lagrangian density for the system is

$$\mathscr{L} = \mathscr{L}_\mathrm{p} + \mathscr{L}_\mathrm{n} + \mathscr{L}_\mathrm{m} + \mathscr{L}_\gamma + \mathscr{L}_\mathrm{s}, \tag{2}$$

in which

$$\mathscr{L}_\mathrm{p} = -\overline{\Psi}_\mathrm{p}\left[\gamma_\mu(\partial_\mu - \mathrm{i}eA_\mu) + m\right]\Psi_\mathrm{p}, \tag{3}$$

$$\mathscr{L}_\mathrm{n} = -\overline{\Psi}_\mathrm{n}\left(\gamma_\mu\partial_\mu + m\right)\Psi_\mathrm{n}, \tag{4}$$

$$\mathscr{L}_\mathrm{m} = -\left[\left(\partial_\mu + \mathrm{i}eA_\mu\right)\Phi^*\left(\partial_\mu - \mathrm{i}eA_\mu\right)\Phi + m_\pi^2\Phi^*\Phi\right], \tag{5}$$

$$\mathscr{L}_\gamma = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right), \tag{6}$$

$$\mathscr{L}_\mathrm{s} = \mathrm{i}\sqrt{2}G\left(\overline{\Psi}_\mathrm{p}\gamma_5\Psi_\mathrm{n}\Phi + \overline{\Psi}_\mathrm{n}\gamma_5\Psi_\mathrm{p}\Phi^*\right) \tag{7}$$

are Lagrangian densities of protons with their electromagnetic interactions, neutrons, $\pi^\pm$-mesons with their electromagnetic interactions, photons, and the strong interactions between related hadrons, respectively. The nature unit system of $\hbar = c = 1$ is used. Symbols are defined in the usual way as given in standard text books, see for example Refs. [13] and [14]. $e$ and $G$ are interaction constants for electromagnetic interaction and strong interaction, respectively, with the corresponding values $\alpha \equiv e^2/4\pi = 1/137$ and $\alpha_\mathrm{s} \equiv G^2/4\pi = 14.6$.

For the reaction (1), a factor of electromagnetic interaction appears always with a factor of strong interaction. The constants $\alpha$ and $\alpha_\mathrm{s}$ always appear together in the form of a product $\alpha\alpha_\mathrm{s}$. Since $\alpha\alpha_\mathrm{s} < 1$, a perturbation treatment for the reaction seems reasonable. The lowest order transition matrix element for the reaction is

$$\langle\sigma_\mathrm{n}, \boldsymbol{q}, \boldsymbol{\kappa}|T|\boldsymbol{k}, e, \boldsymbol{p}, \sigma_\mathrm{p}\rangle = -Ge\frac{\prod_{\mu=0}^{3}\delta(p_\mu + k_\mu - q_\mu - \kappa_\mu)}{\sqrt{2}(2\pi)^2\sqrt{k\kappa_0}}$$

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$$\times \bar{u}_{\sigma_n}(\boldsymbol{q})\gamma_5 \left[\frac{1}{\mathrm{i}\gamma^\mu(p_\mu+k_\mu)+m}\mathrm{i}\gamma^\mu e_\mu + \frac{1}{(p^\mu-q^\mu)(p_\mu-q_\mu)+m_\pi^2}2\kappa^\mu e_\mu\right] u_{\sigma_p}(\boldsymbol{p}), \quad (8)$$

in which $[p_\mu]$, $[q_\mu]$, $[k_\mu]$, and $[\kappa_\mu]$, with $\mu = 0,1,2,3$ are energy–momentum four vectors of proton, neutron, photon, and $\pi^+$-meson, respectively. $[e_\mu]$ with $\mu = 0,1,2,3$ is the polarization four-vector of the photon. $m$ and $m_\pi$ are masses of the nucleon and the charged pion, respectively. $u_{\sigma_n}(\boldsymbol{q})$ is the Dirac spinor of the neutron with spin $\sigma_n$ and momentum $\boldsymbol{q}$, and $u_{\sigma_p}(\boldsymbol{p})$ is that of the proton with spin $\sigma_p$ and momentum $\boldsymbol{p}$. $\delta$-functions in Eq. (8) show energy–momentum conservation in the reaction (1). Together with the energy–momentum relations

$$p_0 = \sqrt{p^2+m^2},\quad q_0 = \sqrt{q^2+m^2},\quad \kappa_0 = \sqrt{\kappa^2+m_\pi^2},$$

and $k_0 = k$, they give the following expressions

$$(A^2-1)\kappa^2 + 2AB\kappa + B^2 - m_\pi^2 = 0, \quad (9)$$

$$A = \frac{k-p}{p_0+k}\cos\theta, \quad B = \frac{m_\pi^2+2k(p_0+p)}{2(p_0+k)}, \quad (10)$$

where $\theta$ is the angle between moving directions of the incident photon and the emitted pion. $\kappa > 0$ is the absolute value of the pion momentum and, therefore, should be the positive root of Eq. (9). This defines the energy $\kappa_0$ as a function of the moving direction $\theta$ for the pion.

Take the Coulomb gauge, in which the contribution from longitudinal and temporal components of the electromagnetic field is collected in the Coulomb energy between charged particles and is negligible when space charge effect being unimportant. Only the contribution from the transverse components of the electromagnetic field will be considered in the following. Let $\boldsymbol{e}_i\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$ with $i = 1,2$ show the transverse plane wave, we have $e_{i0} = 0$, $\boldsymbol{e}_i\cdot\boldsymbol{k} = 0$ for $i = 1,2$, and $\boldsymbol{e}_i^*\cdot\boldsymbol{e}_{i'} = \delta_{ii'}$.

For experiments without measuring spins, all transition probabilities and cross-sections have to be summed up over the final spin states and averaged over the initial spin states. Using the projection operator method, we obtain

$$\frac{1}{4}\sum_{i=1,2}\sum_{\sigma_n=-1,1}\sum_{\sigma_p=-1,1}\left|\bar{u}_{\sigma_n}(\boldsymbol{q})\gamma_5\left[\frac{1}{\mathrm{i}\gamma^\mu(p_\mu+k_\mu)+m}\mathrm{i}\gamma^\mu e_{i\mu} + \frac{1}{(p^\mu-q^\mu)(p_\mu-q_\mu)+m_\pi^2}2\kappa^\mu e_{i\mu}\right]u_{\sigma_p}(\boldsymbol{p})\right|^2 = \frac{X}{p_0 q_0}, \quad (11)$$

$$\begin{aligned} X = {} & \frac{1}{2}f^2\left\{[2k(p+p_0)-\kappa_0(k+p_0)+\kappa(k-p)\cos\theta]\left[k(2p+p_0)-p^2\right]+2pk(p_0+p)(p-k+\kappa\cos\theta)\right. \\ & + \left[p^2+p_0(k-\kappa_0)\right](p-k)^2 - (p^2+p_0k)(p-k)(p-k+\kappa\cos\theta)\left.\right\} + \kappa^2 g(\theta)\sin^2\theta\left\{fk(p+p_0)\right. \\ & \left. + g(\theta)\left[p_0(k-\kappa_0)+p(k-\kappa\cos\theta)\right]\right\}, \end{aligned} \quad (12)$$

with $f = 1/[2k(p_0+p)]$, and $g(\theta) = 1/[2k(\kappa\cos\theta-\kappa_0)]$. The transition probability per-unit time for the pion goes into a differential solid angle $\mathrm{d}\Omega$ is

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 2\alpha_s\alpha\frac{XJ\kappa^2}{k\kappa_0 q_0 p_0}\,\mathrm{d}\Omega\,\mathrm{d}\kappa\,\delta(E_f-E_i). \quad (13)$$

$J = 1/V$ is the incident photon current density in our unit system, and $V$ is the volume of the reaction space. $E_i = p_0+k$ and $E_f = q_0+\kappa_0$ are initial and final energies of the process, respectively. Under fixed initial momenta $\boldsymbol{p}$ and $\boldsymbol{k}$,

$$\mathrm{d}\kappa = \frac{\kappa_0(p_0+k-\kappa_0)}{(p_0+k)\kappa+\kappa_0(p-k)\cos\theta}\,\mathrm{d}(E_f-E_i). \quad (14)$$

We therefore have

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 2\alpha_s\alpha XYJ\lambda_\pi^2\mathrm{d}\Omega. \quad (15)$$

$\lambda_\pi$ is the Compton wavelength of the charged pion, and

$$Y = \frac{\kappa^2 m_\pi^2}{kp_0[(p_0+k)\kappa+\kappa_0(p-k)\cos\theta]}. \quad (16)$$

Therefore, the differential cross-section for the photoproduction of the charged pion on a proton is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = 2\alpha\alpha_s XY\lambda_\pi^2. \quad (17)$$

Notice that, $X$ and $Y$ are dimensionless.

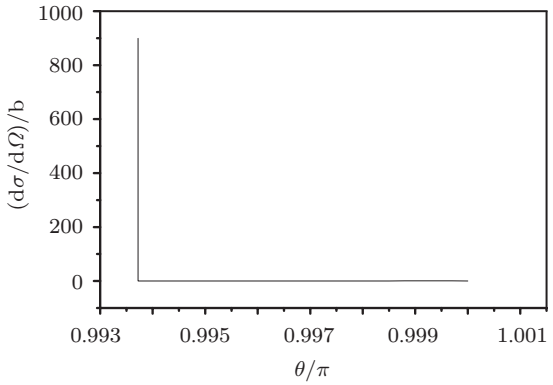



**Fig. 1.** Relation between the differential cross-section in unit of barn for pion emission and the angle $\theta$ in unit of $\pi$, in a head on collision between the 1.4-MeV photon and the 434-GeV proton.

An example of numerical results is shown in Figs. 1 and 2. The angular distribution shown in Fig. 1 is rather characteristic. It is sharply peaked and, therefore, is favorable for

emitting pions into the most probable state. However, the most probable state is not unique. In the example shown in Fig. 1, the most probable emission directions distribute on the surface of a cone, each along a generatrix. The vertex of the cone is at the reaction point and the axis is on the incident line of the proton. The angle between the generatrix and the axis is $0.006278\pi$. In the following, we shall show that a resonance mechanism makes almost all emitted pions go to one selected most probable state and, therefore, generates a $\pi^+$-condensation in free space.

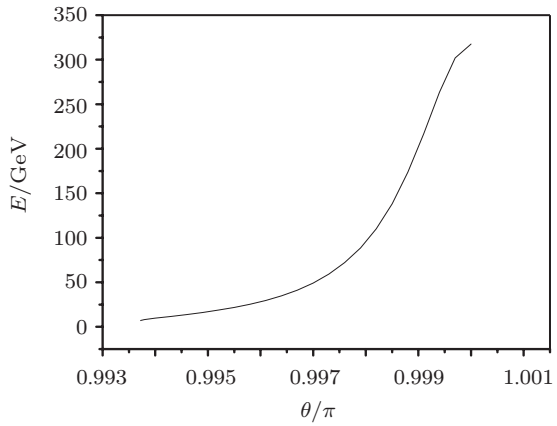



**Fig. 2.** Relation between the emitted pion energy $E$ in unit of GeV and the angle $\theta$ in unit of $\pi$, in a head on collision between the 1.4-MeV photon and the 434-GeV proton.

## 3. Stimulated emission, resonance, and the ring resonator for the $\pi$-condensation

The key ingredient for making a laser is the stimulated emission of radiation. This is also true for making a $\pi$-condensation in free space. If there are already $N$ pions in a state, then the transition probability for emitting one more pion into this state has to be multiplied by an extra factor $N+1$. Equation (15) is, therefore, generalized to

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 2(N+1)\alpha_{\mathrm{s}}\alpha XYJ\lambda_\pi^2 \mathrm{d}\Omega, \tag{18}$$

which includes contributions of both spontaneous and stimulated emissions of pions into a given state. Here, we see a positive feedback between pion numbers of already in and emitted into a given state. The result is a collapse of pion population into the most probable states.

To make emitted pions condense in one state, we need a resonance mechanism. Figure 3 shows a schematic designation of a ring resonator for the stimulated emission of $\pi^+$-mesons in head on collisions between photons and protons. The straight line denotes the incident line and the circle denotes the storage ring. They intersect each other at two gaps on the ring. Photons and protons inject from left and right, respectively, along the incident line, and are designed to collide with each other at the gaps. Elementary geometry tells us that angles between tangents of the circle at two gaps and the incident line equal each other, so that we may design these tangents along the most probable directions for pion emission at both gaps. Therefore, $\pi^+$-mesons emitted on this most probable direction enter the inner space of the ring. This is its central circular channel. They move along the channel under the interaction of an appropriate constant magnetic field perpendicular to the ring plane. A coincidence of colliding photons and protons together with the earlier emitted $\pi^+$-mesons at the gaps is designed, so that the stimulated emissions may happen. The already stored $\pi^+$-mesons stimulate the new $\pi^+$-meson emissions at the gap. The $\pi^+$-mesons are emitted along the most probable direction selected by the ring and enter the storage ring at the gap under the interaction of the magnetic field. Therefore, they are prepared to stimulate the next $\pi^+$-meson emissions at the next gap. In this way, a resonance is formed and a special most probable emission is selected at each gap.

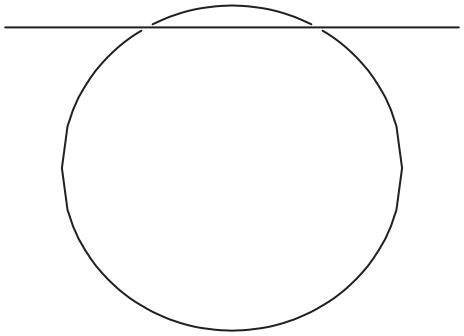


**Fig. 3.** A schematic diagram of the ring resonator for the stimulated emission of $\pi^+$-mesons in head on collisions between photons and protons.

In our example of a head on collision between a 1.4-MeV photon and a 434-GeV proton, the energy of the most probable emitted $\pi^+$-meson is 6.9 GeV, as shown in Fig. 2. In a magnetic field of $B = 1$ T, they would move in a storage ring of radius 23 m. Using the data shown at the end of last section, we see that two gaps on the ring in Fig. 3 open a central angle of $0.012556\pi$ at the center of the ring. Therefore, the length of the arc between these two gaps on the ring of radius 23 m is 0.9 m. One may therefore open many pairs of gaps on the ring to intensify the condensed pion beam many times in one circle. Meanwhile, the half-life time of a 6.9-GeV $\pi^+$-meson is $8.95 \times 10^{-7}$ s. A half of $\pi^+$-mesons in the beam may move 268.33 m before their decay. This is about twice the ring circumference. It seems that we may generate a rather intense condensed pion beam in this way and then store it in the ring. However, there are various interactions between $\pi^+$-mesons. Among them, the long range electromagnetic interaction may be important at not too high density of the meson beam. This is the so called space charge problem. The electric force of the space charge, perpendicular to the meson trajectories, means that there is a nonzero probability for mesons to leave from the resonance orbits and it also limits the beam density. This

offers a saturation mechanism for the $\pi-$condensation in our example. In addition, charged pions running in a ring may lose energy by Bremsstrahlung. However, it may be easily compensated by usual acceleration techniques.

## 4. Conclusions

The $\pi$-condensation may be generated in a similar way to that used in laser generation. First, we need a pion source. In the example proposed here, it is played by the hadronic reaction (1). Various sharply peaked spectra of pion emission appear. These make emissions concentrate to some specified pion states and, therefore, are welcome. We then need a way for realizing the stimulated pion emissions to start the pi-condensation, and need a resonance mechanism to select a special pion state to condense. These are designed in the ring resonator that was shown in the last section.

The reaction (1) was analyzed by the hadron electrodynamics. Hadron-dynamics is not a fundamental theory but is an effective theory. Therefore, we should not rely on its quantitative results. Although we may obtain the quantitative results directly by experiments, some qualitative characters do not depend on the dynamical details. In the derivation and the numerical calculation, we see that the sharply peaked structures of the angular distribution for pion emissions are connected directly with the relativistic energy-momentum conservation relations (9) and (10). This is a result of kinetics governing the reaction and, therefore, is reliable. The designation of the ring resonator is based on fundamental electromagnetism, and is therefore reliable too. This makes us believe that our proposal is worthy to try experimentally.

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