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Optomechanical state transfer between two distant membranes in the presence of non-Markovian environments*

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The quantum state transfer between two membranes in coupled cavities is studied when the system is surrounded by non-Markovian environments. An analytical approach for describing non-Markovian memory effects that impact on the state transfer between distant membranes is presented. We show that quantum state transfer can be implemented with high efficiency by utilizing the experimental spectral density, and the performance of state transfer in non-Markovian environments is much better than that in Markovian environments, especially when the tunneling strength between the two cavities is not very large.

Keywords: optomechanical systems, non-Markovian effect, quantum state transfer

PACS: 03.65.Yz, 03.65.Ud, 42.50.Dv **DOI:** 10.1088/1674-1056/27/12/120302

1. Introduction

Cavity optomechanical system is a well-developed tool to exhibit the squeezing and entanglement phenomena,[1–4] and has recently received much attention[5–7] due to its potential ability to realize the purpose of quantum information processing. The strong coupling between mechanical and optical modes opens up great avenues in efficiently transferring a quantum state between photons and phonons.[8,9] When transferring a quantum state from one site to another,[10] the mechanical oscillator with small decay rate serves as a potential quantum network node for quantum information storage and processing,[11,12] which is also the central goal in quantum networking schemes. Therefore, state transfer in optomechanics is an important task, and it has been extensively studied.[8,13–17] Recently, theoretical research of state transfer between distant mechanical oscillators[18,19] or in optomechanical arrays[20] has also been carried out.

The above-mentioned studies are all focussing on the scenario of memoryless environment, and the corresponding Markov approximation is used to derive the quantum Langevin equations.[21] However, a recent experiment showed that the dynamics of microresonators are non-Markovian.[22] Therefore, it is necessary to explore the non-Markovian effect in optomechanical systems. In recent years, a lot of theoretical research has been carried out on this topic, and it is proved that the backflow of information from the environments into the system known as the memory effect[23] is helpful for entanglement protection,[24,25] ground state cooling,[26,27] and force detection[28] in optomechanical system.

Previous studies have shown that the non-Markovian environments appear to be beneficial to quantum state transfer in distant cavities[29] or qubits.[30] However, it is still unknown whether the non-Markovian memory effect is helpful for state transfer between coupled optomechanical systems. In this paper, the quantum state transfer between two distant membranes is investigated in non-Markovian regime. The experimental mechanical spectral density[22] is used and the exact state transfer efficiency is derived based on the modified Laplace transformation.[31] Our results show that the quantum state transfer between distant membranes can be implemented with high efficiency in the presence of non-Markovian environments. We also find that if the tunneling strength between the two cavities gets smaller, the transfer time required for reaching the maximum value of efficiency increases. At this point, the long-time non-Markovian memory effect becomes significant. Thus, the performance of state transfer in non-Markovian environments will be much better than the Markovian case.

This article is organized as follows. Section 2 describes the model and gives the theoretical description of the dynamics. In Section 3, we investigate in detail the numerical results of quantum state transfer. Finally, the conclusion is given in Section 4.

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2. Model Hamiltonian and theoretical description of the dynamics

We consider a cavity optomechanical system consisting of two membranes, which are placed separately in two distant cavities. The cavities are coupled to each other via an optical fiber. The Hamiltonian of this system can be written as

$$\hat{H}_{\rm S} = \sum_{i=1,2} \hbar[\Omega_i \hat{a}_i^\dagger \hat{a}_i + \omega_i \hat{b}_i^\dagger \hat{b}_i + g_i \hat{a}_i^\dagger \hat{a}_i (\hat{b}_i^\dagger + \hat{b}_i) + E(\mathrm{e}^{-\mathrm{i}\omega_{\rm L} t}\hat{a}_i^\dagger + \mathrm{e}^{\mathrm{i}\omega_{\rm L} t}\hat{a}_i)] + \hbar J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1), \quad (1)$$

where $\hat{a}_i$ and $\hat{b}_i$ are annihilation operators of the i-th cavity field and the membrane, respectively. The cavity fields are coupled to the membranes via radiation pressure with coupling coefficients g_i, and the two cavities are both driven by coherent laser with driving strength E and center frequency $\omega_{\rm L}$. J gives the strength of the tunneling interaction between the two cavities. In the rotating frame with the laser frequency, and when the intracavity field is strong enough, the Hamiltonian of the system can be linearized as[7,25,26]

$$\hat{H}_{\rm S} = \sum_{i=1,2} \hbar[\Delta_i \hat{a}_i^\dagger \hat{a}_i + \omega_i \hat{b}_i^\dagger \hat{b}_i + (G_i \hat{a}_i^\dagger + G_i^* \hat{a}_i)(\hat{b}_i^\dagger + \hat{b}_i)] + \hbar J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1). \quad (2)$$

Here, $G_i = \alpha_i g_i$ is the effective coupling rate, where $\alpha_i = \langle \hat{a}_i \rangle$ is the mean value of the cavity field, and $\Delta_i \approx \Omega_i - \omega_{\rm L} - 2|G_i|^2/\omega_i$ is the optomechanical-coupling modified detuning.[25] In the Born–Markovian approximation, the side-band condition requires that the cavity linewidth is smaller than the mechanical frequency, *i.e.*, $\kappa < \omega_{\rm m}$. However, in the presence of non-Markovian environment, due to the interaction with the environment, the corresponding optical dissipation rates κ_i and the frequencies of the mechanical modes ω_i should be replaced by the modulated parameters, *i.e.*, the effective optical dissipation rate $\kappa_i \to \kappa_{\rm eff}$ and the effective mechanical frequency $\omega_i \to \omega_{\rm eff}$. Here, for simplicity, we assume $\omega_1 = \omega_2 = \omega_{\rm m}$ and $\kappa_1 = \kappa_2 = \kappa$. Thus, the side-band condition can be rewritten as $\kappa_{\rm eff} < \omega_{\rm eff}$. To be specific, the effective optical dissipation rate $\kappa_{\rm eff}$ is determined by the poles localized at the lower half of the complex plane corresponding to the branch cuts B_k.[31] If the non-Markovian effect is not strong enough, then the coupling between the system and environment is much smaller than $\omega_{\rm m}$, we have $\omega_{\rm eff} \approx \omega_{\rm m}$.[27,32] Meanwhile, the effective optical dissipation is approximately equal to the dissipation rate under Markovian approximation, namely, $\kappa_{\rm eff} \approx \mathcal{J}_{\rm c}(\omega_{\rm c})/2$. Thus, this model is valid under the red-detuned regime $\Delta_i = \omega_{\rm m}$ when the effective side-band condition $\kappa_{\rm eff} < \omega_{\rm m}$ is satisfied. In the red-detuned regime, by applying the rotating-wave approximation, the nonresonant term in Eq. (2) can be ignored. The Hamiltonian of the system then reads

$$\hat{H}_{\rm S} = \sum_{i=1,2} \hbar\left[\Delta \hat{a}_i^\dagger \hat{a}_i + \omega_{\rm m} \hat{b}_i^\dagger \hat{b}_i + G \hat{a}_i^\dagger \hat{b}_i + G^* \hat{a}_i \hat{b}_i^\dagger\right] + \hbar J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1). \quad (3)$$

Here, we assume $\Delta_1 = \Delta_2 = \Delta$ and $G_1 = G_2 = G$. The environments can be described by a collection of independent harmonic oscillators, so the Hamiltonian of the environments can be expressed as

$$\hat{H}_{\rm E} = \sum_{i,k} \hbar\Omega_{ik} \hat{a}_{ik}^\dagger \hat{a}_{ik} + \hbar\omega_{ik} \hat{b}_{ik}^\dagger \hat{b}_{ik}, \quad (4)$$

where Ω_{ik} and ω_{ik} are the reservoir frequency of the k-th optical and mechanical mode interacting with the i-th optomechanical system. The interaction between the system and the environment is given by

$$\hat{H}_{\rm I} = \sum_{i,k} \hbar g_{ik}(\hat{a}_{ik}^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_{ik}) + \hbar g'_{ik}(\hat{b}_{ik}^\dagger \hat{b}_i + \hat{b}_i^\dagger \hat{b}_{ik}), \quad (5)$$

where g_{ik} and g'_{ik} are the system–bath coupling strength. Here, the interaction is written in rotating-wave-approximation form, it is valid[33] when the coupling strength is weak compared to the system, *i.e.*, $g_{ik}, g'_{ik} \ll \omega_{\rm m}$.

The Heisenberg equations of the system can be derived from the Hamiltonian given above, which are

$$\begin{aligned}
\dot{\hat{a}}_1 &= -\mathrm{i}\Delta \hat{a}_1 - \mathrm{i}G\hat{b}_1 - \mathrm{i}J\hat{a}_2 - \mathrm{i}\sum_k g_{1k}\hat{a}_{1k}, \\
\dot{\hat{a}}_2 &= -\mathrm{i}\Delta \hat{a}_2 - \mathrm{i}G\hat{b}_2 - \mathrm{i}J\hat{a}_1 - \mathrm{i}\sum_k g_{2k}\hat{a}_{2k}, \\
\dot{\hat{b}}_1 &= -\mathrm{i}\omega_{\rm m}\hat{b}_1 - \mathrm{i}G\hat{a}_1 - \mathrm{i}\sum_k g'_{1k}\hat{b}_{1k}, \\
\dot{\hat{b}}_2 &= -\mathrm{i}\omega_{\rm m}\hat{b}_2 - \mathrm{i}G\hat{a}_2 - \mathrm{i}\sum_k g'_{2k}\hat{b}_{2k}, \quad (6)
\end{aligned}$$

and the Heisenberg equations of the environments read

$$\begin{aligned}
\dot{\hat{a}}_{1k} &= -\mathrm{i}\Omega_{1k}\hat{a}_{1k} - \mathrm{i}g_{1k}\hat{a}_1, \\
\dot{\hat{a}}_{2k} &= -\mathrm{i}\Omega_{2k}\hat{a}_{2k} - \mathrm{i}g_{2k}\hat{a}_2, \\
\dot{\hat{b}}_{1k} &= -\mathrm{i}\omega_{1k}\hat{b}_{1k} - \mathrm{i}g'_{1k}\hat{b}_1, \\
\dot{\hat{b}}_{2k} &= -\mathrm{i}\omega_{2k}\hat{b}_{2k} - \mathrm{i}g'_{2k}\hat{b}_2. \quad (7)
\end{aligned}$$

Solving Eq. (7) for $\hat{a}_{ik}$ and $\hat{b}_{ik}$, then substituting their solutions into Eq. (6), we can obtain the following integro-differential Heisenberg equations

$$\begin{aligned}
\dot{\hat{a}}_1 &= -\mathrm{i}\Delta \hat{a}_1 - \mathrm{i}G\hat{b}_1 - \mathrm{i}J\hat{a}_2 - \int_0^t \mathrm{d}\tau f_{\rm c}(t-\tau)\hat{a}_1(\tau) + \hat{a}_{\rm in}, \\
\dot{\hat{a}}_2 &= -\mathrm{i}\Delta \hat{a}_2 - \mathrm{i}G\hat{b}_2 - \mathrm{i}J\hat{a}_1 - \int_0^t \mathrm{d}\tau f_{\rm c}(t-\tau)\hat{a}_2(\tau) + \hat{a}_{\rm in}, \\
\dot{\hat{b}}_1 &= -\mathrm{i}\omega_{\rm m}\hat{b}_1 - \mathrm{i}G\hat{a}_1 - \int_0^t \mathrm{d}\tau f_{\rm m}(t-\tau)\hat{b}_1(\tau) + \hat{b}_{\rm in}, \\
\dot{\hat{b}}_2 &= -\mathrm{i}\omega_{\rm m}\hat{b}_2 - \mathrm{i}G\hat{a}_2 - \int_0^t \mathrm{d}\tau f_{\rm m}(t-\tau)\hat{b}_2(\tau) + \hat{b}_{\rm in}. \quad (8)
\end{aligned}$$

When deriving Eq. (8), for simplicity, we also assume the identical environments of the cavities and membranes respectively, namely, the cavity–bath/membrane–bath coupling

strength and the cavity/membrane environmental density of states are the same. Therefore, we have $\Omega_{ik}=\Omega_k$, $\omega_{ik}=\omega_k$, $g_{ik}=g_k$, and $g'_{ik}=g'_k$. In Eq. (8), $\hat{a}_{\rm in}=-{\rm i}\sum_k g_k {\rm e}^{-{\rm i}\Omega_k t}\hat{a}_{ik}(0)$ and $\hat{b}_{\rm in}=-{\rm i}\sum_k g'_k {\rm e}^{-{\rm i}\omega_k t}\hat{b}_{ik}(0)$ are the input noise operators which depend on the initial condition of the environment. The non-Markovian effect is fully manifested in Eq. (8) through the non-local time correlation functions $f_{\rm c}(t)=\sum_k g_k^2 {\rm e}^{-{\rm i}\Omega_k t}$ and $f_{\rm m}(t)=\sum_k g'^2_k {\rm e}^{-{\rm i}\omega_k t}$,[24] where g_k and g'_k are assumed to be real. By introducing the spectral density of the reservoir, one can rewrite the correlation functions,

$$f_{\rm c}(t)=\int\frac{{\rm d}\omega}{2\pi}J_{\rm c}(\omega){\rm e}^{-{\rm i}\omega t}$$

and

$$f_{\rm m}(t)=\int\frac{{\rm d}\omega}{2\pi}J_{\rm m}(\omega){\rm e}^{-{\rm i}\omega t},$$

where $J_{\rm c}(\omega)$ and $J_{\rm m}(\omega)$ are the spectral density of the optical and mechanical environment, respectively.

For general bosonic environment, the spectral density can be an Ohmic spectrum with a Poisson-type distribution function.[34] For the cavity, usually the spectral density could be $J_{\rm c}(\omega)=2\pi\eta_{\rm c}\omega(\omega/\tilde{\omega}_{\rm c})^{s_{\rm c}-1}{\rm e}^{-\omega/\tilde{\omega}_{\rm c}}$, where $\eta_{\rm c}$ is a dimensionless coupling strength between system and environment, and $\tilde{\omega}_{\rm c}$ is a high-frequency cutoff.[34,35] The parameter $s_{\rm c}$ classifies the environment as sub-Ohmic $(0<s_{\rm c}<1)$, Ohmic $(s_{\rm c}=1)$, and super-Ohmic $(s_{\rm c}>1)$. Recently, the spectral density of the mechanical environment is measured through experiment.[22] We therefore use this experimental spectral density as the membrane's environment, which can be described by $J_{\rm m}(\omega)=2\pi C\omega^k$, where C is the coupling constant and $k\approx-2.3\pm1.05$. The bandwidth of the reservoir is $\Gamma\approx0.07\omega_{\rm m}$, where $\omega_{\rm m}=914$ kHz is the mechanical resonance frequency. So the frequency region is $\omega\in[\omega_{\rm min},\omega_{\rm max}]$, with $\omega_{\rm min}=885$ kHz and $\omega_{\rm max}=945$ kHz.

Now we rewrite Eq. (8) in a more compact form by defining $\hat{O}(t)\equiv[\hat{a}_1(t),\hat{a}_2(t),\hat{b}_1(t),\hat{a}_2(t)]^{\rm T}$, which are

$$\dot{\hat{O}}(t)=-{\rm i}M\hat{O}(t)-\int_0^t{\rm d}\tau\overline{F}(t-\tau)\hat{O}(\tau)+\hat{\xi}_{\rm in}(t),\qquad(9)$$

where the noise term $\hat{\xi}_{\rm in}(t)=[\hat{a}_{\rm in},\hat{a}_{\rm in},\hat{b}_{\rm in},\hat{b}_{\rm in}]^{\rm T}$, and the matrix M and $\overline{F}(t)$ is given by

$$M=\begin{pmatrix}\Delta & J & G & 0\\ J & \Delta & 0 & G\\ G & 0 & \omega_{\rm m} & 0\\ 0 & G & 0 & \omega_{\rm m}\end{pmatrix},$$
$$\overline{F}(t)=\begin{pmatrix}f_{\rm c}(t) & 0 & 0 & 0\\ 0 & f_{\rm c}(t) & 0 & 0\\ 0 & 0 & f_{\rm m}(t) & 0\\ 0 & 0 & 0 & f_{\rm m}(t)\end{pmatrix}.\qquad(10)$$

Equation (9) can be formally solved by assuming $\hat{O}(t)=\mathcal{U}(t)\hat{O}(0)+\hat{\mathcal{V}}(t)$, where the 4×4 matrix functions $\mathcal{U}(t)$ and $\hat{\mathcal{V}}(t)$ are related to the non-equilibrium Green's functions[36,37] of the system. These Green's functions obey the following Dyson equations:

$$\dot{\mathcal{U}}(t)=-{\rm i}M\mathcal{U}(t)-\int_0^t{\rm d}\tau\overline{F}(t-\tau)\mathcal{U}(\tau),$$
$$\dot{\hat{\mathcal{V}}}(t)=-{\rm i}M\hat{\mathcal{V}}(t)-\int_0^t{\rm d}\tau\overline{F}(t-\tau)\hat{\mathcal{V}}(\tau)+\hat{\xi}_{\rm in}(t).\qquad(11)$$

The integro-differential equations (11) can be solved by utilizing the modified Laplace transformation.[31] Note that $\hat{\mathcal{V}}(t)$ satisfies the initial condition $\hat{\mathcal{V}}(0)=0$, therefore, we can first give the following formal solution:

$$\hat{\mathcal{V}}(t)=\int_0^t{\rm d}\tau\mathcal{U}(t-\tau)\hat{\xi}_{\rm in}(\tau).\qquad(12)$$

Obviously, $\mathcal{U}(t)$ reveals the general properties of non-Markovian dynamics. Using the modified Laplace transform $\mathcal{U}(z)=\mathcal{L}\{\mathcal{U}(t)\}=\int_0^\infty\mathcal{U}(t){\rm e}^{{\rm i}zt}{\rm d}t$ and in view of the initial condition $\mathcal{U}(0)=1$, we have

$$-{\rm i}z\mathcal{U}(z)-I=-{\rm i}M\mathcal{U}(z)-\Sigma(z)\mathcal{U}(z),\qquad(13)$$

where I is the identity, $\Sigma(z)$ is the Laplace transform of the self-energy correction,[31]

$$\Sigma(z)=\mathcal{L}\{\overline{F}(t)\}=\int\frac{{\rm d}\omega}{2\pi}\begin{pmatrix}J_{\rm c}(\omega)/(z-\omega) & 0 & 0 & 0\\ 0 & J_{\rm c}(\omega)/(z-\omega) & 0 & 0\\ 0 & 0 & J_{\rm m}(\omega)/(z-\omega) & 0\\ 0 & 0 & 0 & J_{\rm m}(\omega)/(z-\omega)\end{pmatrix}.\qquad(14)$$

Solving Eq. (13) and applying the inverse Laplace transformation, we obtain the following Bromwich integral:

$$\mathcal{U}(t)=\mathcal{L}^{-1}\{\mathcal{U}(z)\}=\frac{1}{2\pi}\int_{-\infty+{\rm i}\lambda}^{\infty+{\rm i}\lambda}{\rm d}z\frac{{\rm i}\,{\rm e}^{-{\rm i}zt}}{z-M-\Sigma(z)},\qquad(15)$$

where λ is an arbitrary positive number greater than the imaginary part of all the poles of ${\rm i}/(z-M-\Sigma(z))$.

With the solution $\mathcal{U}(t)$, we can exactly describe the dynamical evolution of $\hat{b}_2(t)$, namely, $\hat{b}_2(t)=\mathcal{U}_{41}(t)\hat{a}_1(0)+\mathcal{U}_{42}(t)\hat{a}_2(0)+\mathcal{U}_{43}(t)\hat{b}_1(0)+\mathcal{U}_{44}(t)\hat{b}_2(0)+\hat{\mathcal{V}}_4(t)$. If the state is initially encoded onto the first membrane $\hat{b}_1(0)$ and the second membrane is initially in a vacuum state, the state-independent efficiency[38,39] can be defined as the coefficient in front of $\hat{b}_1(0)$. In order to characterize the performance of the state transfer from the first membrane to the second membrane, we can take $\mathcal{U}_{43}$ as a criterion, and assume the efficiency

$E = |\mathcal{U}_{43}|$, where $\mathcal{U}_{43}$ is given by

$$\mathcal{U}_{43}(t) = \frac{1}{2\pi}\int_{-\infty+i\lambda}^{\infty+i\lambda} dz \frac{iJG^2 e^{-izt}}{(G^2 - K_m K_c)^2 - J^2 K_m^2}, \quad (16)$$

where

$$K_m = z - \omega_m - \int \frac{d\omega}{2\pi}\frac{J_m(\omega)}{z-\omega},$$
$$K_c = z - \Delta - \int \frac{d\omega}{2\pi}\frac{J_c(\omega)}{z-\omega}.$$

3. Numerical results of quantum state transfer

In this section, we numerically solve Eq. (16) to show the performance of the state transfer. First, we discuss the comparison between Markovian and non-Markovian dynamics. The Born–Markovian approximation is usually used when the coupling strength between system and environment is very weak, and the characteristic correlation time of the environment is sufficiently shorter than that of the system.[31,40] Therefore, no memory effect remains. In optomechanical system, the quantum Langevin equations[21] was developed under the Born–Markovian approximation, in which the spectral density of the environment is assumed to be a flat spectrum, as it varies little around the system's frequency. In this case, according to Weisskopf–Wigner approximation,[41] one can replace $J_i(\omega)$ by $J_i(\omega_i)$, where $i = c, m$, and extend the limit of frequency integration to infinite. Then the time integral in Eq. (11) reduces to

$$\int_0^t d\tau \overline{F}(t-\tau)\mathcal{U}(\tau) = \int \frac{d\omega}{2\pi}\begin{pmatrix} J_c(\omega) & 0 & 0 & 0 \\ 0 & J_c(\omega) & 0 & 0 \\ 0 & 0 & J_m(\omega) & 0 \\ 0 & 0 & 0 & J_m(\omega) \end{pmatrix}\int_0^t d\tau e^{-i\omega(t-\tau)}\mathcal{U}(\tau)$$
$$\approx \begin{pmatrix} J_c(\omega_c)/2 & 0 & 0 & 0 \\ 0 & J_c(\omega_c)/2 & 0 & 0 \\ 0 & 0 & J_m(\omega_m)/2 & 0 \\ 0 & 0 & 0 & J_m(\omega_m)/2 \end{pmatrix}\mathcal{U}(t). \quad (17)$$

Thus, for comparison, we choose optical decay rate $\kappa = J_c(\omega_c)/2$ and mechanical decay rate $\gamma_m = J_m(\omega_m)/2$. The corresponding result is shown in Fig. 1. Clearly, the non-Markovian case has better performance of the state transfer than the Markovian case. Thus the non-Markovian memory effect is helpful for state transfer.

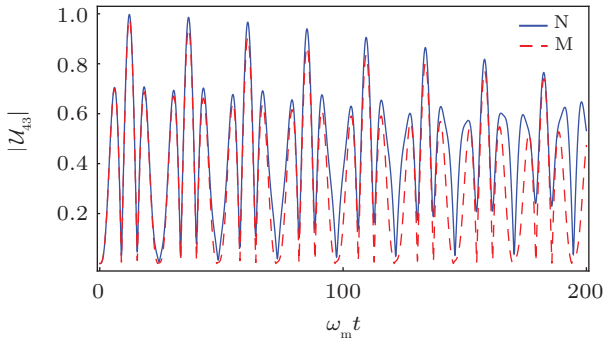


Fig. 1. (color online) The dynamical evolution of $|\mathcal{U}_{43}|$. The non-Markovian regime is denoted by "N", while the Markovian regime is denoted by "M". We choose $\Delta = \omega_m$, $G = 0.5\omega_m$, $J = 0.258\omega_m$, the optical decay rate $\kappa = J_c(\omega_c)/2 \approx 2.5\times10^{-3}\omega_m$, and the mechanical decay rate $\gamma_m = J_m(\omega_m)/2 \approx 9.4\times10^{-4}\omega_m$. The other parameters are $\eta_c = 10^{-4}$, $s_c = 1$, $\tilde{\omega}_c = 300\omega_m$, $C = 3\times10^{-4}$, $k = -2.3$, $\omega_{min} = 885$ kHz, and $\omega_{max} = 945$ kHz.

In Fig. 2, we explore the dynamical evolution of the efficiency by varying the tunneling interaction strength J. The maximum value of efficiency increases with the increase of J. By comparing Figs. 2(a), 2(b), and 2(c), we can also find that the transfer time for reaching the maximum value of efficiency becomes shorter. Meanwhile, in Fig. 2(c), when $\omega_m t \approx 18.6$, the efficiency may reach 99.3%.

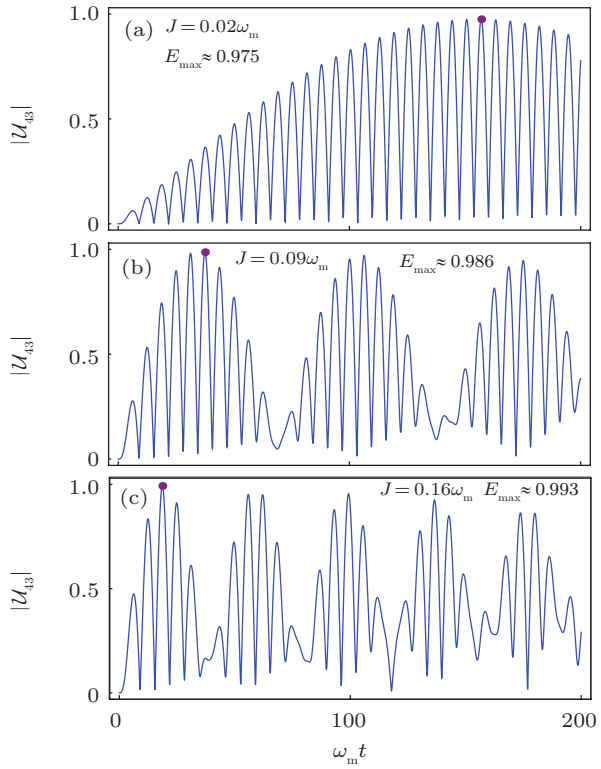


Fig. 2. (color online) The dynamical evolution of $|\mathcal{U}_{43}|$. We choose $\Delta = \omega_m$ and $G = 0.5\omega_m$. From panel (a) to panel (c), we increase the tunneling interaction strength J. The maximum value of efficiency in each figure is marked with a purple point. The other parameters are the same as those in Fig. 1.

In order to have a better understanding of the properties of the efficiency, we explore the maximum value of efficiency via the tunneling interaction strength J and the dimensionless

coupling strength η_c. The result is plotted in Fig. 3. Not surprisingly, η_c plays a negative role for achieving large values of E_{max}, which agrees with the Markovian case. In the presence of non-Markovian environment, as shown in Fig. 3, the high-efficiency state transfer requires an appropriate value of J, and the oscillation becomes apparent when the value of J increases. To show clearly the relation between E_{max} and J, we plot Fig. 4, where we keep $\eta_c = 10^{-4}$. In Fig. 4, we also compare the Markovian and non-Markovian cases. It shows that for both cases, E_{max} non-monotonically increases with the increase of J. However, when J is not very large, the performance of state transfer marked by E_{max} obtained in non-Markovian regime is much better than the one obtained in Markovian regime. If J gets smaller, the transfer time required for E_{max} increases. Then the long-time memory effect plays an important role. Therefore, the results in non-Markovian environments will be much better than the Markovian case. In addition, in the inset of Fig. 4, our numerical result shows that the efficiency can reach 99.7% when $J \approx 0.258\omega_m$.

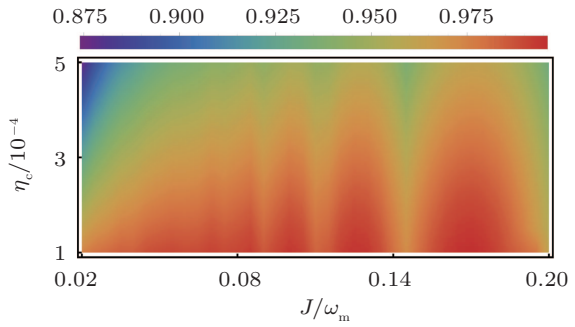


Fig. 3. (color online) Density plot of E_{max} *versus* J and η_c. The parameters are the same as those in Fig. 1.

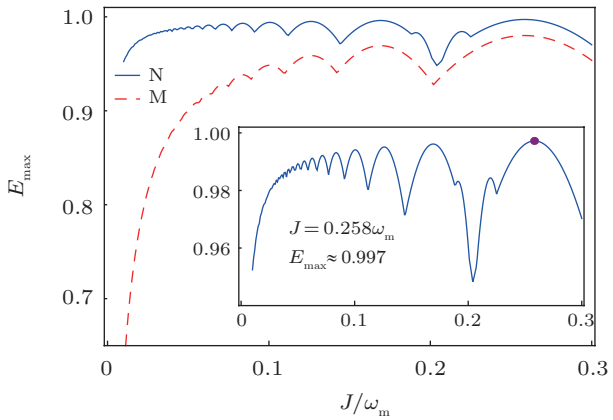


Fig. 4. (color online) The relation for E_{max} *versus* J. We keep $\eta_c = 10^{-4}$. The other parameters are the same as those in Fig. 1.

4. Conclusion

In this paper, we have investigated the quantum state transfer between two distant membranes in non-Markovian regime. We have derived the analytical expression of the efficiency by using the method of modified Laplace transformation. Our results show that quantum state transfer between two membranes can be implemented with high efficiency by using the recently measured experimental mechanical spectral density. Our study might open up a promising perspective for manipulating and transferring the quantum state of optomechanical arrays in memory environments.

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