ISSN 1674-1056
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Quantum metrology with a non-Markovian qubit system

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Citation: Chin. Phys. B . 2018, 27(12): 120301. **doi:** 10.1088/1674-1056/27/12/120301
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Quantum metrology with a non-Markovian qubit system*

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(Received 2 August 2018; revised manuscript received 1 September 2018; published online 10 November 2018)

The dynamics of the quantum Fisher information (QFI) of phase parameter estimation in a non-Markovian dissipative qubit system is investigated within the structure of single and double Lorentzian spectra. We use the time-convolutionless method with fourth-order perturbation expansion to obtain the general forms of QFI for the qubit system in terms of a non-Markovian master equation. We find that the phase parameter estimation can be enhanced in our model within both single and double Lorentzian spectra. What is more, the detuning and spectral width are two significant factors affecting the enhancement of parameter-estimation precision.

Keywords: quantum metrology, quantum Fisher information, time-convolutionless, parameter estimation

PACS: 03.65.Yz, 03.65.Ud, 03.67.Mn **DOI:** 10.1088/1674-1056/27/12/120301

1. Introduction

Quantum metrology[1,2] has played a role in several practical applications of the ultra-precise estimation of parameters bounded by quantum mechanics in recent decades. This aspect of quantum metrology has attracted many researchers to develop its measurement, preservation and application in both theory and experiments. The primary aim of quantum metrology is to enhance the precision of an unknown parameter. Usually, the exploitation of new techniques to enhance the precision of parameter estimation leads to technological advancement and scientific breakthroughs. The fast development of atomic spectroscopy,[3] optical interferometry,[4,5] gravitational wave detection,[6] magnetometry,[7,8] quantum frequency standards improvement,[9] and clock synchronization[10,11] can be attributed to the improvement of quantum metrology. Quantum Fisher information (QFI), which lies at the heart of quantum information theory and quantum estimation theory, can be used to measure the sensitivity of the state with respect to changes in a parameter. According to the quantum estimation theory, in general, the QFI characterized by the quantum Cramér–Rao (QCR) inequality[12] offers the ultimate achievable limit on precision. It imposes the lower bound of any unbiased estimator. A higher value of QFI means that the parameter can be estimated with a higher precision. Therefore, the QFI becomes the central problem to be solved.

Among various kinds of parameter estimation, great attention has been paid to the enhancement of phase estimation because a great deal of tasks are based on the problem of estimating the relative phase. As we known that any quantum system in the real world is in contact with its environment, the unavoidable interaction between the quantum system and its environment will cause decoherence, fluctuations or irreversible dissipative dynamics. Thus the quantum metrological protocols to fight against some kinds of decoherence have been explored in a wider context.[13–28] In this paper, we study phase estimation from an alternative time-convolutionless technique under two conditions of single Lorentzian spectral distribution and non-perfect photonic band gap, respectively. The analytical results of QFI are derived with respect to the fourth-order perturbation expansion. We show that parameter-estimation precision can be greatly preserved in the single Lorentzian spectrum under the non-Markovian case and partially trapped in the photonic band gap with a larger gap width. Moreover, the important factors affecting the enhancement of QFI are explored, and the corresponding physical mechanisms of these phenomena are given.

The paper is organized as follows. In Section 2, we introduce the main aspects of QFI and give the expressions for QFI for pure and mixed states. In Section 3, we first derive the time-convolutionless master equation of a qubit system coupled to a zero temperature environment, and the general forms of QFI with fourth-order perturbation expansion are given. Then we present our ideas of parameter-estimation precision under single and double Lorentzian spectral distributions, respectively. The influences of related factors, such as reservoir correlation time, detuning parameter and width of band gap, on the precision are analyzed exactly. A reasonable physical

*Projects supported by the Natural Science Foundation of Guangdong Province, China (Grant No. 2015A030310354), the Science Foundation for Enhancing School with Innovation of Guangdong Ocean University (Grant Nos. GDOU2017052504 and GDOU2015050207), the Foundation of Excellent-Young-Backbone Teacher of Guangdong Ocean University (Grant No. HDYQ2017005), and the Fund of University Student Innovation and Entrepreneurship Team of Guangdong Ocean University (Grant No. CCTD201823).

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explanation is also given. Finally, we briefly summarize the paper in Section 4.

2. Quantum Fisher information

We briefly introduce some fundamental concepts of QFI. Assume that $p(x_i|\theta)$ is a set probability density with measurement outcomes x_i, where θ is the parameter to be estimated and $\hat{X} = \{x_1, x_2 \ldots, x_N\}$ is an observable random variable. The classical Fisher information, which is used to measure the amount of information contained by $\hat{X}$ is defined as

$$F_\theta = \sum_i p(x_i|\theta) \left[\frac{\partial \ln p(x_i|\theta)}{\partial \theta}\right]^2, \tag{1}$$

which characterizes the inverse variance of the asymptotic normality of a maximum-likelihood estimator. Equation (1) is satisfied with the condition that the observable $\hat{X}$ is discrete; it should be rewritten as an integral if $\hat{X}$ is continuous.

Analogous to the definition of classical Fisher information, the QFI associated with QCR inequality can be defined as

$$F_\theta = \mathrm{Tr}[\rho(\theta)L^2] = \mathrm{Tr}[(\partial_\theta \rho(\theta))L], \tag{2}$$

where $\rho(\theta)$ is the density matrix, θ is the estimating parameter and L is the so-called symmetric logarithmic derivation determined by the equation

$$\partial_\theta \rho(\theta) = \frac{1}{2}[L\rho(\theta) + \rho(\theta)L], \tag{3}$$

where $\partial_\theta = \partial/\partial\theta$. An important feature of QFI is that we can obtain the achievable lower bound of the mean-square error of unbiased estimators for the parameter θ through the QCR theorem

$$(\Delta\theta)^2 \geq (\Delta\theta_{\mathrm{QCR}})^2 = \frac{1}{\nu F_{\mathrm{Q}}}, \tag{4}$$

where $(\Delta\theta)^2$ represents the mean-square error of θ, and ν denotes the number of repeated independent measurements. The QCR theorem above shows that QFI is a measure of certain kinds of information with respect to the precision of estimating the inference parameter. Some previous works[29,30] have been already pointed out the relationship between the QFI and the statistical distinguishability of $\rho(\theta)$ and its neighbor. With the spectrum decomposition $\rho(\theta) = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|$, the QFI can be rewritten as[31]

$$F_\theta = \sum_i \frac{(\partial_\theta \lambda_i)^2}{\lambda_i} + 2\sum_{i\neq j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle\psi_j|\partial_\theta \psi_i\rangle|^2, \tag{5}$$

where $|\psi_i\rangle$ and λ_i are eigenvectors and eigenvalues of ρ, respectively. The first term of Eq. (5) is the classical Fisher information and the second term is referred to as the quantum one. For pure states, equation (5) can be simplified as

$$F_\theta = 4\left[\langle\partial_\theta \psi_i|\partial_\theta \psi_i\rangle\right] - |\langle\psi_i|\partial_\theta \psi_i\rangle|^2, \tag{6}$$

and for mix states we can obtain

$$F_\theta = \sum_{i,j} \frac{2}{\lambda_i + \lambda_j} |\langle\psi_i|\frac{\partial\rho(\theta)}{\partial\theta}|\psi_j\rangle|^2. \tag{7}$$

3. Enhancing the QFI of a qubit using a time-convolutionless method

3.1. Model

We build the model within an interaction between a qubit system and a zero temperature multi-mode reservoir. The Hamiltonian of the system can be written in the following form with a rotating-wave approximation[32]

$$H = \hbar\omega_0 a^\dagger a + \hbar\sum_{k=1}^{N} \omega_k b_k^\dagger b_k + \hbar\sum_{k=1}^{N} g_k (a b_k^\dagger + a^\dagger b_k), \tag{8}$$

where ω_0 is the Bohr frequency of the two-level atom, and $a_k^\dagger(a_k)$ and $b_k^\dagger(b_k)$ represent the atomic raising and the reservoir creation operator, respectively. ω_k is the frequency of the k-th mode and g_k is the coupling constant between the qubit and reservoir. It is proved[33] that $|\Psi\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ is the optimal input state. After applying a singlet-qubit phase gate: $U(\theta) = |g\rangle\langle g| + \mathrm{e}^{\mathrm{i}\theta}|e\rangle\langle e|$, the outcome state is $|\Psi_{\mathrm{out}}\rangle = (|g\rangle + \mathrm{e}^{\mathrm{i}\theta}|e\rangle)/\sqrt{2}$. We assume that initially the qubit is optimal and the reservoir is vacuum $|\Psi(0)\rangle = (1/\sqrt{2})(|g\rangle + \mathrm{e}^{\mathrm{i}\theta}|e\rangle) \otimes |0\rangle$. The exact master equation can be written as[34]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_s(t) = -\frac{\mathrm{i}}{2}S(t)[\sigma_+\sigma_-, \rho_s(t)] + \gamma(t)\left\{\sigma_-\rho_s(t)\sigma_+ - \frac{1}{2}\sigma_+\sigma_-\rho_s(t) - \frac{1}{2}\rho_s(t)\sigma_+\sigma_-\right\}, \tag{9}$$

where $S(t)$ and $\gamma(t)$ are the time-dependent Lamb shift and decay rate. Equation (9) is already of the form of a time-convolutionless master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_s(t) = \kappa_s(t)\rho_s(t), \tag{10}$$

where $\kappa_s(t)$ is the generator. Though the above equation is similar to the Lindblad master equation, the time-dependent coefficients $S(t)$ and $\gamma(t)$ do not provide a quantum dynamical semigroup. After some algebra, we can write the time evolution of the density matrix as

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{10}(t) \\ \rho_{10}^*(t) & 1-\rho_{11}(t) \end{pmatrix}, \tag{11}$$

where

$$\rho_{11}(t) = \frac{1}{2}\exp\left[-\int_0^t \gamma(t')\mathrm{d}t'\right], \tag{12}$$

$$\rho_{10}(t) = \frac{1}{2}\exp(\mathrm{i}\theta)\cdot\exp\left[-\frac{1}{2}\int_0^t [\gamma(t')+\mathrm{i}S(t')]\mathrm{d}t'\right]. \tag{13}$$

It is easy to obtain a more direct expression for QFI by substituting Eq. (11) into Eq. (7)

$$F_\theta = \exp\left[-\int_0^t \gamma(t')\mathrm{d}t'\right]. \tag{14}$$

We develop a perturbation expansion for the time-convolutionless generator in powers of the coupling strength α,[34,35] so that $\gamma(t)$ and $S(t)$ can be expanded in the following form

$$\gamma(t) = \sum_{n=1}^{\infty}\alpha^{2n}\gamma_{2n}(t),$$
$$S(t) = \sum_{n=1}^{\infty}\alpha^{2n}S_{2n}(t), \tag{15}$$

and the two coefficients are connected with the relation

$$\gamma_{2n}(t)+\mathrm{i}S_{2n}(t) = \int_0^t \mathrm{d}t_1\int_0^{t_1}\mathrm{d}t_2\ldots\int_0^{t^{2n-2}}\mathrm{d}t_{2n-1}2(-1)^{n+1}\langle f(t-t_1)f(t_2-t_3)\ldots f(t_{2n-2}-t_{2n-1})\rangle, \tag{16}$$

where $f(t)$ is the correlation function determined by the spectral density $J(\omega)$ of the reservoir

$$f(t) = \int \mathrm{d}\omega J(\omega)\exp[\mathrm{i}(\omega_0-\omega)t], \tag{17}$$

where $J(\omega)$ is the spectral density of an electromagnetic field. The second- and fourth-order contributions for the coefficients of the master equation can be derived according to Eq. (16)

$$\gamma_2(t)+\mathrm{i}S_2(t) = 2\int_0^t \mathrm{d}t_1 f(t-t_1),$$
$$\gamma_4(t)+\mathrm{i}S_4(t) = 2\int_0^t \mathrm{d}t_1\int_0^{t_1}\mathrm{d}t_2\int_0^{t_2}\mathrm{d}t_3[f(t-t_2)f(t_1-t_3)+f(t-t_3)f(t_1-t_2)]. \tag{18}$$

3.2. The single Lorentzian spectral distribution

As the first example, we consider the spectral density $J(\omega)$ with a single Lorentzian spectral structure[31,34]

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0-\Delta-\omega)^2+\lambda^2}, \tag{19}$$

where the parameter λ is the spectral width with relation $\tau_{\mathrm{B}} = \lambda^{-1}$, τ_{B} is the reservoir correlation time and τ_{R} is the relaxation time scale with relation $\tau_{\mathrm{R}} = \gamma_0^{-1}$. Substituting Eq. (19) into Eq. (17), we can solve the correlation function $f(t)$ with the residue theorem

$$f(t) = \frac{\gamma_0\lambda}{2}\exp[-(\lambda-\mathrm{i}\Delta)t]$$
$$= \frac{\gamma_0\lambda}{2}\exp(-\lambda t)[\cos(\Delta t)+\mathrm{i}\sin(\Delta t)]$$
$$= \frac{1}{2}[\Phi(t)+\mathrm{i}\Psi(t)], \tag{20}$$

where the two real functions $\Phi(t) = \gamma_0\lambda\exp(-\lambda t)\cos(\Delta t)$ and $\Psi(t) = \gamma_0\lambda\exp(-\lambda t)\sin(\Delta t)]$ represent the real and image parts of $f(t)$, respectively. It is convenient to use $\Phi(t)$ and $\Psi(t)$ to express $\gamma_{2n}(t)$ and $S_{2n}(t)$. However, in order to derive the analytical results of QFI from Eq. (14), we only need to calculate $\gamma_{2n}(t)$. According to the demands of calculating precision, the fourth-order contribution of the perturbation expansion is a good approximation in the dynamical evolution,[34] such that

$$\gamma^4(t) = \gamma_2(t)+\gamma_4(t), \tag{21}$$

where

$$\gamma_2(t) = \int_0^t \mathrm{d}t_1\Phi(t-t_1) = \frac{\gamma_0\lambda^2}{\lambda^2+\Delta^2}\left[1-\exp(-\lambda t)(\cos(\Delta t)-\frac{\Delta}{\lambda}\sin(\Delta t))\right], \tag{22}$$

$$\gamma_4(t) = \frac{1}{2}\int_0^t \mathrm{d}t_1\int_0^{t_1}\mathrm{d}t_2\int_0^{t_2}\mathrm{d}t_3[\Phi(t-t_2)\Phi(t_1-t_3)+\Phi(t-t_3)\Phi(t_1-t_2)$$
$$-\Psi(t-t_2)\Psi(t_1-t_3)-\Psi(t-t_3)\Psi(t_1-t_2)]$$
$$= \frac{\gamma_0^2\lambda^5\exp(-\lambda t)}{2(\lambda^2+\Delta^2)^3}\left\{\left[1-3\left(\frac{\Delta}{\lambda}\right)^2\right][\exp(\lambda t)-\exp(-\lambda t)\cos(2\Delta t)]-2\left[1-\left(\frac{\Delta}{\lambda}\right)^4\right]\lambda t\cos(\Delta t)\right.$$
$$\left.+4\left[1+\left(\frac{\Delta}{\lambda}\right)^2\right]\Delta t\sin(\Delta t)+\frac{\Delta}{\lambda}\left[3-\left(\frac{\Delta}{\lambda}\right)^2\right]\exp(-\lambda t)\sin(2\Delta t)\right\}. \tag{23}$$

The QFI F_θ of a single Lorentzian spectral density can be calculated with the fourth-order perturbation expansion of $\gamma^4(t)$. Now we shall discuss the effect of enhancing parameter estimation which should be directly reflected from the dynamical evolution of the QFI with respect to Markovian and non-Markovian regimes for different spectral width λ and detuning parameter Δ. In Fig. 1(a), the evolution of QFI is plotted against the dimensionless quantity $\gamma_0 t$ with the optimal initial state. It is clearly shown that the QFI decays to zero rapidly in the Markovian regime $\lambda = 5\gamma_0$ with resonant condition $\Delta = 0$. In this situation the accuracy of parameter estimation is not enhanced. It is easy to understand that the shorter correlation time of the reservoir in the Markovian regime causes the information to flow from the system to its environment monotonically. However, in the non-Markovian regime, the dynamical evolution of QFI is more valuable and interesting. We plot the QFI in the non-Markovian regime for $\lambda = 0.05\gamma_0$ with $\Delta = 0$, $\Delta = 0.5\gamma_0$ and $\Delta = 1.5\gamma_0$, respectively. One can see that the QFI oscillates with time, especially in the non-resonant case. In the non-Markovian regime, the correlation time of the reservoir is greater than the changing time-scale of the system with $\gamma_0 > (\lambda/2)(\tau_R < 2\tau_B)$. Due to the non-Markovian memory effect, information that has flowed into the reservoir will be reabsorbed by the system, which causes the revival behaviors. The phenomenon of oscillation can be regarded as evidence of information reabsorption. Moreover, it is noticed that the QFI in the resonant case is less than in the non-resonant case in the non-Markovian regime. The quantity of the detuning parameter is proportional to the enhancement in the precision of parameter estimation. According to electromagnetic field theory, the interaction between the cavity and the reservoir is strongest in the resonant situation, which implies a more serious decoherence, and vice versa. This is why the greater detuning corresponds with a lower QFI loss.

In the following, we want to investigate the relationship between the detuning and non-Markovian characteristics and QFI preservation. Figure 1(b) is plotted to show the variation behaviors of QFI against the dimensionless quantity Δ/γ_0 for different values of λ with $\gamma_0 t = 10$. The dotted line is for $\lambda = 5\gamma_0$, the blue line is for $\lambda = 0.1\gamma_0$, the green line is for $\lambda = 0.05\gamma_0$, and the red line is for $\lambda = 0.02\gamma_0$, respectively. It is found that the parameter-estimation precision is affected by the correlation time of the reservoir. The quantity of QFI is proportional to λ/γ_0. The longer the correlation time, the stronger the environmental memory effects. And more reversed information can flow back into the system. Hence it is demonstrated that we can enhance the parameter estimation with non-Markovian effects. What is more, the detuning parameter Δ is another important factor affecting the preservation of QFI.

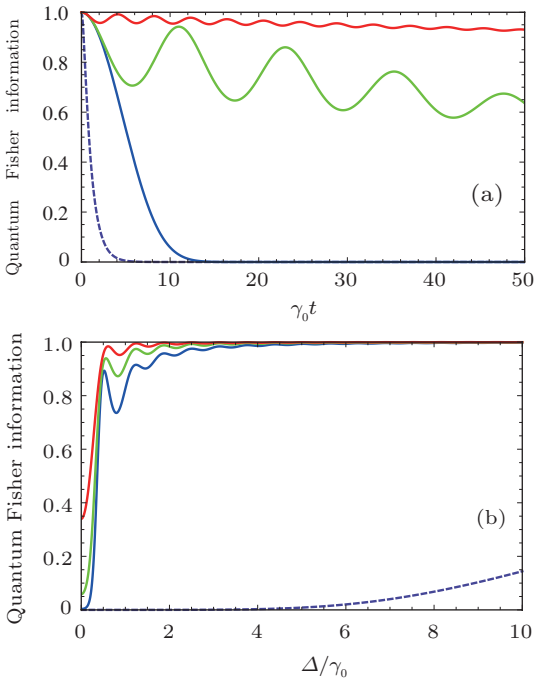


Fig. 1. (color online) QFI is plotted against the dimensionless quantities $\gamma_0 t$ and Δ/γ_0 for the optimal output state $|\psi(0)\rangle_{CR} = (1/\sqrt{2})(|0\rangle_C + e^{i\theta}|1\rangle_C) \otimes |\bar{0}\rangle_R$. (a) The dotted line is the QFI in terms of $\gamma_0 t$ in the Markovian regime with $\lambda = 5\gamma_0$ and $\Delta = 0$. The other three line are the QFI in terms of $\gamma_0 t$ in the non-Markovian regime with $\lambda = 0.05\gamma_0$ and $\Delta = 0$ (the blue line), $\Delta = 0.5\gamma_0$ (the green line) and $\Delta = 1.5\gamma_0$ (the red line). (b) QFI in terms of Δ/γ_0 for different values of λ with $\gamma_0 t = 10$.

3.3. The photonic band gap

Next we consider another model for a qubit system in a photonic band gap, which was first introduced by Garraway[36] in 1997. The double Lorentzian spectral density is of the form

$$J(\omega) = \frac{\gamma_0^2}{2\pi}\left[\frac{W_1\Gamma_1}{(\omega-\omega_0)^2+(\Gamma_1/2)^2} - \frac{W_2\Gamma_2}{(\omega-\omega_0)^2+(\Gamma_2/2)^2}\right], \tag{24}$$

where λ_0^2 denotes the overall coupling strength, and Γ_1 and Γ_2 are the bandwidth of the flat background continuum and the width of the gap, respectively. Due to the positivity of $J(\omega)$, Γ_1 is larger than Γ_2. The positive Lorentzian models a broad, resonant background structure, the negative Lorentzian spectral density represents a dip into the density of states. W_1 and W_2 satisfied the relation $W_1 - W_2 = 1$ giving the relative strength of the background and the gap. We can get the double Lorentzian correlation function from Eq. (17)

$$f(t) = \lambda_0^2\left[W_1 \exp\left(-\frac{\Gamma_1}{2}t\right) - W_2 \exp\left(-\frac{\Gamma_2}{2}t\right)\right]. \tag{25}$$

The two real functions are easy to obtain as follows:

$$\Phi(t) = 2\lambda_0^2\left[W_1 \exp\left(-\frac{\Gamma_1}{2}t\right) - W_2 \exp\left(-\frac{\Gamma_2}{2}t\right)\right],$$
$$\Psi(t) \equiv 0. \tag{26}$$

Repeating similar processes as for the single Lorentzian case, we can easily obtain the analytical result of F_θ with the fourth-order perturbation expansion of $\gamma^4(t)$. However, the analytical result is too complicated, and instead of writing them, we prefer to plot the QFI dynamical evolution in Fig. 2 and discuss its characteristics from two perspectives: the width of the band gap and the limit of $W_2 = 0$ which degenerates into a single Lorentzian state. The red line is for $\Gamma_1/\gamma_0 = 10$, $\Gamma_2/\gamma_0 = 1$, $W_1 = 1.1$, and $W_2 = 0.1$, where Γ_1 is much larger than Γ_2. It displays a two-step decay processes. Firstly there is a rapid decay as a component of the QFI is lost to the background mode.[36] However, the last time of the exponential-style decay is not very long because of the strong coupling. Finally, the QFI is partially trapped in the model with wide band gap. Therefore, asymptotic decay behavior appears because of the slow band gap mode decay rate. The green line is for $\Gamma_1/\gamma_0 = 10$, $\Gamma_2/\gamma_0 = 4$, $W_1 = 1.1$, and $W_2 = 0.1$. Comparing the upper example, it is interesting to note that the QFI preserving time decreases as Γ_2 increases. It is shown[37] that QFI preservation is determined by the modes of the spectrum. The spectral density $J(\omega)$ increases monotonically as Γ_2 increases in the resonant case $\Delta = 0$. Moreover, from the structure of the double Lorentzian we can observe that the negative Lorentzian spectrum can inhibit the spontaneous emission in the region of the dip.[38,39] The wider the band gap, the better the preserving effects. Finally, we consider another extreme situation which contains only one Lorentzian spectrum.[40] The blue line is for $\Gamma_1/\gamma_0 = 10$, $\Gamma_2/\gamma_0 = 1$, $W_1 = 1$, and $W_2 = 0$. If we remove the negative Lorentzian part, it is to be stressed to the model of a Lorentzian. We note that the speed of QFI decreases as the width of the Lorentzian spectral increases on the resonant couplings.

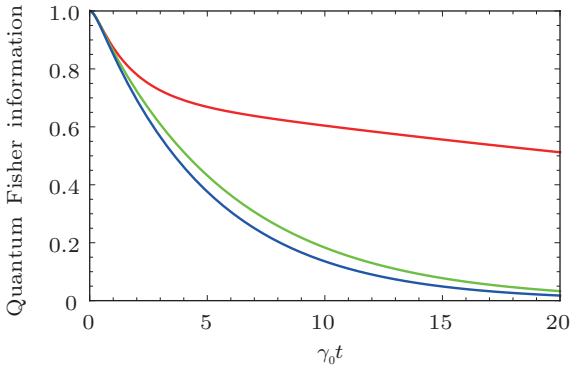


Fig. 2. (color online) QFI is plotted against the dimensionless quantities $\gamma_0 t$ for the optimal output state $|\psi(0)\rangle_{CR} = (1/\sqrt{2})(|0\rangle_C + e^{i\theta}|1\rangle_C) \otimes |\tilde{0}\rangle_R$ in the photonic band gap model.

4. Conclusion

In conclusion, we have proposed a time-convolutionless scheme to enhance the parameter-estimation precision of a dissipative qubit system coupled to zero temperature surroundings. By employing a fourth-order perturbation expansion, we have derived the analytical results of QFI under two different environments: 1) in the single Lorentzian spectral structure, QFI preservation is affected by the detuning and correlation time of the reservoir; and 2) in the double Lorentzian spectral structure, which is called a non-perfect photonic band gap, the preservation of QFI is only affected by the spectral width.

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