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Effects of the planarity and heterogeneity of networks on evolutionary two-player games*

Xu-Sheng Liu(刘旭升), Zhi-Xi Wu(吴枝喜)†, and Jian-Yue Guan(关剑月)‡

Institute of Computational Physics and Complex Systems, Lanzhou University, Lanzhou 730000, *China*

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We study the effects of the planarity and heterogeneity of networks on evolutionary two-player symmetric games by considering four different kinds of networks, including two types of heterogeneous networks: the weighted planar stochastic lattice (a planar scale-free network) and the random uncorrelated scale-free network with the same degree distribution as the weighted planar stochastic lattice; and two types of homogeneous networks: the hexagonal lattice and the random regular network with the same degree $k_0 = 6$ as the hexagonal lattice. Using extensive computer simulations, we found that both the planarity and heterogeneity of the network have a significant influence on the evolution of cooperation, either promotion or inhibition, depending not only on the specific kind of game (the Harmony, Snowdrift, Stag Hunt or Prisoner's Dilemma games), but also on the update rule (the Fermi, replicator or unconditional imitation rules).

Keywords: evolutionary two-player games, planarity, heterogeneity

PACS: 02.50.Le, 87.23.Ge, 07.05.Tp **DOI:** 10.1088/1674-1056/27/12/120203

1. Introduction

The ubiquitous cooperation behavior exhibited in biological, economic, and social systems, which is in conflict with the selfish nature of creatures, has attracted much attention from researchers over several decades.[1–8] The study of the emergence and evolution of cooperative behaviors is one of the most important subjects in the current era. In recent decades, evolutionary game theory has been introduced and has proved to be one of the most successful theoretical frameworks for research into cooperative behaviors.[4,8–10] As a result, many significative models for studying the reasons why we cooperate have been established and abundant research achievements have been made.[11,12]

In particular, it is well recognized that there exist several mechanisms which can promote cooperation effectively, and network reciprocity is an important one of them.[8,13] The members of a population are located on the vertices of networks in which the edges give the neighbors of each player with whom they interact. The interactions of the game lead to payoffs, which are interpreted as fitness. Individuals who receive a higher payoff possess more reproductive success. Network (or spatial) reciprocity,[14,15] together with kin selection,[16–18] direct reciprocity,[19–21] indirect reciprocity,[22–24] and group selection[25–28] have been identified as five general rules[29] that can offset an unfavorable outcome in social dilemmas and lead to the evolution of cooperation.

For the Prisoner's Dilemma game, a replicator equation[15,30] reveals that cooperators cannot exist in the well mixed case. Nowak and May pioneered research into the evolutionary Prisoner's Dilemma game on a square lattice in 1992 and found that cooperators and defectors both persist indefinitely,[31] for the cooperators may survive for the formation of compact clusters in spatially structured populations in which they protect each other against the invasion of defectors. Subsequent research[32–34] indicated that spatial structure can promote the evolution of cooperation. Nevertheless, the excellent work of Christoph Hauert and Michael Doebeli on the evolutionary Snowdrift game on various lattice networks in 2004 tells us that spatial structure may often inhibit the evolution of cooperation.[35]

For the evolutionary game in a network, individuals only interact with their own immediate neighbors. Therefore, there exist no long-range interactions for the game in regular planar structured networks, *i.e.* the interactions are limited to the local. Christoph Hauert and György Szabó, in their work,[36] studied the evolutionary Prisoner's Dilemma game on small world networks and random regular networks, showing that long-range edges can make cooperators perform better than on square lattices. Besides, the heterogeneity of a network's degree distribution may also promote social cooperation. The work of Santos and Pacheco showed that cooperators will dominate the system in scale-free networks.[37–39] They also found that the interconnections among the hubs favor the dominance of cooperation in the Prisoner's Dilemma game.[39] Re-

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†Corresponding author. E-mail: wuzhx@lzu.edu.cn

‡Corresponding author. E-mail: guanjy@lzu.edu.cn

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search by Gómez-Gardeñes *et al.* showed that there exists a single cluster composed of the hub cooperators in scale-free networks, which provides further insight into the cooperative dominance in heterogeneous networks.[40]

According to topological properties, the networks mentioned above can be classified into two categories: planar networks which can be embedded in a Euclidean two-dimensional space (such as square and triangular lattices, Apollonian networks with power-law exponent $\gamma \approx 1.585$,[41] random Apollonian networks with power-law exponent $\gamma = 3$ and large clustering coefficient 0.74 (as proposed by Zhou *et al.* in Ref. [42]), *etc.*) and random networks (such as ER random and random regular graphs, scale-free networks, *etc.*). In most previous studies, almost all planar networks have had homogeneous degree distribution. Most recently, Pierre Buesser and Marco Tomassini studied classical two-player evolutionary games on spatial networks embedded in a Euclidean two-dimensional space with different kinds of degree distributions and topologies. They found that Apollonian networks[41] (a kind of planar scale-free network) will lead to higher levels of cooperation.[43] In 2010, a new kind of planar scale-free network with a steep power-law degree distribution, $P(k) \propto k^{-\gamma}$ in which $\gamma = 5.66$, named the weighted planar stochastic lattice, was proposed by Hassan *et al.* in their work.[44] This allows us to further study the effect of the underlying networks on the evolution of cooperation.

In the present work, we make a detailed exploration of the effects of the planarity of networks (regular or heterogeneous) and the heterogeneity of networks (planar or random) on the evolution of cooperation in four different kinds of networks with evolutionary two-player symmetric games. Three different update rules — the Fermi, replicator and unconditional imitation rules — are considered. The extensive Monte Carlo simulation results indicate that both the planarity and heterogeneity of the network can either facilitate or inhibit the evolution of cooperation, which depends not only on the specific type of game but also on the update rule.

2. Model

In this work, we investigate evolutionary two-player symmetric games on structured populations including planar embedded networks [the weighted planar stochastic lattice (WPSL)[44] and hexagonal lattice (HL)[45]] and random networks [the uncorrelated random scale-free network (URSF)[46] and the random regular network (RRG)[47]]. To study the effect of planarity on the evolution of cooperation in structured populations, we generate the URSF using the uncorrelated configuration method[46] with the same degree distribution as the WPSL, and the RRG with the same degree $k_0 = 6$ as the HL during simulations.

For the WPSL,[44] which is a new scale-free network embedded in the two-dimensional planar space first proposed by Hassan *et al.* in 2010, the construction process is based on the following algorithm. Firstly, a square of unit area (the initiator) is divided into four smaller blocks by two perpendicular lines parallel to the sides through a randomly selected point in the area. The four newly generated blocks are labeled by their corresponding area a_1, a_2, a_3, and a_4 respectively. Then, the general j-th step of the algorithm can be described as follows.

(i) Generate two random numbers $x_{\rm R}$ and $y_{\rm R}$ from [0, 1] and find the block $a_{\rm p}$ which contains the point $(x_{\rm R}, y_{\rm R}R)$.

(ii) Divide the block $a_{\rm p}$ by two perpendicular lines through the point $(x_{\rm R}, y_{\rm R})$ parallel to the sides of $a_{\rm p}$ into four smaller blocks. Note that these two perpendicular lines stop at the sides of block $a_{\rm p}$.

(iii) Label the four newly generated blocks $a_{\rm p}$, a_{3j-1}, a_{3j}, and a_{3j+1} according to their areas respectively.

(iv) Repeat steps (i)–(iii) until the number of blocks meets the requirements.

Finally, the WPSL can be obtained by replacing each block with a node inside it and the common border between blocks with an edge joining the two vertices. The networks obtained using the above algorithm have a power-law degree distribution and it is worth mentioning that the exponent is bigger than 5, significantly larger than those usually found in most real-life networks, which are typically between 2 and 3. According to the algorithm above, it can be found that three new points and eight edges are produced at each step. Therefore, the mean degree of the WPSL tends to 16/3 with increasing time step.

During the interaction, two actions, cooperation and defection denoted by C and D respectively, are available for each player. The payoffs in the two-player games are depicted by the following matrix[48]

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}, \tag{1}$$

in which the entries give payoffs of the row actions. Here, without loss of generality, we let $R = 1$ and $P = 0$. According to the different values of S ($\in [-1, 1]$) and T ($\in [0, 2]$), the two-player game can be divided into four categories (see Fig. 1),[49] namely the Harmony game ($0 < S < 1$ and $0 < T < 1$), Snowdrift game ($0 < S < 1$ and $1 < T < 2$), Stag Hunt game ($-1 < S < 0$ and $0 < T < 1$) and Prisoner's Dilemma game ($-1 < S < 0$ and $1 < T < 2$).

In the model, we consider pure strategy space, which indicates that the strategies are deterministic, *i.e.*, players either cooperate or defect during the evolution. Let $\boldsymbol{\sigma}_i \in \{(1,0)^{\rm T}, (0,1)^{\rm T}\}$, corresponding to action C and D respectively, be a vector as the action profile of player i, and let $\boldsymbol{M}$

denote the payoff matrix of the game [Eq. (1)]. Thus, the cumulative payoff of player i, π_i, can be calculated as

$$\pi_i = \sum_{k \in \boldsymbol{V}_i} \boldsymbol{\sigma}_i^{\mathrm{T}} \, \boldsymbol{M} \, \boldsymbol{\sigma}_k, \tag{2}$$

where $\boldsymbol{V}_i$ is the set of player i's nearest neighbors.

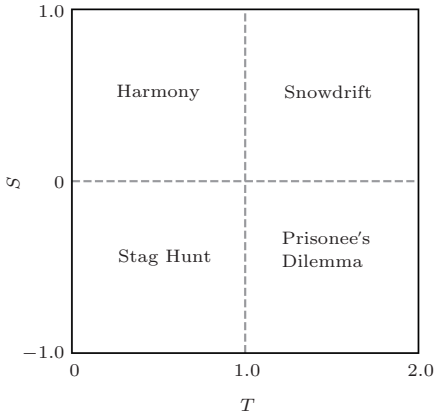


Fig. 1. Schematic diagram for the correspondence between the area in the ST plane and the specific two-player game with $R = 1$ and $P = 0$. For the Harmony game, we have $R > T$ and $S > P$; for Snowdrift, $T > R > S > P$; for Stag Hunt, $R > T > P > S$; and for the Prisoner's Dilemma, $T > R > P > S$.

We consider three imitative rules which have attracted much attention in previous research:[34] the Fermi, replicator, and unconditional imitation rules. The Fermi rule allows one to investigate the effect of the intensity of selection. Let s_i be the strategy of player i, and let k_i denote the degree. With the Fermi rule, player i will adopt the strategy of player j (selected randomly from all neighbors of i) with the probability $W(s_i \to s_j)$, which is assumed to be proportional to their payoff difference $(\pi_j - \pi_i)$. Specifically, the transition probability can be written as[50]

$$W\{s_i \to s_j\} = \frac{1}{1 + \exp[-\beta(\pi_j - \pi_i)]}, \tag{3}$$

where $\beta > 0$ denotes the uncertainty (noise) in the imitation process. From a biological viewpoint, β can also be regarded as the strength of selection.[51,52] In this article, we use $\beta = 2.5$ in our Monte Carlo simulations. Obviously, individuals with higher payoffs are imitated by others with higher probability. It is also worth noting that individuals can also imitate their neighbors who yield lower payoffs, even though with somewhat lower probabilities, which indicates that people may make mistakes in the learning process.

The replicator rule is the standard transplanting version of replicator dynamics[15,30] in finite populations and discrete time. With the replicator rule, the probability of player i adopting the strategy of player j (randomly selected from the neighbors of i) is given by

$$W\{s_i \to s_j\} = \begin{cases} (\pi_j - \pi_i)/\Phi, & \text{if } \pi_j > \pi_i, \\ 0, & \text{if } \pi_j \le \pi_i, \end{cases} \tag{4}$$

in which $\Phi = \max(k_i, k_j)[\max(1, T) - \min(0, S)]$ to ensure the transfer probability $W\{s_i \to s_j\} \in [0, 1]$. It is worth noting that, unlike the Fermi rule, players do not imitate others with lower payoffs according to the replicator rule.

Another frequently used rule is the unconditional imitation rule, in which a player imitates the most successful neighbors who achieve payoffs higher than herself. Note that the unconditional imitation rule is deterministic, in contrast to the above two rules which are stochastic.

During the simulation, each individual in the system has an equal probability to be a cooperator (C) or a defector (D) initially. At each micro time step, an individual is selected randomly from the whole system to update his/her strategy according to the corresponding update rule. The ST plane is divided with a grid interval of 0.02 and the simulation results of each point in the plane are averaged over 50 independent runs. The sizes of all the networks are $N = 10^4$.

3. Results

It is well known that many systems in nature can be described by complex networks and that the structural characteristics of the underlying networks significantly affect the behavior of the dynamics process taking place on them. Here, we think that random networks are closer to the well-mixed case as compared with planar embedded networks. With the four kinds of network above, we can make a detailed study of the effects of the planarity and heterogeneity of networks on the evolution of cooperation in the framework of two-player symmetric games.

Firstly, we make systematic simulations of the evolution of cooperation in the WPSL with three distinct update rules (the Fermi, replicator and unconditional rules) in the whole ST plane respectively. The results are presented in Fig. 2. To study the effect of the planarity of the underlying network with scale-free degree distribution, the steady cooperation frequency in the URSF with the same degree distribution as the WPSL are also calculated in the ST plane under all three update rules, as shown in Fig. 3. Moreover, to investigate the influence of the heterogeneity of the degree distribution in planar and random networks, we also recompute the cooperation level in the HL and RRG with degree $k_0 = 6$ under the same parameters. Figures 4 and 5 exhibit the detailed results in these two structured networks respectively. These results show that cooperation frequency under different update rules is different for all the four structured networks. For each kind of network, the results under the Fermi and replicator rules are similar, for they both consider one pair of nodes and their nearest neighbors during the update. However, the results under the unconditional update rule seem a little more complex, for it considers not only one node and its nearest neighbors but also all the next nearest neighbors during the update.

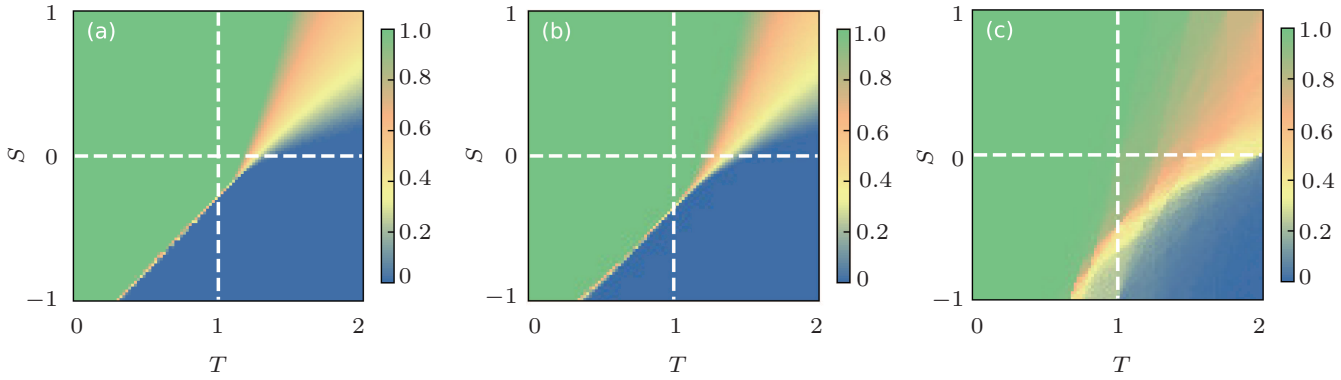


Fig. 2. (color online) Cooperation frequency in the WPSL after the system reached stable status. The update rules are the (a) Fermi, (b) replicator, and (c) unconditional imitation respectively.

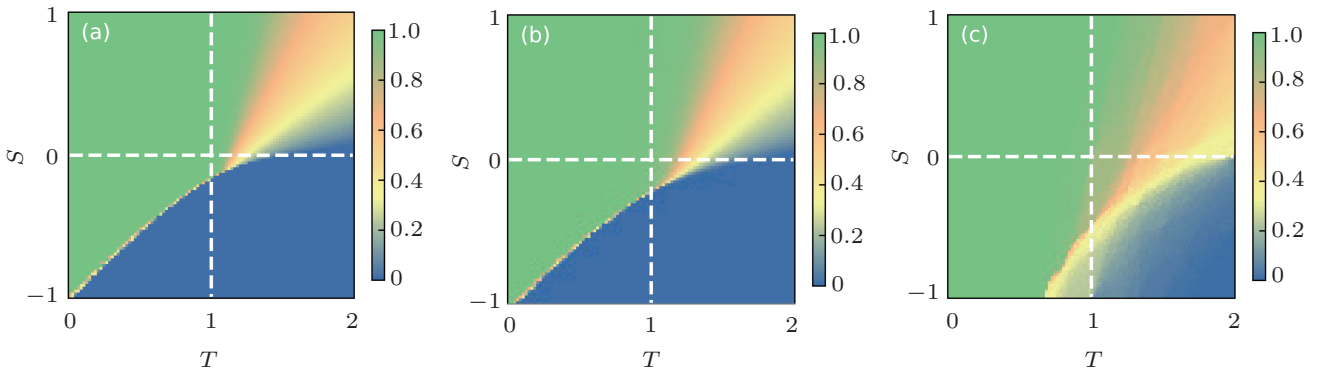


Fig. 3. (color online) Cooperation frequency in the URSF after the system reached stable status. The update rules are the (a) Fermi, (b) replicator, and (c) unconditional imitation respectively.

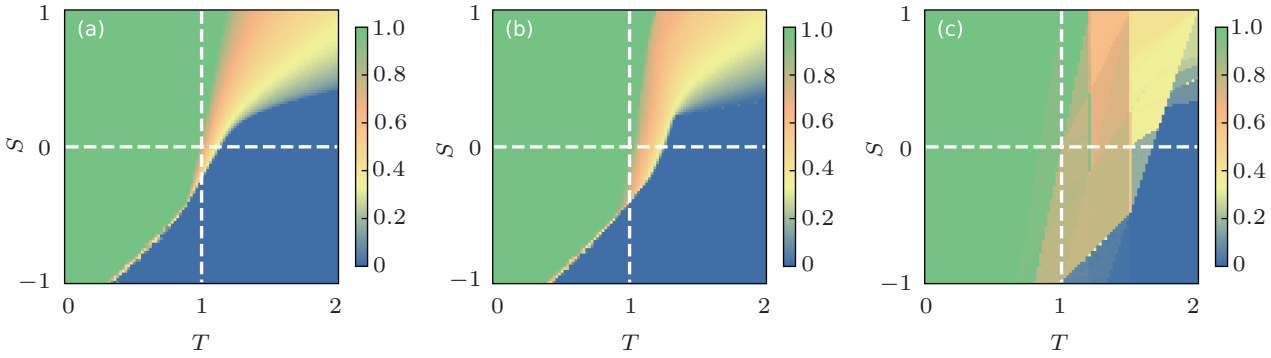


Fig. 4. (color online) Cooperation frequency in the HL after the system reached stable status. The update rules are the (a) Fermi, (b) replicator, and (c) unconditional imitation respectively.

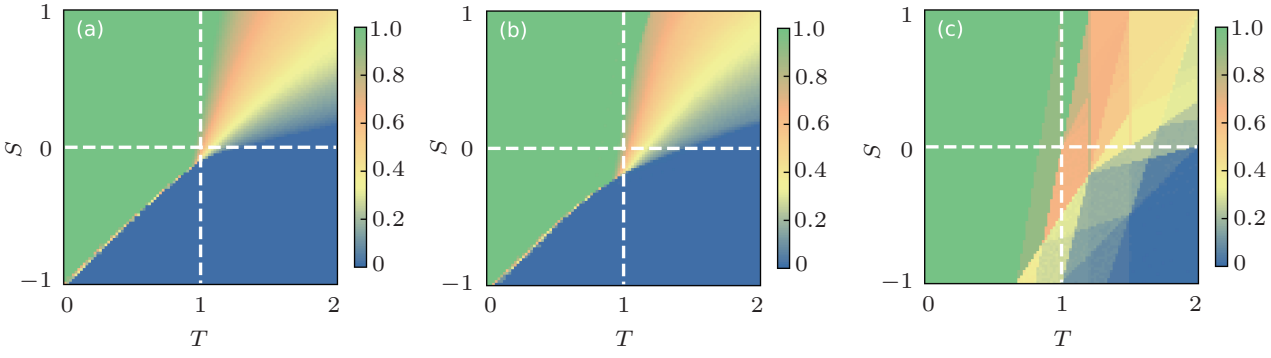


Fig. 5. (color online) Cooperation frequency in the RRG after the system reached stable status. The update rules are the (a) Fermi, (b) replicator, and (c) unconditional imitation respectively.

3.1. Effects of the planarity of networks on the evolution of cooperation

To better present the effect of the planarity of networks, we plot the distinction values of the results between the two kinds of heterogeneous network [WPSL (Fig. 2) and URSF (Fig. 3)] in the upper row of Fig. 6 and between the two kinds of homogeneous network [HL (Fig. 4) and RRG (Fig. 5)] in the lower row of Fig. 6. It can be seen that the difference values depend on the structure of the network, update rules, and the game type. Particularly, for all three update rules there exists little difference between the results in the networks with or without planarity in the area of the Harmony game. Therefore, the influence of the planarity of the network is mainly concentrated in the areas corresponding to the Snowdrift, Stag Hunt and Prisoner's Dilemma games.

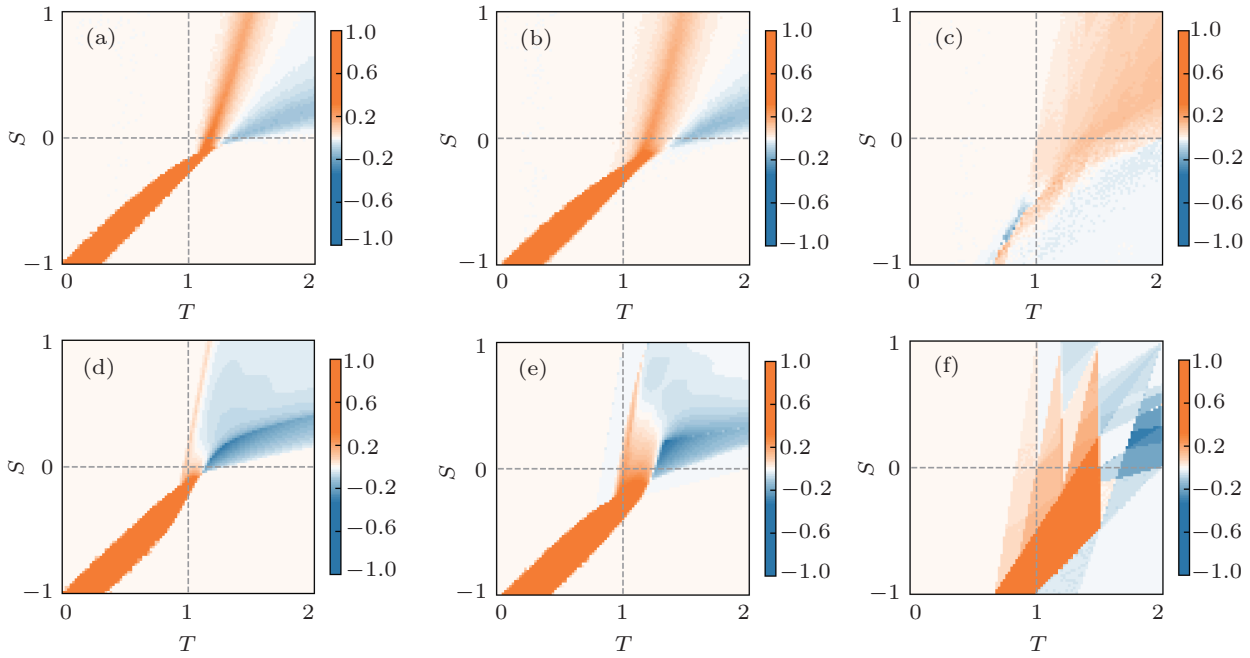


Fig. 6. (color online) Comparison of the results of cooperation levels between the two kinds of heterogeneous network [the results in the WPSL (Fig. 2) minus the results in the URSF (Fig. 3), upper row] and between the two types of homogeneous network [the results in the HL (Fig. 4) minus the results in the RRG (Fig. 5), bottom row] with and without planarity under different update rules. The update rules respectively are (a) and (d) Fermi, (b) and (e) replicator, and (c) and (f) unconditional imitation.

For the case of heterogeneous networks, with the Fermi [Fig. 6(a)] and replicator [Fig. 6(b)] rules the results in the WPSL (planar network) have wider regions of S (cost of cooperation) and T (temptation to defect) for the emergence of cooperation with respect to those in the URSF (random network) in the Stag Hunt and Prisoner's Dilemma game areas [see Figs. 2(a) and 2(b) and Figs. 3(a) and 3(b)]. This indicates that the planarity of the network is effective in promoting cooperation behavior in the system for the two kinds of game, even the networks with a large exponent for the power-law degree distribution. In the plane area of the Snowdrift game, one can observe the different effects of the planarity of the network. It can be clearly seen that the cooperation levels in the WPSL are higher than the results in the URSF when the value of T is small (which means a low temptation to defect) while they are lower when T is large (which indicates a high temptation to defect) for fixed value of S. These phenomena conform well with the conclusions in Ref. [35] in which the authors illustrated that the spatial structure may often inhibit the evolution of cooperation. However, for the unconditional imitation rule [Fig. 6(c)], the planarity of the network has little effect on the evolution of cooperation for the Stag Hunt and Prisoner's Dilemma games, while in the plane area of the Snowdrift game the frequencies of cooperators in the WPSL are always a little higher than the results in the URSF, which presents different results under the Fermi and replicator rules.

For the case of homogeneous networks, with the Fermi [Fig. 6(d)] and replicator [Fig. 6(e)] update rules, the frequency of cooperation is promoted effectively in the areas of the Stag Hunt and Prisoner's Dilemma games, which is identical to the results of the heterogeneous case [Figs. 6(a) and 6(b)]. Nevertheless, the results in the region of the Snowdrift game have a different performance. With the Fermi and replicator update rules, the planarity of the network gives a weak promotion to cooperation only when T is small, then cooperation is inhibited in a wide range of T. Meanwhile, under the unconditional imitation rule [Fig. 6(f)], the situation is also different from the heterogeneous case [Fig. 6(c)]. The planarity of the network gives a significant promotion of cooperation in the regions of the Stag Hunt and Prisoner's Dilemma games. In the region of the Snowdrift game, cooperation frequency varies in a more complex non-monotonic way and there always exist two peaks of the difference values

with increase in T for a fixed S. These results indicate that the effect of the planarity of networks on the evolution of cooperation depends not only on the heterogeneity of the network (heterogeneous and homogeneous) but also on the specific update rules.

3.2. Effects of the heterogeneity of networks on the evolution of cooperation

Next, we study the effects of the heterogeneity of networks on the evolution of cooperation: the difference between the cooperation frequencies in the planar networks [WPSL (Fig. 2) and HL (Fig. 4)] and those in the random networks [URSF (Fig. 3) and RRG (Fig. 5)]. The detailed results are presented in Fig. 7. Similarly to the impact of the planarity of networks (see Fig. 6), it can be seen that for all three update rules the effect of heterogeneity is also negligible in the area of the Harmony game. Consequently, the impact of the heterogeneity of networks is mainly focused on the regions of the Stag Hunt, Prisoner's Dilemma and Snowdrift games for both planar and random networks.

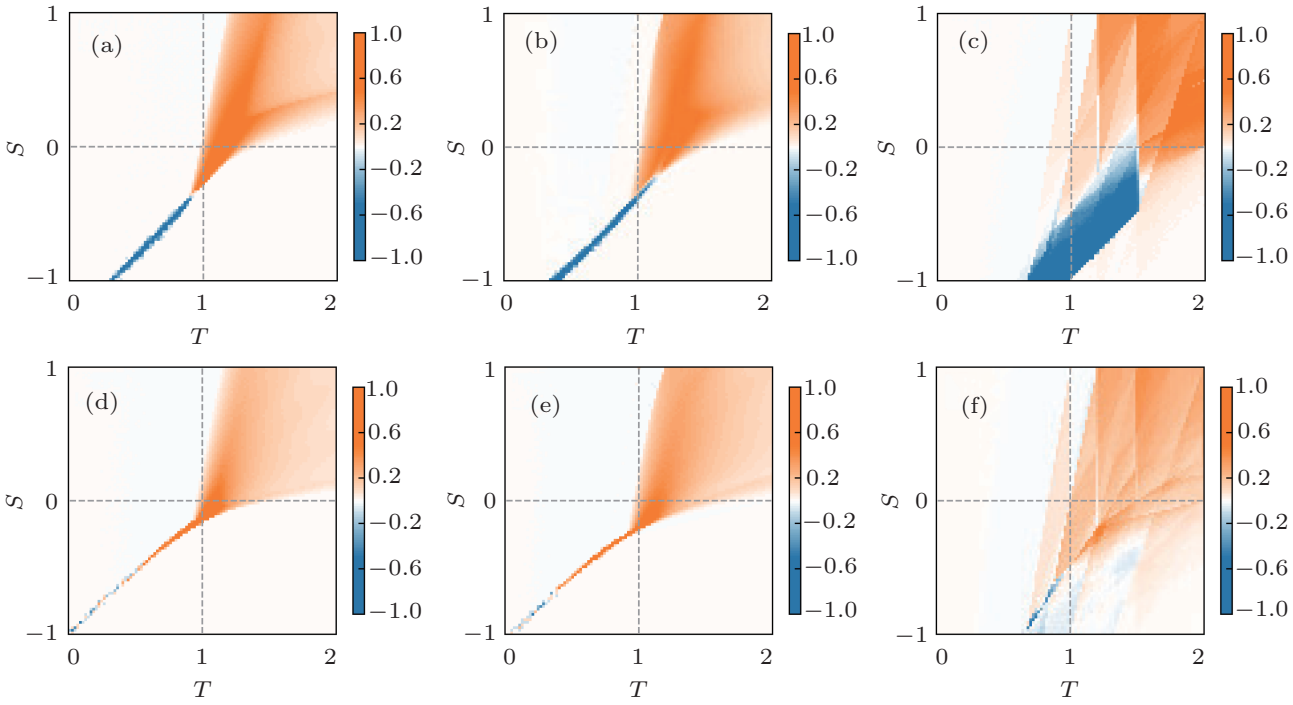


Fig. 7. (color online) Comparison of the results of cooperation levels between the two kinds of planar network [the results in WPSL (Fig. 2) minus the results in HL (Fig. 4), upper row] and between the two types of random network [the results in URSF (Fig. 3) minus the results in RRG (Fig. 5), bottom row] with and without heterogeneity under different update rules. The update rules respectively are (a) and (d) Fermi, (b) and (e) replicator, and (c) and (f) unconditional imitation.

For planar networks, in the region of the Stag Hunt game the affected area of the heterogeneity is narrow and cooperation is inhibited in the WPSL compared to that in the HL for the Fermi and replicator update rules [Figs. 7(a) and 7(b)], which is contrary to the corresponding results in Figs. 6(a) and 6(b). Cooperation is inhibited prominently by the heterogeneity of the network under the unconditional imitation rule [Fig. 7(c)]. In the area of the Prisoner's Dilemma game, cooperation is promoted by the heterogeneity of the network for both the Fermi and replicator update rules, whereas cooperation is inhibited dramatically under the unconditional imitation rule overall, which is contrary to the conclusion of Refs. [37], [38] and [39]. However, in the area of the Snowdrift game, the heterogeneity of the network gives an observable contribution to the promotion of cooperation under all three update rules. From Figs. 7(a) and 7(b), it can be also seen that the promotion of the heterogeneity to the evolution of cooperation exhibits non-monotonic behavior when the value of S exceeds a specific value in the region of the Snowdrift game. Particularly, for a specific value of S, there may arise two peaks of the difference values of cooperation with increase in T.

For random networks, in contrast to the results in the planar case, the influence of the heterogeneity of the network on cooperation is very narrow in the region of the Stag Hunt game under the Fermi, replicator and unconditional imitation rules [Figs. 7(d), 7(e), and 7(f)]. For the regions of the Prisoner's Dilemma and Snowdrift games, the results are similar to the case of planar networks for the Fermi and replicator rules and cooperation is facilitated. For the unconditional imitation rule, the effect of heterogeneity on the evolution of cooperation is very weak in the area of the Stag Hunt game. For clarity, we show a detailed summary of the effects of the planarity and heterogeneity of the network on the evolution of cooperation, promotion or inhibition in Table 1.

Table 1. Summary of the effects of the planarity and heterogeneity of the network on the evolution of cooperation, where +: promotion; −: inhibition; +/−: either promotion or inhibition depending on the value of the payoff parameters; and ∼: no obvious effect. The first column under each planarity header shows the effect of planarity for the heterogeneous networks (He) while the second column shows that for the homogeneous networks (Ho). The first column under each heterogeneity header shows the effect of heterogeneity for the planar networks (P) and the second column shows that for the random networks (R).

Game	Fermi rule				Replicator rule				Unconditional imitation rule			
	Planarity		Heterogeneity		Planarity		Heterogeneity		Planarity		Heterogeneity	
	He	Ho	P	R	He	Ho	P	R	He	Ho	P	R
Harmony	∼	∼	∼	∼	∼	∼	∼	∼	∼	∼	∼	∼
Snowdrift	+/−	+/−	+	+	+/−	+/−	+	+	+	+/−	+	+
Stag Hunt	+	+	−	+	+	+	−	+	∼	+	-	∼
Prisoner's Dilemma	+	+	+	+	+	+	+	+	∼	+	−	+

4. Conclusion

In summary, we have carried out a detailed investigation into evolutionary two-player symmetric games in four different kinds of network, including heterogeneous networks: the WPSL (weighted planar stochastic lattice, a planar scale-free network) and URSF (uncorrelated random scale-free network) with the same degree distribution as the WPSL; and homogeneous networks: the HL (hexagonal lattice), and RRG (random regular network) with degree $k_0 = 6$. By using extensive Monte Carlo simulations, we featured the effect of the planarity (interactions between individuals are local) and heterogeneity (individuals are with various numbers of neighbors) of networks on the evolution of cooperation within evolutionary two-player games (see Table 1). Particularly, both the planarity and the heterogeneity of the network have less impact on cooperation for the area of the Harmony game under all of the three update rules we considered. However, under the Fermi and replicator rules, cooperation can be significantly promoted by the planarity of the network irrespective of the heterogeneity of the network for the Stag Hunt and Prisoner's Dilemma games, in spite of the heterogeneous networks having a high exponent for the power-law degree distribution. Yet, in the area of the Snowdrift game, cooperation is enhanced when the temptation to defect is small, but inhibited when the temptation is large. Under the unconditional imitation rule, planarity only gives a slight promotion in the evolution of cooperation for the heterogeneous networks. Nevertheless, the heterogeneity of the network can facilitate the evolution of cooperation for the Snowdrift and Prisoner's Dilemma games under the Fermi and replicator rules, irrespective of the planarity of the network. While in the area of the Stag Hunt game, only a narrow facilitation and inhibition exist for the planar and random networks respectively. Meanwhile, for the Stag Hunt and Prisoner's Dilemma games, cooperation is inhibited by heterogeneity for the planar networks but slightly enhanced for the random networks under the unconditional imitation rule. Our results illustrate that both the planarity and heterogeneity of networks play an important, and different, role in the evolution of cooperation.

Recently, Press and Dyson have coined the term "zero-determinant" (ZD) strategy in evolutionary games,[53] which allows players to set their opponents' payoffs unilaterally. The relevant research has been well reviewed by Hao *et al.* in Ref. [54]. How the planarity of networks affects the evolution of cooperation with ZD strategy still remains an open question which is worth pursuing further. Besides, Xie *et al.* have proposed a growing network model based on an optimal policy involving both topological and geographical measures controlled by a free parameter α.[55] How the cooperation level is affected by the underlying structure with increasing α is also an interesting issue worth studying seriously.

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