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SPECIAL TOPIC — 80th Anniversary of Northwestern Polytechnical University (NPU)

Effect of stochastic electromagnetic disturbances on autapse neuronal systems*

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With the help of a magnetic flux variable, the effects of stochastic electromagnetic disturbances on autapse Hodgkin–Huxley neuronal systems are studied systematically. Firstly, owing to the autaptic function, the inter-spike interval series of an autapse neuron not only bifurcates, but also presents a quasi-periodic characteristic. Secondly, an irregular mixed-mode oscillation induced by a specific electromagnetic disturbance is analyzed using the coefficient of variation of inter-spike intervals. It is shown that the neuronal discharge activity has certain selectivity to the noise intensity, and the appropriate noise intensity can induce the significant mixed-mode oscillations. Finally, the modulation effects of electromagnetic disturbances on a ring field-coupled neuronal network with autaptic structures are explored quantitatively using the average spiking frequency and the average coefficient of variation. The electromagnetic disturbances can not only destroy the continuous and synchronous discharge state, but also induce the resting neurons to generate the intermittent discharge mode and realize the transmission of neural signals in the neuronal network. The studies can provide some theoretical guidance for applying electromagnetic disturbances to effectively control the propagation of neural signals and treat mental illness.

Keywords: neuronal system, electromagnetic disturbance, autapse, mixed-mode oscillations

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1. Introduction

In 1952, Hodgkin and Huxley proposed a mathematical model of biological neurons by studying the membrane current of squid nerve fibers, which is the so-called Hodgkin–Huxley neuron model.[1] This biological neuron model marked an important breakthrough in the study of biological nervous systems, and resulted in some typical neuron models, such as FitzHugh–Nagumo model,[2] Morris–Lecar model,[3] and Hindmarsh–Rose model.[4,5] References [6] and [7] summarized twenty typical characteristics of biological neuronal discharge activities and compared seventeen representative neuron models, respectively. For computational neurosciences, it is necessary not only to study the mechanisms of various factors such as channels, synapses and electromagnetic fields on the neuronal dynamic behavior, but also to study the relationship between neuronal dynamic behavior and neural information coding, which will help to reveal the neural mechanisms of learning and memory.[8–12]

Anatomical and electrophysiological experiments showed that neurons interact through synapses, which are the key functional structures for achieving signal transmissions and information exchanges between neurons. Autapse is a special synapse structure which enables a neuron to connect with itself through a closed loop and is mathematically depicted by a time-delayed feedback term.[13,14] Autapse divided into electric and chemical autapse is usually studied to explore the effects of autaptic conductance and delay time on neuronal firing patterns.[15–17] The self-adaption of autapse driving can induce different modes of electrical activities in a simple network of neuron-coupled astrocyte.[18] Also, autaptic structures can be considered as a pacemaker which can regulate the neuronal collective behaviors in a forward feedback Hindmarsh–Rose neuronal network.[19] In addition, appropriate autaptic parameters can enhance the propagation of the discharge rhythm and induce more pronounced stochastic resonance in a scale-free Hodgkin–Huxley neuronal network.[20] Physiological anatomic experiments have confirmed the existence of autaptic structures in some neurons of human brain, especially in most cortical pyramidal neurons.[21] In a nearest-neighbor synaptic coupled neuronal network, the time delay can also enhance the neuronal firing rate.[22]

Because of the ubiquity of electromagnetic radiations, it is necessary to study its effects on electrical activities of neurons and neuronal networks. On the other hand, it is necessary to explore how to use electromagnetic radiations to

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treat mental illness. At present, there are mainly two different mechanisms of electromagnetic radiations on nervous systems. Firstly, the electromagnetic radiation absorbed by a neuron can be converted into the power of its own electrical energy, and the neuron model resulting from the electromagnetic radiation can be obtained according to the neuronal energy theory.[23] Secondly, based on the phenomenon of electromagnetic induction induced by ion transmembrane motions, the relationship between magnetic flux and induced current can be established according to the working principle of memristors. Thus, the effects of electromagnetic radiations or disturbances on the discharge behaviors of neurons or neuronal networks can be detected by introducing a magnetic induction current.[24–28] Reference [29] commented on the dynamic behaviors of biological neuron models, summarized the effects of autaptic structures and ion channel noises on neuronal firing activities, and proposed a new coupled mode between neurons.

With the help of a magnetic flux variable, this paper systematically studies the modulations of electromagnetic disturbances to the discharge behaviors of improved Hodgkin–Huxley biological neuronal systems. Firstly, the bifurcation dynamic behavior of an autapse neuron is analyzed to investigate the effect of the self-feedback memory ability induced by its autaptic structure on neuronal electrical activities. Secondly, the stochastic discharge behaviors induced by a stochastic electromagnetic disturbance are studied by using the coefficient of variation of inter-spiking intervals, and the selectivity of the neuronal discharge mode to the noise intensity is detected. Finally, the mechanism of electromagnetic disturbances on a Hodgkin–Huxley neuronal network with autaptic structures is explored by using the average spiking frequency and the average coefficient of variation, and the feasibility of applying electromagnetic disturbances to regulate its dynamic behavior is discussed.

2. Mathematical model

Considering the magnetic induction current caused by ion exchanges or ion concentration fluctuations, a typical 4-dimensional Hodgkin–Huxley neuron model can be modified by introducing the magnetic flux variable. At the same time, considering the autaptic function, an improved 5-dimensional Hodgkin–Huxley neuron model is obtained as follows:

$$\begin{aligned}
C_{\mathrm{m}}\frac{\mathrm{d}V}{\mathrm{d}t} &= g_{\mathrm{Na}}m^3h(V_{\mathrm{Na}}-V)+g_{\mathrm{K}}n^4(V_{\mathrm{k}}-V)\\
&\quad+g_{\mathrm{L}}(V_{\mathrm{L}}-V)+I_{\mathrm{ext}}+I_{\mathrm{aut}}-k\rho(\varphi)V,\\
\frac{\mathrm{d}m}{\mathrm{d}t} &= \alpha_m(V)(1-m)-\beta_m(V)m,\\
\frac{\mathrm{d}h}{\mathrm{d}t} &= \alpha_h(V)(1-h)-\beta_h(V)h,\\
\frac{\mathrm{d}n}{\mathrm{d}t} &= \alpha_n(V)(1-n)-\beta_n(V)n,\\
\frac{\mathrm{d}\varphi}{\mathrm{d}t} &= k_1V-k_2\varphi+\xi(t),
\end{aligned}\tag{1}$$

subject to

$$\begin{aligned}
\alpha_m &= 0.1(25-V)/(\exp((25-V)/10)-1),\\
\beta_m &= 4\exp[-V/18],\\
\alpha_h &= 0.07\exp(-V/20),\\
\beta_h &= 1/(\exp((30-V)/10)+1),\\
\alpha_n &= 0.01(10-V)/(\exp((10-V)/10)-1),\\
\beta_n &= 0.125\exp(-V/80),
\end{aligned}$$

where V is the membrane potential, C_{m} is the capacitance per unit area of the membrane, m, h, n represent the gating variables corresponding to sodium and potassium channels. g_{L} is the leakage conductance, g_{Na} and g_{K} denote the sodium and potassium maximum ionic conductances. V_{Na}, V_{K}, V_{L} represent the reversal potentials for the sodium, potassium, and leakage currents, respectively. I_{ext} denotes the external current stimulation. Because the electrical autapse is selected for this study, the autaptic current I_{aut} is given by $I_{\mathrm{aut}}=g(V(t-\tau)-V(t))$, where g is the autaptic conductance and τ is the delay time. The stochastic disturbance induced by external electromagnetic fields $\xi(t)$ is defined as Gaussian white noise with the statistical relations of $\langle\xi(t)\rangle=0$ and $\langle\xi(t)\xi(t')\rangle=2D\delta(t-t')$, where D is the noise intensity and $\delta(\cdot)$ represents the Dirac delta function. The magnetic induction current can be defined as $I_{\mathrm{ind}}=k\rho(\varphi)V$ with the help of memristor. Here the memductance is given as follows:

$$\rho(\varphi)=\frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi}=\alpha+3\beta\varphi^2,\tag{2}$$

where φ represents the magnetic flux across the memristor and is used to describe the effect of electromagnetic induction, k denotes the feedback gain coefficient, α and β are parameters, and q represents the quantity of electric charges across the memristor. This idea first given in 2016 has a certain degree of rationality and innovation, and can be used not only for Hindmarsh–Rose model but also for Hodgkin–Huxley model.[27]

A ring field-coupled neuronal network with autaptic structures is given as follows:

$$\begin{aligned}
C_{\mathrm{m}}\frac{\mathrm{d}V_i}{\mathrm{d}t} &= g_{\mathrm{K}}n_i^4(V_{\mathrm{K}}-V_i)+g_{\mathrm{N_a}}m_i^3h_i(V_{\mathrm{N_a}}-V_i)+g_{\mathrm{L}}(V_{\mathrm{L}}-V_i)\\
&\quad+I_{\mathrm{ext}}+I_{\mathrm{aut}_i}-k\rho(\varphi_i)V_i,\\
\frac{\mathrm{d}y_i}{\mathrm{d}t} &= \alpha_{y_i}(V_i)(1-y_i)-\beta_{y_i}(V_i)y_i,\ \ y_i=m_i,h_i,n_i,\\
\frac{\mathrm{d}\varphi_i}{\mathrm{d}t} &= k_1V_i-k_2\varphi_i+g_0((\varphi_{i+1}+\varphi_{i-1})-2\varphi_i)+\xi_i(t),\\
&\quad i=1,2,\ldots,N,
\end{aligned}\tag{3}$$

where $\varphi_0 = \varphi_N$ and $\varphi_{N+1} = \varphi_1$. The electromagnetic disturbances $\xi_i(t)$ $(i = 1, 2, \ldots, N)$ are represented with independent Gaussian white noises with the same statistical properties as above, and g_0 denotes the field coupling intensity.[29]

3. Dynamic behaviors of autaptic function

In this study, the second-order stochastic Runge–Kutta algorithm[30] is used and the time step is set as $\Delta t = 0.01$ ms for all numerical computations. In particular, some parameters are also set as the fixed constants, such as $g_{\rm Na} = 120$ mS/cm^2, $g_{\rm K} = 36$ mS/cm^2, $g_{\rm L} = 0.3$ mS/cm^2, $C_{\rm m} = 1.0$ μF/cm^2, $V_{\rm Na} = 115$ mV, $V_{\rm K} = -12$ mV, $V_{\rm L} = 10.6$ mV, $k_1 = 0.01$, $k_2 = 0.1$, $\alpha = 0.1$, $\beta = 0.1$. The initial values for five variables are selected as $V_0 = -64.999801$ mV, $m_0 = 0.052938$, $h_0 = 0.5916$, $n_0 = 0.317726$, $\varphi_0 = 1.0$.

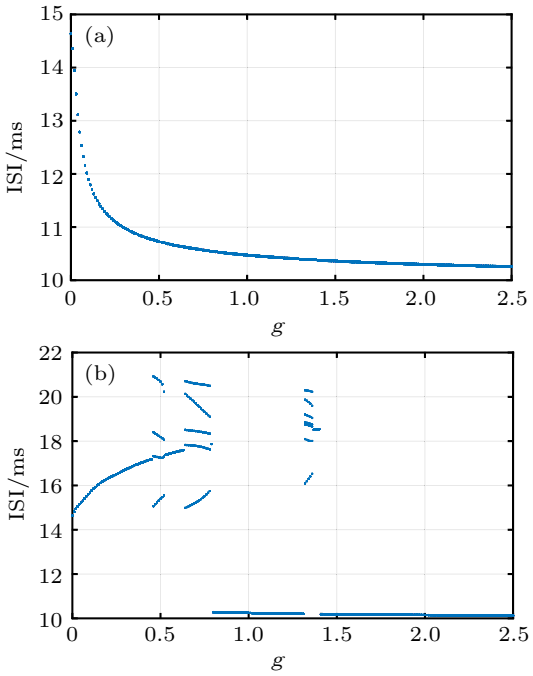


Fig. 1. (color online) Bifurcation diagrams of inter-spike interval versus autaptic conductance for fixed feedback gain coefficient $k = 0$, external forcing current $I_{\rm ext} = 10$ μA/cm^2, delay time (a) $\tau = 10.0$ ms and (b) $\tau = 20.0$ ms. Autapse is activated at $t = 120$ ms and ISI is calculated from $t = 3500$ ms to $t = 5500$ ms.

During the numerical experiments, the autaptic function is activated at $t = 120$ ms, and the inter-spike interval (ISI) series are only recorded from $t = 3500$ ms to $t = 5500$ ms. As can be seen from Fig. 1(a), there exists no bifurcation phenomenon when $\tau = 10.0$ ms, and the inter-spike interval series presents a monotonous decreasing trend as the autaptic conductance increases, that is, with the increase of the autaptic conductance, the period of the neuronal discharge activities gradually becomes smaller. This is mainly because the memory ability induced by the autaptic structure is stronger when the delay time is smaller, and the current discharge activity is more easily affected by the past state corresponding to the delay time. Generally, the neuronal excitability determining the spiking frequency can be enhanced by the autaptic function with a larger autaptic conductance. When $\tau = 20.0$ ms, with the increase of the autaptic conductance, the inter-spike interval series undergoes several special changes and presents a distinct bifurcation process, as shown in Fig. 1(b). When the autaptic conductance is increased to $g = 0.46$, the neuronal discharge mode changes from single-cycle to four-cycle. As the autaptic conductance continues to increase, the inter-spike interval series produces several bifurcation phenomena and experiences several discharge mode transitions between single-cycle, five-cycle and six-cycle. In particular, there occurs a jumping phenomenon in the inter-spike interval series when the autaptic conductance increases to $g = 0.8$, 1.32 and 1.41, respectively, which reflects the local sensitivity of the neuronal discharge activity to the autaptic conductance.

To detect the effect of the delay time on the neuronal dynamic behavior, figure 2 gives bifurcation diagrams of inter-spike interval versus delay time for two different autaptic conductances. Firstly, when the delay time increases from $\tau = 0.2$ ms to $\tau = 50.0$ ms, the two inter-spike interval series experience a cycle-like change process and produce three significant jumping phenomena. The characteristic reflects some sensitivity of the neuronal discharge activity to the delay time. Secondly, in each cycle-like change process, the inter-spike interval usually increases with the increase of the delay time.

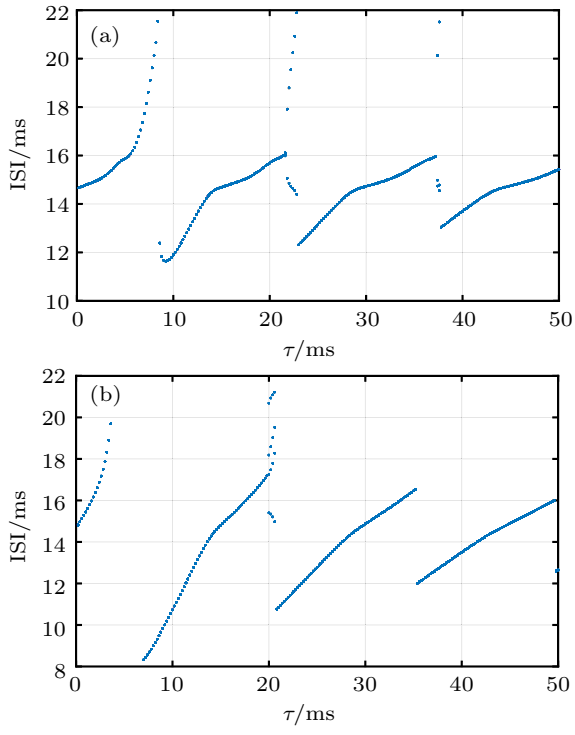


Fig. 2. (color online) Bifurcation diagrams of inter-spike interval versus delay time for fixed feedback gain coefficient $k = 0$, external forcing current $I_{\rm ext} = 10$ μA/cm^2, autaptic conductance (a) $g = 0.1$ and (b) $g = 0.5$. Autapse is activated at $t = 120$ ms and ISI is calculated from $t = 3500$ ms to $t = 5500$ ms.

However, it suddenly decreases when the delay time increases to a certain value, and then restarts the next cycle-like change process. This change should be a reflection of the intrinsic periodicity of the neuron. Finally, for the autaptic conductance $g = 0.1$, the neuronal dynamic behavior has a two-cycle bifurcation phenomenon when the delay time $\tau \in [21.6\ \text{ms}, 22.8\ \text{ms}]$, and a three-cycle bifurcation phenomenon when $\tau \in [37.4\ \text{ms}, 37.6\ \text{ms}]$. For $g = 0.5$, the neuronal discharge activity only produces a four-cycle bifurcation phenomenon when $\tau \in [20.0\ \text{ms}, 20.6\ \text{ms}]$. Thus, the neuronal dynamic behavior depends not only on the autaptic conductance but also on the delay time. Also, only a few specific values of the two parameters can induce the bifurcation phenomena. This shows that the neuronal discharge activity has some selectivity on the autaptic conductance and the delay time.

To explore the mechanism of the autaptic function, we further provide the spiking frequency diagram for different autaptic parameters. As can be seen from Fig. 3, when the external forcing current $I_{\text{ext}} = 10\ \mu\text{A/cm}^2$, the neuronal spiking frequency shows a cycle-like change characteristic with the increase of the delay time when the autaptic conductance is selected and has no significant characteristic when the autaptic conductance is changed and the delay time is fixed. On the other hand, there is a dark blue strip-like region in the lower right corner. The spiking frequency value is discontinuous and can jump especially near the upper boundary, that is to say, the neuronal discharge activity can suddenly change from a resting state to a rapidly oscillating discharge state as the delay time increases. This further reflects the local sensitivity of the neuronal firing activity to two autaptic parameters.

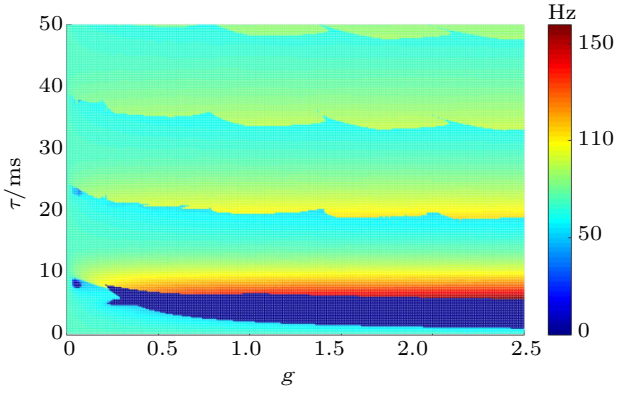


Fig. 3. (color online) Spiking frequency diagram for different values of autaptic conductance and delay time for fixed feedback gain coefficient $k = 0$ and external forcing current $I_{\text{ext}} = 10\ \mu\text{A/cm}^2$. Autapse is activated at $t = 120$ ms and ISI is calculated from $t = 3500$ ms to $t = 5500$ ms.

Thus, the autaptic function can have a great influence on the neuronal dynamic behavior. On the one hand, it can change the neuronal excitability and the neuronal spiking frequency. On the other hand, a bifurcation phenomenon usually occurs when the autaptic function inhibits the neuronal discharge activity. In addition, the effect of the autaptic function on the neuronal dynamic behavior is diverse. This conclusion includes two meanings here, one is that the neuronal discharge activity has certain selectivity to the autaptic parameters, and the different autaptic parameters can induce the different discharge modes. The two is that the influence of the autaptic parameters on the neuronal dynamic behavior is local discontinuous and indicates the local sensitivity of the neuronal firing activity to the autaptic parameters. At last, the bifurcation behavior in the inter-spike interval series is not only affected by the autaptic parameters, but also by the neuronal intrinsic periodicity, which together determine the self-feedback memory ability induced by the autaptic structure. The above analysis could help to apply the autaptic structure in neuronal networks to obtain certain neurophysiological phenomena, such as spiral waves and target waves.

4. Modulation of electromagnetic disturbances to autaptic function

The coefficient of variation of the inter-spike interval series is introduced to measure the effect of electromagnetic disturbances on the neuronal discharge activity, which is defined as follows:

$$\text{CV} = \sqrt{\langle T_{\text{ISI}}^2 \rangle - \langle T_{\text{ISI}} \rangle^2} / \langle T_{\text{ISI}} \rangle, \tag{4}$$

where T_{ISI} represents the inter-spike interval of neuronal membrane potentials, $\langle T_{\text{ISI}} \rangle$ and $\langle T_{\text{ISI}}^2 \rangle$ denote the average and mean-square value of the inter-spike interval series, respectively. Figure 4 shows the variation trends of the coefficient of variation under the interaction of electromagnetic disturbance and autaptic structure. Here, the autaptic function is activated at $t = 120$ ms and the stochastic electromagnetic disturbance is triggered at $t = 500$ ms. In Fig. 4(a), the variation trends of the coefficient of variation are compared for four selected autaptic conductances $g = 0.1$, 0.2, 0.5, and 1.0. For $g = 0.1$ and 0.2, the coefficient of variation will increase first and then decrease as the noise intensity increases. In particular, the coefficient of variation can decay rapidly after reaching its peak value, and then show a weak descending trend when the noise intensity $D > 0.4$. For $g = 0.5$ and 1.0, obvious local oscillatory phenomena exist in the change process of the coefficient of variation. This may be due to the fact that the larger the autaptic conductance, the stronger the self-feedback memory ability induced by the autaptic structure, which can change the synergetic ability of electromagnetic induction and autapse under different noise intensities. Figure 4(b) illustrates the variation trends of the coefficient of variation for four different delay times $\tau = 2.0$ ms, 5.0 ms, 10.0 ms, and 20.0 ms. For the four selected delay times, the corresponding coefficient of

variation shows a clear trend of increasing first and then decreasing and begins to decrease around $D = 0.05$, and gradually stabilizes around 0.54. Particularly, for the delay time $\tau = 10.0$ ms, the coefficient of variation will reach the maximum value about 5.5 when the noise intensity increases from 0 to 0.04, which corresponds to the most extreme coherent resonance phenomenon. In addition, it has been also shown that the proper delay time could enhance the information capacity and energy efficiency.[31]

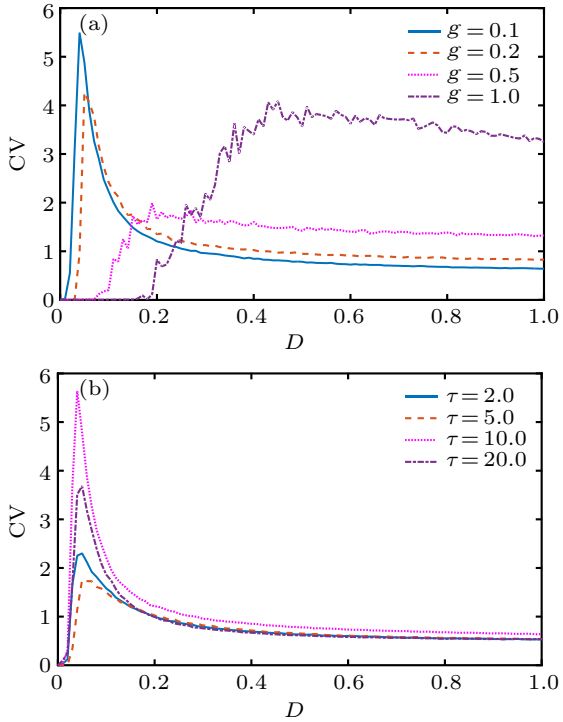


Fig. 4. (color online) Coefficient of variation of ISI series for fixed external forcing current $I_{\rm ext} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, where (a) $\tau = 10.0$ ms and (b) $g = 0.1$. Coefficient variability is calculated from $t = 500$ ms to $t = 10500$ ms when autapse is activated at $t = 120$ ms and electromagnetic disturbance is triggered at $t = 500$ ms.

To further investigate the above extreme coherent resonance phenomenon, the neuronal membrane potentials are simulated by selecting two different noise intensities, as shown in Fig. 5. Obviously, the neuron exhibits a relatively strong spike discharge state after the autaptic function is activated at $t = 120$ ms, but the neuronal discharge mode can be transited to a random-like mode after the electromagnetic disturbance is triggered at $t = 500$ ms. Due to the randomness of the electromagnetic disturbance, the times and peak values of the neuronal spike discharge are unpredictable and almost randomly distributed. When the noise intensity $D = 0.04$, the coefficient of variation just reaches its maximum value and the neuron produces a significant intermittent discharge mode. It is worth noting that this intermittent discharge mode is generally between the resting state and the discharge rhythm of period 1, which not only can be observed in the discharge experiment of a single neuron but also can be generated in Chay model.[32] With the further increase of noise intensity, the intermittent discharge mode becomes not obvious and gradually evolves into a random discharge mode. This shows that the firing activity of the autapse neuron has some selectivity to the noise intensity, that is to say, according to different autaptic conditions, the noise intensity can be selected to cause the neuron to produce a significant intermittent discharge phenomenon.

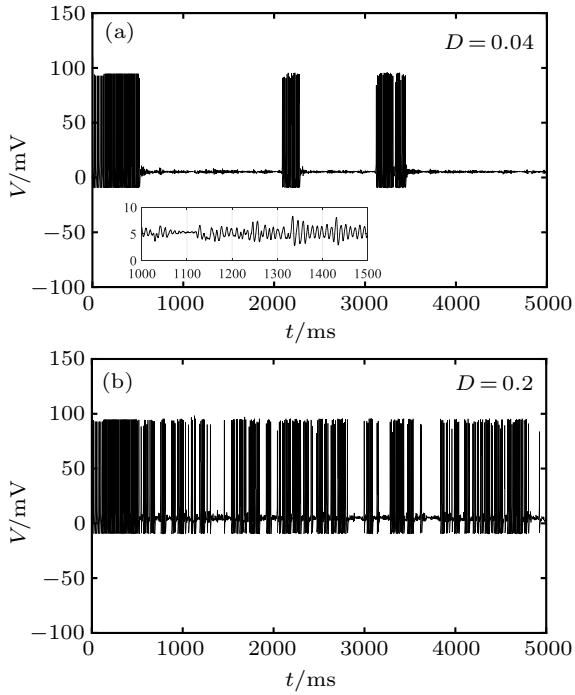


Fig. 5. Sampled time series of membrane potentials under the effects of electromagnetic disturbance and autapse for fixed external forcing current $I_{\rm ext} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms, noise intensity (a) $D = 0.04$ and (b) $D = 0.2$. Autapse is activated at $t = 120$ ms and electromagnetic disturbance is triggered at $t = 500$ ms.

Figure 6 presents the phase portraits on the different phase planes, which corresponds to Fig. 5(a). It can be seen from Fig. 6 that the solution orbits oscillate with different amplitudes near the equilibrium point. The system generates small-amplitude oscillations around the equilibrium point when the electromagnetic disturbance is weak. However, when the electromagnetic disturbance can cause the membrane depolarization to exceed the neuronal discharge threshold, the five-dimensional Hodgkin–Huxley oscillator deviates from the previous orbit near the equilibrium point and begins to oscillate with large amplitudes. Thus, the randomness of the electromagnetic disturbance induces a new discharge phenomenon which is the so-called mixed-mode oscillations (MMOs). The MMOs usually exist in the dynamic systems with fast and slow variables. In neuroscience researches, the special firing mode can be generated not only in the three-dimensional self-coupled or four-dimensional Hodgkin–Huxley neuron models,[33–35] but also in the stochastic FitzHugh–Nagumo

neuron model and in the two-dimensional map-based neuron model through chaotic dynamics.[36,37] In the five-dimensional stochastic Hodgkin–Huxley neuron model studied in this paper, the MMOs can also be generated in the case of the ideal noise intensity.

In summary, the electromagnetic disturbance in the form of Gaussian white noise can cause the transition of neuronal discharge modes. Particularly under the condition of the appropriate noise intensity, the autapse neuron can produce a significant intermittent discharge phenomenon, which can be measured by the coefficient of variation of the inter-spike interval series. According to the characteristics of neuronal firing activities induced by the autaptic function, the autaptic structure could be the cause of some mental states, and the different autaptic conditions could correspond to the different mental states. However, the electromagnetic disturbance can effectively modulate the discharge behavior induced by the autaptic structure and will be helpful for the treatment of mental illness.

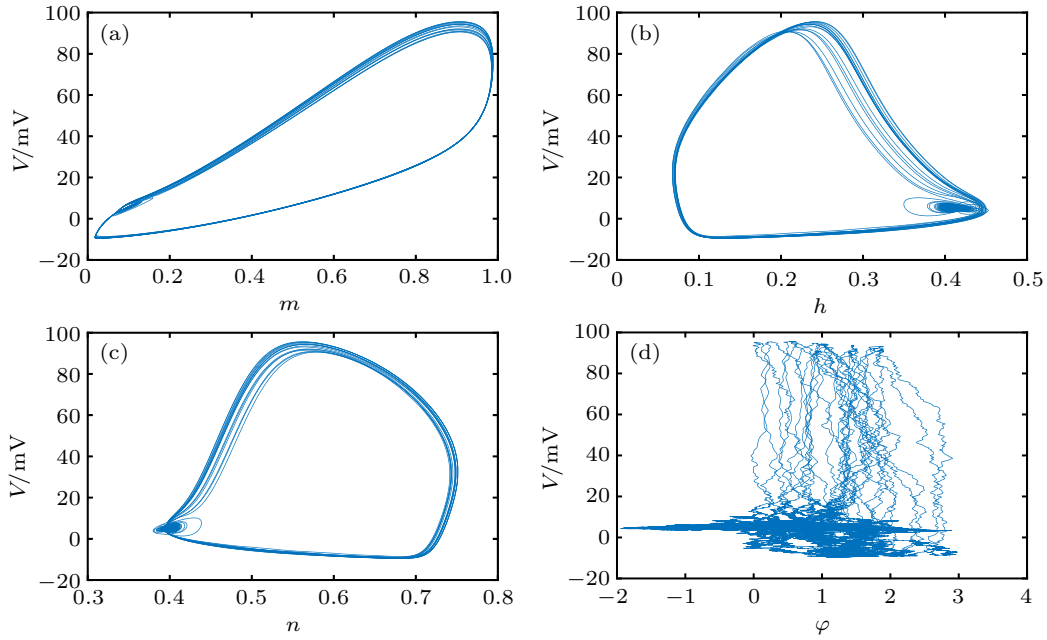


Fig. 6. (color online) Phase portraits on the different phase planes for fixed external forcing current $I_{\rm ext} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms, and noise density $D = 0.04$, where (a) $V - m$, (b) $V - h$, (c) $V - n$, and (d) $V - \varphi$. The solution orbit is calculated from $t = 1000$ ms to $t = 3000$ ms when autapse is activated at $t = 120$ ms and electromagnetic disturbance is triggered at $t = 500$ ms.

5. Modulation of electromagnetic disturbances to an autapse neuronal network

There are mainly three types of connections among neurons in the central nervous system, i.e., convergence, divergence, and ring, and the ring structure chosen here is easily realized in computational neuroscience.[10] In order to study the modulation effect of electromagnetic disturbances on a ring field-coupled neuronal network consisting of 40 neurons, the average coefficient of variation $\overline{\rm CV}$ is introduced to measure the firing regularity of the neuronal network, which is defined as follows:

$$\overline{\rm CV} = \left(\sum_{i=1}^{N} {\rm CV}_i\right)/N, \tag{5}$$

where ${\rm CV}_i$ represents the coefficient of variation of the inter-spike interval series of the i-th neuron.[38] According to the modulation characteristics of electromagnetic disturbances on a single autapse neuron, the autaptic conductance and the delay time are selected as $g = 0.1$ and $\tau = 10.0$ ms, respectively. In the present numerical experiment, the autaptic structures are activated at $t = 120$ ms and the electromagnetic disturbances with independent and identical distribution are triggered at $t = 500$ ms for all neurons. Figure 7 shows the variation trends of the average spiking frequency and the average coefficient of variation for all neurons, where the field coupling intensities are selected as $g_0 = 1.0$. When the noise intensity increases from 0 to 1.0, the average spiking frequency $\overline{f_{\rm out}}$ rapidly decreases from 82.4 Hz to 27.0 Hz, and then rapidly increases and tends to 41.2 Hz, as shown in Fig. 7(a). Thus, in the increasing process of the noise intensity, the neuronal discharge activities in the network can be first suppressed strongly and then promoted by the electromagnetic disturbances. However, the modulation of the stochastic disturbances to the neuronal network has no significant change after the noise intensity increases to a certain extent. In Fig. 7(b), the corresponding average coefficient of variation first increases and then decreases, which trend is just opposite to that of the average spiking frequency. Also, the average coefficient of variation reaches its maximum value $\overline{\rm CV} = 3.8$ when the noise intensity $D = 0.21$. When $g_0 = 0.2$, the variation trends of the two measurements

have no remarkable difference from $g_0 = 1.0$, so the field coupling intensity is set as $g_0 = 1.0$ for the following experiments.

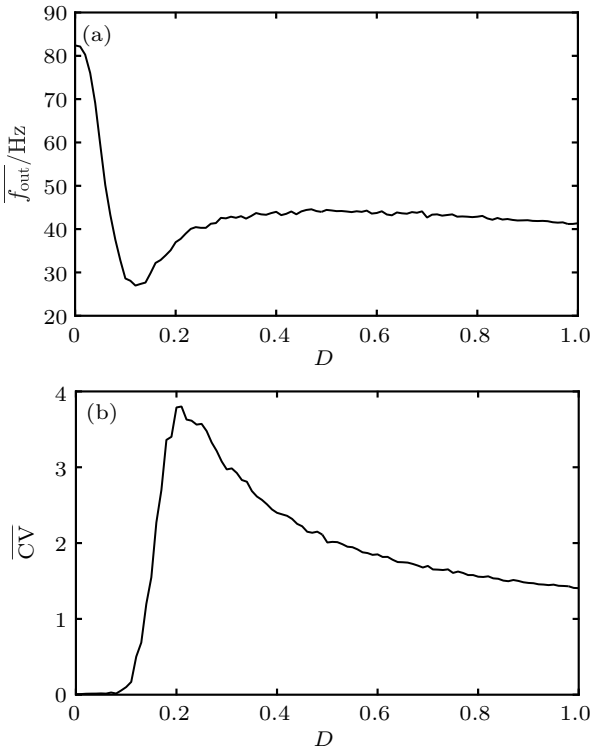


Fig. 7. Average spiking frequency and average coefficient of variation for different values of noise intensity for fixed external forcing current $I_{\rm ext} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms and field coupling intensity $g_0 = 1.0$, where (a) average spiking frequency and (b) average coefficient of variation. Average spiking frequency and average coefficient of variation are calculated from $t = 5000$ ms to $t = 10000$ ms when autapses are activated at $t = 120$ ms and electromagnetic disturbances are triggered at $t = 500$ ms.

To explore the sensitivity of the discharge pattern to the noise intensity, figure 8 presents the spatiotemporal patterns for four different noise intensities $D = 0.12$, 0.21, 0.5, and 1.0. Here, $D = 0.12$ and 0.21 just make the average spiking frequency and the average coefficient of variation reach the minimum and maximum value, respectively. In the case of synchronization of the initial neuronal network, the autaptic structures cannot change the neuronal synchronous discharge behaviors. However, after the stochastic disturbances are triggered at $t = 500$ ms, the synchronous state of the network is destroyed and the desynchronization phenomenon can be observed. When $D = 0.12$, the average spiking frequency is the smallest. In addition, some neurons in the network produce relatively long continuous discharge phenomenon, and some other neurons can remain the resting state after a period of time. When $D = 0.21$ corresponding to the largest average coefficient of variation, the neurons in the network produce a long and discontinuous periodic spiking discharge phenomenon, which is similar to the intermittent discharge mode. When the noise intensity increases to $D = 1.0$, the long spiking discharge mode is completely destroyed, and all the neurons present an intermittency-like random discharge pattern characterized by a short duration and a frequent distribution. In conclusion, as the noise intensity increases, the long and continuous discharge state gradually evolves into the intermittency-like random discharge pattern, and the synchronization of the neuronal network completely disappears. This indicates that the external electromagnetic disturbances can significantly affect the transmembrane motion of charged ions and play an important role in the neuronal system. These conclusions could provide some valuable theoretical guidance for synchronization or chaos control of neuronal networks.[39,40]

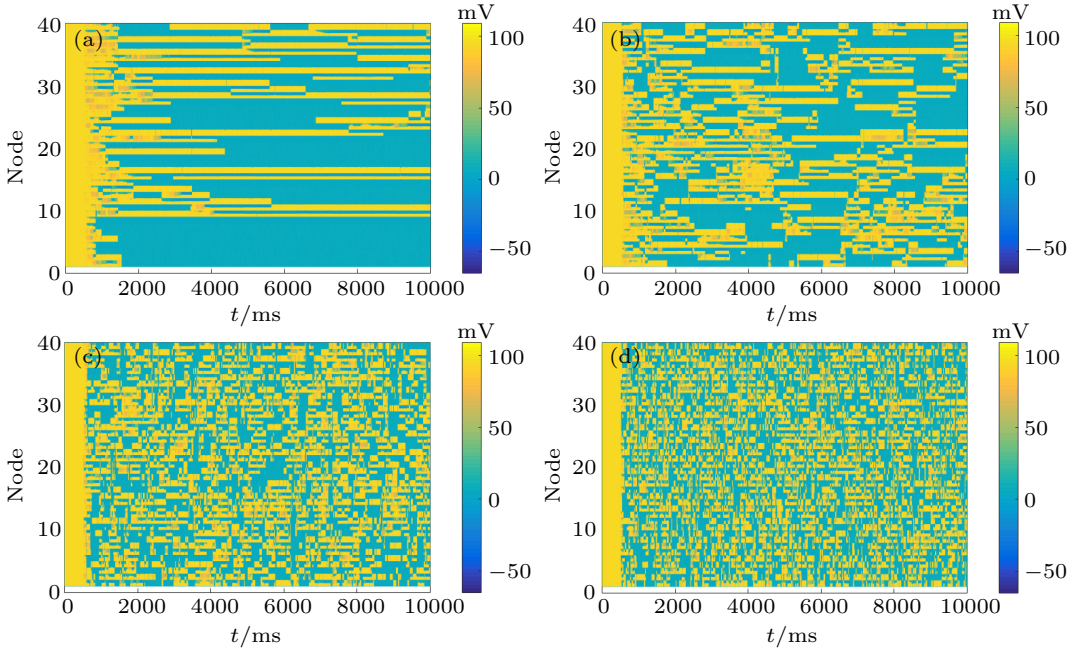


Fig. 8. (color online) Spatiotemporal patterns of ring coupled neuronal network for different values of noise intensity for fixed external forcing current $I_{\rm ext} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms, field coupling intensity $g_0 = 1.0$, noise intensity (a) $D = 0.12$, (b) $D = 0.21$, (c) $D = 0.5$, and (d) $D = 1.0$. Autapses are activated at $t = 120$ ms and electromagnetic disturbances are triggered at $t = 500$ ms.

To further study the effects of electromagnetic disturbances on neuronal discharge behavior and neural signal propagation in the network, only four neurons in the network are given the autaptic structures, that is, only if $i = 19$, 20, 21, and 22, then $I_{\mathrm{aut}_i} = g(V_i(t-\tau) - V_i(t))$, else $I_{\mathrm{aut}_i} = 0$. In addition, the initial state of the four neurons is the same as above, while the others remain the resting state in the beginning. Figure 9 gives two schematic diagrams of the average spiking frequency and the average coefficient of variation as a function of noise intensity, and figure 10 is the corresponding spatiotemporal patterns of the ring field-coupled neuronal network for four different noise intensities. When there is no disturbance or the noise intensity is very small, the action of field coupling cannot cause the other 36 neurons to generate spiking discharge phenomena. When $D = 0.1$, it can be seen from Fig. 10(b) that the electromagnetic disturbances start to have a pronounced effect on the neuronal network and almost make the average spiking frequency reach the minimum value. Moreover, the continuous discharge state of the four autapse neurons is changed, and the other neurons occasionally produce the intermittent firing phenomena. When the noise intensity increases to $D = 0.2$, the 36 neurons without autaptic structures can present a more significant intermittent spiking discharge behavior, and the average coefficient of variation reaches its peak value $\overline{\mathrm{CV}} = 2.46$. As the noise intensity further increases in Fig. 9, the average spiking frequency of the network keeps increasing and gradually stabilizes around 27.2 Hz, while the average coefficient of variation almost presents a decreasing trend and gradually stabilizes around 0.98. The main feature of the corresponding spatiotemporal pattern is that the 36 neurons without autaptic structures generate more and more frequent intermittency-like discharge modes, as shown in Fig. 10(d).

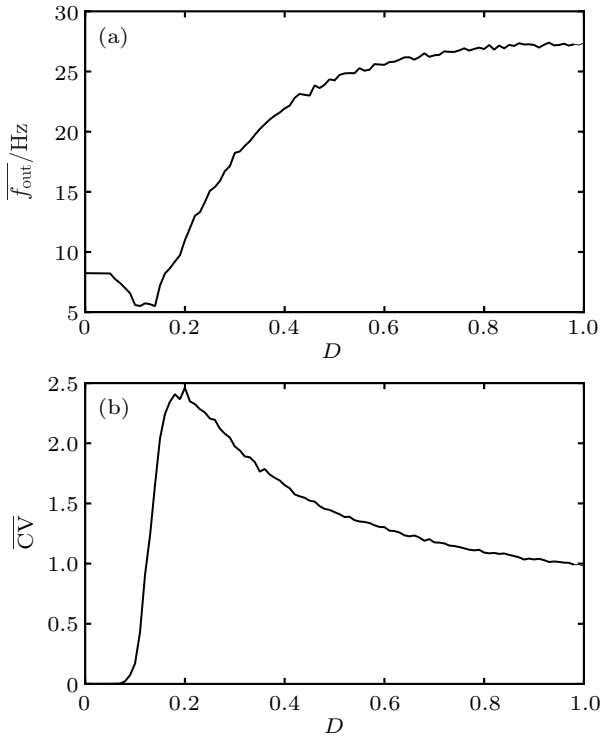


Fig. 9. Average spiking frequency and average coefficient of variation for different values of noise intensity for fixed external forcing current $I_{\mathrm{ext}} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms, and field coupling intensity $g_0 = 1.0$, where (a) average spiking frequency and (b) average coefficient of variation. Average spiking frequency and average coefficient of variation are calculated from $t = 5000$ ms to $t = 10000$ ms when autapses are activated at $t = 120$ ms for only four nodes 19–22 and electromagnetic disturbances are triggered at $t = 500$ ms for all nodes.

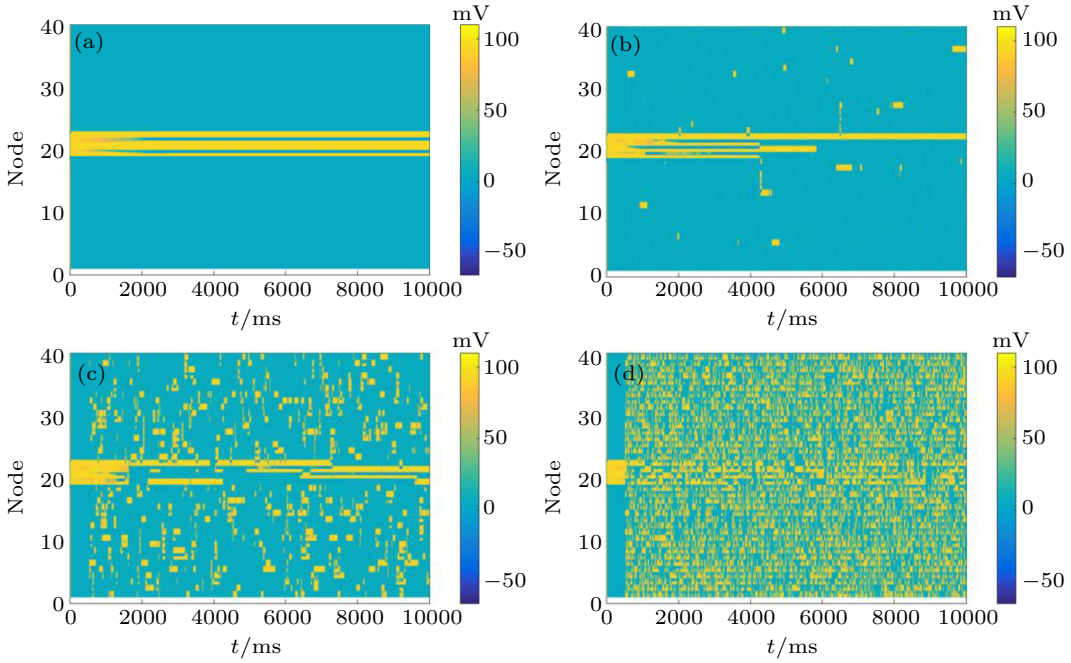


Fig. 10. (color online) Spatiotemporal patterns of ring coupled neuronal network for different values of noise intensity for fixed external forcing current $I_{\mathrm{ext}} = 10$ μA/cm^2, feedback gain coefficient $k = 0.5$, autaptic conductance $g = 0.1$, delay time $\tau = 10.0$ ms, field coupling intensity $g_0 = 1.0$, noise intensity (a) $D = 0.0$, (b) $D = 0.1$, (c) $D = 0.2$, and (d) $D = 1.0$. Autapses are activated at $t = 120$ ms for only four nodes 19–22 and electromagnetic disturbances are triggered at $t = 500$ ms for all nodes.

In summary, when some neurons in the neuronal network have autaptic structures, the electromagnetic disturbances can act as a bridge or a catalyst to some extent, not only affecting the ability of electromagnetic field coupling between neurons in the network, but also changing the self-feedback effect of neurons with autaptic structures. Furthermore, the stochastic disturbances can help to release the neural electrical signals carried by these neurons, and then excite the resting neurons in the network to produce the intermittent firing activities. Thus, the electromagnetic disturbances can help to achieve the purpose of controlling the signal transmission in the neuronal network.

6. Conclusion

The modulation effects of electromagnetic disturbances on the dynamic behaviors of autapse Hodgkin–Huxley neurons and neuronal networks are studied systematically, which can be converted into induced current by introducing a magnetic flux variable.

First, due to the self-feedback memory ability induced by an autaptic structure and the neuronal intrinsic periodic property, the neuronal excitability can change and different autaptic parameters can induce different discharge modes. In addition, the neuronal discharge activity changes discontinuously, and its dynamic behavior not only presents bifurcation phenomena but also has a periodic characteristic. Second, the discharge activity of the autapse neuron has some selectivity to the noise intensity, and the appropriate noise intensity can induce MMOs. Numerical experiments show that the neuronal discharge modes can be transited after the electromagnetic disturbance is triggered. In particular, under the condition of the appropriate noise intensity, the neuron can produce a more significant intermittent firing mode. Finally, for the ring field-coupled neuronal network with autaptic structures, the electromagnetic disturbances can change the continuous and synchronous discharge state of neurons in the network, and result in a desynchronization phenomenon. With the increase of the noise intensity, the average spiking frequency and the average coefficient of variation of all neurons show the opposite change trends and have the significant extremum points, which indicate that the dynamic behavior of the network has certain dependence on the noise intensity. Especially, when only some neurons in the network have the autaptic structures, the electromagnetic disturbances can have some impact on the ability of the electromagnetic field coupling between neurons and the self-feedback memory function induced by the special structures, which can excite the resting neurons in the network to generate an intermittent firing mode and ultimately ensure the transmission of neural signals between neurons.

The above studies can deepen our understanding of the mechanism of electromagnetic disturbances on the dynamic behaviors of neuronal systems. Furthermore, the studies can also provide a valuable reference for effectively controlling the propagation of neural signals in the network and applying electromagnetic disturbances to treat mental illness.

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