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SPECIAL TOPIC — 80th Anniversary of Northwestern Polytechnical University (NPU)

Effect of the fluctuant acoustic channel on the gain of a linear array in the ocean waveguide*

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The inhomogenous ocean waveguide, which leads the amplitude and phase of the signal arriving at a hydrophone array to fluctuate, is one of the causes that make the array gain deviate from its ideal value. The relationship between the array gain and the fluctuant acoustic channel is studied theoretically. The analytical expression of the array gain is derived via an acoustic channel transfer function on the assumption that the ambient noise field is isotropic. The expression is expanded via the Euler formula to give an insight into the effect of the fluctuant acoustic channel on the array gain. The result demonstrates that the amplitude fluctuation of the acoustic channel transfer functions has a slight effect on the array gain; however, the uniformity of the phase difference between the weighting coefficient and the channel transfer function on all the hydrophones in the array is a major factor that leads the array gain to further deviate from its ideal value. The numerical verification is conducted in the downslope waveguide, in which the gain of a horizontal uniform linear array (HLA) with a wide-aperture operating in the continental slope area is considered. Numerical result is consistent with the theoretical analysis.

Keywords: acoustic channel fluctuation, array gain, phase difference, downslope

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1. Introduction

Array gain is defined as the improvement in the signal-to-noise ratio (SNR) obtained at the array output over that at a single hydrophone.[1,2] It is one of the most important measures of the sonar system performance which is affected by the ocean waveguide. The array gain reaches its ideal value $10\lg M$ (M is the number of the hydrophones in the array) when the signals arrive as plane waves and the noises are uniform and uncorrelated between the receivers. However, the sound waves propagating in the ocean are bound to be affected by acoustic channels. And subject to the topography and acoustic properties of sea-floor, as well as eddies, tides, internal wave, and surface gravity waves, the ocean waveguides are always randomly inhomogeneous, which leads the acoustic channels to suffer from spatiotemporal variation. Besides, in the continental slope area,[3] the ocean bottom is range-dependent, which will result in the more complex acoustic channels as manifested by inhomogenous amplitude and corrugated wavefront. A distorted wavefront or the multipath effect leads the amplitude and phase of the received signals (not the plane waves) to fluctuate. Consequently, the array gain will deviate from its ideal value[4] especially for a wide-aperture uniform linear array, which ultimately fails to achieve the design performance of a sonar system in the practical application.

Earlier researches on the array gain influenced by the amplitude and phase fluctuations of the signals were based on the hypothesis that the fluctuations conform to a statistical regularity. The expression of the array gain for a linear array can be obtained if the phase fluctuation is consistent with the multidimensional normal distribution.[5–7] Kleinberg[8] investigated the effect of the amplitude and phase fluctuations on the array gain when the fluctuations were normally distributed. Using signal correlations to characterize the signal amplitude and phase fluctuations, Cox[9] and Green[10] have derived the expression of the array gain in the uncorrelated ambient noise field when the signal correlations were presented as exponential attenuation and linear attenuation, respectively. However, the amplitude and phase fluctuations and the signal correlation attenuation are affected by the ocean waveguide, which cannot always be described by the regular rules.

With the development of the underwater acoustic field modeling, the acoustic channel can be modeled by different acoustic field theories such as ray-based model, normal-mode model, wavenumber integration method, and parabolic-equation method. It provides a new approach to studying the influence of the acoustic channel fluctuation on the array gain. Recently, the majority of researches of the array

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gain focused on the linear array in the shallow water[11–17] and deep ocean,[18] both of which are the range-independent ocean waveguide, based on the normal-mode model.[19] In the range-independent waveguide, the signal field is characterized by a set of discrete normal modes of propagation, and the array gain can be formulated by normal modes. But in the continental slope area,[3] the acoustic wave propagation will span the different seas. In this case, the ocean waveguide is not horizontally layered, but it is dependent on the range, which leads the modes to be coupled seriously. Hence, the analytical methods of the array gain in the range-independent waveguide based on the normal-mode model are not applicable in the case of the range-dependent waveguide.

In Ref. [3] , we have discussed the gain of a linear array relating to the structure of the acoustic field from the acoustic field correlation and the transmission loss. In the present paper, we will provide a new theoretical method to further analyze the array gain affected by the fluctuant acoustic channel, which can be applied to the range-dependent and range-independent waveguides. Assuming that the ambient noise is isotropic, the expression of the array gain is derived as a function of the acoustic channel transfer functions, and then the expression is expanded via the Euler formula to investigate the relationship between the array gain and the acoustic channel. The rest of this paper is organized as follows. The expression of the array gain considering the acoustic channel transfer function is derived in Section 2. Then, in Section 3, the effect of the acoustic channel fluctuation (amplitude and phase fluctuations) on the array gain is analyzed. Finally, the array gain varying with the receiving position is numerically investigated in the continental downslope waveguide, and the results verify the theoretical analysis in Section 3.

2. Array gain in ocean waveguide

2.1. Definitions

The array gain of an arbitrary array of hydrophones is defined as

$$g_{\mathrm{a}} = \frac{\mathrm{SNR}_{\mathrm{array}}}{\mathrm{SNR}_{\mathrm{hyp}}}, \tag{1}$$

where $\mathrm{SNR}_{\mathrm{array}}$ is the SNR at the array output, and $\mathrm{SNR}_{\mathrm{hyp}}$ is the SNR at a single hydrophone. In a practical ocean waveguide, however, both the signal field and the noise field will vary over range and depth, and the signals and noises at different hydrophones will have different values. For this reason, we take the average of the signal-to-noise ratios at all hydrophones to replace the single hydrophone reference in the definition of Eq. (1), and the array gain is redefined as

$$g_{\mathrm{a}} = \frac{\mathrm{SNR}_{\mathrm{array}}}{\mathrm{SNR}_{\mathrm{average_hyp}}}, \tag{2}$$

where $\mathrm{SNR}_{\mathrm{average_hyp}}$ is the average of the SNRs at all hydrophones.

The effects of the array processing on signal and on noise can be considered separately.[12] The improvement on the signal power obtained by using an array is defined as the array signal gain, and improvement on the noise power as the array noise gain, denoted as $g_{\mathrm{as}} = S_{\mathrm{array}}/S_{\mathrm{average_hyp}}$ and $g_{\mathrm{an}} = N_{\mathrm{array}}/N_{\mathrm{average_hyp}}$, respectively, where S_{array} and $S_{\mathrm{average_hyp}}$ are the signal power at the array output and the average signal power of all hydrophones, N_{array} and $N_{\mathrm{average_hyp}}$ are the noise power at the array output and the average noise power of all hydrophones, respectively. Then the array gain in Eq. (2) can be expressed as

$$\begin{aligned} g_{\mathrm{a}} &= \frac{\mathrm{SNR}_{\mathrm{array}}}{\mathrm{SNR}_{\mathrm{average_hyp}}} = \frac{(S/N)_{\mathrm{array}}}{(S/N)_{\mathrm{average_hyp}}} \\ &= \frac{S_{\mathrm{array}}}{S_{\mathrm{average_hyp}}} \Big/ \frac{N_{\mathrm{array}}}{N_{\mathrm{average_hyp}}} = \frac{g_{\mathrm{as}}}{g_{\mathrm{an}}}. \end{aligned} \tag{3}$$

Convert Eq. (3) into decibels, then we will have

$$G_{\mathrm{A}} = 10\lg\left(\frac{g_{\mathrm{as}}}{g_{\mathrm{an}}}\right) = 10\lg(g_{\mathrm{as}}) - 10\lg(g_{\mathrm{an}}). \tag{4}$$

We can see that the array gain can be described as either the ratio of array signal gain to array noise gain as expressed in Eq. (3), or the difference between them in dB as indicated in Eq. (4). Next, we will study the array gain in the ocean waveguide by investigating the array signal gain and the array noise gain separately.

2.2. Array signal gain and array noise gain

The propagation of sound waves in the ocean can be described by the wave equations. For a short durative pulse, the acoustic channel can be characterized by the impulse response $h(t)$ in the time domain or the transfer function $H(f)$ in the frequency domain. The acoustic channel transfer functions can be calculated by solving the wave equation through utilizing different acoustic field theories[19] for range-dependent and range-independent waveguides.

Assume that the source radiates a continuous-wave (CW) signal with a frequency of f_0. Then the receiving signal in the frequency domain at the i-th hydrophone can be described as

$$X_i(f) = S(f)H_i(f), \tag{5}$$

where $S(f)$ is the source signal in the frequency domain, and $H_i(f)$ is the acoustic channel transfer function between the source and the i-th hydrophone.

According to Parseval's theorem, the signal energy at the i-th hydrophone is

$$\begin{aligned} E_i &= \int_{-\infty}^{\infty} x^2(t)\,\mathrm{d}t = \int_{-\infty}^{\infty} |X_i(f)|^2\,\mathrm{d}f \\ &= \int_{-\infty}^{\infty} |S(f)H_i(f)|^2\,\mathrm{d}f. \end{aligned} \tag{6}$$

For a CW signal, the signal energy is concentrated on the frequency f_0, so

$$E_i \approx |S(f_0)H_i(f_0)|^2, \tag{7}$$

and the signal power $P_i(s)$ at the i-th hydrophone is

$$P_i(s) = \frac{|S(f_0)H_i(f_0)|^2}{T_s}, \tag{8}$$

where T_s is the duration of the signal. And then the average signal power at all hydrophones is

$$\begin{aligned} S_{\text{average_hyp}} &= \frac{1}{M}\sum_{i=1}^{M}\frac{|S(f_0)H_i(f_0)|^2}{T_s} \\ &= \frac{|S(f_0)|^2}{MT_s}\sum_{i=1}^{M}|H_i(f_0)|^2, \end{aligned} \tag{9}$$

and the signal power at the array output is

$$\begin{aligned} S_{\text{array}} &= \left|\sum_{i=1}^{M}\frac{w_iS(f_0)H_i(f_0)}{T_s}\right|^2 \\ &= \frac{|S(f_0)|^2}{T_s}\left|\sum_{i=1}^{M}w_iH_i(f_0)\right|^2, \end{aligned} \tag{10}$$

where w_i is the weighting coefficient of the i-th hydrophone.

Therefore, the array signal gain can be expressed as

$$\begin{aligned} g_{\text{as}} &= \frac{S_{\text{array}}}{S_{\text{average_hyp}}} = \frac{\frac{|S(f_0)|^2}{T_s}\left|\sum_{i=1}^{M}w_iH_i(f_0)\right|^2}{\frac{|S(f_0)|^2}{MT_s}\sum_{i=1}^{M}|H_i(f_0)|^2} \\ &= M\left|\sum_{i=1}^{M}w_iH_i(f_0)\right|^2 \Big/ \sum_{i=1}^{M}|H_i(f_0)|^2. \end{aligned} \tag{11}$$

From Eq. (11), it can be seen that the array signal gain is determined by the number of hydrophones in the array, the acoustic channel transfer functions, and the weighting coefficients. For a given array with a fixed number of hydrophones, the array signal gain depends only on the weighting coefficients and the channel transfer functions.

The array noise gain of the array with an equal inter-element spacing d in an isotropic noise environment has been derived as[3]

$$\begin{aligned} g_{\text{an}} &= \frac{N_{\text{array}}}{N_{\text{average_hyp}}} \\ &= \sum_{i=1}^{M}w_i^2 + 2\sum_{i=1}^{M-1}\sum_{k=i+1}^{M}w_iw_k^*\frac{\sin[2\pi(k-i)d/\lambda]}{2\pi(k-i)d/\lambda}, \end{aligned} \tag{12}$$

where the superscript "$*$" denotes the complex conjugate operation. When d is a multiple of $\lambda/2$ and the weighting coefficients satisfy the normalization condition of $\sum_{i=1}^{M}w_i^2 = M$, equation (12) will be equal to a constant M, the number of hydrophones in the array. In this paper, we shall only focus on this special case.

2.3. Formula of the array gain

Substituting Eqs. (11) and (12) into Eq. (3), we obtain the array gain, with the effect of acoustic channel taken into account, to be

$$\begin{aligned} g_{\text{a}} &= \frac{g_{\text{as}}}{g_{\text{an}}} = \frac{M\left|\sum_{i=1}^{M}H_i(f_0)\right|^2}{\sum_{i=1}^{M}|H_i(f_0)|^2} \\ &\quad\times\left(\sum_{i=1}^{M}w_i^2 + 2\sum_{i=1}^{M-1}\sum_{k=i+1}^{M}w_iw_j^*\frac{\sin[2\pi(i-k)d/\lambda]}{2\pi(i-k)d/\lambda}\right)^{-1}. \end{aligned} \tag{13}$$

For the case of $d = \lambda/2$, we have

$$g_{\text{a}} = \left|\sum_{i=1}^{M}w_iH_i(f_0)\right|^2 \Big/ \sum_{i=1}^{M}|H_i(f_0)|^2, \tag{14}$$

which will reaches the ideal value when the amplitudes of the channel transfer functions are uniform and w_i is matched with H_i by compensating for the phase difference for all the hydrophones in the array. However, due to the variations in the acoustic channels, the amplitudes and phases of the channel transfer functions are unpredictable. And the mismatches between the weighting coefficients of a beamformer and the channel transfer functions often happen in the ocean waveguide, which in turn leads the array gain to degrade.

3. Effect of acoustic channel on array gain

To investigate the relationship between the array gain and the fluctuant acoustic channels, we expand Eq. (14) by using Euler formula to obtain an insight into the array gain affected by the amplitude and phase fluctuations of the acoustic channels.

The acoustic channel transfer function and the weighting coefficient corresponding to the i-th hydrophone can be respectively expressed by complex numbers as

$$H_i(f_0) = A_i e^{-j\psi_i}, \quad w_i = e^{j\theta_i}, \tag{15}$$

where A_i and ψ_i are the amplitude and the phase of $H_i(f_0)$, and θ_i is the phase of the weighting coefficient. The amplitudes of the weighting coefficients are set to be 1 throughout the paper, i.e., a rectangle window is assumed.

Substituting Eq. (15) into Eq. (14) yields

$$\begin{aligned} g_{\text{a}} &= \frac{\left|\sum_{i=1}^{M}w_iH_i(f_0)\right|^2}{\sum_{i=1}^{M}|H_i(f_0)|^2} = \frac{\left|\sum_{i=1}^{M}A_ie^{-j\psi_i}e^{j\theta_i}\right|^2}{\sum_{i=1}^{M}|A_i|^2} \\ &= \left|\sum_{i=1}^{M}A_ie^{j\Delta_i}\right|^2 \Big/ \sum_{i=1}^{M}|A_i|^2, \end{aligned} \tag{16}$$

where $\Delta_i = \theta_i - \psi_i$ is the phase difference between the channel transfer function and the weighting coefficient of the i-th hydrophone. From Euler formula, we have

$$e^{j\Delta_i} = \cos(\Delta_i) + j\sin(\Delta_i). \tag{17}$$

Substituting Eq. (17) into Eq. (16) yields

$$\begin{aligned} g_a &= \frac{\left|\sum_{i=1}^{M}[A_i\cos(\Delta_i) + jA_i\sin(\Delta_i)]\right|^2}{\sum_{i=1}^{M}|A_i|^2} \\ &= \frac{\left|\sum_{i=1}^{M}A_i\cos(\Delta_i)\right|^2 + \left|\sum_{i=1}^{M}A_i\sin(\Delta_i)\right|^2}{\sum_{i=1}^{M}|A_i|^2}. \end{aligned} \tag{18}$$

After some manipulations, we obtain the expression of the array gain as follows:

$$\begin{aligned} g_a &= \frac{\sum_{i=1}^{M}\sum_{k=1}^{M}A_i\cos(\Delta_i)A_k\cos(\Delta_k) + \sum_{i=1}^{M}\sum_{k=1}^{M}A_i\sin(\Delta_i)A_k\sin(\Delta_k)}{\sum_{i=1}^{M}|A_i|^2} \\ &= \sum_{i=1}^{M}\sum_{k=1}^{M}A_iA_k\cos(\Delta_i - \Delta_k)\Big/\sum_{i=1}^{M}|A_i|^2. \end{aligned} \tag{19}$$

Comparing with Eq. (11), it is further noted that the array gain is determined by the amplitudes of the channel transfer functions and the phase differences between the channel transfer functions and the weighting coefficients at all hydrophones. We now discuss the array gain given in Eq. (19) in more detail.

1) When the phase differences on all the hydrophones in the array are uniform, i.e., Δ_i and Δ_k satisfy the relationship $\Delta_i - \Delta_k = 2n\pi$, where n is an arbitrary integer, equation (19) can be simplified into

$$g_a = \left|\sum_{i=1}^{M}A_i\right|^2\Big/\sum_{i=1}^{M}|A_i|^2. \tag{20}$$

In this case, the array gain only relates to the amplitudes of the channel transfer functions corresponding to each hydrophone, which means that the array gain is closely related to the fluctuation in transmission loss. According to the Cauchy–Schwarz inequality

$$\left|\sum_{i=1}^{M}A_i\right|^2 \le M, \tag{21}$$

one observes that the array gain can reach the ideal value when the amplitudes of all the channel transfer functions are identical, such as those of a plane wave propagating in a free space. However, the array gain will be smaller than the ideal value for the acoustic channels with fluctuant amplitudes which always holds true in the ocean waveguide.

2) When the phase differences are nonuniform, in which case $\Delta_i - \Delta_k \neq 2n\pi$, one can obtain

$$g_a = \frac{\sum_{i=1}^{M}\sum_{k=1}^{M}A_iA_k\cos(\Delta_i - \Delta_k)}{\sum_{i=1}^{M}|A_i|^2} < \left|\sum_{i=1}^{M}A_i\right|^2\Big/\sum_{i=1}^{M}|A_i|^2. \tag{22}$$

It is noted that the non-uniformity of phase differences leads the array gain to further deviate from the ideal value.

Generally, for an array in the ocean waveguide, the amplitude fluctuation of the acoustic channel transfer function is relatively small within the scope of the whole array. Therefore, the array gain will deviate from the ideal value slightly if it is only affected by the amplitude fluctuation. However, the array gain suffers more the phase fluctuation of the channel transfer function. Especially, if the differences between two arbitrary Δ_s are in the second quadrant or the third quadrant, i.e., $0.5\pi + 2n\pi < \Delta_i - \Delta_k < \pi + 2n\pi$ or $-\pi + 2n\pi < \Delta_i - \Delta_k < -0.5\pi + 2n\pi$, in the expansion of the summation formula in the numerator of Eq. (19), there will appear a negative cross term. And the array gain will deviate from the ideal value seriously because of the above-mentioned reasons. Hence, the nonuniformity of the phase differences between the weighting coefficients and between the channel transfer functions on all the hydrophones of the array is a major factor that will lead the array gain to seriously deviate from the ideal value in the ocean waveguide. The effects of amplitude fluctuation and phase fluctuation on the array gain will be verified in the next section with numerical simulation.

4. Numerical simulation and verification

Assuming that the ambient noise field is isotropic, we focus on investigating the relationship between the array gain and the acoustic channel transfer function in a continental downslope waveguide and also on verifying the theoretical analysis in Section 3. The acoustic channel transfer function can be obtained numerically by using RAM that is a code based on the split-step Padé algorithm[20] for the wide-angle parabolic equation,[20–23] which has proved to be an exact model for the range-dependent waveguide. The array gains of a horizontal uniform linear array (HLA) varying with the receiving depth and the source–receiver range are discussed, and the corresponding amplitudes and phases of the acoustic channel transfer functions are analyzed comparatively to verify the theoretical analysis.

4.1. Downslope waveguide

Numerical simulations are conducted in a downslope waveguide, and the results are shown in Fig. 1 and can be di-

vided into three different regions: the shallow water, a continental slope area, and an abyssal plain. The first 20 km is for the shallow water with a depth of 229 m. We assume that the oblique angle of the slope is 3.5°, the continental slope covers a range from 20 km to 98 km with the water depth varying from 229 m to 5000 m. A 2 km distance in the abyssal plain is considered with a water depth of 5000 m, making the whole distance in the three regions be 100 km in simulations. The bottom absorption coefficient for each of all three regions is assumed to be the same of 0.5 dB/λ. The source is at a fixed depth of 110 m in the shallow water, radiating a CW signal of 190 Hz. An HLA with 100 hydrophones ($M = 100$) is suspended in the slope region with varying depth and distance to the source. The spacing between adjacent hydrophones in the array is 4 m, approximately equal to half the wavelength corresponding to 190 Hz.

The sound-speed profile (SSP) of the shallow water has a negative gradient as shown by the blue solid line in Fig. 2(a), while the SSP of the deep ocean is the Munk profile with a deep sound channel axial depth of 1300 m as shown by the red dotted line in Fig. 2(a). The SSPs in the slope area are generated by interpolating every 10 km utilizing RAM. Figure 2(b) shows the transmission losses calculated by RAM in the downslope waveguide with the given parameters. One observes a downslope enhancement near the deep sound channel, resulting from that acoustic energy ducted in the sound channel is not subjected to the losses associated with repeated bottom interactions.

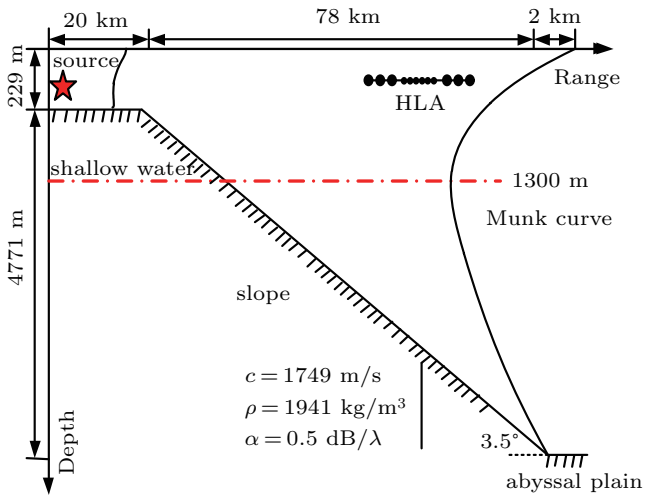


Fig. 1. (color online) Environment and parameters used in the simulation.

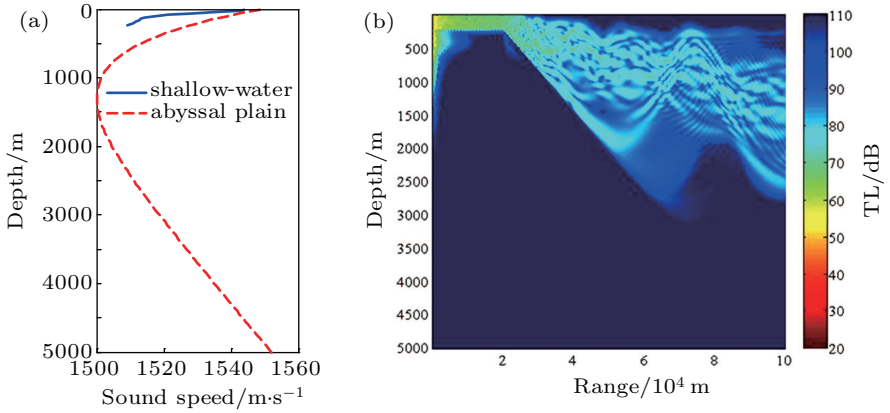


Fig. 2. (color online) Downslope acoustic propagation, showing (a) sound speed profiles of the shallow water (blue solid line) and abyssal plain (red dotted line), and (b) transmission losses, when the source frequency is 190 Hz and depth is 110 m.

4.2. Array gains of HLA

In the simulation, the depth of the array is changed in a range of 10–1300 m and the array gain is calculated every 20 m; the source–receiver range is changed from 20 km to 90 km and the array gain is calculated every 200 m.

Firstly, we consider an ideal case that the weighted phases can compensate for the phase differences for all the hydrophones, and investigate the effect of amplitudes fluctuation on the array gain. Assuming that the phases of the acoustic channel transfer functions are known, we take the conjugate phases of the channel transfer functions as the phases of the weighting coefficients, i.e., $\theta_i = \psi_i$ and $w_i = \mathrm{e}^{\mathrm{j}\psi_i}$, in which case the array gain can be expressed as Eq. (20). Then the array gains are plotted in Fig. 3 as a function of the depth and the source–receiver range. The brown shaded portion is the slope-bottom.

As shown in Fig. 3, the array gains in the whole region of interest are greater than 19 dB, which are very close to the ideal value. It indicates that the array gain is slightly affected by the amplitude fluctuation of the channel transfer function, which is consistent with the theoretical analysis result in Section 3.

Next, consider a general case. We take conventional beamformer (CBF) for example to study the influence of the fluctuant acoustic channel on the array gain. The weighting

vector of CBF is

$$\boldsymbol{w}_{\mathrm{CBF}} = [1, \mathrm{e}^{\mathrm{j}k_0\cos(\phi_0)d}, \ldots, \mathrm{e}^{\mathrm{j}k_0\cos(\phi_0)(M-1)d}]^{\mathrm{T}}, \quad (23)$$

where ϕ_0 is the source bearing, and k_0 is the wavenumber.

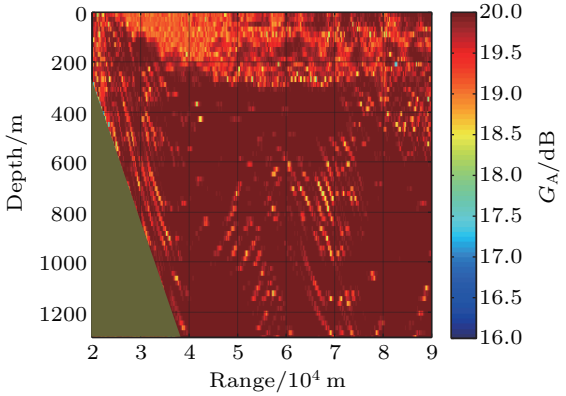


Fig. 3. (color online) Array gains affected only by the amplitude fluctuation of the channel transfer function in the downslope waveguide, with the source frequency being 190 Hz and source depth being 110 m.

The simulation parameters are the same as the above, the array gains of CBF are plotted in Fig. 4. It is noted that there is a region in the slope area where the array gains seriously deviate from the ideal value, indicated by a white rectangle in Fig. 4. One observes that the region covers roughly the receiving depth 0–17 m and the source–receiver range 28–50 km.

Comparing Fig. 3 with Fig. 4, it is noted that the array gains deviate from the ideal value more seriously, especially in the white rectangle region. It indicates that the array gain will further deviate from the ideal value, if the phases are mismatched between the CBF weighting coefficients and the acoustic channel transfer functions.

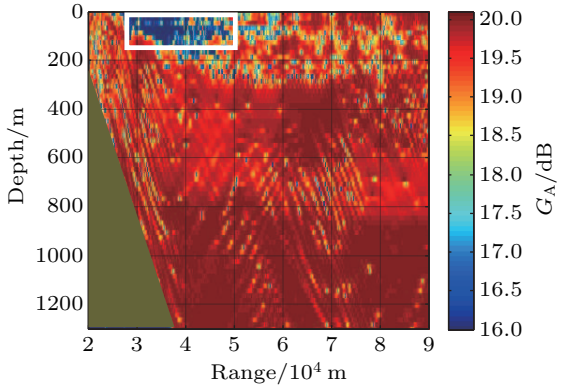


Fig. 4. (color online) Array gains of CBF varying with receiving position in the downslope waveguide, with the source frequency being 190 Hz and source depth being 110 m.

For further analyses, we choose the array gains respectively at the receiving-depth of 100 m (in the white rectangle) and 300 m (out of the white rectangle) to investigate the effect of the fluctuant acoustic channel on the array gain. We present the array gain as a function of source–receiver range changing from 25 km to 90 km. Figure 5 shows the array gains of CBF at two depths. It is noted that the array gains at a depth of 100 m are more fluctuant and much less than those at the depth of 300 m. In addition, the array gains in a range of 34–50 km seriously deviate from the ideal value at the depth of 100 m; however, the phenomenon disappears at the depth of 300 m.

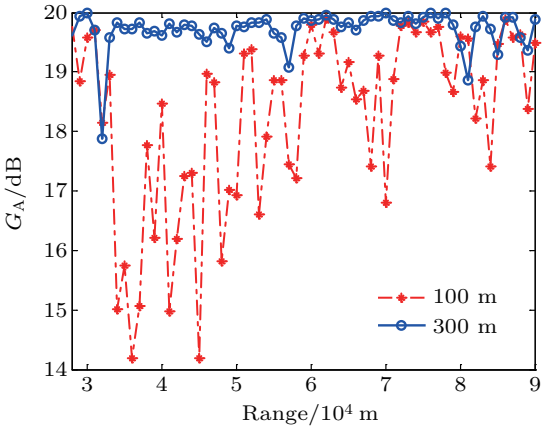


Fig. 5. (color online) Gains of CBF varying with the source–receiver range at depths of 100 m and 300 m, with the source frequency being 190 Hz and depth being 110 m.

4.2.1. At receiving-depth 100 m

We choose the array gains when the source–receiver ranges are 36 km (in the white rectangle) and 54 km (out of the white rectangle) to analyze the effects of the fluctuant acoustic channel on the array gain. Figure 6 shows the amplitudes

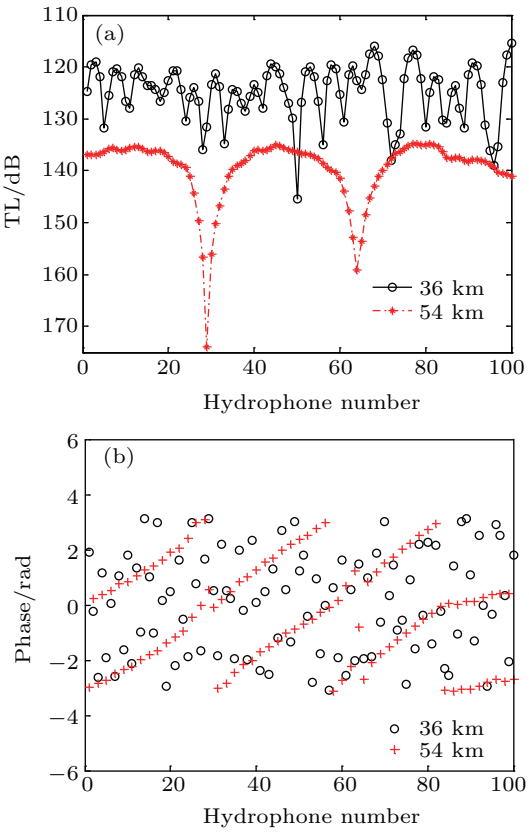


Fig. 6. (color online) (a) Amplitude and (b) phase fluctuations of acoustic channels on the HLA, with the receiving depth being 10 m and source–receiver ranges being 36 km and 54 km.

and phases of the acoustic channel transfer functions on each hydrophone. The amplitude of the acoustic channel transfer function is converted into the transmission loss.[19] One observes that the amplitudes of the acoustic channels at these two source–receiver ranges are fluctuated with the hydrophone number seriously; the phase fluctuation of the acoustic channels is larger when the source–receiver range is 36 km than that when the source–receiver range is 54 km.

The phase differences (Δ_s) between the weighting coefficients of CBF and the channel transfer functions are correspondingly shown in Fig. 7. It is noted that the uniformity of Δ shown in Fig. 7(a) is higher than that in Fig. 7(b), but the difference of two arbitrary Δ_s is not equal to 2π. The corresponding CBF gains are 14.2 dB and 17.9 dB as shown in Fig. 5. It indicates that the array gain of CBF will deviate from the ideal value seriously if Δ is not uniform; and the array gain will be higher if Δ is more uniform.

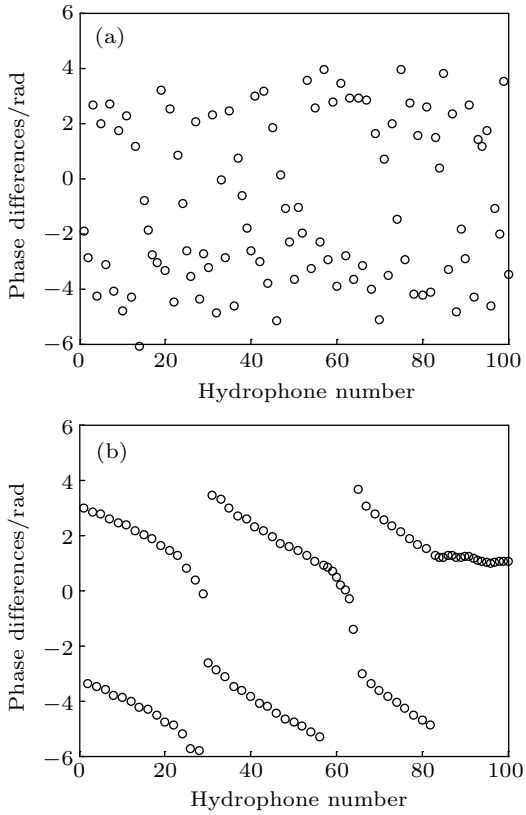


Fig. 7. Phase differences on hydrophones, with the receiving depth being 100 m and ranges being (a) 36 km and (b) 54 km.

4.2.2. At receiving-depth 300 m

We also analyze the fluctuant acoustic channel and array gain when the source–receiver ranges are 36 km and 54 km (the same as the above values). The amplitudes and phases of the acoustic channel transfer functions on each hydrophone are plotted in Figs. 8(a) and 8(b), respectively. It is noted that the amplitude fluctuation of the acoustic channel at a range of 36 km is larger than at a range of 54 km. The phase fluctuations of the acoustic channel at ranges of 36 km and 54 km are similar. Then, the results in the case of Δ_s varying with the hydrophone number are plotted in Fig. 9. One observes that the phase differences in Figs. 9(a) and 9(b) are uniform, and the difference between two arbitrary Δ_s approximates to 0 or $\pm 2\pi$.

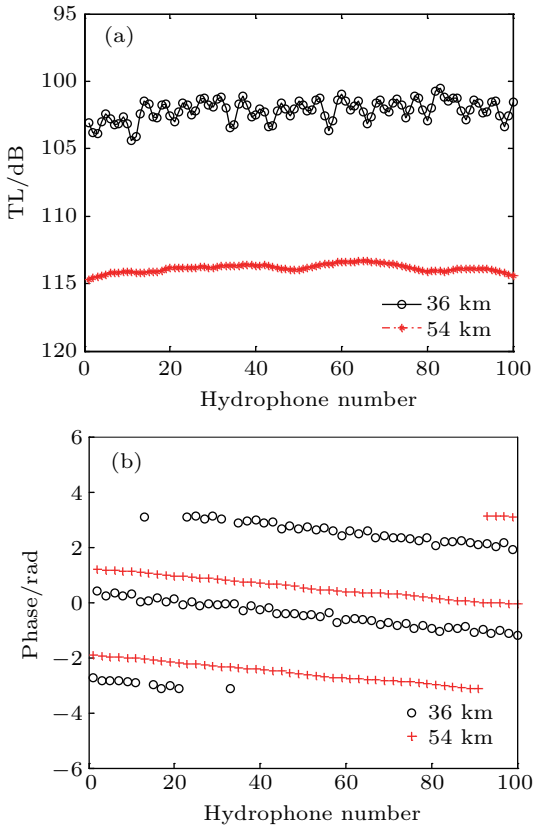


Fig. 8. (color online) (a) Amplitude and (b) phase fluctuations of acoustic channels on HLA, with the receiving depth being 30 m and source–receiver ranges being 36 km and 54 km.

The corresponding array gains of CBF are 19.7 dB and 19.9 dB, which are very close to the ideal value as shown in Fig. 5. It results from the fact that the phase differences between the weighting coefficients of CBF and the acoustic channel transfer functions are uniform as shown in Fig. 9. In this case, the array gains can be expressed as Eq. (20), which means that only the amplitude fluctuation of the channel transfer function affects the array gain.

In addition, despite the amplitude fluctuation on the range of 36 km being larger than that on the range of 54 km, the corresponding array gain has a value almost the same as that resulting from the case of uniform Δ_s. It also verifies that the phase fluctuation of the acoustic channel transfer function has a main influence on the array gain.

From the analyses in this section, it concludes that the amplitude fluctuation has a slight influence on the array gain, but the uniformity of Δ_s is a major factor that leads the array gain to deviate from the ideal value. The uniform enhancement of Δ_s will increase the array gain. Furthermore, the array gain will be close to the ideal value if the difference between

two arbitrary Δ_s approximates to $2n\pi$, in which case only the amplitude fluctuation reacts on the array gain. The numerical results verify the theoretical analyses in Section 3.

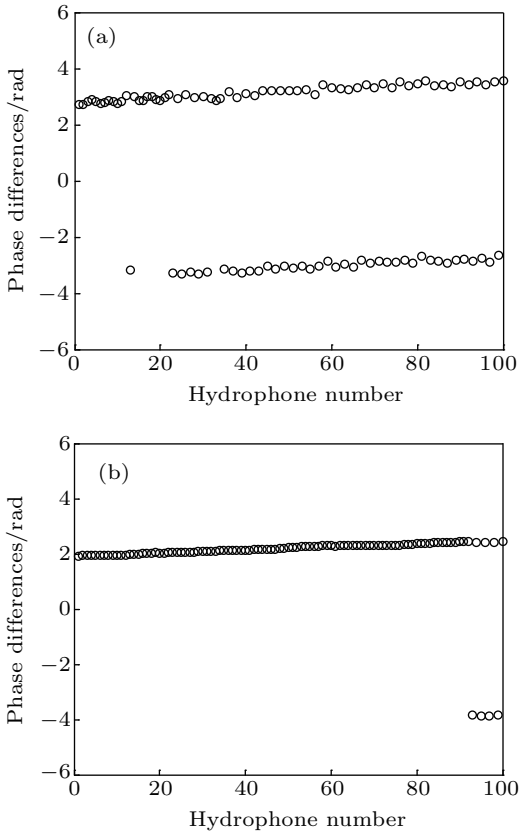


Fig. 9. Phase differences on hydrophones, with the receiving depth being 300 m and ranges being (a) 36 km and (b) 54 km.

5. Conclusions

The array gain that deviates from the ideal value in an ocean waveguide is investigated from the fluctuant acoustic channel. The analytical expression of array gain considering the acoustic channel is derived on the assumption that the ambient noise field is isotropic. The discussion provides an insight into the physics of the problem as well as enabling quantitative estimates of the array performance in the ocean waveguide.

The fluctuant acoustic channel can be considered by the amplitude fluctuation and the phase fluctuation of the channel transfer function, then we derive the expressions for the array gain as given by Eqs. (14) and (19) which can be applied to the range-independent and range-dependent waveguides, respectively. We conclude that the amplitude fluctuation has a slight effect on the array gain; however, the non-uniformity of Δ can lead the array gain to further deviate from the ideal value.

To verify the theoretical analysis, the gains at the depths of 100 m and 300 m in the downslope waveguide are chosen to contrastively analyze the effect of acoustic channel fluctuation on the array gain. Both the theoretical and numerical results show that the uniformity of Δ has a main influence on the array gain. Furthermore, the array gain will be close to the ideal value if the difference between two arbitrary Δ_s is approximately $2n\pi$.

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