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Generation of breathing solitons in the propagation and interactions of Airy–Gaussian beams in a cubic–quintic nonlinear medium*

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Using the split-step Fourier transform method, we numerically investigate the generation of breathing solitons in the propagation and interactions of Airy–Gaussian (AiG) beams in a cubic–quintic nonlinear medium in one transverse dimension. We show that the propagation of single AiG beams can generate stable breathing solitons that do not accelerate within a certain initial power range. The propagation direction of these breathing solitons can be controlled by introducing a launch angle to the incident AiG beams. When two AiG beams accelerated in opposite directions interact with each other, different breathing solitons and soliton pairs are observed by adjusting the phase shift, the beam interval, the amplitudes, and the light field distribution of the initial AiG beams.

Keywords: cubic–quintic nonlinear medium, Airy–Gaussian beams, propagation and interactions, breathing solitons

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1. Introduction

Self-accelerating Airy beams have become the focus of many theoretical and experimental investigations[1,2] due to their possible applications in particle clearing,[3] light bullets,[4] curved plasma channel generation,[5] optical routing,[6] optical interconnects,[7] image signal transmission,[8] vacuum electron acceleration,[9] abruptly autofocusing beams,[10] and so on. In the last decade, besides their use in linear media, such beams have also been extensively researched in many nonlinear materials, such as Kerr nonlinear dielectrics,[11,12] photorefractive media,[13–16] nonlocal nonlinear media,[17] Bose–Einstein condensates,[18] metal surfaces,[19] quadratic media,[20] highly non-instantaneous cubic media,[21] and photonic crystals.[22] In particular, solitons can be generated from Airy beams because of the existence of nonlinearity,[11–15] which propagate without transverse acceleration and perform a breathing state. The interactions between Airy beams can also result in the generation of single solitons or soliton pairs.[17,23–26] More recently, such solitons have been observed experimentally in photorefractive media.[13,15]

As a generalized form of Airy beams, Airy–Gaussian (AiG) beams provide a more realistic approach to the description of Airy beams, which carry finite power and retain non-diffracting properties within a finite propagation distance, and can achieve a very good approximation in experiment.[27] So far, many researchers have theoretically and experimentally studied the propagation and interactions of AiG beams.[28–37] For example, Deng *et al.* investigated the propagation and properties of AiG beams in a Kerr medium,[28] quadratic-index medium,[29] strongly nonlocal nonlinear media,[30] and uniaxial crystals.[31] They also discussed the interactions of AiG beams in Kerr and saturable media.[32,33] Furthermore, Zhang reported the bound states of breathing AiG beams in a nonlocal nonlinear medium.[34] Shi *et al.*[35] discussed the interactions of AiG beams in photonic lattices with defects. Gao *et al.*[36] analyzed the radiation force functions of two AiG beams on a cylindrical particle. We also studied in detail the propagation and interactions of AiG beams in saturable nonlinear media in a recent article.[37]

However, due to strong focusing nonlinearity, unstable soliton propagation may occur in cubic media for intense excitation at the input plane.[12] So, how can we obtain stable solitons during the propagation of AiG beams? How can we control the propagation of these breathing solitons? What factors can affect the propagation and interactions of AiG beams? These questions are very interesting.

In this paper, we introduce a higher-order nonlinearity, such as the defocusing quintic type, which can suppress the strong cubic nonlinearity so as to obtain stable breathing solitons. In one transverse dimension, we first numerically study the propagation of single AiG beams in a cubic–quintic nonlinear medium, and then the stability and the propagation control of the solitons produced by the AiG beams. In addition, we briefly investigate the interactions of two AiG beams by adjusting their initial amplitudes, phase shift, beam interval, and the initial light distribution factor.

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2. Theoretical model and basic equations

Under paraxial approximation, we consider the generalized nonlinear Schrödinger equation in the normalized one transverse dimension form[38]

$$\mathrm{i}\frac{\partial q}{\partial \xi}+\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2}+F\left(|q|^2\right)q=0, \tag{1}$$

where q is the complex amplitude of the electric field, $\eta = x/x_0$ and $\xi = z/(kx_0^2)$ are the dimensionless transverse coordinate and the propagation distance scaled by some characteristic transverse width and the corresponding Rayleigh range, respectively. The real function $F(|q|^2)$ accounts for the nonlinear change in the index of refraction. Here we take the type of nonlinearity to be in the normalized form $F(|q|^2) = 3|q|^2+\varepsilon|q|^4$, so that the quintic term describes a correction to the Kerr law, namely enhanced ($\varepsilon=+1$, focusing quintic) or competing ($\varepsilon=-1$, defocusing quintic) nonlinearity. To investigate the propagation dynamics of AiG beams, we address the case of a truncated AiG beam at the entrance plane $\xi=0$ of the medium. Such a beam occurs naturally if a broad Gaussian beam is propagated by imposing a cubic phase modulation. So we consider the input field distribution to be

$$q(\eta,\xi=0)=A_0\mathrm{Airy}(\eta)\exp(a\eta)\exp\left(-g\eta^2\right), \tag{2}$$

where A_0 denotes the initial amplitude controlling the beam power, Airy$(\cdot)$ represents the Airy function, and a is a small positive parameter named the 'truncation coefficient' which ensures that the optical beam carries finite energy. For simplicity, we take $a=0.2$ throughout this paper. The parameter g is an initial light field distribution factor adjusting the beam that will tend to the Gaussian beam with a larger value of g and the Airy beam with a smaller value, as illustrated in Fig. 1(a). Evidently, the AiG main lobe intensity profiles look quite similar to the fundamental soliton, and a considerable part of the energy of the AiG beam is located in the main lobe. Figure 1(b) shows the evolution of the AiG beam in free space when $g=0.01$. It can be seen that such a beam carries finite power and retains the non-diffracting property within a finite propagation distance.

We have shown the single beam solutions in Eq. (2). In fact, to investigate the interactions of AiG beams, we should construct more complex incident beams, composed of two shifted counter-propagating single AiG beams with a relative phase shift and different amplitudes,[23–25,35,37]

$$\begin{aligned} &q(\eta,\xi=0)\\ &=A_1\mathrm{Airy}(\eta-B)\exp[a(\eta-B)]\exp\left[-g(\eta-B)^2\right]\\ &\quad+A_2\exp(\mathrm{i}\delta\phi)\,\mathrm{Airy}(-\eta-B)\\ &\quad\times\exp[-a(\eta+B)]\exp\left[-g(\eta+B)^2\right], \end{aligned} \tag{3}$$

where A_1 and A_2 are the amplitudes of the two AiG beams respectively, B controls the transverse separations in the η direction, and $\delta\phi$ is the parameter that controls the phase shift with $\delta\phi=0$ and $\delta\phi=\pi$ describing in-phase and out-of-phase AiG beams respectively.

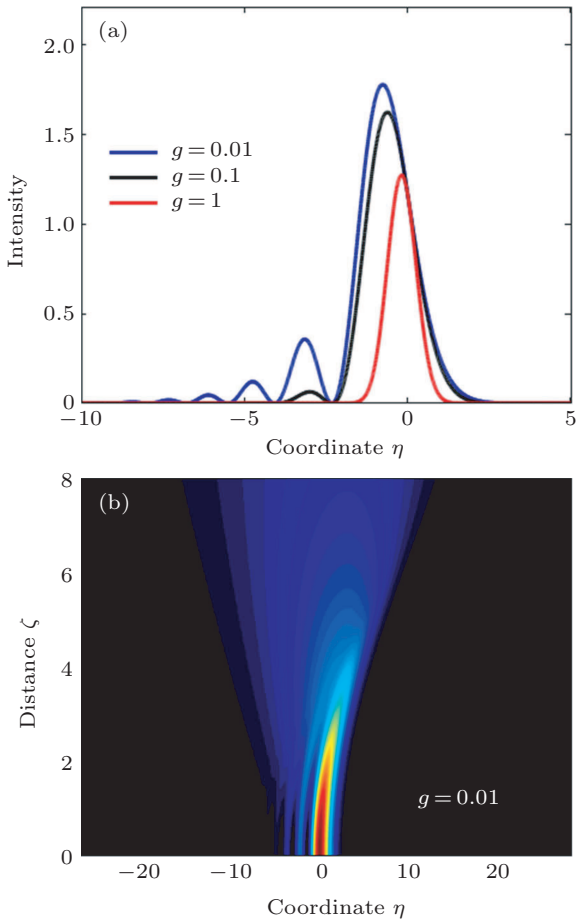


Fig. 1. (color online) (a) Intensity distributions of AiG beams with $g=0.01$, 0.1, and 1. (b) Propagation of AiG beam in free space for $g=0.01$.

3. Propagation of a single AiG beam

To investigate the propagation of a single AiG beam in a cubic–quintic nonlinear medium, we have implemented the well-known split-step Fourier transform methods to solve Eq. (1) by varying the initial amplitude A_0 and the field distribution factor g. The corresponding results are shown in Fig. 2. Firstly, we consider a typical case of pure Kerr nonlinearity with $\varepsilon=0$ and $g=0.01$. For a small initial amplitude A_0 (low power), the AiG beam accelerates in the transverse direction and eventually succumbs to diffraction in Fig. 2(a1). With an increase of A_0 in Fig. 2(a2), it is shown that a single breathing soliton can emerge from the centered energy around the AiG main lobe, and it moves along a straight line. Actually, this phenomenon has been reported in Refs. [12] and [17]. Unfortunately, when A_0 continues to increase, it is clear that the soliton width quickly narrows in Fig. 2(a3), and the soliton can even fail to form for intense excitation at the input plane in Fig. 2(a4) due to the strong focusing nonlinearity, which means that the formation of a breathing soliton should also impose an upper threshold on the initial amplitude.[39]

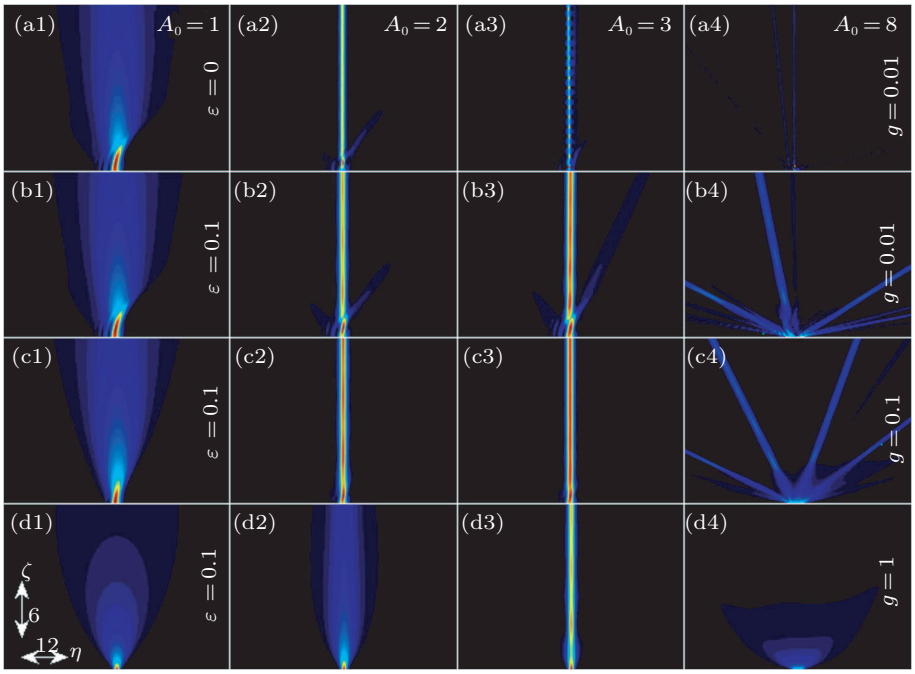


Fig. 2. (color online) Soliton generation in the propagation of a single AiG beam in a cubic–quintic nonlinear medium with changing values of A_0, g, and ε.

In this paper, our focus is on the case of $\varepsilon = -1$ where the defocusing quintic nonlinearity will weaken the focusing cubic nonlinearity, so as to strengthen the stability of the breathing soliton. As seen in Figs. 2(b1)–2(b4), although a stable breathing soliton always requires a larger initial amplitude and no soliton exists for sufficiently high initial amplitude, a notable difference is that the soliton width can remain almost invariant because of the suppression of quintic nonlinearity. It is worth mentioning that the soliton intensity will be weakened for higher values of g in Figs. 2(c1)–2(c4). When g is large enough, a stable soliton needs a larger initial amplitude (see Figs. 2(d1)–2(d4)). That is to say, the formation of a soliton is easier when the AiG beam is close to the Airy distribution than when it is close to the Gaussian distribution.

We have examined the relations between the soliton oscillations and the incident AiG beam amplitude. One can see in Fig. 3(a) that the soliton breathing period decreases monotonically with the increase of A_0. Here, the soliton breathing period is defined as the distance between adjacent peaks or troughs of the oscillating soliton envelope. In Fig. 3(b), the oscillations of the soliton maximum intensity are shown as functions of the propagation distance for selected values of A_0 corresponding to the red circles in Fig. 3(a). As can be seen, the soliton exhibits damped oscillations in the soliton maximum intensity. The maximum intensity increases with increasing initial amplitude. Similar behavior in Kerr or saturable nonlinear media has also been reported in Refs. [12] and [37].

To check the stability properties of these breathing solitons, we simulate their evolution under some random perturbations. As an example, figures 4(a1)—4(c1) depict the initial AiG intensities perturbed by 20% random noise perturbations for $g = 0.01$, 0.1, and 1, and the corresponding evolution simulation results are shown in Figs. 4(a2)–4(c2). It is evident that the solitons propagate robustly against perturbations (as far as 100 Rayleigh range), and hence they are certainly stable.

In order to control the propagation direction of the breathing soliton, we introduce a launch angle to the incident AiG

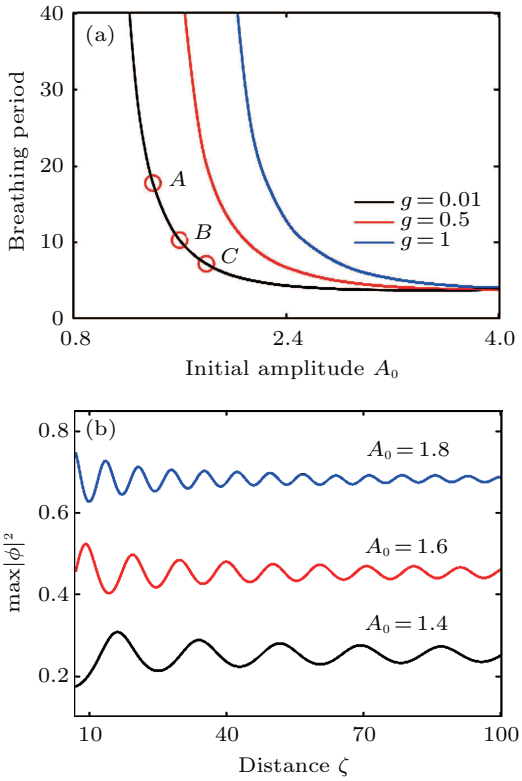


Fig. 3. (color online) (a) Breathing period of the soliton as a function of the initial amplitude A_0 for selected light field distribution factor $g = 0.01$, 0.5, and 1. (b) Oscillations of the soliton maximum intensity with three different initial amplitudes A_0 for $g = 0.01$.

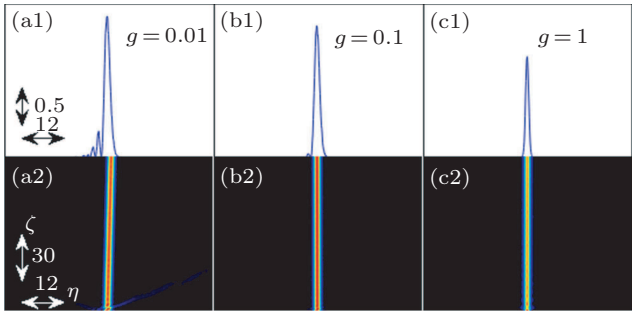


Fig. 4. (color online) Intensity evolution of AiG beams under 20% random noise perturbations with $A_0 = 3$ for selected $g = 0.01$, 0.1, and 1. (a1)—(c1) The input AiG beam intensities. (a2)—(c2) The evolution with perturbation corresponding to panels (a1)—(c1), respectively.

beam. For computer modeling of this process, we solved Eq. (1) with the initial condition that[40,41]

$$q(\eta, \xi = 0) = A_0 \mathrm{Airy}(\eta) \exp(a\eta) \exp\left(-g\eta^2\right) \exp(\mathrm{i}v\eta), \quad (4)$$

where v is associated with the initial launch angle θ of the AiG beam, $\theta = v/(kx_0)$, which can be controlled by varying the transverse displacement of the imaging lens with respect to the axis of the system in experiment.[40]

In Figs. 5(a1)–5(d1), the dynamics of an AiG beam with different values of v is presented for given values of $A_0 = 2$ and $g = 0.01$. We plot the evolution of the AiG beam intensity in order to see the influence of the launch angle on the propagation of the single breathing soliton. The propagation direction of the soliton will tilt to the left when $v < 0$ and to the right when $v > 0$. The bigger the value of $|v|$, the greater the tilt angle of the soliton is. Therefore, the propagation direction of the breathing soliton can be adjusted by controlling the launch angle of the AiG beam. We also noticed that the change of launch angle has no effect on the intensity and width of the soliton. Figures 5(a2)–5(d2) show the soliton intensity profiles when they are propagated at $\xi = 10$ (corresponding to the position of the red dashed line in Figs. 5(a1)–5(d1)). As a result, except for the transverse position, the intensity and width of the soliton remain unchanged with the change of the launch angle.

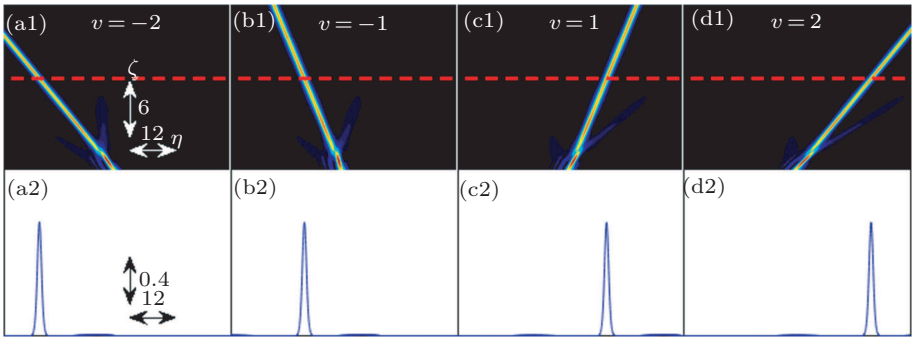


Fig. 5. (color online) Intensity evolution of an AiG beam with different launch angles with $A_0 = 2$, $g = 0.01$, $v =$ (a1) -2, (b1) -1, (c1) 1, and (d1) 2. (a2)–(d2) The intensity of the soliton at propagation distance $\xi = 10$ corresponding to the location of the red dashed line in panels (a1)–(d1), respectively.

4. Interactions of AiG beams

In this section, we will focus on interactions of AiG beams in a cubic–quintic nonlinear medium. The split-step Fourier method is used to solve Eq. (1) with competing nonlinearity under the initial condition of Eq. (3).

Firstly, when $A_1 = A_2 = 1.4$ and $g = 0.01$, numerical simulation results with different phase shifts $\delta\phi$ and beam intervals B (relatively small) are depicted in Fig. 6. Obviously, the in-phase AiG beams ($\delta\phi = 0$ in Figs. 6(a1)–6(e1)) attract each other and single solitons can form, while the out-of-phase AiG beams ($\delta\varphi = \pi$ in Figs. 6(a3)–6(e3)) repel each other and soliton pairs can form. Similar behavior has been reported in Refs. [23] and [24]. It is interesting to note that in Figs. 6(a2)–6(e2), when $\delta\phi = \pi/2$, asymmetric soliton pairs are visible for selected values of B. If $B \leq 0$, the intensity of the soliton on the right side is stronger than that on the left side. However, if $B > 0$, the intensity of the soliton on the right side is weaker than that on the left side. With a decreasing interval $|B|$, the intensity of the weak side soliton becomes weaker and almost disappears. A contrary result appears in Figs. 6(a4)–6(e4) with the case of $\delta\phi = 3\pi/2$. Thus, we can control the interaction of AiG beams to obtain different breathing solitons or soliton pairs by adjusting the phase shift and interval between the two AiG beams.

Secondly, as an example, for $B = 3$ in the in-phase case, we consider the interaction of two AiG beams for different A_1 and A_2 in cubic–quintic nonlinear media, as shown in Fig. 7. When $g = 0.01$, the initial light fields of the two AiG beams approach the Airy distribution. After shedding some radiation from the main lobes of the two AiG beams initially, single breathing solitons can form in the center of the two launched AiG beams, and symmetric soliton pairs can also be generated on both sides of the beam center (Figs. 7(a1)–7(a4)). It is important to note that in Fig. 7(a1) a soliton pair cannot form for small initial amplitudes because of the weak nonlinear focusing effect. With increasing initial amplitudes, the focusing nonlinearity becomes stronger so that one soliton pair can form in Fig. 7(a2), and the intensity of the single soliton is

higher than that of the soliton pair. With further increase in the initial amplitudes, we can see in Figs. 7(a3) and 7(a4) that the intensity of the soliton pair gradually becomes larger and the intensity of the single soliton becomes smaller. When $g = 0.1$,

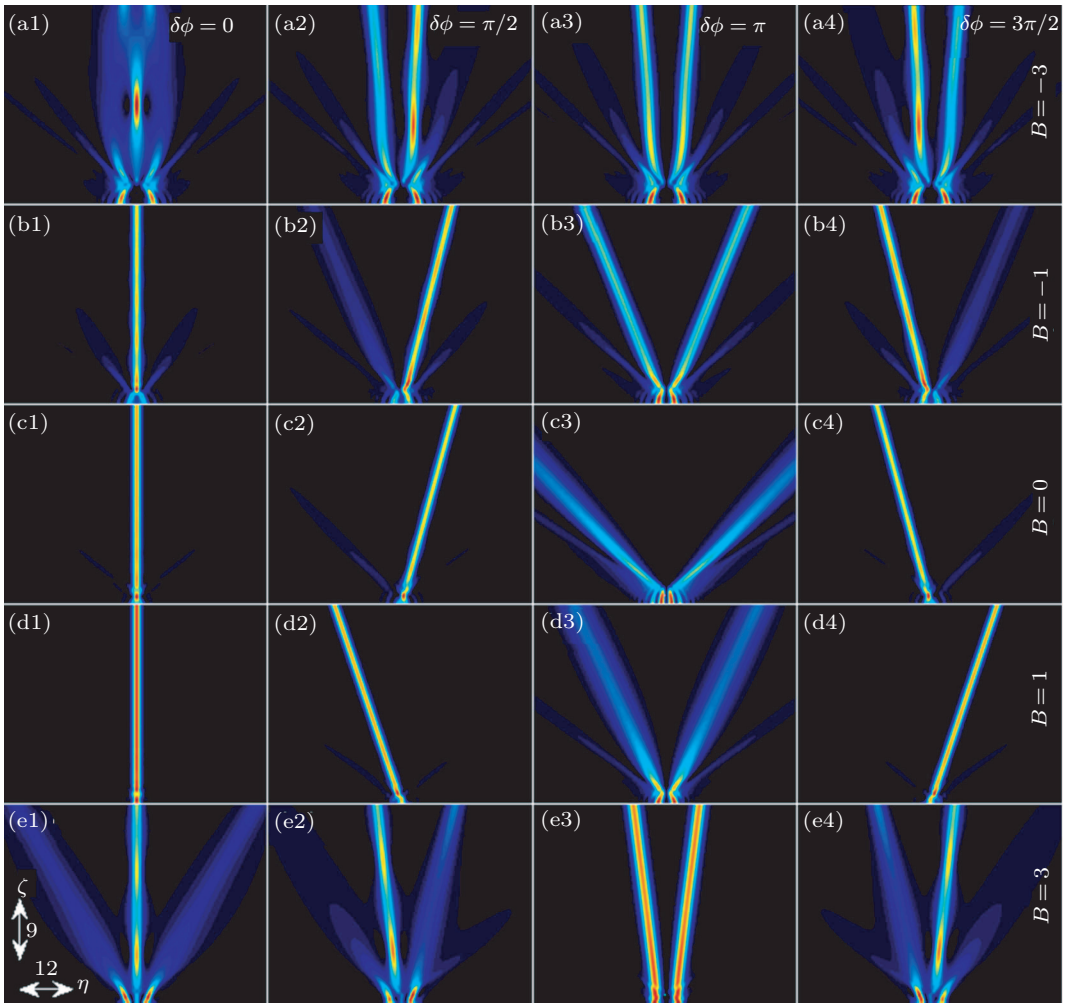


Fig. 6. (color online) The interactions of two AiG beams with different phase shifts and beam intervals. Other parameters are $A_1 = A_2 = 1.4$ and $g = 0.01$.

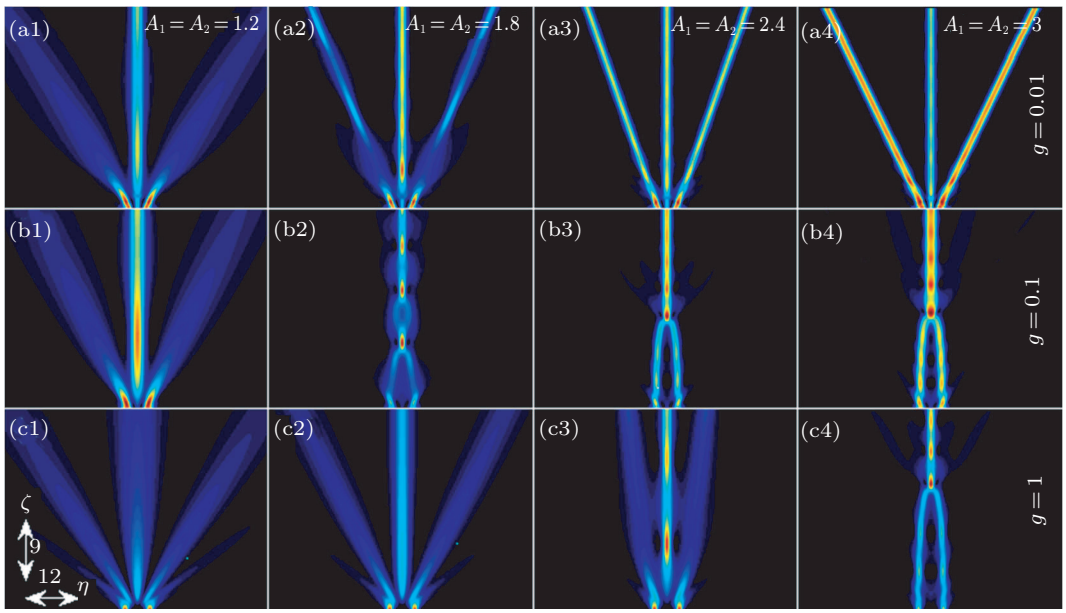


Fig. 7. (color online) The interactions of two AiG beams with different initial amplitudes and light field distribution factors. Other parameters are $\delta\phi = 0$ and $B = 3$.

only single breathing solitons can form. The bigger the initial amplitudes are, the larger the soliton intensity is and the faster the soliton breathing rate is (Figs. 7(b1)–7(b4)). When $g = 1$, the initial light fields of the two AiG beams are close to the Gaussian distribution, the initial amplitudes at which stable single breathing solitons can be formed are higher than those when g is smaller (Figs. 7(c1)–7(c4)).

5. Conclusion

We have numerically investigated the generation of breathing solitons in the propagation and interactions of AiG beams in a cubic–quintic nonlinear medium in one transverse dimension by using the split-step Fourier transform method. We find that stable breathing solitons can be produced in the propagation of single AiG beams due to the compensation of the defocusing quintic nonlinearity on the focusing cubic nonlinearity within a certain range of the initial amplitude A_0. The soliton breathing period decreases and the maximum intensity of the soliton increases monotonically with increasing A_0. The propagation direction of the solitons can be adjusted by controlling the launch angle of the AiG beams. In addition, numerical simulations of the interactions between two AiG beams are carried out in a cubic–quintic nonlinear medium.The results show that different single breathing solitons or soliton pairs can be obtained by adjusting the phase shift and interval between two AiG beams for given values of the initial amplitude. When the initial amplitudes are changed, the interactions of the two AiG beams can lead to single breathing solitons and symmetrical soliton pairs when the initial light field distribution tends to be the Airy distribution. The intensity of the symmetrical soliton pair increases and the intensity of the single breathing soliton decreases with increase of the initial amplitudes. When the initial light field distribution is close to the Gaussian distribution, only single breathing solitons can form. The bigger the initial amplitudes are, the larger the soliton intensity is and the faster the soliton breathing rate is.

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